

# Why is the rate of single-parenthood lower in Canada than in the U.S.? A dynamic equilibrium analysis of welfare policies

Nezih Guner *Department of Economics, Universidad Carlos III de Madrid*

John Knowles *Economics Division, School of Social Sciences, University of Southampton*

*Abstract.* A critical question in the design of welfare policies is whether to target aid according to household composition, as was done in the U.S. under the Aid to Families with Dependent Children (AFDC) program, or to rely exclusively on means-testing, as in Canada. Restricting aid to single mothers, for instance, has the potential to distort behaviour along three demographic margins: marriage, fertility, and divorce. We contrast the Canadian and the U.S. policies within an equilibrium model of household formation and human capital investment on children. Policy differences we consider are eligibility, dependence of transfers on the number of children, and generosity of transfers. Our simulations indicate that the policy differences can account for the higher rate of single-parenthood in the U.S. They also show that Canadian welfare policy is more effective for fostering human capital accumulation among children from poor families. Interestingly, a majority of agents in our benchmark economy prefers a welfare system that targets single mothers (as the U.S. system does), yet (unlike the U.S. system) does not make transfers dependent on the number of children.

*Pourquoi est-ce que le taux de monoparentalité est plus bas au Canada qu'aux Etats-Unis? Une analyse d'équilibre dynamique des politiques de bien-être.* Une question critique dans le design des politiques sociales est à savoir s'il faut cibler l'aide selon la composition du ménage, comme on le fait aux Etats-Unis dans le cadre du programme Aid to Families with Dependent Children (AFDC) ou s'en remettre exclusivement à une enquête sur les ressources disponibles. Limiter l'aide aux mères monoparentales, par exemple, peut influencer le comportement à la marge selon trois axes démographiques: mariage, fécondité, divorce. On compare les politiques canadienne et américaine dans le cadre d'un modèle d'équilibre de formation des ménages et d'investissement en capital humain dans les enfants. Les différences dans les politiques portent sur l'éligibilité, la dépendance des transferts sur le nombre d'enfants, et la générosité des transferts. Les simulations indiquent que

Nezih Guner is also affiliated with CEPR and IZA. John Knowles is also affiliated with IZA. The authors thank Andrew Rice for helpful comments and acknowledge financial support from Statistics Canada. An earlier version of this paper was titled 'Marriage, fertility and divorce: a dynamic equilibrium analysis of social policy in Canada.' Email: [nguner@eco.uc3m.es](mailto:nguner@eco.uc3m.es); [J.A.Knowles@soton.ac.uk](mailto:J.A.Knowles@soton.ac.uk)

les différences dans les politiques peuvent expliquer le plus haut taux de monoparentalité aux Etats-Unis. On montre aussi que la politique canadienne est plus effective pour encourager l'accumulation du capital humain dans les enfants des familles pauvres. On note qu'une majorité des agents dans l'économie de référence préfère une politique qui cible les mères monoparentales (comme on le fait aux Etats-Unis) mais qui (contrairement à ce qui se fait aux Etats-Unis) ne rend pas les transferts dépendants du nombre d'enfants.

## **1. Introduction**

A recurring question in the design of social policy is whether programs that target aid to those who need it most might be less effective in reducing long-run poverty than programs that offer aid to a wider population.<sup>1</sup> Targeting disadvantaged families on the basis of characteristics that make poverty particularly dire, such as single motherhood, can provide larger average benefits for any given cost. However, since single-motherhood reflects decisions concerning marriage, fertility and divorce, targeting aid weakens the incentives to avoid single motherhood. Thus, welfare payments designed to help the children of single parents can, at least in principle, increase the fraction of children who are born to single parents. The impact of a targeted system on the fraction of single-parent households depends critically on the relative responsiveness of potential recipients along the margins targeted.

