Managers and Productivity Differences

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Abstract

We document that mean earnings of managers grow faster than for non managers over the life cycle for a group of high-income countries. Furthermore, we find that the growth of earnings for managers (relative to non managers) is positively correlated with output per worker across these countries. We interpret this evidence through the lens of an equilibrium span-of-control model where managers invest in their skills. Central to our analysis is a complementarity between skills and investments in the production of new managerial skills that amplifies initial differences in skills over the life cycle. We discipline model parameters with observations on managerial earnings and the size-distribution of plants in the United States, and then use our framework to quantify the importance of (i) lower exogenous productivity differences, and (ii) the size-dependent distortions emphasized in recent literature. Our findings show that both of these factors reduce managerial investments and lead to a lower earnings growth of managers relative to non managers. We also specialize the framework to evaluate the relative contribution of exogenous productivity versus size-dependent distortions for output and plant-size differences between the U.S. and Japan. Our results show that exogenous productivity differences account for about 80% of the output gap between these two countries. Size-dependent distortions are responsible for nearly all differences in plant size.

Key Words: Managers, Distortions, Size, Skill Investments, Productivity Differences.

JEL Classification: E23, E24, J24, M11, O43, O47.

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1 Introduction

Understanding cross-country income differences is still the Holy Grail of economics. Development accounting exercises conclude that understanding productivity differences across countries is the key (Klenow and Rodriguez-Clare, 1997; Prescott, 1998; Hall and Jones, 1999; Caselli, 2005). What does determine cross country productivity differences?

A growing literature emphasizes differences in management practices as a source of productivity differences (Bloom and Van Reenen 2011 and Bloom, Sadun and Van Reenen, 2014). Management practices differ greatly, both across countries and across firms within a given country, and better management practices are associated with better performance (total factor productivity, profitability, survival etc.). U.S. firms on average have the best management practices, and the quality of management declines rather sharply as one moves to poorer countries.

In this paper, we first document that age-earnings profiles of managers also differ non trivially across countries. Using micro data for a set of high-income countries, we show that incomes of managers grow much faster than the earnings of individuals who have non-managerial occupations in most countries. In the United States, the earnings of managers grow by about 75% during prime-age years (between ages 25-29 to 50-54), while the earnings growth for non-managers is about 40%. This gap is weaker in other countries in our sample. In Belgium, for instance, earnings growth of managers in prime working years is about 65% whereas earnings growth of non-managers is similar to the U.S. On the other extreme, in Spain the earnings of non-managers grow more than those of managers over the life-cycle.

We subsequently document that there is a strong positive relation between the steepness of age-earnings profiles and GDP per worker: managerial incomes grow faster in higher income countries than they do in poorer ones. The correlation coefficient between the earnings growth of managers relative to the earnings growth of non managers and log-GDP per worker is about 0.49, and stable across several robustness checks on our data. Since better management practices and the GDP per worker are positively correlated in the data, there is also a very strong positive relation between the earnings growth of managers relative to the earnings growth of non managers and the quality of management practices across countries.

It is of course an open question how to interpret differences in managerial practices. In this paper, we assume that better management practices reflect the fact that some individuals
have better managerial skills as in Lucas (1978). We allow, however, managerial abilities to change over time as managers invest in their skills. Hence, we place incentives of managers to invest in their skills and the resulting endogenous skill distribution of managers and their incomes at the center of income and productivity differences across countries. 

We study a span-of-control model with a life-cycle structure along a balanced growth path. Every period, a large number of finitely-lived agents are born. These agents are heterogeneous in terms of their initial endowment of managerial skills. The objective of each agent is to maximize the lifetime utility from consumption. In the first period of their lives, agents make an irreversible decision to be either workers or managers. If an agent chooses to be a worker, her managerial skills are of no use and she earns the market wage in every period until retirement. If an agent chooses to be a manager, she can use her managerial skills to operate a plant by employing labor and capital to produce output and collect the net proceeds (after paying labor and capital) as managerial income. Moreover, managers invest resources in skill formation, and as a result managerial skills grow over the life cycle. This implies that a manager can grow the size of her production unit and managerial income by investing a part of her current income in skill formation each period.

In the model, the evolution of managerial skills and hence plant size depends not only on initially endowed skills, but also on skill investment decisions. These investment decisions reflect the costs (resources that have to be spent rather than being consumed) and the benefits (the future rewards associated with being endowed with better managerial skills). Since consumption goods are an input for skill investments, a lower level of aggregate productivity results in lower incentives for managers to invest in their skills. We assume that economy-wide productivity that affects all managers grows at a constant rate, and show that the model economy exhibits a balanced growth path as long as the managerial ability of successive generations grows at a constant rate.

A central component of our model is the complementarity between available skills and investments in the production of new managerial skills. More skilled managers at a given age invest more in their skills, which propagates and amplifies initial differences in skills over the life cycle. This allows the model to endogenously generate a concentrated distribution of managerial skills. As in equilibrium more skilled managers operate larger production units, the model has the potential to account for the highly concentrated distribution of plant size in data.
We calibrate the model to match a host of facts from the U.S. economy: macroeconomic statistics, cross sectional features of establishment data as well as the age-earnings profiles of managers. We assume for these purposes that the U.S. economy is relatively free of distortions. We find that the model can indeed capture central features of the U.S. plant size distribution, including the upper and lower tails. It also does an excellent job in matching age-earnings profiles of managers that we observe in the data.

We then proceed to introduce size-dependent distortions as in the literature on misallocation in economic development. We model size-dependent distortions as progressive taxes on the output of a plant and do so via a simple parametric function, which was proposed originally by Benabou (2002). In our specification, there are two parameters that determine the level and the progressivity (or size-dependency) of distortions. In the model economy, size-dependent distortions have two effects. First, a standard reallocation effect, as the enactment of distortions implies that capital and labor services flow from distorted to undistorted production units. Second, a skill accumulation effect, as distortions affect the patterns of skill accumulation and thus, the overall distribution of managerial ability (which manifests itself in the distribution of plant level productivity). The model economy provides us with a natural framework to study how differences among countries in aggregate exogenous productivity and distortions can account not only for differences in output per worker but also for differences in managerial quality, size distribution of establishments and age-earnings profiles of managers.

We first show, consistent with the facts mentioned above, that exogenous differences in economy-wide productivity result both in lower managerial ability as well as in flatter relative age-earnings profiles. A 20% decline in exogenous aggregate productivity lowers investment in skills by managers by nearly 47%, leading to a decline in the average quality of managers of about 4%. With less investment, managerial incomes grow at a slower rate over the life-cycle, generating the positive relation between output per worker and steepness of age-income profiles that we observe in the data. Lower investment by managers magnifies the effects of lower aggregate productivity, and output per worker declines by about 29%.

We then consider a menu of distortions and evaluate their effects on output, plant size, notions of productivity, and age-earnings profiles of managers. We show that for distortions to have a bite, it is critical that they are correlated with productivity of production units. When we impose a 10% output tax on all establishments, output per worker declines by about
5.5%, but the effects on size distribution of establishments, investment in skills, managerial quality, age-earnings profiles are quite trivial. If we make instead distortions more dependent on output (size), while keeping the overall tax burden the same, the effects are dramatic. We measure the size-dependency of distortions as the ratio of 1 minus the output tax (or take-home rate) for an establishment that produces 5 times the mean output level to the 1 minus the tax rate for an establishment that produces mean output. When this wedge is 90% (i.e. take-home rate of smaller establishment is 10% higher), investment in skills and average managerial ability decline by nearly 58%. Size-dependent distortions also result in misallocation of resources as large efficient establishments shrink and less efficient ones expand. As a result of both misallocation and skill investment effects, average plant size declines significantly, from about 17.4 workers in the benchmark economy to about 8.2 workers. Output per worker declines by 12.3%.

We finally use the model to assess the combined effects of distortions and exogenous variation in economy-wide productivity. For these purposes, we force the model economy to reproduce Japanese data via size-dependent distortions and exogenous aggregate productivity variation. We find that our model can account for properties of the Japanese size distribution very well. It also captures well the flatter age-earnings profiles in Japan. In order to account for the features of the Japanese economy, the model requires an exogenous economy-wide productivity level that is about 12.5% below the U.S. level, and average distortions (taxes on output) that are of about 6.2% on average. Our findings indicate that size-dependent distortions account for the bulk of plant-size size differences between the U.S. and Japan, while differences in economy-wide productivity are responsible for about 80% of the differences in output per worker between the two countries.

