Altruism and Risk Sharing in Networks

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Introduction

- Informal transfers are still prevalent in our 21^{st} century economies.
  - Transfers in cash, kind, or time between individuals and households and which are not market transactions.
- Key stylized facts:
  - Quantitatively large, even in high-income countries.
  - Interact with the business cycle and with public transfers.
  - Flow through social networks.
  - Generate (not too) inefficient informal insurance.

Introduction: quantitatively large

- In low-income countries, remittances often on par with the formal economy.
  - In 2009, Lebanon: 22% of GDP; Jordan: 16%; Philippines 12%, Worldbank (2011).
  - Greater than financial aid and close to foreign direct investment, Yang (JEP 2011).
  - Remarkably stable following the crisis. Between 2008 and 2009, dropped by 5% while FDI dropped by 40%.
- Also important in high-income countries.
  - In France, family transfers increased from 2% of GDP before the crisis to 4% after, Le Monde (2014).
  - In the US, interhousehold transfers estimated at 1.2% of GDP in 2003, National Transfer Accounts (2011).

Introduction: interact with public transfers

- Large empirical evidence on *crowding out*.
  - Increase in public transfers may decrease private transfers. Reduces the impact of public interventions.
  - In a study on the Philippines, between 30% to 80% of crowding out for those in the lowest income quintile, Cox, Hansen & Jimenez (JPubE 2004).
  - "attempts to aid the poor could be thwarted by private responses, which leak benefits to richer households in the form of lighter burdens of support for less fortunate kin."
- Recent evidence that transfers to the poor indirectly benefit socially connected households.
  - Angelucci & De Giorgi (AER 2009), Angelucci, De Giorgi & Rasul (WP 2012).

Introduction: social networks

- Expanding empirical literature studying bilateral transfers.
  - Dercon & De Weerdt (JDE 2006); Fafchamps & Lund (JDE 2003); Fafchamps & Gubert (JDE 2007).
  - Recent work by Arun Chandrasekhar, Pascaline Dupas and others.
- Even in small rural communities, it is not true that everyone is helping everyone else.
  - Rather, informal transfers are structured through social networks: close and distant relatives, friends, neighbors.

Introduction: inefficient insurance

- Following Towsend (ECA 1994), large empirical literature that tests efficient insurance on consumption data.
  - Typically rejected at the village level. Still, does not seem too inefficient: consumption little affected by individual shocks.
  - Often interpreted as a sign that informal risk-sharing works quite well.
- Mazzocco & Shaini (AER 2012) develop new versions of the tests to account for preference heterogeneity.
  - On data on rural India, reject efficient insurance at the village level but not at the subcaste level.

Introduction: motives

- Why do people give? Three main explanations.
  - Exchange: Mutually beneficial informal insurance contract.
  - Altruism: People give to others they care about.
  - Social pressure: People give because they feel obliged to.
- Identifying the motives empirically is challenging.
  - Large literature following Cox (JPE 1987). Recent studies based on clever experiments.
  - Evidence that the three motives are at work, although maybe across different types of ties and circumstances.

Introduction: theory

- Growing theoretical literature rationalizing these four facts.
  - Models of informal transfers in networks.
- Links as social collateral, Ambrus, Mobius & Szeidl (AER 2014).
  - Links have values constraining how much money can flow through them.
  - Characterize Pareto-constrained risk-sharing arrangements.
- Local information constraints, Ambrus, Gao & Milan (WP 2017).
  - How to reach these Pareto-constrained arrangements?

Introduction: theory

- Altruism in networks, Bourlès, Bramoullé & Perez-Richet (ECA 2017).
  - Altruism à la Becker, structured through a network.
  - Characterize Nash equilibria of the game of transfers, for non-stochastic incomes.
- Even in the absence of risk, altruism generates informal redistribution.
  - Not true under informal insurance contracts.

Introduction: altruism in networks

- Advances the economics of altruism.
  - Following Becker (JPE 1974) and Barro (JPE 1974).
  - Large literature but irrealistic structures: Small groups of completely connected agents or linear dynasties.
- However, family ties form complex networks.
  - Well-known from human genealogy.
  - Argued early on by Bernheim & Bagwell (JPE 1988) but had not been explored by economists.