Until recently, the main U.S. welfare program, Aid to Families with Dependent Children (AFDC), via its rules governing eligibility and benefits, penalized women for marriage and rewarded them for non-marital fertility. Could it be that such policies actually increased poverty and single-motherhood? Central issues in the empirical estimation of these effects are the possibility of reverse causality, that is, that welfare policies are responses to poverty as well as vice versa; how to account for the interactions between welfare and labour supply; marriage and fertility decisions; and how to capture forward-looking behaviour. These issues are discussed in detail by Moffit (1997) and Keane and Wolpin (2002). Although the general conclusion in recent reviews by Moffit (1997, 2003) is that welfare is likely to affect family structure, it is still not clear whether the response of family decisions to the incentives implied by welfare programs is large enough to have significant effects on either the cost of welfare policies or the distribution of children across household types.<sup>2</sup>

Canadian welfare programs are much less biased against marriage and less responsive to higher fertility.<sup>3</sup> They are also more generous on average than U.S. programs. While AFDC in the U.S. was largely limited to single mothers,

1 See Akerlof (1978) for an earlier analysis of targeted and universal programs and Atkinson (1995) for a recent review.

2 This literature on incentive effects of the U.S. welfare system can be traced back at least as far as discussion of the negative income tax by Friedman (1962).

3 There are surprisingly few papers on the effects of Canadian welfare system. In an early paper Allen (1993) found large and significant effects of welfare benefits on single-motherhood. A

Canadian welfare programs, in principle at least, required that recipients be neither single nor parents; these programs also benefit married parents as well as married and unmarried childless adults. Our empirical analysis shows that a single parent with no earnings and one child receives about 82% higher transfers in Canada than in the U.S. Also, under the U.S. system the effect of an extra child on single parents' income is more significant.

Our empirical analysis also shows that there are important differences between the U.S. and Canada in terms of single-motherhood. In 1994, about 24% of children below age 8 were living with single mothers in the U.S., compared with 17% in Canada. For older children (between ages 9 and 18), the difference was even more striking: 25% versus 15%. Hence, both single-parenthood and marital instability were significantly more prevalent in the U.S. than in Canada. Such differences raise a natural question: how much of them can be explained by differences in welfare programs?

We use a model of marriage, fertility, and investment in children to simulate the long-run outcomes of a change from universal to targeted welfare policies. We ask what would have been the impact on the number of children with single parents in Canada had Canada adopted a social policy similar to that which prevailed until recently in the U.S. In order to answer this question, we first simulate our model economy so that income inequality and welfare reciprocity in the steady-state equilibrium follow the same patterns with respect to family structure as in the Canadian data. We then simulate the Canadian economy under alternative social policies and compare the long-term distribution of children across different family types in economies that are identical except for the parameters of the government transfer policies.

The basic policy differences we consider are (1) eligibility, (2) dependence of transfers on the number of children, and (3) average level of transfers. We find that these policy differences can account for the gap between Canada and the U.S. in the proportion of children with single parents. We then ask which type of policy is more effective in helping children from poor families. Our results show that the Canadian policy is more effective than a U.S.-type policy in making poor children better off. Most of the disadvantage of a U.S.-type policy comes from the implicit subsidy of single women's fertility. Furthermore, the subsidy to fertility has a disequalizing effect: as shown in Knowles (1999), even small increases in the fertility differential between poor and rich parents will have strong effects on the steady-state income distribution when productivity levels are persistent across generations.

Finally, we ask which one of the policies that we consider is the one most preferred by different income groups in the economy. Although children from

recent paper by Fortin, Lacroix, and Drolet (2004) also reports significant effects of welfare benefits on duration of welfare spells. Recently, the Canadian Self-Sufficiency project, an experimental reform to Welfare in Canada, generated a large body of literature; see, among others, Bitler, Gelbach, and Hoynes (2008), Zabel, Schwartz, and Donal (2006), and Wilk et al. (2006).

poor families receive higher human capital investment under the Canadian policy, this policy is more expensive, so it requires a higher tax rate. The system preferred by the majority turns out to be a compromise; except for the poorest 20% of the population, agents in our model economy prefer a welfare system that targets single mothers (as the U.S. system does), yet (unlike the U.S. system) does not make transfers dependent on the number of children.

