

Sincere Voting with Cardinal Preferences: Approval Voting*

Miguel A. Ballester[†] Pedro Rey-Biel[‡]

Universitat Autònoma de Barcelona

Abstract

We discuss sincere voting when voters have cardinal preferences over three alternatives. We define sincerity as opposed to strategic voting, and thus define sincerity as the optimal behaviour when conditions to behave strategically are diminished. When voting mechanisms allow for only one *message type* a definition of sincerity is shown to be intuitive. However, when voting mechanisms allow for multiple *message types*, such as in approval voting, a definition of sincerity is more complex. We prove that in approval voting, when there is no information on other voters' preferences and as the size of the electorate increases, voters' optimal strategy tends to a previously provided definition of sincerity, consisting in voting for those alternatives that yield more than the average of cardinal utilities.

First draft: July 5, 2006

JEL codes: *D63, D71, D72, D80*

Keywords: *sincere and strategic voting, approval voting*

*Financial support from Ministerio de Ciencia y Tecnología (BEC 2003-01132), SEJ2005-01481/ECON, Generalitat de Catalunya (2001 SGR-00162) and Barcelona Economics, CREA is gratefully acknowledged.

[†]Miguel Ángel Ballester. Universitat Autònoma de Barcelona. Department d'Economia i d'Història Econòmica. Unitat de Fonaments de l'Anàlisi Econòmica. Desp. B3-158, Edifici B - Campus de Bellaterra. 08193 Bellaterra (Cerdanyola del Vallés), Barcelona, Spain. Tel: (00 34) 935814739. E-mail: mballester@pareto.uab.es.

[‡]Pedro Rey-Biel. Universitat Autònoma de Barcelona. Department d'Economia i d'Història Econòmica. Unitat de Fonaments de l'Anàlisi Econòmica. Desp. B3-166, Edifici B - Campus de Bellaterra. 08193 Bellaterra (Cerdanyola del Vallés), Barcelona, Spain. Tel: (00 34) 93582113. E-mail: pedro.rey@uab.es.

1 Introduction

In this paper we discuss what sincere voting means under different voting mechanisms when voters have cardinal preferences over alternatives. A definition of sincerity is important since it allows to compare the properties of different voting rules with respect to voters' strategic behavior. Under different voting mechanisms, and given voters' preferences for alternatives, they may be better able to favour the election of preferred outcomes by behaving strategically instead of sincerely and thus, manipulate the voting mechanism.¹ In order to provide a general definition of sincerity, our approach is to consider this strategic component of voting and eliminate it.

There exists ample literature on the definition of sincere voting for different voting mechanisms and on which voting rules achieve it.² Brams (1978) defines sincere voting as non-strategic behavior in which individuals vote “directly in accordance with their preferences”. The problem arises because the translation of preferences over alternatives to sincere votes may not be direct under some voting rules, since voting mechanisms may demand to structure votes in a different format than preferences are specified.

The majority of the voting literature simplifies the analysis by assuming that voters have ordinal preferences over alternatives.³ On the other hand, it seems plausible to assume that voters are able to quantify differences between alternatives and thus, they may have cardinal preferences over them. Under cardinal preferences, if a voting mechanism exactly required all cardinal information, the definition of “sincere voting” would be straightforward. A sincere “vote” would just be the declaration of the cardinal utility that each alternative gives to a voter. However, the majority of voting mechanisms only use ordinal information and thus a definition of sincere voting is more complex.

Votes can be understood as messages since they transmit information on the desirability of the alternatives for the voters. The translation of cardinal utilities to non-cardinal votes depends on the number (and type) of messages each voting mechanism allows. If the voting mechanism only allows for one possible message type then identifying sincere behavior is not so problematic. A sincere vote would be the one that intuitively “best represents” the order of the cardinal preferences, given the restrictions of the voting mechanism. For example, the plurality rule is a clear case of a voting rule that allows for only one message type, since voters can only choose between singletons (with the meaning of a superior alternative, since the aggregation process will consider positively such singletons). Thus sincere voting under Plurality Rule (PR) would intuitively fit with voting (in the top set) for the alternative that gives highest utility to the voter.

There are however several voting rules that allow for more than one message type. In such cases, there is no clear intuition of what the best representation of cardinal preferences

¹Any voting rule is subject to strategic voting behaviour when there are at least three alternatives and there are no dictators (Gibbard (1973), Satterwhaite (1975), Dutta et al. (2001)).

²Starting with Farquharson (1969).

³See, for example, Arrow (1951), Fishburn (1973) and Nurmi (1987).

would be. A paradigmatic example of such rules is Approval Voting (AV), since the decision of whether to include an alternative among the “approved” ones or not may naturally depend on the difference in cardinal utility between alternatives.

Consider the following example. There are three alternatives x, y and z that yield the following utilities to a voter: $U(x) = 0.8$, $U(y) = 0.5$ and $U(z) = 0.1$. A voting rule that required all cardinal information would have associated as “sincere voting” the revelation of utilities 0.8, 0.5 and 0.1 respectively. A non-cardinal voting rule allowing for only one message type, for example PR, only allows singletons as messages. A sincere voter under PR would then declare her real preferences by voting for alternative $\{x\}$. Intuitively, any other possible message, for example, $\{z\}$ would be a worse representation of the voter’s real cardinal preferences and thus, would not be sincere. Finally, under a voting mechanism allowing for different message types, the problem of identifying the best representation arises. Consider the case of AV, where any subset of alternatives is allowed as a message. If the voter was only allowed to approve her “best alternative” (to choose from the set of singletons of 2^X) then voting (approving) $\{x\}$ and not voting for the other two alternatives would intuitively be sincere as it best fits with her cardinal preferences. If on the contrary, the voter was only allowed to vote for pairs of alternatives (which is what Negative voting does) then voting for $\{x, y\}$ would intuitively be sincere. However, given that AV allows the voter to specify any subset of alternatives as the set of approved options, it is not clear whether voting for $\{x\}$ or for $\{x, y\}$ is sincere.

At least two definitions of sincerity under AV have been provided. The first one⁴ imposes that if one alternative is voted in the top set (approved), all alternatives that provide higher cardinal utility to the individual should also be included in the top set. Notice that this definition is somewhat weak as there are several messages considered as sincere. In our example, $\{x, y, z\}$ (meaning “all alternatives are approved”), ϕ (meaning “all alternatives are disapproved”), $\{x\}$ and $\{x, y\}$ would all be considered sincere under this weak definition. A second and more restrictive definition considers sincerity as voting for those alternatives that give the individual more cardinal utility than the average of all alternatives.⁵ In our numerical example, the only sincere voting representation would then be to vote for both x and y (i.e. $\{x, y\}$) since both provide more cardinal utility than the average ($0.8 > 0.47$ and $0.5 > 0.47$).