Introduction: altruism and risk sharing

- In this new paper, we study the risk sharing implications of altruism networks.
  - Incomes are stochastic, transfers conditional on incomes as in BBP (2017).
  - Becker (JPE 1974)'s early intuition: "The head's concern about the welfare of other members provides each, including the head, with some insurance against disasters."
  - Never studied in a network context.
- We find that altruism networks have a strong impact on risk.

Introduction: altruism and risk sharing

- Informal insurance tends to be better if the network has lower average path length.
- We characterize what happens for small shocks.
  - Partially insured by endogenous risk-sharing communities.
- We show that large shocks tend to be well-insured.
- Throughout, we contrast outcomes under altruism and under social collateral.
- We uncover complex structural effects.
  - A new link may decrease or increase the risk faced by others.

Model: informal transfers

- Agent i has income y_i^0 and may give $t_{ij} \geq 0$ to agent j.
  - Matrix $\mathbf{T} = (t_{ij})$ represents the network of informal transfers.
- Consumption y_i is equal to

$$y_i = y_i^0 - \sum_j t_{ij} + \sum_k t_{ki}$$

- Aggregate income is conserved: $\sum_i y_i = \sum_i y_i^0$.

Model: altruism in networks

- Agents care about others' well-being:

$$v_i(\mathbf{y}) = u_i(y_i) + \sum_j \alpha_{ij} u_j(y_j)$$

- Coefficient $\alpha_{ij} \in [0, 1]$ measures the strength of the altruistic link from i to j.
  - Network of altruism (α_{ij}) describing the structure of social preferences.
- i may care about j but not about j's friends. Interests of a giver and a receiver may be misaligned.

Model: altruism in networks

- Noncooperative game: Agents makes transfers to maximize their altruistic utilities.
  - Transfers by an agent depend on transfers made by others.
- **T** is a Nash equilibrium iff (1) $t_{ij} > 0 \Rightarrow u_i'(y_i) = \alpha_{ij} u_j'(y_j)$ and (2) $\forall i, j, u_i'(y_i) \geq \alpha_{ij} u_j'(y_j)$.
  - Under CARA $u_i(y) = -e^{-Ay}$,
    (1) $t_{ij} > 0 \Rightarrow y_i = y_j + (-\ln(\alpha_{ij}))/A$ and (2)
    $\forall i, j, y_i \leq y_j + (-\ln(\alpha_{ij}))/A$.
- An agent does not let the consumption of a poorer friend become too much lower than his own.

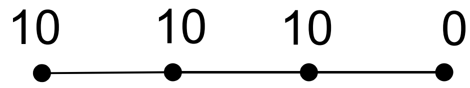
Model: altruism in networks

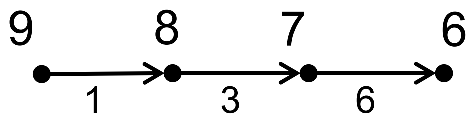
Theorem (BBP 2017) *A Nash equilibrium always exists. Equilibrium consumption* $\mathbf{y}$ *is unique. Generically in* $\boldsymbol{\alpha}$*, there is a unique Nash equilibrium* $\mathbf{T}$.

- Emergence of transfer intermediaries in equilibrium.
  - Give to poorer friends part of the money received from richer friends.
  - Shocks propagate in the altruism network. Example on the line.

Equilibria on the line

u CARA, links have same strength: $-ln(\alpha)/A=1$





Altruism and risk

- Suppose now that incomes are stochastic.
  - How do altruistic transfers in networks affect risk?
- Interplay of two countervailing forces: (1) Conditional on $\mathbf{y}^0_{-i}$, altruistic transfers reduce risk.

Proposition *Conditional on* $\mathbf{y}^0_{-i}$, $y_i - E(y_i)$ *Second-Order Stochastically Dominates* $y_i^0 - E(y_i^0)$.

- Proof: y_i is weakly increasing in y_i^0 but $y_i - y_i^0$ is weakly decreasing in y_i^0
- Changes in transfers made or received caused by changes in own income tend to smooth own consumption.

Altruism and risk

- (2) Conditional on y_i^0, altruistic transfers tend to increase risk. Bear part of the income risk of others.
- To give examples, useful benchmark with no redistribution in expectation.