It is important to note that we solve for the equilibrium of the marriage market under each policy. Thus, we not only account for how the marriage and divorce decisions of women might respond to welfare programs, but also incorporate the response of men to the effects of welfare on marriage prospects. In the model, family structure affects children's outcomes by changing the optimal shares of time and income devoted to investment in children's human capital. For understanding poverty, an important feature of our model is that it distinguishes among children of two-parent families, children from divorced parents, and those whose parents were never married. This is critical for the exercise in question because empirical studies, for example, McLanahan and Sandefur (1994), suggest that children's outcomes as adults (such as employment and wages) and teenage fertility depend at least as much on family structure as they do on family income.<sup>4</sup>

We focus in this paper on policies that prevailed until the mid-1990s. There have been significant changes in social policies on both sides of the border since the mid-1990's. A major change in the U.S. was the introduction in 1996 of lifetime time limits on welfare reciprocity. Each state now has more freedom in policy design, which makes an aggregate portrait of the current system muddier than it was before the reforms. In Canada, the welfare reform wasn't as clear-cut as it was in the U.S., although the welfare system reputedly became 'leaner and meaner' during the last decade (see National Council of Welfare 1997; Mendelson 2001).

Because our analysis is confined to steady states, we cannot draw from it any predictions regarding the immediate effects of such changes.<sup>5</sup> Greenwood, Guner, and Knowles (2000) show, however, that when a similar theory is used to model the transition path between social policies, the effects of AFDC on fertility and investment in children's human capital induce substantial inertia in welfare dependence and human-capital levels. This suggests that the type of simulation-based analysis developed here may be essential for the design of social policy, as it may take many years for the effects of real-life policies to become evident.

Our approach is complementary to the standard empirical approach in that we build into the model the types of response that are difficult to observe directly and see whether the model's output is consistent with the relationships we observe in the data. While our formulation of the policy differences is simplistic, our approach treats the effects of different policy regimes on the behaviour

4 In a related paper, Greenwood, Guner, and Knowles (2000) show that parental investment in children's human capital is central for understanding why large increases in rates of single-motherhood persisted long after welfare payments had stabilized in the U.S.

5 For evaluations of these recent reforms see Meyer and Sullivan (2004) for the U.S. and Brzozowski (2007) for Canada.

and composition of households in terms of the optimal responses of individual agents, both to the policy parameters and to each other. The emphasis on marriage-market equilibrium makes it difficult to incorporate more realistic policies, but it allows us to address directly the incentive issues that have surrounded public debate regarding the effect of these programs on the composition of the population by household structure.

The model here is based on the general framework developed in Greenwood, Guner, and Knowles (2003). The current paper is part of the growing literature on quantitative models of family; see, among others, Aiyagari, Greenwood, and Guner (2000), Regalia and Rios-Rull (1999), Erosa, Fuster, and Restuccia (2002), Chade and Ventura (2002), Fernandez, Guner, and Knowles (2005), Da-Rocha and Fuster (2006), Knowles (2008), and Guner, Kaygusuz, and Ventura (2007). This paper is also related to recent papers that focus on the labour-market effects of social policies in an equilibrium framework. Heckman, Lochner, and Taber (1998), for example, analyze the effects of tuition subsidies within a general equilibrium model with an explicit college enrolment margin and show that the effects can be much smaller than the ones reported in previous partial equilibrium analysis. Lise, Seitz, and Smith (2004) evaluate the labour-market effects of the Canadian Self-Sufficiency Project and show that partial and general equilibrium effects of such a program can be quite different.

In the next section, we compare income inequality, social policies, and family structures in the two countries. This is followed first by a formal development of the model and then by a description of the procedure used to calibrate the model to Canadian data. We then evaluate the effects of introducing an AFDC-style policy.

## **2. Income and family structure in the U.S. and Canada**

It is well documented that income distribution in the U.S. is more unequal than in Canada. According to Gottshalk and Smeeding (2000), the Gini coefficient for disposable household income per adult equivalent was 0.368 in the U.S. and 0.287 in Canada in 1994. They show that the households at the bottom 10% of the income distribution had about 34% of median income in the U.S., compared with 47% in Canada. Social policies play an important role in these differences; while the earnings distributions for the U.S. and Canada are similar for poor households, their post-tax transfer income is much higher in Canada.

