Aplications: labor supply and intertemporal choice

17 de noviembre de 2011

6.1

1. Optimal conditions:

$$\begin{cases} p \cdot c \leq M + w \underbrace{(H-h)}_{l} \quad \Rightarrow p \cdot c + w \cdot h \leq M + w \cdot H \\ \\ MRS = -\frac{w}{p} \end{cases}$$

Then:

$$-\frac{3ch^2}{h^3} = -\frac{w}{p}$$

$$\frac{3c}{h} = \frac{w}{p}$$

$$c = \frac{w}{p} \cdot \frac{h}{3}$$

Substituting in the budget constraint:

$$p\left(\frac{w}{p} \cdot \frac{h}{3}\right) + wh = M + w \cdot H$$

$$w \cdot \frac{h}{3} + wh = M + wH$$

$$h\left(w + \frac{w}{3}\right) = M + wH$$

$$h^*(M,H,w) = \frac{M+wH}{\left(w+\frac{w}{3}\right)} = \frac{3(M+wH)}{4w}$$

$$c^*(M,H,w) = \frac{w}{p} \cdot \frac{\left(\frac{3(M+wH)}{4w}\right)}{3} = \frac{3(M+wH)}{12p}$$

Substituting the values:

$$\begin{cases} h^* = 21{,}75 \\ c^* = 7{,}25 \end{cases}$$

We obtained the quantity demanded of leisure and consumption, however the statement asks the consumer quantity demande and quantity supplied of work:

$$\begin{cases} l^* = H - h^* = 24 - 21{,}75 = 2{,}25 \\ c^* = 7{,}25 \end{cases}$$

2.

$$\frac{\partial h}{\partial (M+wH)} \lesseqgtr 0?$$

$$\frac{\partial h}{\partial (M+wH)} = \frac{3}{4w} > 0$$

If income increases, leisure increases. Leisure is a normal good.

3.
Just we have to change $H = 25$ by $H = 10$ in:

$$\begin{cases} h^*(M,H,w) = \frac{3(M+wH)}{4w} = 11{,}25 \\ c^*(M,H,w) = \frac{3(M+wH)}{12p} = 3{,}75 \end{cases}$$

Now we get the quantity of labor:

$$\begin{cases} l^* = H - h^* = 24 - 11{,}25 = 12{,}75 \\ c^* = 3{,}75 \end{cases}$$

4.

- Efects on consumption of variations in w:

$$\frac{\partial c}{\partial w} \lesseqgtr 0?$$

$$\frac{\partial c}{\partial w} = \frac{3H}{12p} = \frac{H}{4p} > 0$$

A wage increase have a positive effect on consumption:

- Efects on leisure of variations in w:

$$\frac{\partial h}{\partial w} \lesseqgtr 0?$$

$$\frac{\partial h}{\partial w} = \frac{3h \cdot 4w - 4\,(3M + 3wH)}{4w^2} = -\frac{3M}{4w^2} < 0$$

A wage increase have a negative effcet on leisure and therefore have a positive effcet on hours worked.

5.

Remember that in first section we found :