Insincere behavior may appear because given a voting rule and how other individuals vote, a voter may be able to influence the outcome of the election by voting for alternatives that do not maintain the order of her cardinal preferences but that provide her higher utility than when voting sincerely. This will happen in cases were a sincere vote would not affect the election of an alternative that provides even lower utility than the alternative elected when the voter behaves strategically. Strategic voting implies balancing the relative preference for the different alternatives against the relative likelihood of influencing the outcome of the

⁴Brams and Fishburn (1981), Niemi (1984) .

⁵See, for instance, Merrill (1983), Merrill and Nagel (1987) and Hoffman (1982) who present some basic results characterizing this behavior under a very restrictive set of assumptions.

election.⁶ Notice that whether a voter assesses that her vote may affect the outcome depends on how she thinks other voters will vote. Strategic behavior may thus be enhanced the more information voters have on the preferences of other voters. Weber (1978) goes as far as claiming that in settings where voters have little access to information concerning either the preferences of other voters or their intended behavior, voters can be presumed to vote sincerely, since the lack of information means there is no basis for voting “in some clever strategic way”. Our first result shows what Weber’s (1978) would define as sincere voting behaviour in settings with three alternatives and for any voting rules allowing for only one message type. We show that the best strategy of a voter with no information on other voters’ preferences is unique and independent of the size of the electorate. We thus define sincere voting behaviour as this best strategy for voting rules that allow for only one message type.

However, when voting rules allow for more than one message type things become more complex. Taking AV as our benchmark example of such rules, our second result shows that the optimal strategy for any voter under AV when information on other’ preferences is eliminated depends on the size of the electorate. Thus, it cannot be the case that we consider this behavior as a precise definition of sincerity, since how sincere a vote is should not vary with the number of voters. We also show, however, that the optimal strategy with no information on others’ preferences always satisfies (for any size of the electorate) the weak definition of sincerity in AV previously discussed.

We thus introduce new conditions that may make diminish strategic behaviour. A natural intuition is that the larger the electorate the lower the manipulative effect of a strategic vote on the outcome of the election may be. Therefore, we check what the best strategy in AV is when the size of the electorate increases (and there is no information on others’ preferences). Thus, we define sincere voting as the best strategy under such conditions. Our third result shows that the best strategy with no information and when the size of the electorate tends to infinite coincides with the second and stronger definition of sincerity previously discussed. Our paper thus provides new support for the idea that sincerity in AV implies adding to the top set all alternatives that yield (cardinal) utility above the average of the utilities generated by all alternatives. Although this definition has been previously discussed, ours is the result of a new methodology consisting in eliminating those features of the problem that generate strategic behaviour.

We have focused in the case of three alternatives x, y and z . Although this case is of course special, it is the simplest case that allows to differentiate between different voting rules and, at the same time, the simplest case in which strategic voting may occur.⁷

The rest of the paper is organized as follows. Section 2 shows the notation and the basic assumptions made. Section 3 discusses sincerity in simple voting mechanisms (Result 1), while Section 4 discusses sincerity in complex voting mechanisms, in particular in approval voting (Results 2 and 3). Section 5 concludes.

⁶See Fisher and Myatt (2002).

⁷See Myerson and Weber (1993), Myerson (2002) and Dhillon and Lockwood (2004).

2 Notation and Definitions

Consider a set of agents $j = 1, 2, \dots, n$ and a set of three alternatives X with $X = \{x, y, z\}$. A Society is a $n \times 3$ matrix of cardinal utilities $U = (U_j(k))$ with $k \in X$, $U_j(k) \in [0, 1]$. For the elegance of the exposition, assume that there are not two alternatives providing the same utility to each agent.⁸

Assume there exists a set of messages M from which each voter has to choose one. Such message transmits information on her preferences. Given a message $m \in M$, $[m]$ denotes the set of all messages that can be obtained by using any bijective mapping $\sigma : X \rightarrow X$. We refer to $[m]$ as a *message type*. In this paper we consider sets of messages for which $m \in M \Rightarrow [m] \subseteq M$ and such that they are either linear orders over X or subsets of X .⁹ A voting mechanism V can be naturally defined as the composition of a set of messages (among which the voters can signal one) and an aggregation process of the collected messages such as some alternatives are chosen.

A voting mechanism may have associated a set of messages with several message types. As we will show below, the existence of several message types is crucial to define sincere voting. We first classify voting mechanisms according to the number of message types associated to them. The crucial property to study sincerity will be whether voting mechanisms have a single or several message types associated to them.

Definition 1 *A voting mechanism is said to be simple if $M = [m]$ for some $m \in M$. Otherwise, it is said to be complex.*

In order to discuss sincerity, we frequently refer to some classical voting mechanisms as examples. We here define three of them. According to our previous definitions, the first two are simple mechanisms, the first one using linear orders while the second uses subsets, while the third one is complex.¹⁰

Definition 2 *A voting mechanism is the Borda Rule if $M = \{\text{linear orders over } X\}$ and the selected alternatives are those that maximize the number of binary comparisons in which they are the dominating alternative.*

Definition 3 *A voting mechanism V is the Plurality Rule if $M = \{\{x\}, \{y\}, \{z\}\}$ and the selected alternatives are those that maximize the number of messages in which they appear.*

Definition 4 *A voting mechanism is Approval Voting if $M = 2^X$ and the selected alternatives are those that maximize the number of messages in which they appear.¹¹*

⁸Parallel results are obtained without such assumption, although proofs become tedious without adding further insights.

⁹Messages on linear orders or subsets of alternatives are the most common approach to voting.

¹⁰Notice that there is no complex voting mechanism with linear orders as associated messages.

¹¹Merril and Nagel (1987) differentiate between balloting methods and the decision rules that produce an outcome. In that spirit, they would claim that AV is our balloting method, while given our definition, the outcome of the election is decided under Plurality Rule. Our definitions consider both characteristics of voting rules, i.e., the set of available messages and the way to aggregate them.

Once we have discussed messages, we now briefly refer to the aggregation process. In particular, we establish minimal conditions on how voting mechanisms aggregate messages to select alternatives. The following three definitions establish minimal conditions for voting mechanisms to be admissible.

Definition 5 *A voting mechanism V is neutral in alternatives if for any permutation σ of the set of alternatives $V\left(\{\sigma(m_j)\}_{j=1\dots n}\right) = \sigma\left(V\left(\{m_j\}_{j=1\dots n}\right)\right)$.*

Neutrality in alternatives implies that the names of the alternatives do not affect the selection of alternatives. Notice that if neutrality in alternatives were not satisfied a group of alternatives could be favoured without taking into account agents' preferences.