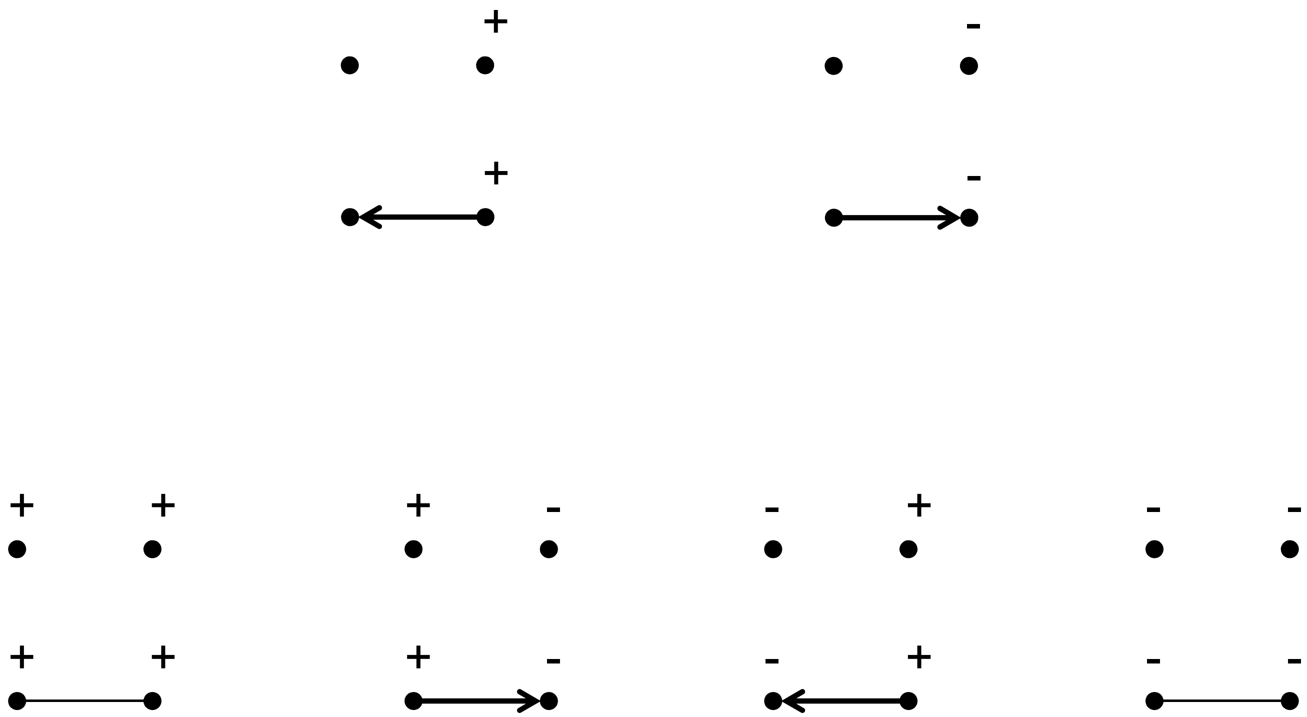
Proposition *Under CARA, symmetric incomes and undirected ties* $\alpha_{ij} = \alpha_{ji}$, $\forall i$, $Ey_i = Ey_i^0$.

- Proof: If $\mathbf{T}$ equilibrium transfers for $E(\mathbf{y}^0) + \varepsilon$, reverse transfers $\mathbf{T}^t$ equilibrium for $E(\mathbf{y}^0) - \varepsilon$.

Altruism and risk

- Simple example with 2 agents and iid binary shocks.
- When i has non-stochastic income, y_i more risky than y_i^0 while y_j less risky.
- When both agents have stochastic incomes, consumption less risky for both.
  - Here, the first force dominates the second.

Altruism and risk sharing: 2 agents



Model: efficient insurance

Definition *Informal transfers yield efficient insurance if* $\exists \boldsymbol{\lambda} \gneqq \mathbf{0}$ *such that consumption* $\mathbf{y}$ *solves*

$$\max_{\mathbf{y}} \sum_i \lambda_i E u_i(y_i)$$

subject to $\sum_i y_i = \sum_i y_i^0$.