Comparing social policies in the U.S. and Canada is a complex task, partly because there are many different ways policies might vary on paper, but also because poor families can benefit from a multitude of social programs, some of which are national in scope, such as food stamps in the U.S. and child tax credits in Canada, while others, like welfare payments, vary according to the local jurisdictions, that is, city, state, or province. Furthermore, policies that are similar on paper may be administered quite differently across different jurisdictions, so that assembling an

accurate picture of the social policy within each country is actually an ill-defined task. Nevertheless, it is clear that the differences across countries are much larger than the differences within countries, so some abstraction is justified.<sup>6</sup>

In this section, we proceed by measuring the social policies in terms of the transfer income actually reported by households in representative household surveys. Social transfers include old-age or retirement benefits, child or family allowances, training allowances, unemployment benefits and non-cash benefits, such as food and housing and means-tested social assistance, such as welfare.

Given household survey data for both countries, our approach is to estimate how transfer income depends on the earnings, marital status, and family size of the recipients. This procedure results in an aggregate portrait of transfer payments in each country, which we use to parameterize the relevant differences in social policy across the countries, and to evaluate our simulation results for Canada.

The data are taken from the 1994 household surveys disseminated by the Luxembourg Income Study (LIS). The U.S. data are extracted from the 1994 Current Population Survey and the Canadian data from the 1994 Survey of Consumer Finances.<sup>7</sup> More recent data are available, but the U.S. system has been changing rapidly over the last few years as support for welfare reform has grown, and many states changed their policies even before the reformed welfare system Temporary Assistance for Needy Families (TANF) replaced AFDC in 1996. Hence, the year 1994 was chosen because it seemed more likely to reflect a longer-run outcome from the characteristic welfare system of the U.S., rather than the new policy. In all calculations, Canadian dollars were converted to U.S. dollars by dividing by 1.2, a number drawn from the 1994 purchasing power parity (PPP) index disseminated by the World Bank.

Table 1 shows basic characteristics of the household samples for each country. Households were included in our sample if they had children. Our data include the ages of the three youngest children. The total number of children in the household is available, though not the total children ever born to each parent.<sup>8</sup> In table 1 households were classified as belonging to Period 0 if the age of the youngest child was less than 8. If the youngest child was between 9 and 18, then the household was classified as Period 1. This division reflects the compressed life cycle structure of the model to be developed here. Because Canadian data do not distinguish between divorced women and never-married mothers, the marital status of parents was partitioned between married and single, the latter comprising widows, never-married women, and divorced women.

6 This view is also supported by the work of Blank and Hanratty (1993). They show that while there exists substantial variation in social programs within each country, these intra-country differences generate only small changes in poverty rates. The potential effects of cross-country differences, on the other hand, are much more significant.

7 These are stratified samples, so the data analysis is based on the household weights included with each survey.

8 In order to more accurately reflect the implications of social policy for children, the samples were reweighted by taking the product of the household weight and the number of children.

TABLE 1  
Average sample characteristics

	Married parents				Single parents			
	Period 1		Period 2		Period 1		Period 2	
	U.S.	Canada	U.S.	Canada	U.S.	Canada	U.S.	Canada
Observations*	10174	4635	4118	5279	3059	970	1396	949
% of children	76.88	82.69	74.68	84.76	23.12	17.31	25.32	15.24
<i>Family</i>								
DPI	40201.67	37372	45485	40698.87	16725	17963	21041	21105
(std. dev.)	26111.65	17747	27969	21853.23	12435	10570	14365	12036
Fam. earnings	47609.55	41739	54664	45840.8	11237	10077	17491	13349
(std. dev.)	38684.35	28061	40642	36560.26	16076	14329	17627	17084
Govt. transfers	1743.22	4441.9	1786.7	4350.42	5964.3	8434	4005.4	7962.3
(std. dev.)	4311.29	5305.1	4478.7	5492.46	5766.4	5811.7	5191.1	6353.8
Children	2.62	2.03	2.11	2.64	2.77	1.91	1.95	2.32
(std. dev.)	1.24692	0.8842	0.8396	1.15082	1.5093	0.8455	45.767	22.491
<i>Mother</i>								
Age	33.66	31.77	40.41	38.19	31.26	30.16	38.41	37.39
(std. dev.)	6.0202	5.1372	5.4107	5.0156	6.8529	6.3745	5.7859	5.6299
Educ.	12.94	13.3	13.09	12.84	11.64	12.12	12.38	12.29
(std. dev.)	2.9024	2.319	2.6619	2.474	2.4041	2.4449	2.4859	2.3981
Weekly hours	22.22	18.81	27.45	21.9	22.52	13.78	30.53	17.95
(std. dev.)	18.6678	18.193	17.339	18.1763	19.675	18.469	17.687	19.371
Wage	11.97	11.63	12.35	11.76	9.71	11.99	10.92	12.81
(std. dev.)	8.7898	7.0344	9.89	7.6254	10.285	6.9128	7.4095	14.633
<i>Father</i>								
Age	35.93	34.28	42.67	40.83	–	–	–	–
(std. dev.)	6.8999	5.7613	6.5831	5.9225	–	–	–	–
Educ.	13.11	13.36	13.42	12.93	–	–	–	–
(std. dev.)	3.2667	2.4845	3.1827	2.7371	–	–	–	–
Weekly hours	43.25	37.37	42.59	37.54	–	–	–	–
(std. dev.)	14.1447	17.558	15.006	18.009	–	–	–	–
Wage	16.83	16.08	18.52	17.54	–	–	–	–
(std. dev.)	13.461	8.4992	12.698	12.363	–	–	–	–