Definition 6 *A voting mechanism V is neutral in agents if for any permutation π of the set of agents $V\left(\{m_{\pi(j)}\}_{j=1\dots n}\right) = V\left(\{m_j\}_{j=1\dots n}\right)$*

Neutrality in agents implies that agents' identity does not affect the selection of alternatives. Notice that if neutrality in agents were not satisfied some agents' preferences could be irrelevant when aggregating messages.

Finally, our last definition refers to the monotonicity of the aggregation process. We need to distinguish between voting mechanisms composed by linear orders or subsets as messages.

Definition 7 *A voting mechanism V is weakly monotonic if for any alternative x and for any pair of messages' collections $\{m_j\}_{j=1\dots n}$ and $\{m'_j\}_{j=1\dots n}$ such that:*

- *If messages are linear orders, they only differ in the position of alternative x (higher ranked in $\{m'_j\}_{j=1\dots n}$ than in $\{m_j\}_{j=1\dots n}$),*
- *If messages are subsets of alternatives, they only differ in the presence of alternative x (always present in $\{m'_j\}_{j=1\dots n}$ when present in $\{m_j\}_{j=1\dots n}$),*

then:

$$x \in V\left(\{m_j\}_{j=1\dots n}\right) \implies x \in V\left(\{m'_j\}_{j=1\dots n}\right).$$

$$\{x\} = V\left(\{m_j\}_{j=1\dots n}\right) \implies \{x\} = V\left(\{m'_j\}_{j=1\dots n}\right).$$

Our Monotonicity condition is mild. It just implies that if an agent's message is modified such that it favours an alternative x , the voting mechanism responds accordingly. Thus, if x was in the elected set before modifying agent's message then it is also elected under the new message. Similarly, if x is the only elected alternative then it must also be the only elected alternative under the new message.

Using the above definitions, our goal is to define sincere voting behaviour for voting mechanisms. We understand sincere voting behaviour as opposed to strategic behaviour. The latter comprises the possibility of favouring the election of preferred outcomes by misrepresenting *sincere* messages. There exist some conditions that facilitate the appearance of strategic behaviour. For instance, the influence of an individual agent's message on the

outcome of the election or the amount of information agents have on others' preferences over alternatives. Our approach is to define sincere voting as the best voting strategy when the conditions that ease strategic behaviour are diminished. Thus, we focus on agents' best responses under such conditions instead of on possible equilibria of the voting game. Since such approach requires to study how agents react to uncertainty, we impose the following two assumptions.

Assumption 1 In the absence of information on other agents' preferences over alternatives, agents believe that any possible combination of others' messages is equally probable. Formally, $\forall j, \forall m_i \in M$, $p_j(m_1, \dots, m_{j-1}, m_{j+1}, \dots, m_n) = \frac{1}{\#M}$ where $p_j(m_1, \dots, m_{j-1}, m_{j+1}, \dots, m_n)$ is the probability with which agent j believes other agents will transmit messages m_i with $i \neq j$.

The subjective probability each agent assigns to any combination of messages by other agents clearly depends on the cardinality of the set of messages. In particular, for the case of AV, $\forall j, \forall m_i \in M$, $p_j(m_1, \dots, m_{j-1}, m_{j+1}, \dots, m_n) = \left(\frac{1}{2^{\#X}}\right)^{n-1}$.

Assumption 2 Given agents' beliefs, they maximize their expected utility over alternatives.

Assumptions 1 and 2 are a simple way for voters to resolve the uncertainty about others' preferences. Notice that we aim to strength conditions that eliminate strategic voting and thus, our assumptions refer to cases in which agents can not form clear expectations about how others will vote. Moreover, these assumptions may have a behavioural support. Both assumptions are the common starting point to define *k-levels of rationality* in the literature on degrees of cognitive complexity which has found certain experimental validity.¹²

3 Sincerity in simple voting mechanisms

Along this paper we are studying sincere voting behaviour when agents have cardinal preferences over alternatives. Following our previously mentioned approach, we aim to define sincerity for simple voting mechanisms as the best response strategy when the possibility of strategic behaviour is diminished. Theorem 1 confirms the intuition that under such mechanisms sincerity implies transmitting pieces of ordinal information contained in agents' cardinal preferences over alternatives.

Theorem 1: *Consider there is no information on agents' preferences over alternatives $X = \{x, y, z\}$ and assumptions 1 and 2 hold. For any number of agents n and any simple voting mechanism satisfying Neutrality in agents and alternatives and Weak Monotonicity then agent i 's best response (sincere behaviour) is:*

- For $M = \{\text{linear orders over } X\}$, the linear order such that $x \succ y \succ z \Leftrightarrow U_i(x) > U_i(y) > U_i(z)$.

¹²See Stahl (1993), Stahl and Wilson (1994, 1995), McKelvey and Palfrey (1995), Broseta, Costa-Gomes and Crawford (2001) and Goeree and Holt (2004).

- For $M = [m]$ with m being a subset of alternatives, the subset of the $\#m$ alternatives which provide highest utility to agent i .

Proof: We proceed to prove separately the cases in which the set of messages is the set of linear orders and the cases in which the set of messages is a collection of subsets of X .

- We first consider the case in which $M = \{\text{linear orders over } X\}$. Consider wlog. $U_i(x) > U_i(y) > U_i(z)$. Denote the linear order $x \succ y \succ z$ by m . We have to prove that m is agent i 's best response independently of the number of agents in society.

We show that m is a better response than $m' = y \succ_i x \succ_i z$. To see this, let us analyze all the possible situations in which transmitting m' could be beneficial for agent i . Consider any combination of messages by the other agents in society, m_{-i} . Then, given that the voting mechanism is Weakly Monotonic, we now that $x \in V(m_{-i}, m') \Rightarrow x \in V(m_{-i}, m)$ and $y \in V(m_{-i}, m) \Rightarrow y \in V(m_{-i}, m')$. We also know that $\{x\} = V(m_{-i}, m') \Rightarrow \{x\} = V(m_{-i}, m)$ and $\{y\} = V(m_{-i}, m) \Rightarrow \{y\} = V(m_{-i}, m')$. The following table specifies all possible outcomes of the election in which declaring m' instead of m may be beneficial for agent i . Any other combination of others' messages always yields a worse outcome when declaring m' . For instance, outcome $\{x, y\}$ whenever i states m and outcome $\{x, z\}$ whenever i states m' yields lower utility since $\frac{U_i(z)+U_i(x)}{2} < \frac{U_i(y)+U_i(x)}{2}$ and thus declaring m' would not be beneficial. Notice also that not every pair of outcomes can be associated to messages m and m' . For example, the outcome $\{y, z\}$ whenever i states m and outcome $\{x, y\}$ whenever i states m' is not possible since $x \in V(m_{-i}, m')$ but $x \notin V(m_{-i}, m)$.