- Classical notion underlying empirical analysis following Townsend (1994).
- With common utilities and equal Pareto weights, leads to equal income sharing $\forall i, y_i = \bar{y}^0$.
  - In general, $u_i'(y_i)/u_j'(y_j) = \lambda_j/\lambda_i$ in every state of the world.
  - Under CARA, $y_i = \bar{y}^0 + \ln(\lambda_i)/A$.

Efficient insurance and perfect altruism

Proposition *Equilibrium transfers induce efficient insurance with equal Pareto weights if any two agents are indirectly connected through a path of altruistic links of strength* 1.

- Proof: If $\alpha_{ij} = 1$, $u_i'(y_i) \geq u_j'(y_j)$. Path from i to j and path from j to i yield $u_i'(y_i) = u_j'(y_j)$.
- Perfect altruism between pairs aggregate up into efficient insurance.
  - Equal income sharing between pairs leads to overall equal sharing in a connected network.
  - Even when sparse: stars, circle or line.
  - Agents act *as if* they were altruistic towards their friends' friends.

Insurance and imperfect altruism

- How far does society get from efficient insurance when altruism is imperfect?
  - Following Ambrus, Mobius & Szeidl, introduce distance from equal income sharing: $DISP(\mathbf{y}) = E\frac{1}{n}\sum_i |y_i - \bar{y}^0|$.
- Define $c_{ij} = -\ln(\alpha_{ij})$ virtual cost of link ij and $\hat{c}_{ij} =$ least-cost of paths connecting i to j.
  - If the network is binary and $\alpha_{ij} \in \{0, \alpha\}$, $\hat{c}_{ij} = cd_{ij}$ where d_{ij} = network distance between i and j.
  - Let $\bar{d} = \frac{1}{n(n-1)}\sum_{i \neq j} d_{ij}$ be the average path length.
  - $\hat{c}_{ij}$ = extension of network distance to weighted networks.

Insurance and imperfect altruism

Proposition

Under CARA and undirected ties $\alpha_{ij} = \alpha_{ji}$,

$$DISP(\mathbf{y}) \leq \frac{1}{A}\frac{1}{n^2}\sum_{i,j}\hat{c}_{ij}$$

- For binary networks $\alpha_{ij} \in \{0, \alpha\}$,

$$DISP(\mathbf{y}) \leq \frac{-\ln(\alpha)}{A}\frac{n(n-1)}{n^2}\bar{d}$$

- Proof: Sum of equilibrium inequalities $y_i \leq y_j + (-\ln(\hat{\alpha}_{ij}))/A$.

Insurance and imperfect altruism

- Under CARA, informal insurance induced by altruism tends to be better when average path length is lower.
  - Contrasts to the role played by expansiveness under social collateral.
- Informal insurance subject to *small-world effects* under altruism but not under social collateral.
  - A few links between communities may have a strong impact.
- Bound extends to directed networks, other measures of distance and other utilities.
  - Dispersion of path lengths around the mean may also matter.

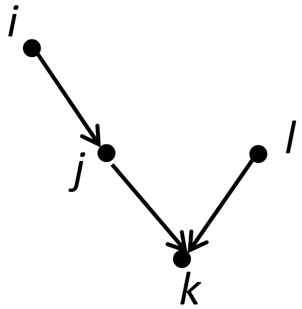
Efficient insurance and small shocks

- Are there other circumstances where efficient insurance is achieved?
  - Perhaps surprisingly, the answer is yes.
  - We next characterize what happens for small shocks.
- From equilibrium $\mathbf{T}$, define the *graph of transfers* $\mathbf{G}$ as $g_{ij} = 1$ if $t_{ij} > 0$ and $g_{ij} = 0$ otherwise.
  - Consider the (weak) components of $\mathbf{G}$.
  - Generically in $\boldsymbol{\alpha}$ and $\mathbf{y}^0$, small shocks $\mathbf{y}^0 + \varepsilon$ yield the same graph of transfers.