1.1 Related Literature

The current paper builds on recent literature that studies how misallocation of resources at the micro level can lead to aggregate income and productivity differences; see Hopenhayn (2014), Restuccia and Rogerson (2013) and Restuccia (2013) for recent reviews. Following Guner, Ventura and Yi (2008) and Restuccia and Rogerson (2008), we focus in this paper on implicit, size-dependent distortions as a source of misallocation.¹

¹Other papers have dealt with explicit policies in practice. Garcia-Santana and Pijoan-Mas (2014) and Garicano, Lelarge and Van Reenen (2013) study examples of size-dependent policies in India and France,
We model explicitly, however, how distortions and economy-wide productivity differences affect managers’ incentives to invest in their skills and generate an endogenous distribution of skills. This naturally links our paper to the empirical literature on differences in management practices (see Bloom and Van Reenen 2010, and Bloom, Lemos, Sadun, Scur and Van Reenen 2014 for recent surveys), as well as to the recent development and trade literature that considers amplification effects of productivity differences or distortions due to investments in skills and R&D. Examples of these papers are Manuelli and Seshadri (2010), Erosa, Koreshkova and Restuccia (2010), Rubini (2011), Atkeson and Burstein (2010, 2011), Cubas, Ravikumar and Ventura (2013), and Gabler and Poschke (2013), among others.

The importance of management and misallocation of managerial ability for cross-country income differences have been emphasized by others before. Caselli and Gennaioli (2013) was possibly the first paper that highlighted the importance of managers for cross-country income differences. Recent work by Bhattacharya, Guner, and Ventura (2013), Roys and Seshadri (2013) and Alder (2014), among others, also study how managers and their incentives matter for aggregate productivity and the size distribution of plants and firms. Our paper is also connected to work that documents cross-country differences in plant and firm-level productivity and size. Hsieh and Klenow (2009), Bartelsman, Haltiwanger, and Scarpetta (2013), Hsieh and Klenow (2013), and Garcia-Santana and Ramos (2013) are examples of this line of work. Poschke (2014) builds a model of occupational choice with skill-biased change in managerial technology — managers with better skills benefit more from technological change — to account for cross-country differences in firm size distribution. Finally, our paper is related to recent paper that emphasize the link between managerial incentives, allocation of talent and income inequality. Celik (2015) studies how income inequality can affect the allocation of talent between routine production and innovation in an overlapping generations models in which agents can spend resources productively to enhance their skills, or unproductively to create signals about their skills. More closely related to our paper, Jones and Kim (2014) study a model in which heterogeneous entrepreneurs exert effort to generate growth in their incomes and how such effort can create a Pareto-tail for top incomes.

Our paper is organized as follows. Section 2 documents facts on age-earnings profiles for a set of high income countries. Section 3 presents the model, the modeling of distortions respectively. Buera, Kaboski and Shi (2011), Cole, Greenwood and Sanchez (2012), and Midrigan and Xu (2014) focus on the role of financial frictions in leading to misallocation of resources.
and discusses the model implications and connection with the literature. Section 4 discusses
the calibration of the benchmark model. Section 5 presents the findings associated to the
introduction of differences in exogenous economy-wide productivity and size-dependent dis-
tortions. Section 6 shows how a combination of lower aggregate productivity and distortions
can replicate the key features of Japanese economy. Finally, section 7 concludes.

2 Managerial Earnings over the Life Cycle

In this section, we present age-earnings profiles for managers and non-managers for a group
of high-income countries. Since individuals with managerial occupations constitute a small
group, it is not possible to construct age-earnings profiles for managers using panel data
data sets. As a result, we conduct our analysis with large cross-sectional data sets pertaining to
different countries.

We use four data sources: The Integrated Public Use Microdata Series-USA (IPUMS-
USA), IPUMS-International, Luxembourg Income Study (LIS), and the European Union
Statistics on Income and Living Conditions (EU-SILC). IPUMS-International provides har-
monized Census data for a large set of countries. Only few international censuses, however,
contain information both on incomes and occupations. LIS is another harmonized inter-
national data set that contains cross-sectional individual level data on income and other
socioeconomic characteristics. Finally, the EU-SILC contains both cross-sectional and lon-
gitudinal microdata data for European Union countries on income, work, poverty, social
exclusion and living conditions.

Our final sample consists of 20 countries: Australia, Austria, Belgium, Canada, Denmark,
Finland, France, Germany, Iceland, Ireland, Israel, Italy, Luxembourg, Netherlands, Norway,
Spain, Sweden, Switzerland, the United Kingdom, and the United States. Table A1 in
Appendix I shows survey years, data sources, and the number of observations for each
country.

We construct age-earnings profiles by estimating earnings equations as a function of age,
controlling for year effects and educational attainment. Specifically, for each country we
estimate the following regression:

$$\ln y_{it} = \alpha + \beta_1 a_{it} + \beta_2 a_{it}^2 + \gamma_t + \phi e_i + \varepsilon_{it}$$  \hspace{1cm} (1)
where \(y_{it}\) is earnings and \(a_{it}\) is age of individual \(i\) in year \(t\). The coefficients \(\beta_1\) and \(\beta_2\) capture the non-linear relationship between age and earnings, while \(\gamma_t\) represents year fixed-effects. Finally, \(e_i\) is an individual dummy variable capturing college education: it is equal to 1 if the individual has a bachelor’s degree or higher, and zero otherwise. In this way we account, for the fact that countries differ in the educational attainment of their population and could differ the returns to education. We estimate this equation for individuals with managerial and non-managerial occupations separately.

To estimate equation (1), we restrict the samples to ages 25 to 64, and group all ages into eight 5-year age groups: 25-29, 30-34, ..., 60-64. Individuals are classified as managers and non-managers based on their reported occupations. Table A2 in Appendix I documents how managers are defined in different data sets. Whenever it is possible, we stick to the occupational classification by International Labor Organization.\(^2\) The sample is further restricted to individuals who report positive earnings. Earnings are defined as the sum of wage & salary income and self-employment income. Most individuals in our samples earn either wages or self-employment income. However, the samples contain a small number of managers and non-managers who report positive amounts for both types of income.

Figures 1 and 2 report age-earnings profiles for managers and non-managers for a subset of countries. Two central facts emerge from these figures. First, in most countries, managers have steeper age-earnings profiles than non-managers, i.e. mean earnings of managers grow faster with age than mean earnings of non-managers. In the United States, managerial incomes grow by a factor of about 1.6 in prime working years – between ages 25-29 and 50-54 – whereas incomes of non-managers only rise by a factor of 1.4. Second, there are important differences between countries in age-earnings profiles. Let the relative income growth, \(\hat{g}\) be defined as

\[
\hat{g} = \ln \left( \frac{\text{income manager, 50-54}}{\text{income manager, 25-29}} \right) / \left( \frac{\text{income non manager, 50-54}}{\text{income non manager, 25-29}} \right)
\]

(2)

Our key finding is the positive relationship between GDP per worker and the life cycle growth of earnings of managers relative to the growth of non-managerial earnings.\(^3\) We

\(^2\)Codes 11 ("Legislators, senior officials and managers"), 12 ("Corporate Managers"), and 13 ("General Managers") in the International Standard Classification of Occupations (ISCO-88). We do not use the more recent ISCO-08, since most of our observations are dated earlier than 2008. Source: http://www.ilo.org/public/english/bureau/stat/isco/isco88/major.htm

\(^3\)We use the data on GDP per worker in year 2000 from Penn World Tables 7.1, Heston et al (2012)
report this relationship in Figure 3, where we plot the notion of relative income growth ($\dot{g}$) against the log of GDP per worker. While the results should be viewed with some caution due to small sample size, the relationship between log-GDP per worker and the steepness of managerial age-earning profiles is remarkably strong. The slope of the fitted lines is about 0.57, and the correlation is 0.49.\textsuperscript{4} Since higher GDP per worker is also associated with better management practices, there is also a very strong relation between the steepness of managerial age-earning profiles and management practices. This relation is shown in Figure 4. In countries with better management practices, such as the US or Germany, managers enjoy much higher earnings growth compared to managers in countries with poor management practices, such as Italy.\textsuperscript{5}

Robustness We next perform several robustness checks regarding the composition of the sample and the regression equation. First, we exclude the self-employed from the whole sample, i.e. both from managers and non-managers, as well as only from the non-managers category. In the data, self-employed individuals are either those who state that their main source of income is self-employment, or the ones who have positive self-employment income and no wage an salary income. Many self-employed, especially those who report a non-managerial occupation, have both managerial and non-managerial duties and hence do not easily fit into our categorization. Figures 5.1 and 5.2 show that our results are robust to exclusion of all self-employed and self-employed non-managers. Second, we narrow the definition of earnings to be wage and salary income only. Under this restriction, the self-employed who earn positive wage and salary income – either as managers or non-managers – are in the sample. However, their income from self-employment is not counted as part of their earnings. Figure 5.3 displays that dropping the self-employment income does not change our results markedly.