Messages	Outcome							
m	$\{x, z\}$	$\{x, y, z\}$	$\{x, z\}$	$\{x, z\}$	$\{x, y, z\}$	$\{y, z\}$	$\{z\}$	$\{z\}$
m'	$\{x, y\}$	$\{x, y\}$	$\{x, y, z\}$	$\{y\}$	$\{y\}$	$\{y\}$	$\{y\}$	$\{y, z\}$
Cases	1)	2)	3)	4)	5)	6)	7)	8)

Notice that under cases 3), 4) and 5) m' yields higher expected utility than m only when $U_i(y) > \frac{U_i(x)+U_i(z)}{2}$.

In order to prove that message m is a better response than m' , we show that, for any of the previous cases (associated to a combination of messages by the others), there exists another combination of messages by the others such that:

1. Its probability of occurrence is larger.
2. The benefit from transmitting m instead of m' is larger than the benefit from transmitting m' instead of m in the initial case.

Consider the bijection $\sigma : X \Rightarrow X$, where $\sigma(x) = y, \sigma(y) = x$ and $\sigma(z) = z$. For case $k, k \in \{1, \dots, 8\}$, consider the combination of others' messages $m_j^k, j \neq i$ which makes

transmitting m' beneficial with respect to m . Consider also the combination of others' messages $\sigma(m_j^k)$, $j \neq i$. By Assumption 1, individual i assigns the same probability to messages $\sigma(m_j^k)$ as to m_j^k . Since $\sigma(m) = m'$ and $\sigma(m')$ = m , by neutrality in alternatives, it must be that $V\left(m, \left\{\sigma\left(m_j^k\right)\right\}_{j \neq i}\right) = \sigma\left(V\left(m', \left\{m_j\right\}_{j=1 \dots n}\right)\right)$ and $V\left(m', \left\{\sigma\left(m_j^k\right)\right\}_{j \neq i}\right) = \sigma\left(V\left(m, \left\{m_j\right\}_{j=1 \dots n}\right)\right)$. Thus, we can compute parallel cases to those of the previous table, with equal probability. The outcomes of the voting mechanism now are:

Messages	Outcome							
m	$\{x, y\}$	$\{x, y\}$	$\{x, y, z\}$	$\{x\}$	$\{x\}$	$\{x\}$	$\{x\}$	$\{x, z\}$
m'	$\{y, z\}$	$\{x, y, z\}$	$\{y, z\}$	$\{y, z\}$	$\{x, y, z\}$	$\{x, z\}$	$\{z\}$	$\{z\}$
Cases	1')	2')	3')	4')	5')	6')	7')	8')

For cases k' , $k \in \{1, \dots, 8\}$, the benefit obtained from declaring m instead of m' is, in all the cases, at least as large as the loss for the corresponding case k . Given that any of these cases has the same probability as its counterpart, m guarantees a expected utility at least as large as m' . For m to be strictly preferred to m' , notice that $U_i(x) > U_i(y)$ implies that, for cases 1) and 1'), m strictly dominates m' since $\frac{U_i(x)+U_i(z)}{2} > \frac{U_i(y)+U_i(z)}{2}$.

Showing that any other message m'' yields lower expected utility than m follows exactly the same reasoning.¹³ Thus, m is agent i 's best response.

- We now consider situations in which M is a family of subsets of X . In order to have a simple voting mechanism, there only exist four possibilities:

$$M_1 = \{\phi\}, M_2 = \{X\}, M_3 = \{\{x\}, \{y\}, \{z\}\} \text{ and } M_4 = \{\{x, y\}, \{x, z\}, \{y, z\}\}.$$

M_1 and M_2 are trivial cases given that agents can not decide which message to transmit. Plurality Rule is a prime example of a voting mechanism using M_3 . Negative Voting (or Antiplurality) is an example of a voting mechanism using M_4 .¹⁴ We here prove the result for M_3 and leave the analogous proof for M_4 for the reader.¹⁵

Consider $M_3 = \{\{x\}, \{y\}, \{z\}\}$ and wlog. $U_i(x) > U_i(y) > U_i(z)$. We first show that transmitting $\{x\}$ is better than transmitting $\{y\}$. Consider any combination of messages in society, m_{-i} . Then, given that the voting mechanism is Weakly Monotonic, we now that $x \in V(m_{-i}, \{y\}) \Rightarrow x \in V(m_{-i}, \{x\})$ and $y \in V(m_{-i}, \{x\}) \Rightarrow y \in V(m_{-i}, \{y\})$. Additionally,

¹³Since all the proofs rely in the same construction, for simplicity we explicitly exclude them. They are, however, available upon request.

¹⁴One is tempted to think that Negative Voting also uses M_3 , given that agents transmit their least preferred alternative. However, for Negative Voting to satisfy weak monotonicity, its messages must be interpreted as transmitting all the alternatives but the least preferred one.

¹⁵We consider this theorem as a baseline for the results of the following section and thus, we explicitly avoid this part of the proof that can be easily derived from the one presented.

$\{x\} = V(m_{-i}, \{y\}) \Rightarrow \{x\} = V(m_{-i}, \{x\})$ and $\{y\} = V(m_{-i}, \{x\}) \Rightarrow \{y\} = V(m_{-i}, \{y\})$. The following table, which is in fact equivalent to the case of linear orders, specifies all possible outcomes in which transmitting $\{y\}$ may be beneficial for agent i .

Messages	Outcome							
$\{x\}$	$\{x, y\}$	$\{x, y\}$	$\{x, y, z\}$	$\{x\}$	$\{x\}$	$\{x\}$	$\{x\}$	$\{x, z\}$
$\{y\}$	$\{y, z\}$	$\{x, y, z\}$	$\{y, z\}$	$\{y, z\}$	$\{x, y, z\}$	$\{x, z\}$	$\{z\}$	$\{z\}$
Cases	1)	2)	3)	4)	5)	6)	7)	8)

The analysis is parallel to the case of linear orders, by proving that $\{x\}$ strictly yields a larger expected utility than $\{y\}$. Reproducing the analysis with strategies $\{y\}$ and $\{z\}$ it can be shown that $\{y\}$ strictly yields a larger expected payoff than $\{z\}$. Thus, transmitting $\{x\}$ strictly yields a larger expected utility than $\{y\}$ and $\{z\}$, concluding the proof for M_3 . \square

We have therefore shown what sincere voting behaviour is under simple voting mechanisms. Notice that sincere voting behaviour in simple mechanisms does not depend on the influence of an individual agent's message on the outcome of the election. Voters' optimal strategy under no information conditions is to assign votes in a manner that maintains some ordinal information of their true preferences. However, in the next section we show that the absence of information is not enough to guarantee a precise definition of sincerity for complex voting mechanisms since best responses will depend, for instance, on the number of agents participating in the election.