Efficient insurance and small shocks

- If i indirectly connected to j in $\mathbf{G}$, define $\bar{c}_{ij} = \sum_s \varepsilon_{i_s i_{s+1}} c_{i_s i_{s+1}}$ where $\varepsilon_{i_s i_{s+1}} > 0$ if $t_{i_s i_{s+1}} > 0$ and $\varepsilon_{i_s i_{s+1}} < 0$ if $t_{i_s i_{s+1}} < 0$ in path from i to j.
- Net virtual cost of indirect connection from i to j.
  - Example. If $\alpha_{ij} = \alpha_{ji} \in \{0, \alpha\}$, $\bar{c}_{ij}$ = directed distance between i and j in $\mathbf{G}$.
  - Satisfies $\bar{c}_{ji} = -\bar{c}_{ji}$ and triangular equality $\bar{c}_{ij} + \bar{c}_{jk} = \bar{c}_{ik}$.

Directed cost



$$\bar{c}_{il} = c_{ij} + c_{jk} - c_{kl}$$

Efficient insurance and small shocks

Theorem

(1) Generically in $\boldsymbol{\alpha}$ *and* $\mathbf{y}^0$*: Consider small shocks around* $\mathbf{y}^0$*. Then, equilibrium transfers induce efficient insurance within components of* $\mathbf{G}$ *for Pareto weights* $\lambda_i = \exp(\frac{1}{n_C}\sum_{j \in C} \bar{c}_{ij})$.

(2) Generically in $\boldsymbol{\alpha}$*: Suppose that society is partitioned in communities, equilibrium transfers induce efficient insurance within communities and* y_i^0 *is continuous with positive density over its support. Then the graph of transfers is constant across income realizations and its components are the communities.*

Efficient insurance and small shocks

- Idea of the proof: (1).
  - Assemble Nash conditions on positive transfers. Yields $\ln(u_i') - \ln(u_j') = -\bar{c}_{ij}$ for any i,j in a component of $\mathbf{G}$.
  - We then check that $\ln(\lambda_i) - \ln(\lambda_j) = \bar{c}_{ij}$. In addition, $\sum_i \ln(\lambda_i) = 0$.
- Idea of the proof: (2).
  - Consider open set with an equilibrium transfer graph $\mathbf{G}$. By (1), $\boldsymbol{\lambda}(\mathbf{G}) = \boldsymbol{\lambda}$.
  - Generically in $\boldsymbol{\alpha}$, the mapping $\mathbf{G} \rightarrow \boldsymbol{\lambda}(\mathbf{G})$ is injective.

Efficient insurance and small shocks

- Components of **G** constitute endogenous risk sharing communities.
  - Extends Theorem 3 in BBP (2017) on income pooling to risk sharing. Characterizes functions f_i.
- *Within components, equilibrium behavior equivalent to a planner's program*.
  - Pareto weight $\lambda_i = \exp(\frac{1}{n_C} \sum_{j \in C} \bar{c}_{ij})$ increasing in average net distance between i and other members of the component.
  - Tends to be larger for givers, smaller for receivers.

Efficient insurance and small shocks

- Quality of insurance depends on the connectedness of $\mathbf{G}$.
- *Small shocks are efficiently insured if* $\mathbf{G}$ *is connected.*
  - Happens if α is connected and i very rich or very poor.
  - Money then flows from i to everyone or from everyone to i.
  - Adjustements in altruistic transfers generate efficient insurance.

Efficient insurance and small shocks

- *Small shocks are not insured if* **G** *is empty.*
  - Happens with similar incomes $y_i^0 \approx y_j^0$.
  - Agents then bear all the risk associated with small shocks.
- In general, the quality of insurance depends on the components' sizes.
  - Under CARA, $y_i = \bar{y}_C^0 + \frac{1}{A}\ln(\lambda_i)$. With iid incomes, $Var(y_i) = \frac{1}{n_C}Var(y_i^0)$.

Large shocks

- Consider opposite benchmark, when an agent is subject to large shocks.
  - Recall, $\hat{c}_{ij}$ = least cost of indirect connection between i and j.