$$h^*(H, M, w) = \frac{3(M + wH)}{4w}$$

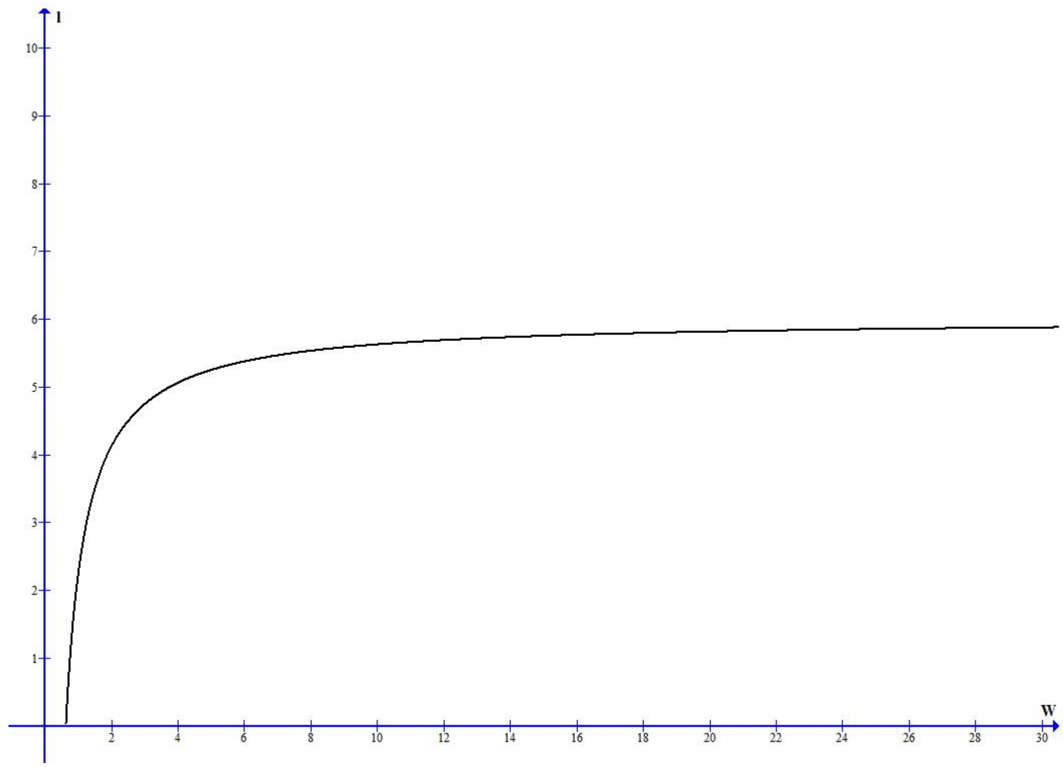
In the other hand we know that:

$$l = H - h$$

Then:

$$l^*(H, M, w) = H - \frac{3(M + wH)}{4w} = \frac{4wH - 3M - 3wH}{4w} = \frac{wH - 3M}{4w}$$

Graphical representacion of the labour supply when $H = 24$ and $M = 5$:



6.2

1. Optimal conditions:

$$\begin{cases} p \cdot c + w \cdot h \leq M + w \cdot H \\ \\ MRS = -\frac{w}{p} \end{cases}$$

Then:

$$RMS = -\frac{w}{p}$$

$$-\frac{\frac{1}{2}(2h)^{-1/2} \cdot 2}{1} = -\frac{w}{p}$$

$$(2h)^{-1/2} = \frac{w}{p}$$

$$\frac{1}{(2h)^{1/2}} = \frac{w}{p}$$

$$\frac{p}{w} \cdot 1 = (2h)^{1/2}$$

$$\left(\frac{p}{w}\right)^2 = 2h$$

$$h = \frac{1}{2}\left(\frac{p}{w}\right)^2$$

We know that:

$$l = H - h$$

Then we have our labor supply function:

$$l(H, p, w) = H - \left(\frac{p}{w}\right)^2 \cdot \frac{1}{2}$$

2.

$$l > 0$$

$$H - \left(\frac{p}{w}\right)^2 \cdot \frac{1}{2} > 0$$

$$H > \left(\frac{p}{w}\right)^2 \cdot \frac{1}{2}$$

$$2H > \left(\frac{p}{w}\right)^2$$

$$(2H)^{1/2} > \frac{p}{w}$$

$$w > \frac{p}{(2H)^{1/2}}$$

3. A small reduction in income tax is like a small increase in income, so we will use the derivative to analyse how it will affect his labor supply:

$$\frac{\partial l}{\partial w} = -\frac{1}{2} \cdot 2 \left(\frac{p}{w}\right) \left(-\frac{p}{w^2}\right) = \frac{p^2}{w^3} > 0$$

He will increase his labor supply.

4. Is like a reduction in price, we will also use the derivative:

$$\frac{\partial l}{\partial p} = -\frac{1}{2} \cdot 2 \left(\frac{p}{w}\right) \left(\frac{w}{w^2}\right) = -\frac{p}{w^2} < 0$$