4 Complex Voting mechanisms: Approval Voting

We consider in this section voting mechanisms which allow for several message types, i.e., $M = \bigcup_{t=1}^T [m_t]$. Following Theorem 1, for each message type t we could obtain a best response message, m_t^* within each message type. Therefore, under the absence of information on other agents' preferences, the best response belongs to set $\{m_1^*, m_2^*, \dots, m_T^*\}$. Approval Voting is a prototypical case of complex voting mechanisms. Under AV agents can transmit a large variety of messages. For example, in the case of three alternatives and $U_i(x) > U_i(y) > U_i(z)$, AV allows for the following four different message types:

1. Support all three alternatives, $[m_1] = \{X\}$ and, clearly, $m_1^* = X$.
2. Support two alternatives, $[m_2] = \{\{x, y\}, \{x, z\}, \{y, z\}\}$ and, clearly, $m_2^* = \{x, y\}$.
3. Support just one alternative, $[m_3] = \{\{x\}, \{y\}, \{z\}\}$ and, clearly, $m_3^* = \{x\}$.
4. Support no alternative, $[m_4] = \{\phi\}$ and, clearly, $m_4^* = \phi$.

The message an agent chooses, and thus the message type used, naturally depends on the cardinal utility alternatives yield. A definition of sincere voting behaviour is thus more complicated. Brams and Fishburn (1981) and Niemi (1984) have previously defined sincerity in AV as given that an agent supports an alternative, she must also support all alternatives

that are preferred to that one. Translating this argument to cardinal utilities and using our notation, we establish a definition of *weak sincerity*:

Definition 8 *Agent i 's message m is weak sincere under AV if for all x, y , $U_i(x) > U_i(y)$ and $y \in m$ implies $x \in m$.*

Clearly, given three alternatives $X = \{x, y, z\}$ and $U_i(x) > U_i(y) > U_i(z)$, the set of weak sincere messages under AV is precisely $\{m_1^*.m_2^*, \dots, m_4^*\} = \{X, \{x, y\}, \{x\}, \phi\}$. Notice the connection between the above referred implications of Theorem 1 for complex voting mechanisms and the set of weak sincere messages. For each message type we can find a unique message that is at the same time a best response within the type and weakly sincere. However, we refer to such definition as *weak* because it does not determine a unique message as sincere, which may be an appealing property.

Weber (1978), Merrill (1983), Merrill and Nagel (1987) and Hoffman (1982) provide a stronger definition of sincerity in AV that uses cardinal utilities and uniquely determines a message as sincere. Using our notation, such definition is as follows:

Definition 9 *Agent i 's message m is strong sincere under AV if for all $x \in X, U_i(x) \geq \sum_{y \in X} U_i(y) / \#X \Leftrightarrow x \in m$.*

The strong definition of sincere voting under AV implies voting for those alternatives that yield more utility than the average of utilities. This definition, although intuitively appealing, has not been given a formal justification. In particular, it has been defined under a restrictive set of assumptions, such as imposing specific probabilities on the number of votes each alternative receives. As in the previous subsections, we obtain our results by evaluating these probabilities as a result of a cognitive process based only on initial beliefs over individual votes. In the remains of the paper, we show that the best response of an agent under conditions that diminish the possibility of behaving strategically is precisely voting for those alternatives that yield more than the average of utilities. Therefore, we provide stronger support for this second definition of sincerity, which uniquely determines which messages are sincere in AV.

4.1 Dependence on the size of the electorate

We proceed here to obtain voter behavior in the absence of information in order to achieve a proper definition of sincerity in AV. Theorem 2 shows, however, that voter behavior is dependent on the number of individuals in a society, thus making impossible to achieve a non-contingent definition of sincerity. However, it also shows that any behavior (for any size of the electorate) satisfies the weak definition of sincerity in AV.

Theorem 2: *Consider wlog. $U_i(x) > U_i(y) > U_i(z)$. Then agent i transmits message $\{x, y\}$ if and only if $U_i(y) \geq \lambda(n)U_i(x) + (1 - \lambda(n))U_i(z)$ with $\lambda(n) \in (0, 1)$, otherwise agent i transmits message $\{x\}$. In particular, this behavior satisfies the weak definition of sincerity in Approval Voting.*

Proof: Consider wlog. $U_i(x) > U_i(y) > U_i(z)$. Using Theorem 1, the only messages worth considering are $\{m_1^*, m_2^*, m_3^*, m_4^*\} = \{X, \{x, y\}, \{x\}, \phi\}$. Notice that voting $\{x\}$ is always better than voting X (respectively ϕ). Suppose that voting X (respectively ϕ) leads to have $S \neq \{x\}$ as the set of elected alternatives.¹⁶ If $x \in S \neq \{x\}$, then transmitting $m_3^* = \{x\}$ leads to have x as the unique elected outcome, which obviously dominates S for the agent. If $x \notin S$, transmitting $m_3^* = \{x\}$ leads either to have S as the set of elected outcomes or to have $S \cup \{x\}$ as the set of elected outcomes. This is clearly preferable to the outcome obtained when transmitting X (respectively ϕ). Thus we focus on m_2^* and m_3^* . We present here the situations in which m_2^* and m_3^* could yield different outcomes:

Messages	Outcome					
m_3^*	$\{x\}$	$\{z\}$	$\{x, y\}$	$\{x, z\}$	$\{y, z\}$	$\{x, y, z\}$
m_2^*	$\{x, y\}$	$\{y, z\}$	$\{y\}$	$\{x, y, z\}$	$\{y\}$	$\{y\}$
Cases	1)	2)	3)	4)	5)	6)

The previous table shows all the possible combinations of others agents' messages in which messages m_2^* and m_3^* yield different outcomes. In order for these situations to occur, the distribution of other agents' messages must satisfy the following conditions:

$$\begin{aligned}
1) & a_x = a_y > a_z - 1 & 4) & a_x + 1 = a_y + 1 = a_z \\
2) & a_x + 1 < a_y + 1 = a_z & 5) & a_x + 1 < a_y = a_z \\
3) & a_x = a_y - 1 > a_z - 1 & 6) & a_x + 1 = a_y = a_z,
\end{aligned}$$

where a_k represents the number of times alternative k appears in other agents' messages, excluding agent i .