We subsequently try alternative specifications of the regression equation (1). First, we repeat the analysis without controlling for individual educational attainment. Figure 6.1 shows that our results become slightly weaker, suggesting that cross-country differences in

\textsuperscript{4}Lagakos, Moll, Porzio and Qian (2013) study differences in age-earnings profiles, i.e. return to experience, across countries and show that they are flatter in poorer countries. Their findings are consistent with ours, although we focus on a particular group (managers).

\textsuperscript{5}The data on management practices is taken from Nicholas, Genakos, Sadun, and Van Reenen (2012), Table 2.
educational attainment partly account for the relationship between GDP per worker and relative growth of managers’ earnings. Second, we drop year fixed effects from the regression equation. The results of this exercise is shown in Figures 6.2. Dropping year fixed effects does not change our results substantially.\(^6\)

**Are managers different?** The main result in this section (Figure 3) indicate that earnings of managers grow faster relative to non-managers in richer countries. In the next section, we build a model economy in which steeper age earnings profiles of managers in richer countries emerge as the result of higher investments that managers make to enhance their skills over the life-cycle in countries with either higher aggregate productivity or lower distortions. There are of course other professions for which human capital investment play an essential role. Do we observe a similar relation between the relative steepness of age earnings profiles and the GDP per worker for other professions? Figure 7.1 replicates our exercise in Figure 3 for professionals, e.g. lawyers, engineers, doctors, etc., since professionals are likely to be more similar to managers in terms of their incentives to invest in skills.\(^7\) We again look at the earnings growth for professional relative to earnings growth of workers (those who have non-professional, non-managerial occupations) versus the GDP per worker. Surprisingly, there is no positive relation between GDP per worker and the relative earnings growth of professionals over their life-cycles. In Figure 7.2, we repeat the same exercise for self-employed individuals – who are often used in applied work to capture the size of entrepreneurial activity in a country. Again, there is no systematic relation between the earnings growth for self employed individuals relative to workers (those who are not self-employed and have non-managerial occupations). These results suggest that forces that affect age earnings profiles of managers relative to workers/non-managers is rather specific to the incentives they face, and is unlikely to be due to factors that are likely to affect the whole population such as non-linear income taxation.

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\(^6\) We also run a regression in which, on top of the time fixed effect, we introduce a cohort fixed effect following a procedure proposed by Deaton (1997). However, due to the short time span of most of our samples (e.g. 2005-2012 for most European countries), this procedure yields unreliable estimates in our case.

\(^7\) We define professionals as individuals who hold occupations in Group 2 in ISCO-88. Source: http://www.iilo.org/public/english/bureau/stat/isco/isco88/major.htm
3 Model

We develop a life-cycle version of Lucas (1978) span-of-control model. Time is discrete. Each period, an overlapping generation of heterogeneous agents are born into economy that lives for $J$ periods. The objective of each agent is to maximize the present value of lifetime utility from consumption

$$\sum_{j=1}^{J} \beta^{j-1} \log(c_j),$$

where $\beta \in (0, 1)$ and $c_j$ is the consumption of an age-$j$ agent at date $t$.

Each agent is born with an initial endowment of managerial ability. We denote managerial ability by $z$. We assume that initial (age-1) abilities of an agent born at date $t$ are given by $z_1(t) = \tilde{z}(t)z$, and $z$ is drawn from an exogenous distribution with cdf $F(z)$ and density $f(z)$ on $[0, z_{\text{max}}]$. That is, individuals are heterogenous in initial managerial ability, and abilities for newborns are shifted in each date by the factor $\tilde{z}(t)$. We assume that $\tilde{z}(t)$ grows at the constant (gross) rate $1 + g_z$.

Each agent is also endowed with one unit of time which she supplies inelastically as a manager or as a worker. In the very first period of their lives, agents must choose to be either workers or managers. This decision is irreversible. If an individual chooses to be a worker, her managerial efficiency units are foregone, and she supplies one efficiency unit of labor at each age $j$. Retirement occurs exogenously at age $J_R$. The decision problem of a worker is to choose how much to consume and save every period.

If an individual chooses instead to be a manager, she has access to a technology to produce output, which requires managerial ability in conjunction with capital and labor services. Hence, given factor prices, she decides how much labor and capital to employ every period. In addition, in every period, a manager decides how much of his/her net income to allocate towards current consumption, savings and investments in improving her/his managerial skills. Retirement for managers also occurs exogenously at age $J_R$.

We assume that each cohort is $1 + n$ bigger than the previous one. These demographic patterns are stationary so that age-$j$ agents are a fraction $\mu_j$ of the population at any point in time. The weights are normalized to add up to one, and obey the recursion, $\mu_{j+1} = \mu_j/(1+n)$. 
Technology  Each manager has access to a span-of-control technology. A plant at date \( t \) comprises of a manager with ability \( z \) along with labor and capital,

\[
y(t) = A(t)z^{1-\gamma} (k^\alpha n^{1-\alpha})^\gamma,
\]

where \( \gamma \) is the span-of-control parameter and \( \alpha \gamma \) is the share of capital.\(^8\) The term \( A(t) \) is productivity term that is common to all establishments, and given by \( A(t) = \bar{A} G(t) \), where \( G(t) \) grows at the (gross) rate \( 1 + g_A \). Thus, \( \bar{A} \) controls the level of exogenous productivity.

Every manager can enhance her future skills by investing current income in skill accumulation. The law of motion for managerial skills is given by

\[
z_{j+1}(t+1) = z_j(t) + g(z_j(t), x_j(t), j) = z_j(t) + B(j)z_j(t)^{\theta_1} x_j(t)^{\theta_2},
\]

where \( x_j(t) \) is goods invested in skill accumulation by a manager of age \( j \) in period \( t \). We assume that \( \theta_1 \in (0, 1) \) and \( \theta_2 \in (0, 1) \). \( B(j) \) is the overall efficiency of investment in skills at age \( j \). The skill accumulation technology described above satisfies three important properties, of which the first two follow from the functional form and the last one is an assumption. First, the technology shows complementarities between current ability and investments in next period’s ability; i.e. \( g_{xx} > 0 \). Second, \( g(z, 0, j) = 0 \). That is, investments are essential to increase the stock of managerial skills. Finally, since \( \theta_2 < 1 \), there are diminishing returns to skill investments, i.e. \( g_{xx} < 0 \). Furthermore, we assume that \( B(j) = (1 - \delta_0)B(j - 1) \) with \( B(1) = \theta \).

3.1 Decisions

Let factor prices be denoted by \( R(t) \) and \( w(t) \) for capital and labor services, respectively. Let \( a_j(t) \) denote assets at age \( j \) and date \( t \) that pay the risk-free rate of return \( r(t) = R(t) - \delta \).

Managers  We assume that there are no borrowing constraints. As a result, factor demands and per-period managerial income (profits) are age-independent, and only depend on her ability \( z \) and factor prices. The income of a manager with ability \( z \) at date \( t \) is given by

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\(^8\)In referring to production units, we use the terms establishment and plant interchangeably.
\[ \pi(z, r, w, A, t) \equiv \max_{n, k} \{ A(t) z^{1-\gamma} \left( k^n n^{1-\alpha} \right)^\gamma - w(t) n - (r(t) + \delta) k \}. \]

Factor demands are given by

\[ k(z, r, w, A, t) = (A(t)(1 - \alpha)\gamma)^{\frac{1}{1 - \gamma}} \left( \frac{\alpha}{1 - \alpha} \right)^{\frac{1-\gamma}{1-\gamma}} \left( \frac{1}{r(t) + \delta} \right)^{\frac{1-\gamma}{1-\gamma}} \left( \frac{1}{w(t)} \right)^{\frac{1-\alpha}{1-\gamma}} z, \tag{4} \]

and

\[ n(z, r, w, A, t) = (A(t)(1 - \alpha)\gamma)^{\frac{1}{1 - \gamma}} \left( \frac{\alpha}{1 - \alpha} \right)^{\frac{1-\gamma}{1-\gamma}} \left( \frac{1}{r(t) + \delta} \right)^{\frac{1-\gamma}{1-\gamma}} \left( \frac{1}{w(t)} \right)^{\frac{1-\alpha}{1-\gamma}} z. \tag{5} \]