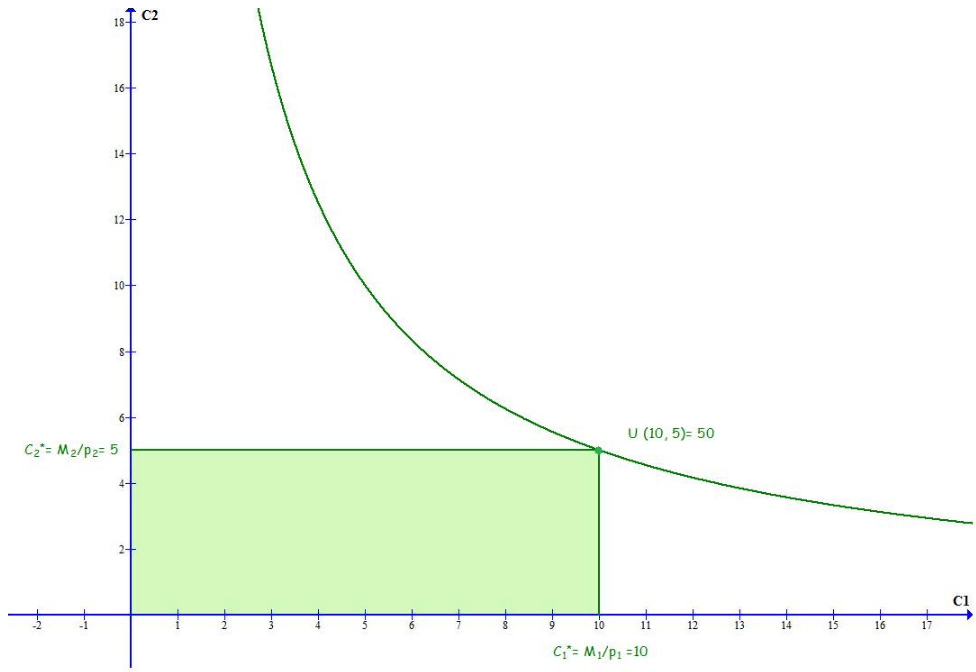
He will decrease his labor supply.

6.3

1.

Budget constraint period 1: $p_1 c_1 = M_1 \longmapsto c_1^* = \frac{M_1}{p_1}$

Budget constraint period 2: $p_2 c_2 = M_2 \longmapsto c_2^* = \frac{M_2}{p_2}$



2.
Budget constraint:

$$p_1 c_1 + \frac{p_2 c_2}{(1+r)} \leq m_1 + \frac{m_2}{(1+r)}$$

Optimal:

$$MRS = -\frac{p_1}{p_2}$$

$$-\frac{c_2}{c_1} = -\frac{p_1}{p_2}$$

$$c_2 = \frac{p_1}{p_2} \cdot c_1$$

Substituting in budget constraint:

$$p_1 c_1 + \frac{p_2 \left(\frac{p_1}{p_2} \cdot c_1 \right)}{(1+r)} = M_1 + \frac{M_2}{(1+r)}$$

$$p_1 c_1 + \frac{p_1 c_1}{(1+r)} = M_1 + \frac{M_2}{(1+r)}$$

$$p_1 c_1 \left(1 + \frac{1}{(1+r)} \right) = M_1 + \frac{M_2}{(1+r)}$$

$$p_1 c_1 \left(\frac{2+r}{1+r} \right) = \frac{M_1(1+r) + M_2}{(1+r)}$$

$$c_1 p_1 (2+r) = M_1(1+r) + M_2$$

$$c_1 = \frac{M_1(1+r) + M_2}{p_1(2+r)}$$

Substituting M_1 and M_2:

$$c_1^*(p_1, r) = \frac{10(1+r)+5}{p_1(2+r)}$$

Then:

$$c_2^*(p_2, r) = \frac{10(1+r)+5}{p_2(2+r)}$$

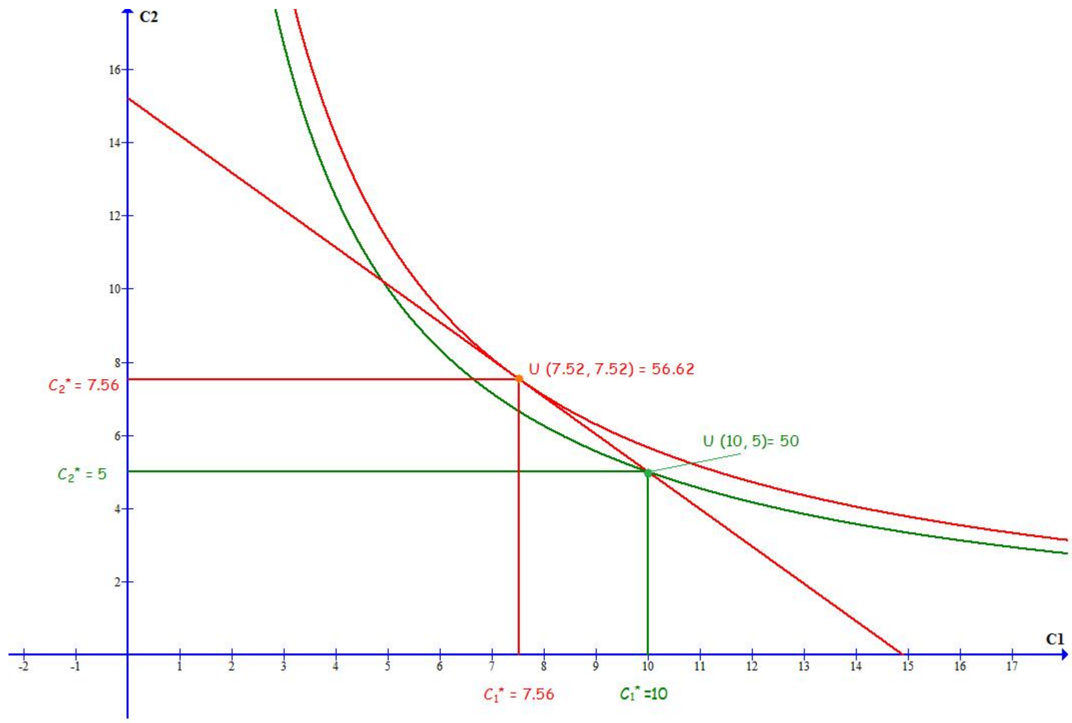
⇒ Is there a savings?

$$\underbrace{\frac{M_1(1+r)+M_2}{p_1(2+r)}}_{c_1\, with\, credit\, market} < \underbrace{\frac{M_1}{p_1}}_{c_1\, without\, credit\, market}$$

$$\underbrace{\frac{M_1(1+r)+M_2}{p_2(2+r)}}_{c_2\, with\, credit\, market} > \underbrace{\frac{M_2}{p_2}}_{c_2\, without\, credit\, market}$$

Yes, savings does exists.

⇒Graphical representation for $p_1 = p_2 = 1$ and $r = 0{,}02$, (in green we have the case witout credit market and in red we have the case with credit market):



3.
In our previous example:

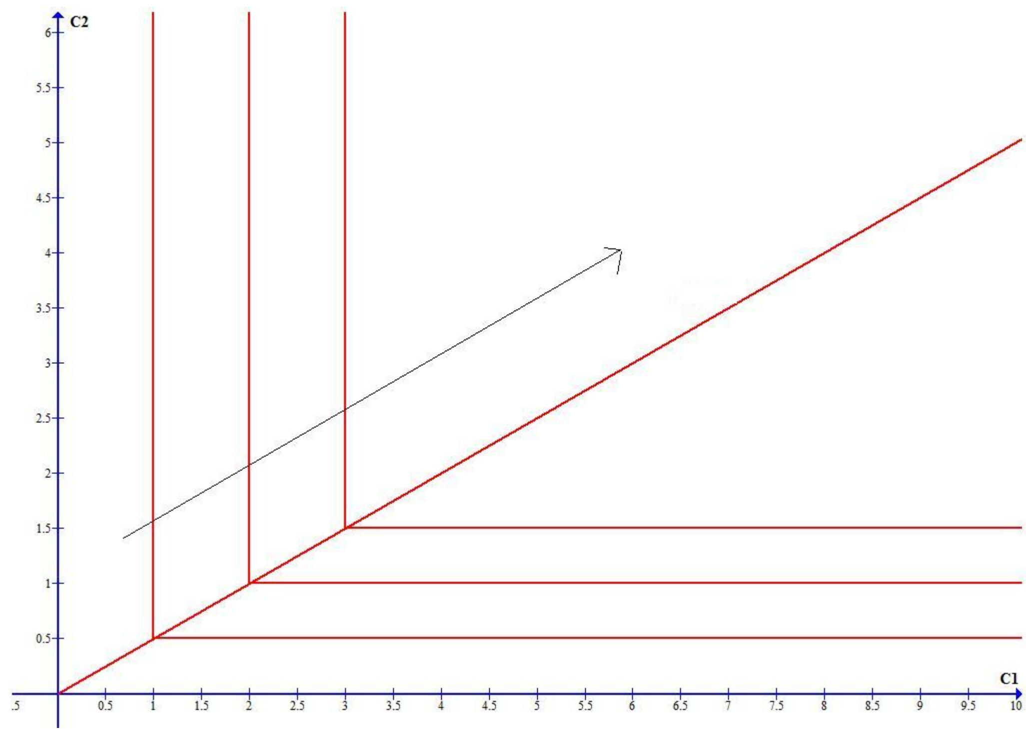
$$u(7{,}52,\ 7{,}52) > u(10,\ 5)$$

Yes, credit market promotes consumer welfare.