- Under the non-information assumptions, the probabilities P_k of each of these six conditions are:

$$\begin{aligned}
P_1 &= \frac{\sum_{t=0}^{n-1} \sum_{s=0}^t \binom{n-1}{t} \binom{n-1}{t} \binom{n-1}{s}}{(2^3)^{n-1}} & P_2 &= \frac{\sum_{t=2}^{n-1} \sum_{s=0}^{t-2} \binom{n-1}{s} \binom{n-1}{t-1} \binom{n-1}{t}}{(2^3)^{n-1}} \\
P_3 &= \frac{\sum_{t=1}^{n-1} \sum_{s=0}^{t-1} \binom{n-1}{t-1} \binom{n-1}{t} \binom{n-1}{s}}{(2^3)^{n-1}} & P_4 &= \frac{\sum_{t=1}^{n-1} \binom{n-1}{t-1} \binom{n-1}{t-1} \binom{n-1}{t}}{(2^3)^{n-1}} \\
P_5 &= \frac{\sum_{t=2}^{n-1} \sum_{s=0}^{t-2} \binom{n-1}{s} \binom{n-1}{t} \binom{n-1}{t}}{(2^3)^{n-1}} & P_6 &= \frac{\sum_{t=1}^{n-1} \binom{n-1}{t-1} \binom{n-1}{t} \binom{n-1}{t}}{(2^3)^{n-1}}
\end{aligned}$$

¹⁶Obviously if $S = \{x\}$ then transmitting $\{x\}$ has the same effect on the election of outcomes.

Hence, the expected utility of messages m_3^* and m_2^* under the non-information assumption can be expressed in terms of these probabilities. For message m_3^* , the expected utility equals:

$$P_1U_i(x) + P_2U_i(z) + P_3\left(\frac{U_i(x) + U_i(y)}{2}\right) + P_4\left(\frac{U_i(x) + U_i(z)}{2}\right) \\ + P_5\left(\frac{U_i(y) + U_i(z)}{2}\right) + P_6\left(\frac{U_i(x) + U_i(y) + U_i(z)}{3}\right)$$

whereas for message m_2^* the expected utility equals:

$$P_1\left(\frac{U_i(x) + U_i(y)}{2}\right) + P_2\left(\frac{U_i(y) + U_i(z)}{2}\right) + P_3U_i(y) + \\ P_4\left(\frac{U_i(x) + U_i(y) + U_i(z)}{3}\right) + P_5U_i(y) + P_6U_i(y)$$

Therefore, the condition for preferring to transmit m_2^* has to be that it yields a higher expected value than transmitting m_3^* , i.e.,

$$\left(\frac{P_1}{2} + \frac{P_3}{2} + \frac{P_4}{6} + \frac{P_6}{3}\right)U_i(x) + \left(\frac{P_2}{2} + \frac{P_4}{6} + \frac{P_5}{2} + \frac{P_6}{3}\right)U_i(z) \leq \left(\frac{P_1}{2} + \frac{P_2}{2} + \frac{P_3}{2} + \frac{P_4}{3} + \frac{P_5}{2} + \frac{2P_6}{3}\right)U_i(y)$$

Denoting:

$$f(n) = \left(\frac{P_1}{2} + \frac{P_3}{2} + \frac{P_4}{6} + \frac{P_6}{3}\right) \\ g(n) = \left(\frac{P_2}{2} + \frac{P_4}{6} + \frac{P_5}{2} + \frac{P_6}{3}\right) \\ h(n) = \left(\frac{P_1}{2} + \frac{P_2}{2} + \frac{P_3}{2} + \frac{P_4}{3} + \frac{P_5}{2} + \frac{2P_6}{3}\right),$$

we can express the previous inequality as $\frac{f(n)}{h(n)}U_i(x) + \frac{g(n)}{h(n)}U_i(z) \leq U_i(y)$. We only need to consider the function $\lambda(n) = \frac{f(n)}{h(n)}$ to conclude the proof. Since any $P_t : t = 1, \dots, 6$ is different from zero, it follows that $\lambda(n) \neq 1$ and $\lambda(n) \neq 0$. Finally, given that the only optimal messages are m_3^* and m_2^* , it easily follows that the weak definition of sincerity always holds. \square

Whether the alternative yielding second highest utility to an agent is included in her transmitted message on its *relative* cardinal utility with respect to the utilities yielded by the most and least preferred alternatives. Such dependence rests on the weights measured by the function $\lambda(n)$, which varies with the size of the electorate n . For example, if $U(x) = 0.9, U(y) = 0.7$ and $U(z) = 0.1$, basic calculus shows that an agent transmits message $\{x\}$ when the size of the electorate is 2. On the other hand, the same agent transmits message $\{x, y\}$ when the size of the electorate is 3. However, it seems unreasonable that how sincere a voting strategy is depends on the size of the electorate. Thus, in the following subsection we further impose conditions to diminish the possibility of strategic voting in order to obtain sincere behaviour.

4.2 Sincerity in Approval Voting

The influence of an individual agent's vote on the outcome of an election diminishes the bigger the size of an electorate. Our last result shows that when the number of agents tends to infinite the previously defined strong definition of sincere approval voting arises as the best response of any agent.

In Theorem 2, we showed agents' best response under the absence of information. Under such conditions, including in the transmitted message the alternative yielding the second highest utility partially depends on the size of the electorate through the weighting function $\lambda(n)$. Theorem 3 determines the value that $\lambda(n)$ takes when we strength the conditions making more difficult the appearance of strategic behaviour, by increasing up to infinity the size of the electorate.

Theorem 3: *When agents have no information on the preferences of other voters and form uniform beliefs about them, then $\lambda(n) \xrightarrow{n \rightarrow \infty} \frac{1}{2}$.*

Proof: Following notation introduced in the proof of Theorem 2, we want to prove that $\lambda(n) = \frac{f(n)}{h(n)} \xrightarrow{n \rightarrow \infty} \frac{1}{2}$. Given that

$$f(n) + g(n) = h(n)$$

this is equivalent to proving,

$$\frac{f(n) - g(n)}{h(n)} \xrightarrow{n \rightarrow \infty} 0.$$

Substituting values and after basic calculus,

$$\frac{f(n) - g(n)}{h(n)} = \frac{\frac{1}{2}(P_1 - P_5)}{h(n)} + \frac{\frac{1}{2}(P_3 - P_2)}{h(n)} \leq \frac{\frac{1}{2}(P_1 - P_5)}{\frac{1}{2}P_5} + \frac{\frac{1}{2}(P_3 - P_2)}{\frac{1}{2}P_2} = \frac{P_1 - P_5}{P_5} + \frac{P_3 - P_2}{P_2}.$$

Hence, proving

$$\frac{P_1 - P_5}{P_5} + \frac{P_3 - P_2}{P_2} \xrightarrow{n \rightarrow \infty} 0,$$

implies $\frac{f(n) - g(n)}{h(n)} \xrightarrow{n \rightarrow \infty} 0$.