Substituting these into the profit function, one can show that managerial income is given by

\[ \pi(z, r, w, A, t) = A(t)^{\frac{1}{1 - \gamma}} \Omega \left( \frac{1}{r(t) + \delta} \right)^{\frac{\gamma}{1 - \gamma}} \left( \frac{1}{w(t)} \right)^{\frac{\gamma}{1 - \gamma}} z, \tag{6} \]

where \( \Omega \) is a constant equal to

\[ \Omega \equiv (1 - \alpha)^{\frac{\gamma(1-\alpha)}{(1-\gamma)}} \alpha^{\frac{\gamma}{1-\gamma}} (1 - \gamma)^{\frac{1}{1-\gamma}}. \tag{7} \]

Note that since profits are linear function of managerial ability, \( z \), the impact of additional skills on profits is independent of \( z \), and a function only of parameters, exogenous productivity and prices. Also note that given two managers, with ability levels \( z \) and \( z' \), we have

\[ \frac{k(z', r, w, A, t)}{k(z, r, w, A, t)} = \frac{n(z', r, w, A, t)}{n(z, r, w, A, t)} = \frac{\pi(z', r, w, A, t)}{\pi(z, r, w, A, t)} = \frac{z'}{z}. \]

Hence, differences in managerial abilities map one-to-one to differences in establishments sizes and managerial incomes.

The problem of a manager is to maximize (3), subject to

\[ c_j(t) + x_j(t) + a_{j+1}(t + 1) = \pi(z, r, w, A, t) + (1 + r(t)) a_j(t) \quad \forall \ 1 \leq j \leq J_R - 1, \tag{8} \]

\[ c_j(t) + a_{j+1}(t + 1) = (1 + r(t)) a_j(t) \quad \forall \ j \geq J_R, \tag{9} \]
and

\[ z_{j+1}(t+1) = z_j(t) + g(z_j(t), x_j(t), j) = z_j(t) + B(j)z_j(t)^{\theta_1}x_j(t)^{\theta_2} \quad \forall 1 \leq j < J_R - 1, \]  

(10)

with \( a_{J+1}(.) = 0 \). The manager chooses consumption at each age, assets and investments in skill formation. Let the value of solving this problem be given by \( V(z(t), t) \).

The solution to the problem of a manager is characterized by two conditions. First, the solution for next-period assets implies a standard Euler equation for asset accumulation

\[
\frac{1}{c_j(t)} = \beta(1 + r(t + 1)) \frac{1}{c_{j+1}(t + 1)}, \quad \forall 1 \leq j < J
\]  

(11)

Second, the optimality condition for skill investments \((x)\) and (11) imply the following no-arbitrage condition for investing in physical capital and skills

\[
\frac{(1 + r(t + 1))}{\text{marginal cost}} = \pi_z(t + 1) \frac{g_x(t)}{g_x(t + 1)} + \frac{g_x(t)}{g_x(t + 1)} [1 + g_z(t + 1)] \quad \forall 1 \leq j < J_R - 2,
\]  

(12)

For age \( j = J_R - 2 \), we have

\[
\frac{(1 + r(t + 1))}{\text{marginal cost}} = \pi_z(t + 1) \frac{g_x(t)}{\text{marginal benefit}},
\]  

(13)

The left-hand side of equation (13) is next period’s gain in income from one unit of current savings. The manager can also use this one unit as an investment on her skills. Hence, the term \( g_x(.) \) on the right-hand side stands for the additional skills available next period from an additional unit of investment in the current period. The term \( \pi_z(.) \) is the additional profit generated from an additional unit of managerial skills. Therefore, the right-hand side is the income again captured by the manager in his last working-age from investing one unit of the current consumption good in skill accumulation. It follows that one period before retirement, the manager must be indifferent at the margin between investing in assets and skills.

For ages less than \( j = J_R - 2 \), the marginal benefit incorporates an additional term as equation (12) shows. This term appears as an extra unit of investment also relaxes the skill accumulation constraint in the subsequent period.
**Workers**  The problem of an age-\( j \) worker is to maximize (3) by choice of consumption and assets at each age, subject to

\[
c_j(t) + a_{j+1}(t + 1) = w(t) + (1 + r(t))a_j(t) \quad \forall \ 1 \leq j \leq J_R - 1
\]

and

\[
c_j(t) + a_{j+1}(t + 1) = (1 + r(t))a_j(t) \quad \forall j \geq J_R,
\]

with \( a_{J+1}(.) = 0 \). Like managers, workers can borrow and lend without any constraint as long as they do not die with negative assets. Let the value of solving the worker’s problem be given by \( W(t) \).

**Occupational Choice**  Let \( z^*(t) \) be the ability level at which a 1-year old agent is indifferent between being a manager and a worker. This threshold level of \( z \) is given by (as agents are born with no assets)

\[
V(z^*(t), t) = W(t).
\]

Given all the assumptions made, \( V \) is a continuous and a strictly increasing function of \( z \). Therefore, (16) has a well-defined solution, \( z^*(t) \), for all \( t \).

### 3.2 Balanced Growth

We focus from now on the balanced growth scenario. In this case, the rate of return to assets and the fraction of managers are constant, and all variables grow in the long run at specified rates, driven ultimately by the two sources of growth in the environment: exogenous productivity growth and exogenous growth in the managerial skills of newborns. In Appendix 2, we show that our economy has a balanced growth path if and only if initial skills growth takes place at a given rate. We show that the growth rate in output per person \( (g) \) along a the balanced growth path is given by

\[
1 + g = (1 + g_A)^\psi,
\]

where \( \psi \)

\[
\psi \equiv \frac{1 - \theta_1}{\gamma(1 - \alpha) + (1 - \theta_2)(1 - \gamma) - \theta_1(1 - \alpha \gamma)}.
\]
3.3 Equilibrium

We outline now what constitutes an equilibrium for an economy in the stationary case, i.e. along a balanced growth path. We normalize variables to account for stationary growth. Define the growth factor $D(t) \equiv (1 + g)^t$. Hence, we normalize variables wage rates, managerial income, individual consumption, asset holdings and factor demands by $D(t)$, and denote normalized variables by the “$^\sim$” symbol (i.e. $a_j = a_j(t)/D(t)$). Regarding managerial abilities, recall that managerial ability levels of members of each new cohort are given by $z_1(t) = \tilde{z}(t)z$, with a common component that grows over time at the rate $g_z$, and a random draw, $z$, distributed with cdf $F(z)$ and density $f(z)$ on $[0, z_{\max}]$. Hence, the normalized component is simply $z$ for each individual. After the age-1, and given the stationary threshold value $z^*$, the distribution of managerial abilities is endogenous as it depends on investment decisions of managers over their life-cycle.

Let managerial abilities take values in set $\mathcal{Z} = [z^*, \tilde{z}]$ with the endogenous upper bound $\tilde{z}$. Similarly, let $\mathcal{A} = [0, \tilde{a}]$ denote the possible asset levels. Let $\psi_j(\hat{a}, z)$ be the mass of age-$j$ agents with assets $\hat{a}$ and skill level $z$. Given $\psi_j(\hat{a}, z)$, let

$$f_j(z) = \int \psi_j(\hat{a}, z) d\hat{a},$$

be the skill distribution for age-$j$ agents. Note that $f_1(z) = f(z)$ by construction.

Each period those agents whose initial ability is above $z^*$ work as managers, whereas the rest are workers. Then, in a stationary equilibrium with given prices, $(r, \hat{w})$, labor, capital and goods markets must clear. The labor market equilibrium condition can be written as

$$\sum_{j=1}^{J_R-1} \mu_j \int_{z^*}^\tilde{z} \hat{n}(z, r, \hat{w}, \bar{A}) f_j(z) dz = F(z^*) \sum_{j=1}^{J_R-1} \mu_j$$

where $\mu_j$ is the total mass of cohort $j$. The left-hand side is the labor demand from $J_R - 1$ different cohorts of managers. A manager with ability level $z$ demands $\hat{n}(z, r, \hat{w}, \bar{A})$ units of labor and there are $f_j(z)$ of these agents. The right-hand side is the fraction of each cohort employed as workers.

In the capital market, the demand for capital services must equal the aggregate value of the capital stock. Hence,
where $\hat{K}$ is the normalized, per person stock of capital and $\hat{k}(z, r, \hat{w}, \hat{A})$ is capital demand from a manager with ability $z$. The goods market equilibrium condition requires that the sum of undepreciated capital stock and aggregate output produced in all plants in the economy is equal to the sum of aggregate consumption and savings across all cohorts plus skill investments by all managers across all cohorts.