6.4

1. Consumer preferences can be write like:

$$u(c_1, c_2) = Min\,\{c_1, 2c_2\}$$



2.
The consumer can consume:

$$c_2 = \frac{m_1}{p_2}(1+r)$$

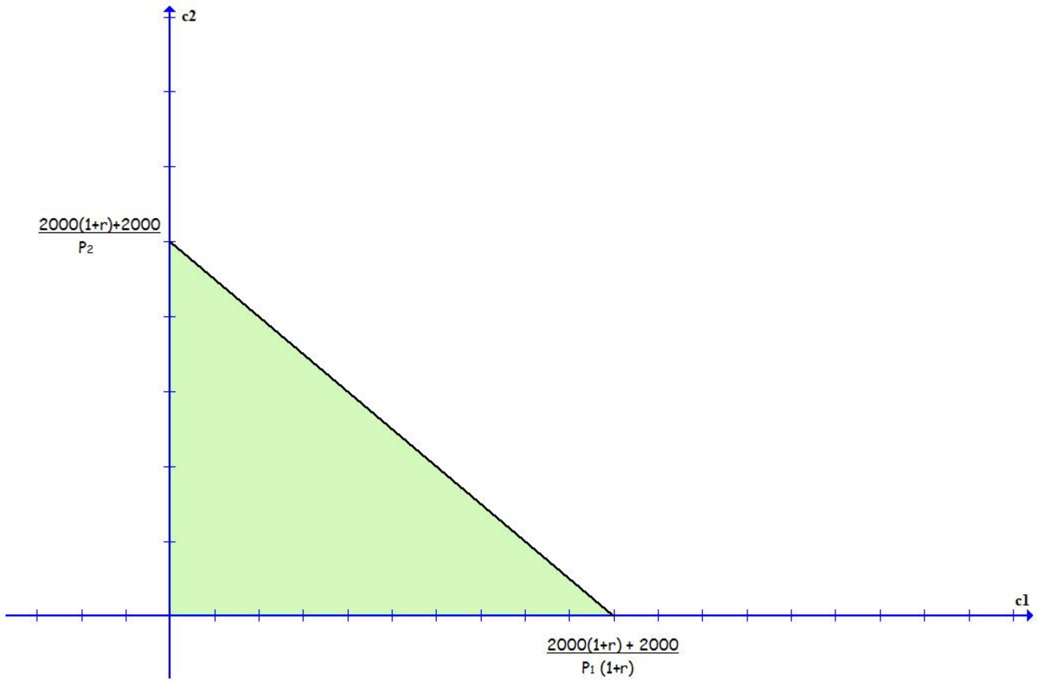
3.
Maximal loan is:

$$\frac{m_2}{(1+r)}$$

4.
Budget constraint:

$$p_1 c_1 + \frac{p_2 c_2}{(1+r)} \leq 2000 + \frac{2000}{(1+r)}$$

Budget set:



Let's calculate how much consumed in each period:

$$\begin{cases} p_1c_1 + \frac{p_2c_2}{(1+r)} \leq 2000 + \frac{2000}{(1+r)} \\ \\ c_1 = 2c_2 \end{cases}$$

Substituting second equation in first:

$$p_1 2c_2 + \frac{p_2c_2}{(1+r)} = 2000 + \frac{2000}{(1+r)}$$

$$c_2\left(2p_1 + \frac{p_2}{(1+r)}\right) = 2000 + \frac{2000}{(1+r)}$$

$$c_2\left(\frac{2p_1(1+r) + p_2}{(1+r)}\right) = \frac{2000(1+r) + 2000}{(1+r)}$$

$$c_2\left(2p_1(1+r)+p_2\right)=2000(1+r)+2000$$

$$c_2^*(p_1,p_2,r)=\frac{2000(1+r)+2000}{2p_1(1+r)+p_2}$$

Then:

$$c_1^*(p_1,p_2,r)=\frac{2000(1+r)+2000}{2\left[2p_1(1+r)+p_2\right]}=\frac{2000(1+r)+2000}{4p_1(1+r)+2p_2}$$

5.

Because the consumer want to keep the proporcion $c_1=2c_2$ and therefore any modification of his income (due to a variation of interest rate or prices) will increase/decrease that proportion.