Actually, we here prove that $\frac{P_1 - P_5}{P_5} \xrightarrow{n \rightarrow \infty} 0$ and $\frac{P_3 - P_2}{P_2} \xrightarrow{n \rightarrow \infty} 0$, which is stronger than what is needed. We start by proving that $\frac{P_1 - P_5}{P_5} \xrightarrow{n \rightarrow \infty} 0$.

Consider the following two standard properties of combinatorial numbers which apply to any non-negative integers k, i for $k \geq i$:

Property 1: $\binom{k}{i} + \binom{k}{i-1} = \binom{k+1}{i}$.

Property 2 (symmetry): $\binom{k}{i} = \binom{k}{k-i}$.

By Property 1,

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{P_1 - P_5}{P_5} &= \lim_{n \rightarrow \infty} \frac{\sum_{t=2}^{n-1} \left[\binom{n-1}{t}^2 \left[\binom{n-1}{t} + \binom{n-1}{t-1} \right] \right]}{\sum_{t=2}^{n-1} \binom{n-1}{t}^2 \sum_{s=0}^{t-2} \binom{n-1}{s}} = \\ &= \lim_{n \rightarrow \infty} \frac{\sum_{t=2}^{n-1} \left[\binom{n-1}{t}^2 \left[\binom{n}{t} \right] \right]}{\sum_{t=2}^{n-1} \binom{n-1}{t}^2 \sum_{s=0}^{t-2} \binom{n-1}{s}}. \end{aligned}$$

We only consider the cases in which n is even (a similar reasoning would follow for the case in which n is odd). From the last expression and Property 2 we can derive,

$$\lim_{n \rightarrow \infty} \frac{P_1 - P_5}{P_5} = \lim_{n \rightarrow \infty} \frac{\sum_{t=2}^{n-1} \left[\binom{n-1}{t}^2 \left[\binom{n}{t} \right] \right]}{\sum_{t=2}^{n-1} \binom{n-1}{t}^2 \sum_{s=0}^{t-2} \binom{n-1}{s}} = \lim_{n \rightarrow \infty} \frac{A}{B}$$

where

$$A = \sum_{t=2}^{\frac{n-2}{2}} \binom{n-1}{t}^2 \left[\binom{n}{t} + \binom{n}{n-t-1} \right] + \binom{n-1}{n-2}^2 \binom{n}{n-2} + \binom{n-1}{n-1}^2 \binom{n}{n-1}$$

and

$$\begin{aligned} B &= \sum_{t=2}^{\frac{n-2}{2}} \binom{n-1}{t}^2 \left[\sum_{s=0}^{t-2} \binom{n-1}{s} + \sum_{s=0}^{n-t-3} \binom{n-1}{s} \right] \\ &\quad + \binom{n-1}{n-2}^2 \sum_{s=0}^{n-4} \binom{n-1}{s} + \binom{n-1}{n-2}^2 \sum_{s=0}^{n-3} \binom{n-1}{s}. \end{aligned}$$

Notice that by Property 2, $\binom{n}{n-t-1} = \binom{n}{t+1}$. Applying Properties 1 and 2 to A , we obtain:

$$A = \sum_{t=2}^{\frac{n-2}{2}} \binom{n-1}{t}^2 \binom{n+1}{t+1} + \binom{n-1}{n-2}^2 \binom{n}{n-2} + \binom{n-1}{n-1}^2 \binom{n}{n-1}.$$

Notice that $\lim_{n \rightarrow \infty} \frac{A}{B}$ can be expressed as $\lim_{n \rightarrow \infty} \frac{\sum_t a_t(n)}{\sum_t b_t(n)}$. By taking into account that

$\lim_{n \rightarrow \infty} \frac{\sum_t a_t(n)}{\sum_t b_t(n)} \leq \lim_{n \rightarrow \infty} \frac{a_{k_n}}{b_{k_n}}$ where k_n is the value that maximizes $\frac{a_t(n)}{b_t(n)}$ for dimension n , it is sufficient to

prove that $\lim_{n \rightarrow \infty} \frac{a_{kn}}{b_{kn}} = 0$.

In our expression, it is $\frac{a_t}{b_t} = \frac{\binom{n-1}{t}^2 \binom{n+1}{t+1}}{\binom{n-1}{t}^2 \left[\sum_{s=0}^{t-2} \binom{n-1}{s} + \sum_{s=0}^{n-t-3} \binom{n-1}{s} \right]}$ and therefore,

$$\frac{a_{t+1}}{b_{t+1}} = \frac{\binom{n-1}{t+1}^2 \binom{n+1}{t+2}}{\binom{n-1}{t+1}^2 \left[\sum_{s=0}^{t-1} \binom{n-1}{s} + \sum_{s=0}^{n-t-4} \binom{n-1}{s} \right]}.$$

Notice that $t+1 \leq \frac{n-2}{2}$ implies $t+2 \leq \frac{n}{2} \leq \frac{n+1}{2} \implies \binom{n+1}{t+2} \geq \binom{n+1}{t+1}$ and

$$b_t - b_{t-1} = \binom{n-1}{t+2} - \binom{n-1}{t-1} > 0,$$

$$\begin{aligned} \implies \frac{a_t}{b_t} \leq \frac{a_{t+1}}{b_{t+1}} &\iff \frac{\binom{n+1}{t+1}}{\binom{n-1}{t}^2 \left[\sum_{s=0}^{t-2} \binom{n-1}{s} + \sum_{s=0}^{n-t-3} \binom{n-1}{s} \right]} \leq \\ &\frac{\binom{n+1}{t+2}}{\binom{n-1}{t+1}^2 \left[\sum_{s=0}^{t-1} \binom{n-1}{s} + \sum_{s=0}^{n-t-4} \binom{n-1}{s} \right]}. \end{aligned}$$

Therefore,

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{a_t}{b_t} &\leq \lim_{n \rightarrow \infty} \frac{\binom{n-1}{\frac{n-2}{2}}^2 \binom{n+1}{\frac{n}{2}}}{\binom{n-1}{\frac{n-2}{2}}^2 \left[\sum_{s=0}^{\frac{n-6}{2}} \binom{n-1}{s} + \sum_{s=0}^{\frac{n-4}{2}} \binom{n-1}{s} \right]} = \\ &\lim_{n \rightarrow \infty} \frac{\binom{n-1}{\frac{n}{2}}}{\sum_{s=0}^{n-1} \binom{n-1}{s} - \binom{n-1}{\frac{n-2}{2}} - \binom{n-1}{\frac{n}{2}} - \binom{n-1}{\frac{n+2}{2}}} = \\ &\lim_{n \rightarrow \infty} \frac{\binom{n+1}{\frac{n}{2}}}{2^{n-1} - \binom{n-1}{\frac{n-2}{2}} - \binom{n-1}{\frac{n}{2}} - \binom{n-1}{\frac{n+2}{2}}} = 0. \end{aligned}$$

The proof for P_3 and P_2 is similar, and thus we omit it. This concludes the proof. \square

Thus, as the size of the electorate increases, agents' best response consists in voting for those alternatives that yield more than the average of utilities. Given that we have eliminated the most important components of strategic behavior, namely information on others' preferences over alternatives and the weight of an individual vote in determining the outcome, we interpret such best response as sincere voting behaviour under approval voting.