### 3.4 Size-Dependent Distortions

Consider now the environment in which managers face distortions to operate production plants. We model these distortions as size-dependent output taxes. In particular, we assume an establishment with output $y$ faces an average tax rate $T(y) = 1 - \lambda y^{-\tau}$. This tax function, initially proposed by Benabou (2002), has a very intuitive interpretation: when $\tau = 0$, distortions are the same for all establishments and they all face an output tax of $(1 - \lambda)$. For $\tau > 0$, the distortions are size-dependent, i.e. larger establishments face higher distortions than smaller ones. Hence, $\tau$ controls how dependent on size the distortions are.

With distortions, profits are given by

$$
\pi(z, r, w, A, t) = \max_{n, k} \left\{ \lambda A(t)^{1-\tau} z^{(1-\gamma)(1-\tau)} \left( k^\alpha n^{1-\alpha} \right)^{\gamma(1-\tau)} - w(t)n - (r(t) + \delta)k \right\}
$$

From the first order conditions, the factor demands are now given by

$$
n(z, r, w, A, t) = \left[ \lambda A(t)^{1-\tau} \gamma(1-\alpha)(1-\tau) \right]^{\frac{1}{1-\gamma(1-\tau)}} \times
$$

$$
\times \left( \frac{1}{r(t) + \delta} \right)^{\gamma(1-\tau) \frac{1-\gamma(1-\tau)}{1-\gamma(1-\tau)}} \left( \frac{\alpha}{1-\alpha} \right)^{\gamma(1-\tau) \frac{1-\gamma(1-\tau)}{1-\gamma(1-\tau)}} \left( \frac{1}{w(t)} \right)^{\frac{1-\gamma(1-\tau)}{1-\gamma(1-\tau)}} \frac{1-\gamma(1-\tau)}{1-\gamma(1-\tau)} z^{\frac{1-\gamma(1-\tau)}{1-\gamma(1-\tau)}},
$$

and

$$
k(z, r, w, A, t) = \left[ \lambda A(t)^{1-\tau} \gamma(1-\alpha)(1-\tau) \right]^{\frac{1}{1-\gamma(1-\tau)}} \times
$$

$$
\times \left( \frac{1}{r(t) + \delta} \right)^{1-\gamma(1-\alpha)(1-\tau) \frac{1-\gamma(1-\tau)}{1-\gamma(1-\tau)}} \left( \frac{\alpha}{1-\alpha} \right)^{1-\gamma(1-\alpha)(1-\tau) \frac{1-\gamma(1-\tau)}{1-\gamma(1-\tau)}} \left( \frac{1}{w(t)} \right)^{\frac{1-\gamma(1-\tau)}{1-\gamma(1-\tau)}} \frac{1-\gamma(1-\tau)}{1-\gamma(1-\tau)} z^{\frac{1-\gamma(1-\tau)}{1-\gamma(1-\tau)}}.
$$
Using the factor demands 19 and 20, we can write the profit function as

$$
\pi(z, r, w, A, t) = (\lambda A(t)^{1-\tau})^{1 \over \tau} \tilde{\Omega} \left( \frac{1}{r(t) + \delta} \right)^{\alpha \gamma (1-\alpha) \gamma (1-\tau)} z^{\gamma (1-\gamma)(1-\tau)}
$$

(21)

where

$$
\tilde{\Omega} \equiv (1 - \gamma (1 - \tau)) \alpha^{\gamma \alpha (1-\alpha) \gamma (1-\tau)} (1 - \alpha)^{\gamma (1-\gamma)(1-\tau)} (\gamma (1 - \tau))^{\gamma (1-\gamma)(1-\tau)}.
$$

Note that for any \( z \) and \( z' \), we now have

$$
k(z', r, w, A, t) = n(z', r, w, A, t) = \pi(z', r, w, A, t) = \pi(z, r, w, A, t) = \left( \frac{z'}{z} \right)^{\gamma (1-\gamma)(1-\tau)}
$$

where

$$
\frac{(1 - \gamma)(1 - \tau)}{1 - \gamma (1 - \tau)} < 1,
$$

as long as \( \tau > 0 \). That is, for a given distribution of managerial abilities, size-dependent distortions produce a more compressed size distribution of establishments and managerial incomes.

Similarly, for any \( z \) and \( z' \) the optimal skill investment is now characterized by

$$
\frac{x'_j(t)}{x_j(t)} = \left( \frac{z'_j(t)}{z_j(t)} \right)^{\gamma (1-\gamma)(1-\tau)}.
$$

It is easy to show that the exponent in the expression is decreasing with respect to the parameter \( \tau \) governing size dependency. Hence, size-dependent distortions also reduce incentives of higher-ability managers to invest in their skills.

### 4 Parameter Values

We assume that the U.S. economy is distortion free and calibrate the benchmark model parameters to match central aggregate and cross sectional features of the U.S. data on plants as well as managerial incomes. In particular, we force our economy to reproduce the earnings of managers relative to non-managers over the life cycle estimated in section 2.

For observations on the U.S. plant size data, we use the 2004 U.S. Economic Census. The average plant size is about 17.9, and the distribution of employment across plants is quite skewed. About 72.5% of plants in the economy employ less than 10 workers, but account for
only 15% of the total employment. On the other hand, less than 2.7% of plants employ more than 100 employees but account for about 46% of total employment. From our findings in section 2, managerial incomes (relative to non-managers) grow by about 18% between ages 25-29 to 40-44, and by about 24% by ages 60-64.

We assume that the exogenous skill distribution of newborn agents, $z_1$, follows a log normal distribution. Specifically, we assume that $\log(z_1)$ is normally distributed with parameters $\mu_z$ and $\sigma_z$. We let the model period correspond to 5 years. Each cohort of agents enter the model at age 25 and live until they are 85 years old. Agents retire at age 65. Hence, in the model agents live for 12 model periods; 8 as workers or managers and 4 as retirees. There is a total of 12 parameters to calibrate, as listed in Table 1. The product of two of these parameters, importance of capital ($\alpha$) and returns to scale ($\gamma$), determine the share of capital in output. We determine the values of capital share in output and the depreciation rate from the data. A measure of capital consistent with the current model on business plants should include capital accounted for by the business sector. Similarly a measure of output consistent with our definition of capital should only include output accounted for by the business sector. The measure of capital and output discussed in Guner et al (2008) is consistent with the current plant size distribution model. Hence we use the value of capital output ratio and the capital share reported in that paper. These values are 2.325 (at the annual level) and 0.326, respectively, with a corresponding investment to output ratio of about 0.178. Given a capital output ratio and an investment ratio, our (stationary) law of motion of capital implies a depreciation rate of about 4% at the annual level. We choose the population growth rate in the model such that the annual population growth rate is 1.1%. Finally, the economy grows by about 2.6% a year for our measure of output.

After calibrating the depreciation rate, the population growth rate, and the growth rate of output, we have 9 more parameters to calibrate: importance of capital, the parameter governing returns to scale, the discount factor, two parameters of the skill accumulation technology and the mean and variance of the skill distribution. Note that the capital share in the model is given by $\gamma \alpha$, and since this value has to be equal to 0.317, a calibrated value for $\gamma$ determines $\alpha$ as well. Hence we have indeed 8 parameters to determine: $\gamma, \beta, \theta, \theta_1, \theta_2, \delta, \mu_z, \sigma_z$. The resulting parameter values are displayed in Table 1.

At the aggregate level, we want the benchmark model to replicate the capital output ratio in the U.S. economy. At the cross sectional level, the model implied distribution of plants
should capture some of the important features of the U.S. plant size distribution discussed in the beginning of this section. At the same time, we would like the model economy to generate age-earning profiles for managers that are consistent with the data. We normalize the mean of the skill distribution to zero and jointly calibrate the 7 remaining parameters to match the following 7 moments of the U.S. plant size distribution: mean plant size, the fraction of plants with less than 10 workers, the fraction of plants with more than 100 workers, the fraction of the labor force employed in plants with 100 or more employees, the growth of managerial incomes relative to those of non-managers between ages 25-29 and 40-44, the growth of managerial incomes relative to those of non-managers between ages 25-29 to 60-64, and the aggregate capital output ratio. These moments together with their model counterparts are given in Table 2.