Proposition *Under CARA and $\boldsymbol{\alpha}$ connected, for any $\mathbf{y}^0_{-i}$ there exist $Y_H > Y_L$ such that:*

$$y_i^0 \geq Y_H \Rightarrow y_i = \bar{y}^0 + \frac{1}{A}\frac{1}{n}\sum_j \hat{c}_{ij}$$
$$y_i^0 \leq Y_L \Rightarrow y_i = \bar{y}^0 - \frac{1}{A}\frac{1}{n}\sum_j \hat{c}_{ji}.$$

- If i's income is very low or very high and $\boldsymbol{\alpha}$ connected, everyone involved in informal transfers.
  - Except in the middle range, y_i varies linearly in y_i^0 with slope $1/n$. As with efficient insurance.

Large shocks

Proposition *Consider the model of social collateral with* $t_{ij} \leq \kappa_{ij}$*. Under CARA, for any* $\mathbf{y}^0_{-i}$ *there exist* $Y_H > Y_L$ *such that*

$$y_i^0 \geq Y_H \Rightarrow y_i = y_i^0 - c_H \text{ and } y_i^0 \leq Y_L \Rightarrow y_i = y_i^0 + c_L$$

- If shocks are large, informal transfers saturate links' maximum capacity.
  - Except in the middle range, y_i varies linearly in y_i^0 with slope 1. As without insurance.

Large shocks

- Say that shocks on i become arbitrarily large if for any Δ, $prob(|y_i^0 - E(y_i^0)| \leq \Delta) \to 0$.
  - $DISP(y_i) = E|y_i - \bar{y}_0|$.

Corollary *Suppose u CARA and $\boldsymbol{\alpha}$ connected. When shocks on i become arbitrarily large, $DISP(y_i)$ stays bounded under altruism but becomes arbitrarily large under social collateral.*

- Large shocks well-covered under altruism but not under social collateral.
  - Around equal incomes, the opposite happens for small shocks.