Notice that previous attempts¹⁷ to define sincere behaviour under approval voting did not differentiate between the implications of our Theorems 2 and 3. The reason is that they assumed that the probability of a tie between the number of votes that two alternatives received was equal to the probability of one of the alternatives surpassing the other by just one vote. As our by-product of our Theorem 3, we show that such assumption only holds true in the limit, i.e., as the size of the electorate increases.

5 Discussion

Identifying sincere voting behaviour under a variety of voting rules is an important starting point in the discussion of adopting new voting mechanisms. A definition of sincerity is almost straightforward when simple voting mechanisms are considered. However, we have seen that under a complex voting mechanism such as Approval voting defining sincerity is cumbersome.

Approval Voting is a paradigmatic voting mechanism to study how cardinal utilities over alternatives affect sincere behaviour. We conjecture that the difficulty in defining sincerity arises as a consequence of the presence of several message types in complex mechanisms. Our intuition should be confirmed by studying other complex voting mechanisms.

Our approach to define sincere voting behaviour consists in opposing sincere behavior to strategic behavior. We methodologically contribute by omitting the elements that facilitate strategic behavior, namely by increasing the size of the electorate and by eliminating information on other agents' preferences over alternatives, to obtain a formal definition of sincerity. The optimal behavior obtained under such conditions is thus what we define as sincere voting behaviour. We have shown that under Approval Voting sincere agents vote for those alternatives that yield more than the average of the utilities.

Notice that following our approach, the definition of sincerity coincides with the previously provided strong definition of sincerity. Our technical contribution consists in calculating the optimal voting behaviour by assessing explicitly the probability of each of the possible races between alternatives that can occur instead of assuming they all have the same probability. Therefore, we have provided stronger support to an intuitive definition of sincerity when agents have cardinal utilities over alternatives.

¹⁷Hoffman (1982).

Our aim in this paper has been to provide a definition of sincere voting behaviour. Nevertheless, this is not equivalent to identifying sincere voting from the results of an election. Knowledge on the cardinal value that the alternatives yield to the voters is required in empirical tests of our results. An experiment controlling for such utility values may be a worthwhile avenue to explore how individuals vote when informational conditions or on the influence of their votes are changed.

6 References

Aragonés, E., Gilboa, I., Weiss, A., (2005). “Making Statements in Approval Voting”. Cowles Foundation Discussion Paper No. 1531. Yale University.

Arrow, J. K., (1951). *Social Choice and Individual Values*. New York: John Wiley & Sons, rev. ed. 1963.

Brams, S. J., Fishburn, P. C., (1978). “Approval Voting”. *The American Political Science Review* 72: 831-847.

Brams, S. J., Fishburn, P. C., (1981). “Approval Voting, Condorcet’s Principle, and Runoff Elections”. *Public Choice* 36: 89-114.

Broseta, B, Costa-Gomes, M., Crawford, B., (2001). “Cognition and Behavior in Normal Form Games: an Experimental Study”. *Econometrica* 68, 1193-1235.

Dhillon, A., Lockwood, B. (2004). “When are plurality rule voting games dominance-solvable?”. *Games and Economic Behavior* 46, 55-75.

Dutta, B., Jackson M., Le Breton, M., (2001). “Strategic Candidacy and Voting Procedures”. *Econometrica* 69, 1-22.

Farquharson, R., (1969). *Theory of Voting*. New haven, Connecticut. Yale University Press.

Fishburn, P. C., (1973). *The Theory of Social Choice*. Princeton, New Jersey. Princeton University Press.

Fisher, S., Myatt, D., (2002). “Strategic Voting Experiments”. Nuffield College Politics Working Paper 2002.W4.

Gibbard, A., (1973). “Manipulation of Voting Systems: A General Result”. *Econometrica* 41, 587-601.

Goeree, J., Holt, C., (2004). “A Model of Noisy Introspection”. *Games and Economic Behavior* 46(2), 365-382.

Hoffman, D., (1982). “A model for Strategic Voting”. *SIAM journal on Applied Mathematics* 42(4), 751-761.

McKelvey, R., Palfrey, T., (1995). “Quantal Response Equilibrium for Normal Form Games”. *Games and Economic Behavior* 10, 6-38.

Merrill, S., (1983). *Making Multicandidate Elections more Democratic*. Princeton. Princeton University Press.

Merrill, S., Nagel, J., (1987). “The Effect of Approval Balloting on Strategic Voting Under Alternative Decision Rules”. *The American Political Science Review* 81, 509-524.

- Myerson, R., (2002). "Comparison of Scoring Rules in Poisson Voting Games". *Journal of Economic Theory* 103, 219-251.
- Myerson, R. B., Weber, R. J., (1993). "A Theory of Voting equilibria". *American Political Science Review* 87, 102-114.
- Niemi, R. G., (1984). "The Problem of Strategic Behavior under Approval Voting". *The American Political Science Review* 78, 952-958.
- Nurmi, H., (1987). *Comparing Voting Systems*. Theory and Decision Library. Series A: Philosophy and Methodology of the Social Sciences. D. Reidel Publishing Company.
- Satterthwaite, M., (1975). "Strategy Proofness and Arrow's Conditions". *Journal of Economic Theory* 10, 187-217.
- Stahl, D., (1993). "Evolution of Smart Players". *Games and Economic Behavior* 5, 604-617.
- Stahl, D., Wilson, P., (1994). "Experimental Evidence on Players' Models of Other Players". *Journal of Economic Behavior and Organization* 25, 309-327.
- Stahl, D., Wilson, P., (1995). "On Players' Models of Other Players: Theory and Experimental Evidence". *Games and Economic Behavior* 10, 218-254.
- Weber, R.K., (1978). "Comparison of Voting Systems". Cowles Foundation Discussion Paper No. 498. Yale University.