The benchmark model is able to replicate properties of the entire plant size distribution fairly well, as illustrated in Figures 8 and 9. The model also does an excellent job in replicating age-income profiles for managers (Figure 10). The success of the skill accumulation model in accounting for the upper tail of the plant-size distribution is important, since the bulk of employment is concentrated there.

In our calibration, the fraction of resources that are invested in skill accumulation is of about 0.33% of GDP. Despite the relatively small fraction of resources devoted to the improvement of managerial skills, the incomes of managers grow significantly with age.

5 Economy-Wide Productivity Differences and Size-Dependent Distortions

In this section, we present and discuss the first central quantitative findings of the paper. We first explore the implied responses of our model economy to variations in economy-wide productivity. Subsequently, we introduce distortions as described in section 3.4.

5.1 Variation in Economy-wide Productivity

We now consider the effects of changes in economy-wide productivity levels; the term $\bar{A}$ that is common to all establishments. We do this for multiple reasons. First, there is substantial variation in the size of establishments across countries that is correlated to the level of
Second, differences in productivity across countries can have effects in skill accumulation decisions, and thus their role in development can be therefore be amplified. If productivity differences affect the accumulation of managerial skills, variation in productivity can contribute to account for cross-country differences in establishment size.

Table 3 documents the results when we lower \( \bar{A} \) relative to the benchmark economy. Not surprisingly, reducing exogenous productivity leads to a reduction in output across steady states. When \( \bar{A} \) is lowered by 10% (20%), GDP per worker declines by about 15% (29%). Managers also invest less in their skills and as a result, age-earnings profiles become flatter as shown in Figure 11. Therefore, the model naturally generates the positive relation between GDP per worker and the steepness of age-earnings profiles documented in Section 2 (see Figure 3).

Changes in exogenous productivity, as modeled here, do not generate size differences in a growth model with a Lucas (1978) span-of-control technology, as changing \( \bar{A} \) has no effect on occupational decisions.\(^9\) The consequences of changing aggregate productivity, however, are different in the current setup. As productivity drops, both wage rates and managerial rents drop as in the standard span-of-control model. But a productivity drop also reduces the marginal benefit associated to an extra unit of income invested in skill accumulation (see equations 12 and 13). As a result, managerial ability drops across steady states as well, which translates into further reductions in labor demand and therefore, on the wage rate. The net result is a reduction in the value of becoming a worker relative to a manager at the start of life, which leads in turn to an increase in the number of managers (establishments). Quantitatively, however, these effects are small as Table 3 demonstrates.

Overall, changes in aggregate productivity concentrate employment at smaller production units as size-dependent distortions do. As a result, they can matter in quantitatively accounting for the observed cross-country differences. In Section 6, we explore in detail the interplay between distortions and exogenous productivity differences for the case of Japan.

Given the results in Table 3 and Figure 11, it is natural to ask the extent to which the model can reproduce the relation between GDP per worker and the relative earnings growth

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\(^9\)While the mean size of establishments in the U.S. is about 17.9 employees in the U.S. and 15.0 in Norway, it amounts to about 9.7 in Japan. The size of typical establishments is substantially lower in poorer countries. For instance, mean establishment size is 7.5 employees in Turkey, 4.3 in Jordan and 4.4 in India. See Bhattacharya (2010) for a documentation of differences in establishment size across countries.

\(^{10}\)This requires a Cobb-Douglas specification as modeled here.
for managers that we observe in the data. To this end, for each of 20 countries in Figure 3, we select a value of $\bar{A}$ such that our model economy reproduces GDP per worker of that country relative to the U.S. We keep all other parameters fixed at their benchmark values. Figure 12 reproduces Figure 3 together with a line that shows how GDP per worker and earnings growth of managers are related in the model economies. While the model is able to generate the positive relation between GDP per worker and earnings growth of managers, the relation is weaker in the model than it is in the data. This should not be surprising as several other factors contribute to the differences in age-earnings profiles across countries.

Quantitatively, the effects associated to reductions in exogenous productivity on managerial ability are not substantial, despite the large effects on skill investments that Table 3 shows. Reducing exogenous productivity by 30% leads to a substantial reduction in output – about 42% – but a drop in managerial ability of only about 8.9%.

5.2 Effects of Size-Dependent Distortions

The distortions in the model economy are governed by two parameters: $\lambda$ and $\tau$. While $\lambda$ determines the average level of distortions, $\tau$ determines its progressivity. A natural measure of progressivity is given by

$$\frac{1 - T(y')}{1 - T(y)} = \frac{(y')^{-\tau}}{(y)^{-\tau}} = \left(\frac{y}{y'}\right)^{\tau},$$

which is always less than 1 for $y' > y$ as long as $\tau > 0$. When $\tau = 0$, this wedge is 1. When $\tau > 0$, larger establishments face higher distortions, the wedge takes values less than 1.

We first set $\lambda = 0.9$ and $\tau = 0$, i.e. each establishment faces a 10% output tax. The effects of this policy are shown in the third column of Table 4. Consistent with previous work (e.g. Guner et al, 2008 and Restuccia and Rogerson, 2008), distortions that are not correlated with the productivity of production units do not have significant effects on aggregates and productivity.

Then we increase $\tau$, and adjust $\lambda$ so that on average we collect the same amount of taxes as we did with $\lambda = 0.9$ and $\tau = 0$. In particular, we increase $\tau$ such that for $y' = 5\bar{y}$ and $y = \bar{y}$ (the mean level of output), the progressivity wedge is equal to 0.95 and 0.9. As Table 4 demonstrates, the effects of size-dependent distortions are dramatic. When the progressivity wedge is equal to 0.95 (0.9), mean size falls from the benchmark value of 17.4 employees to
about 11.8 (8.2) and the number of managers increase by 46.7% (103.4%). This occurs as with the introduction of distortions that are correlated with size, relatively large, distorted establishments reduce their demand for capital and labor services, leading to a reduction in the wage rate. This prompts the emergence of smaller production units, as individuals with low initial managerial ability become managers. At the same time, investment in skills decline significantly, and age-earnings profiles become much flatter. As a result of these changes, output per worker drops across steady states by 8% (12.3%).

6 Distortions versus Productivity Differences

In previous sections, we showed the quantitative implications of the model in terms of output, productivity measures and the size distribution of establishments. In this section, we use data from the Japanese economy to evaluate the performance of the model in several dimensions, and provide quantitative estimates of economy-wide productivity differences between Japan and the United States.

Japan is quite a relevant case to consider from the perspective of this paper. First, Japan’s output per worker is only about 70% of the U.S. level and it has been at that level for roughly twenty years. Second, there are substantial differences in the size distribution of establishments in relation to the United States. Using data from the Japanese Establishment and Enterprise Census, we calculate that mean establishment size in Japan is substantially below the U.S. level: 9.7 versus 17.9 employees. Not surprisingly, production in Japan is effectively concentrated in small units. As we documented earlier, the fraction of small establishments (less than 10 workers) in the U.S. is 72.5%, the fraction of large establishments (100 workers or more) is about 2.6% and large establishments account for about 46.2% of employment. The corresponding values for Japan differ non trivially: 79.1%, 1.0% and 26.0%, respectively. Finally, Japan is a case that fits well with the case of distortions that are correlated with the size of establishments. As documented in Guner, Ventura and Yi (2006, 2008), Japan regulates severely size of the retail sector at the national level, with policies that go back to the pre-World War II era.

Using the calibrated parameters of the benchmark economy, we proceed to find the levels of economy-wide productivity ($\bar{A}$) and distortions ($\lambda$, $\tau$), in order to reproduce, in a stationary equilibrium, three targets: (i) the level of output of Japan relative to the United
States; (ii) fraction of small (0 to 9 workers) establishments; iii) the employment share of large (100+) establishments. We then contrast the model implications for other properties of the size distribution with Japanese data, and quantify the importance of aggregate productivity differences vis-a-vis idiosyncratic distortions. We also report how well the model economy does in generating flatter age-earnings profiles in Japan.

Findings  The findings from the experiment are shown in Table 5. The model is successful in generating the quantitative properties of the Japanese size-distribution data. The model implies a fraction of small (large) establishments of about 79.0% (1.5%) and a share of employment at large establishments of 26.0%. As mentioned earlier, the corresponding values from the data are 79.1% (1.1%) and 26.0%, respectively. We view these findings as important since they illustrate the capacity of our framework to account for size observations; they give us confidence to take the quantitative findings of the model seriously.