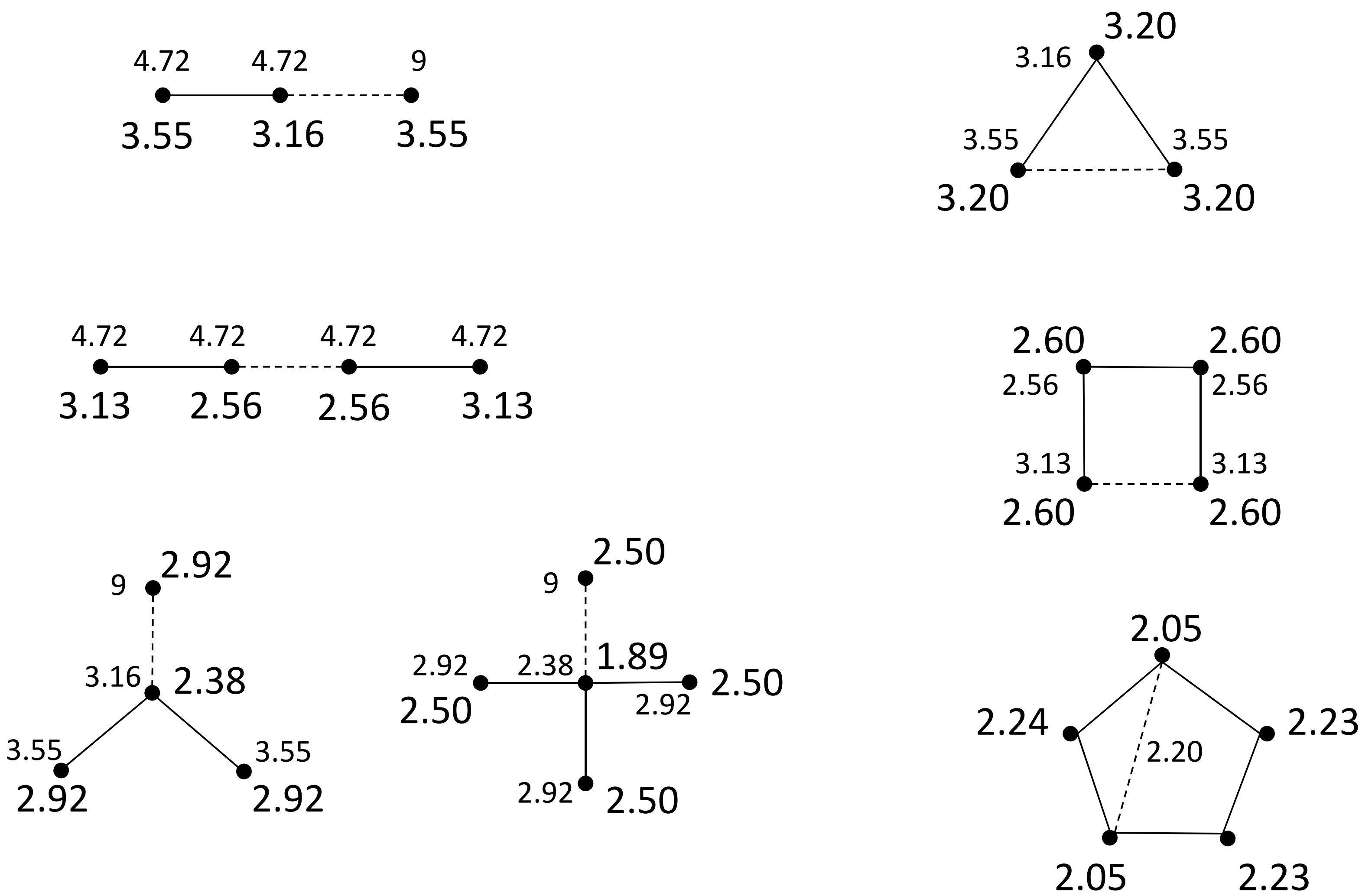
Network structure and insurance

- Impact of new link on risk faced by others?
  - Consider i connected to j and form new link between j and k. Impact on i?
  - Two countervailing forces. k is a source of indirect support. Could reduce the risk faced by i.
  - k is also a competitor for j's help. Could increase i's risk.
- From numerical simulations, we see the two situations emerging and in different circumstances.
  - Simulations with u CARA, y_0 normally distributed iid $N(10, \sigma = 3)$ and $-\ln(\alpha)/A = 1$.
  - $100,000$ runs per network to recover the full consumption distribution.

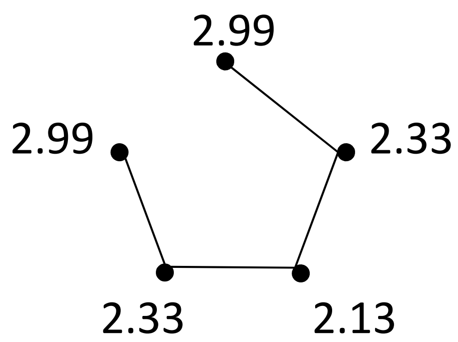
Network structure and insurance

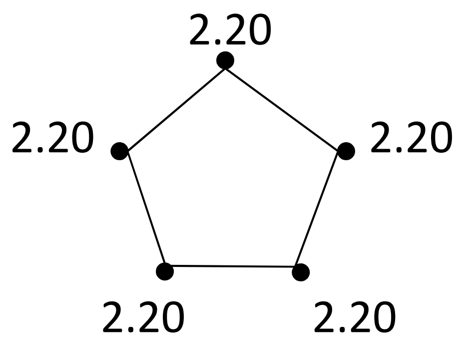
- A new link between separate communities helps.
- By contrast, new link in connected nets often increases consumption variance of indirect neighbors.
  - Line to circle: variance reduction for agents close to the new link and variance increase for agents far from it.
- We also find that more central agents tend to have lower variance.
- With correlated incomes, a new link between separate communities may increase risk.