The model generates the Japanese facts with about an aggregate, economy-wide productivity level of about 0.875 (versus benchmark value 1.0), and distortions that amount to an average implicit tax rate of 6.1%. It is worth noting here the implied effects on the relative earnings of managers to non managers. While in the benchmark economy, relative earnings grow by about 25% over the entire working life cycle, they grow just by 11.2% under the distortions and economy-wide productivity that reproduce the Japanese data.

How large is the contribution of exogenous productivity differences vis-a-vis distortions in order to generate the results in Table 5? The last column in the Table answers this question. Using the previously found values of $\overline{A}$, we compute the effects of changes in exogenous productivity only. The results reveal that differences in $\overline{A}$ play a non-trivial role: they capture most of the changes in output. The last column in Table 5 shows that nearly 80% of the changes in output are accounted for by changes in economy-wide productivity. On the other hand, the share of employment at large establishments drops only marginally to 42.3% due to the reduction in exogenous productivity. From these findings, we conclude that distortions as modeled here matter: exogenous differences in aggregate productivity alone cannot account for the differences between Japan and the United States. Indeed, as the results demonstrate, distortions appear to be the main driver in accounting for differences in the size of establishments between the U.S. and Japan. Differences in economy-wide productivity account for the bulk of output per-worker differences between the countries.


7 Conclusion

[TO BE COMPLETED]
References


Table 1: Parameter Values (annualized)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population Growth Rate ($n$)</td>
<td>0.011</td>
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<tr>
<td>Productivity Growth Rate ($g$)</td>
<td>0.0255</td>
</tr>
<tr>
<td>Depreciation Rate ($\delta$)</td>
<td>0.040</td>
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<tr>
<td>Importance of Capital ($\alpha$)</td>
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<tr>
<td>Returns to Scale ($\gamma$)</td>
<td>0.77</td>
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<tr>
<td>Mean Log-managerial Ability ($\mu_z$)</td>
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<td>Dispersion in Log-managerial Ability ($\sigma_z$)</td>
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<td>Skill accumulation technology ($\theta$)</td>
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<td>Skill accumulation technology ($\theta_2$)</td>
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</table>

Note: Entries show model parameters used to simulate the benchmark economy.
Table 2: Empirical Targets: Model and Data

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<tr>
<th>Statistic</th>
<th>Data</th>
<th>Model</th>
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</thead>
<tbody>
<tr>
<td>Mean Size</td>
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<td>17.75</td>
</tr>
<tr>
<td>Capital Output Ratio</td>
<td>2.325</td>
<td>2.314</td>
</tr>
<tr>
<td>Fraction of Small (0-9 workers) establishments</td>
<td>0.725</td>
<td>0.719</td>
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<td>Fraction of Large (100+ workers) establishments</td>
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<td>Employment Share of Large establishments</td>
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<tr>
<td>Managerial Income (40-44/25-29)</td>
<td>1.183</td>
<td>1.198</td>
</tr>
<tr>
<td>Managerial Income (60-64/25-29)</td>
<td>1.247</td>
<td>1.253</td>
</tr>
</tbody>
</table>

Note: Entries show the empirical targets used in the quantitative analysis and the model’s performance.
Table 3: Effect of Economy-Wide Productivity

<table>
<thead>
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<tr>
<td>Mean Size</td>
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<td>Average Managerial Ability</td>
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<td>1.144</td>
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<td>1.217</td>
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<td>1.149</td>
</tr>
</tbody>
</table>

Note: Entries show the effects on displayed variables associated to exogenous reductions in the level of economy-wide productivity.
Table 4: Effect of Size-Dependent Distortions

<table>
<thead>
<tr>
<th></th>
<th>Level ($\lambda$)</th>
<th>1</th>
<th>0.9</th>
<th>1.038</th>
<th>1.165</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size Dependency ($\tau$)</td>
<td>0</td>
<td>0</td>
<td>0.032</td>
<td>0.066</td>
<td></td>
</tr>
<tr>
<td>Wedge $(1 - T(5\bar{y}))/ (1 - T(\bar{y}))$</td>
<td>1</td>
<td>1</td>
<td>0.95</td>
<td>0.9</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statistic</th>
<th>GDP per worker</th>
<th>100</th>
<th>94.48</th>
<th>91.96</th>
<th>87.66</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Size</td>
<td>17.74</td>
<td>17.74</td>
<td>17.74</td>
<td>11.78</td>
<td>8.23</td>
</tr>
<tr>
<td>Investment in Skills</td>
<td>100</td>
<td>74.28</td>
<td>55.45</td>
<td>42.21</td>
<td></td>
</tr>
<tr>
<td>Number of Managers</td>
<td>100</td>
<td>100.00</td>
<td>146.71</td>
<td>203.41</td>
<td></td>
</tr>
<tr>
<td>Average Managerial Ability</td>
<td>100</td>
<td>97.99</td>
<td>68.07</td>
<td>49.46</td>
<td></td>
</tr>
</tbody>
</table>

Note: Entries show the effects on displayed variables associated to size-dependent distortions. Column 2 report benchmark values. Column 3 reports the effects of a 10% proportional tax on output. Columns 4-5 report the effects of size-dependent distortions that tax more heavily larger establishments. Experiments in columns 4-5 are revenue neutral, i.e. the total tax collection is the same as in column 3.
<table>
<thead>
<tr>
<th>Variable</th>
<th>BM (U.S.)</th>
<th>Japan Data</th>
<th>Japan Model</th>
<th>Japan Model (No Dist.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economy-Wide Productivity</td>
<td>1</td>
<td>-</td>
<td>0.875</td>
<td>0.875</td>
</tr>
<tr>
<td>Mean tax rate on output (%)</td>
<td>0</td>
<td>-</td>
<td>6.09</td>
<td>0</td>
</tr>
<tr>
<td>GDP per worker</td>
<td>100</td>
<td>0.76</td>
<td>0.76</td>
<td>0.81</td>
</tr>
<tr>
<td>Frac. of Small (0-9) estab.</td>
<td>0.719</td>
<td>0.793</td>
<td>0.788</td>
<td>0.716</td>
</tr>
<tr>
<td>Emp. in Large (100+) establ.</td>
<td>0.444</td>
<td>0.260</td>
<td>0.257</td>
<td>0.439</td>
</tr>
<tr>
<td>Mean Size</td>
<td>17.75</td>
<td>9.7</td>
<td>10.82</td>
<td>17.75</td>
</tr>
<tr>
<td>Investment in Skills</td>
<td>100</td>
<td>-</td>
<td>51.76</td>
<td>85.25</td>
</tr>
<tr>
<td>Average Managerial Ability</td>
<td>100</td>
<td>-</td>
<td>62.34</td>
<td>97.5</td>
</tr>
<tr>
<td>Frac. of Large (100+) estab.</td>
<td>0.030</td>
<td>0.011</td>
<td>0.014</td>
<td>0.031</td>
</tr>
<tr>
<td>Managerial Inc. (40-44/25-29)</td>
<td>1.198</td>
<td>1.089</td>
<td>1.163</td>
<td></td>
</tr>
<tr>
<td>Managerial Inc. (60-64/25-29)</td>
<td>1.253</td>
<td>1.112</td>
<td>1.208</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** Entries show the effects on displayed variables when the model is applied to the case of Japan. The fourth column shows the effects when the distortions and exogenous economy-wide productivity are varied to match Japan’s output relative to the U.S., fraction of small (0-9 workers) establishments and employment share of large (100+) establishments. The last column shows the effects when only the exogenous economy-wide productivity is lowered to its value in column 4.
### Appendix I: Data on Managerial Incomes

#### Table A1: Data Sources

<table>
<thead>
<tr>
<th>Country</th>
<th>Years</th>
<th>Source</th>
<th>No. of Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>2004-2012</td>
<td>EU-SILC</td>
<td>44,426</td>
</tr>
<tr>
<td>Belgium</td>
<td>2004-2011</td>
<td>EU-SILC</td>
<td>37,231</td>
</tr>
<tr>
<td>Denmark</td>
<td>2004-2012</td>
<td>EU-SILC</td>
<td>59,241</td>
</tr>
<tr>
<td>Finland</td>
<td>2004-2010, 2012</td>
<td>EU-SILC</td>
<td>97,390</td>
</tr>
<tr>
<td>Germany</td>
<td>2005-2012</td>
<td>EU-SILC</td>
<td>76,978</td>
</tr>
<tr>
<td>Iceland</td>
<td>2004-2010, 2012</td>
<td>EU-SILC</td>
<td>30,181</td>
</tr>
<tr>
<td>Ireland</td>
<td>2004-2010</td>
<td>EU-SILC</td>
<td>24,015</td>
</tr>
<tr>
<td>Italy</td>
<td>2007-2010, 2012</td>
<td>EU-SILC</td>
<td>89,420</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>2004-2010, 2012</td>
<td>EU-SILC</td>
<td>32,105</td>
</tr>
<tr>
<td>Netherlands</td>
<td>2005-2010, 2012</td>
<td>EU-SILC</td>
<td>58,233</td>
</tr>
<tr>
<td>Norway</td>
<td>2004-2010, 2012</td>
<td>EU-SILC</td>
<td>49,038</td>
</tr>
<tr>
<td>Spain</td>
<td>2006-2012</td>
<td>EU-SILC</td>
<td>77,196</td>
</tr>
<tr>
<td>Sweden</td>
<td>2004-2010, 2012</td>
<td>EU-SILC</td>
<td>53,589</td>
</tr>
<tr>
<td>Switzerland</td>
<td>2011-2012</td>
<td>EU-SILC</td>
<td>13,105</td>
</tr>
<tr>
<td>UK</td>
<td>2005-2010, 2012</td>
<td>EU-SILC</td>
<td>47,197</td>
</tr>
</tbody>
</table>
Table A2: Managerial Occupations

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>Legislators, senior officials and managers, Corporate managers, Managers of small enterprises</td>
<td>Managers and administrators</td>
</tr>
<tr>
<td>Austria, Belgium, Canada, Denmark, Finland, France, Germany, Iceland, Ireland, Israel, Italy, Luxembourg, Netherlands, Norway, Spain, Sweden, Switzerland, UK</td>
<td>International Standard Classification of Occupations (ISCO-88), Codes 11-13</td>
<td>Legislators, senior officials and managers, Corporate managers, Managers of small enterprises</td>
</tr>
<tr>
<td>US</td>
<td>IPUMS-USA 1990 Occupation Codes 004-022, Chief executives and public administrators, Financial managers, Human resources and labor relations managers, Managers and Specialists in marketing, advertising, and public relations, Managers in education and related fields, Managers of medicine and health occupations, Postmasters and mail superintendents, Managers of food services and lodging occupations, Managers of properties and real estate, Funeral directors, Managers of service organizations, Managers and administrators</td>
<td></td>
</tr>
</tbody>
</table>
Appendix II: Balanced Growth

Along a balanced growth path (i) growth rates are constant; (ii) the growth rate in output equals the growth rate in labor and managerial income; (iii) growth in aggregate skill investment is the same as the growth rate in output; (iv) the capital-output ratio is constant; (v) the fractions of managers and workers are constant (i.e. \( z^*(t) = z^* \) for all \( t \)); (vi) factor prices are constant.

We find the growth rate in output per person \( (g) \) and initial managerial skills \( (g_z) \) consistent with (i)-(vi), given a growth rate in exogenous productivity \( (g_A) \). Specifically, we show that there is a balanced growth path if and only if initial managerial skills grow at a specific rate determined by exogenous productivity growth.

From the properties of the plant’s technology, it follows that

\[
1 + g = (1 + g_A)(1 + g_z)^{1-\gamma}(1 + g_k)^{\alpha\gamma},
\]

where \( g_k \) stands for the growth rate of capital per person. It follows that

\[
1 + g = (1 + g_A) \frac{1}{1-\gamma} (1 + g_z)^{\frac{1-\gamma}{1-\alpha}}
\]

(22)

We proceed now to find the rate of growth of managerial skills that is consistent with a balanced-growth path. We denote by \( g_z^* \) such growth rate. Note that if such path exists, then the age profile is shifted by a time-invariant factor \((1 + g_z^*)\). That is,

\[
\frac{z_j(t + 1)}{z_j(t)} = (1 + g_z^*)
\]

for all \( j = 1, \ldots J_R - 1 \). It follows that we can infer the value of \( g_z^* \) from the first-order conditions for skill investments of two cohorts of age \( j \leq J_R - 2 \), at two consecutive dates. In particular, the first-order condition for decisions at the penultimate period of the working life cycle must hold along a balanced-growth path. From (13), it follows:

\[
\left( \frac{1}{1 + g_z^*} \right)^{\theta_1} \left( \frac{1}{1 + g} \right)^{\gamma \frac{1-\alpha}{1-\gamma}} = (1 + g_A) \frac{1}{1-g} \left( \frac{1}{1 + g} \right)^{\gamma \frac{1-\alpha}{1-\gamma}}.
\]

(23)
In deriving the expression above, we used the fact that along a balanced growth path, the rate of return is constant and that the growth in output per capita, $g$, equals the growth rate in skill investments and the growth rate in wage rates. Solving for $g^*_z$ in (23), we obtain:

$$1 + g^*_z = (1 + g) \frac{\gamma(1-\alpha) + (1-\theta_2)(1-\gamma)}{\varphi_1(1-\gamma)} \left(\frac{1}{1 + g_A}\right)^{\frac{1}{\varphi_1(1-\gamma)}}$$  \hspace{1cm} (24)

Substituting (24) in (22), after algebra we obtain

$$1 + g = (1 + g_A)^\psi,$$

where $\psi$

$$\psi \equiv \frac{1 - \theta_1}{\gamma(1-\alpha) + (1-\theta_2)(1-\gamma) - \theta_1(1-\alpha\gamma)}.$$  \hspace{1cm} (25)

Several issues are worth noting from the expression above. First, there is balanced growth path with positive growth in per capita output as long as $\theta_1 \in [0, 1)$. Second, all the same, the growth rate in output per capita increases with $\theta_2$: as the importance of investments in the production of new skills increases, the growth rate in output per capita increases as well. Indeed, as $\theta_2 \to 0$,

$$\psi \to \frac{1}{1-\alpha\gamma}.$$  

That is, the growth rate approaches the growth rate with exogenous skill investments. Finally, as $\theta_1 \to 0$ (i.e. no complementarities),

$$\psi \to \frac{1}{(1-\alpha\gamma) + (1-\theta_2)(1-\gamma)}.$$  

The growth rate in output is still affected by the importance of skill investments.
Figure 1. Age-earnings profiles for managers

Canada

US

Belgium

Spain

Figure 2. Age-earnings profiles for non-managers

Canada

US

Belgium

Spain
Figure 3. GDP per Worker and Relative Income Growth

Log (GDP per hour worked, 2000)

Log (Income Growth, 50−54/25−29)

Slope: 0.569. Corr: 0.489
Figure 4. Management Practices and Relative Income Growth


Slope: 0.297. Corr: 0.602
GDP per Worker and Relative Income Growth

Figure 5.1. Excluding Self-Employed

Figure 5.2. Excluding Self-Employed Non-Managers

Figure 5.3. Wage and Salary Income only
GDP per Worker and Relative Income Growth

Figure 6.1. Without Education Controls

Slope: 0.410. Corr: 0.389

Figure 6.2. Without Year Fixed Effects

Slope: 0.575. Corr: 0.497
Figure 7.1. Professionals vs Non-Managers

Figure 7.2. Self-Employed vs Non-Managers

GDP per Worker and Relative Income Growth
Figure 8: Size Distribution

Plant Size Classes

- less than 10
- 10-19
- 20-49
- 50-99
- 100-499
- 500+

Data
Model
Figure 9: Employment Shares

- **Data**
- **Model**

<table>
<thead>
<tr>
<th>Plant Size Classes</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>less than 10</td>
<td>0.15</td>
<td>0.17</td>
</tr>
<tr>
<td>10-19</td>
<td>0.1</td>
<td>0.12</td>
</tr>
<tr>
<td>20-49</td>
<td>0.15</td>
<td>0.14</td>
</tr>
<tr>
<td>50-99</td>
<td>0.13</td>
<td>0.14</td>
</tr>
<tr>
<td>100-499</td>
<td>0.25</td>
<td>0.3</td>
</tr>
<tr>
<td>500+</td>
<td>0.2</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Figure 10: Age-Earning Profiles, Managers
Figure 11: Aggregate TFP and Age-Earnings Profiles of Managers
Figure 12. GDP per Worker and Relative Income Growth

Data vs Model

Log (Income Growth, 50-54/25-29) vs Log (GDP per Worker, 2000)

- Australia
- Austria
- Belgium
- Canada
- Denmark
- Finland
- France
- Germany
- Iceland
- Ireland
- Israel
- Italy
- Luxembourg
- Netherlands
- Norway
- Sweden
- Switzerland
- UK
- US
- Finland
- Iceland
- Spain
- Denmark

Data vs Model

- Red line: data
- Green line: model