Link ij reduces $Var(y_k)$ | Link ij increases $Var(y_k)$

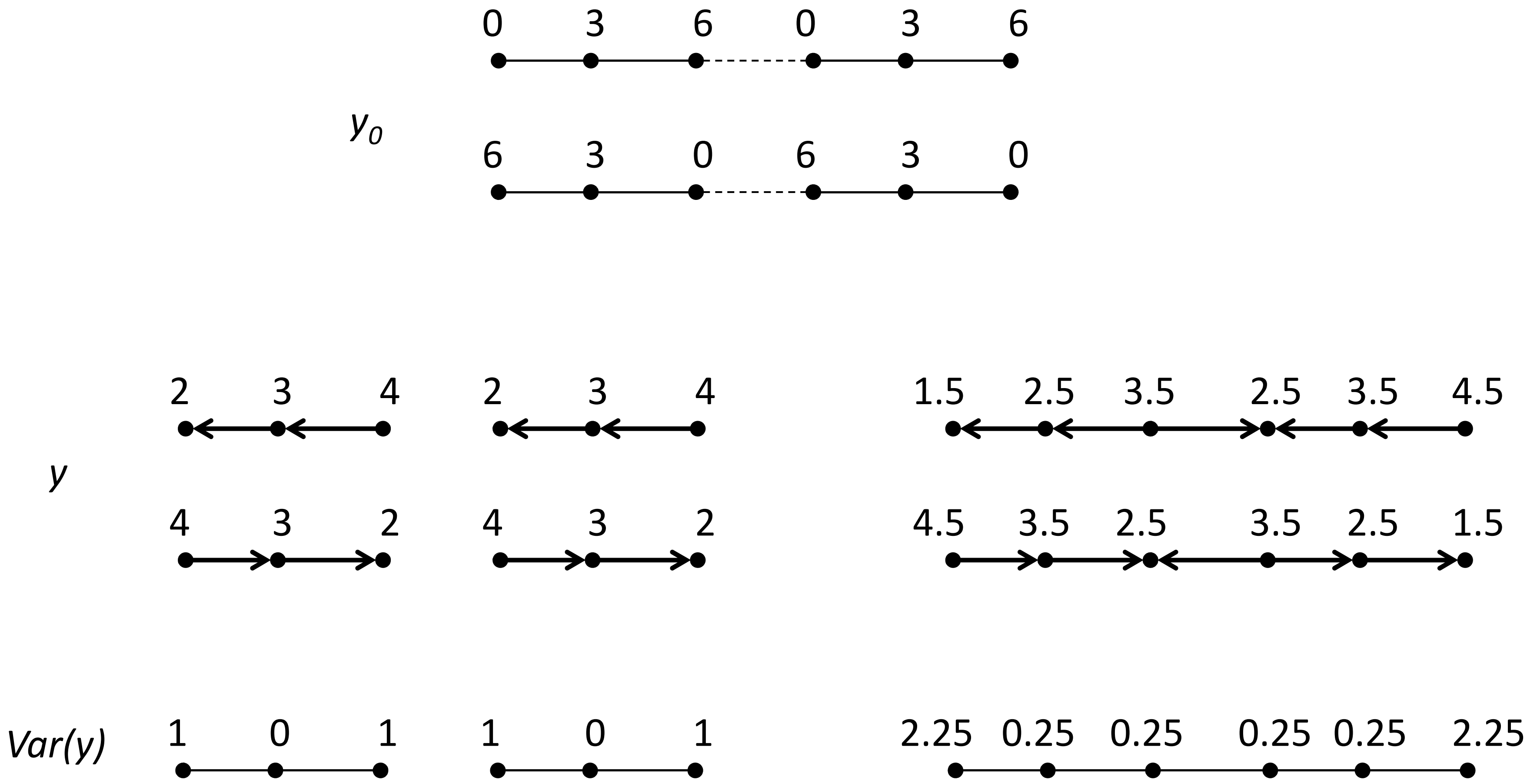


A new link may decrease variance for some indirect neighbors and increase it for others





A link between communities can increase risk for indirect neighbors



Conclusion: summary

- We analyze the risk sharing properties of altruism networks.
  - Informal insurance tends to be better when the average path length is lower.
  - Small shocks efficiently insured if the graph of transfers is connected.
  - Large shocks tend to be well-insured.
  - Rich structural effects.
- Outcomes quite distinct from the model of social collateral.

Conclusion: future research

- Endogenous networks: Bramoullé & Kranton (JEBO 2007), Ambrus, Chandrasekhar & Elliott (WP 2015).
  - Risk sharing affects marriage, Rosenzweig & Stark (JPE 1989).
- Altruism and incentives in networks.
  - Altruism may help solve moral hazard problems, Alger & Weibull (AER 2010).
- Structural estimations of models of informal transfers in networks.
  - Or even simply detailed empirical investigation of intermediation. If i is linked with j who is linked with k, how does a shock on k affects i?