\*Sample observations unweighted; percentages reflect household weights.

SOURCE: Luxembourg Income Study, 1994

The table reveals a number of significant differences between the two countries. The key differences between countries concern the distribution of children across family structure. In the U.S., 23% of Period-0 children live with single parents, compared with 17% in Canada. Even more striking is the growth in the share of U.S. kids with single parents as the children age: 25% of U.S. children over the age of 9 live with single parents, compared with 15% in Canada. Thus, not only is single-parenthood more common in the U.S., but children in two-parent families are at a higher risk of suffering a household breakup in the U.S. The income of single-parent families is roughly the same in both countries. The average level of transfers to these families is higher in Canada, but this difference is not statistically significant, owing to the high standard deviation of this statistic. Married families,

TABLE 2  
Social policy regression results

Variables		USA	Canada
Intercept	Estimate	709.56	4760
	(t-ratio)	6.42	31.09
Children	Estimate	858.04	1281.84
	(t-ratio)	30.94	30.33
Single mother	Estimate	2779	1886.76
	(t-ratio)	30.07	13.05
<i>N</i>		16197	11421

NOTES: Dependent variable = total public transfers received. The regressions also include age and earnings controls.

SOURCE: Authors calculations from Luxembourg Income Study

however, receive on average a much lower amount of income from government transfers in the U.S. than in Canada.

In assessing the significance of these income differences, it is important to bear in mind that both parental income and family structure have significant effects on the future income of the children. In the U.S., for instance, Stokey (1998) argues, on the basis of a number of empirical studies, the intergenerational correlation of income is on the order of 0.7. For Canada, Corak (2006) reports higher degrees of mobility across generations, particularly in Canada. In the U.S., children inherit about 50% of their parent's earnings variation, compared with about 20% in Canada. Corak (2006) also notes that, in both countries, mobility is substantially less among low-income families; among children born to low-income families, about one-half in the U.S. and about one-third in Canada become low-income adults. McLanahan and Sandefur (1994) find that U.S. children from single-parent families are about twice as likely to drop out of high school or to become pregnant as teenagers. They report that only about one-half of these effects are explained by the lower income of single-parent families. It seems important, therefore, for any theory of income inequality to account for the independent impact of family structure.

The observed patterns of social policy so far do not imply that U.S. policy favours single parents at the expense of married: it may be simply that married parents, having higher incomes, are much less likely to apply for welfare. This point is addressed in table 2, which displays the estimated coefficients for each country for a linear regression model of social-transfer income on household characteristics. It is clear that the Canadian system is much more generous than the U.S. system. The average transfer income of a single parent with no earnings and one child, for example, is about 82% higher in Canada. Furthermore, the share of transfer income associated with children is much higher in the U.S.; the coefficient on number on children is greater than the average payment to non-parents, while in Canada, this coefficient is only 26%. Finally, being single results in a higher transfer in the U.S. than in Canada.

TABLE 3  
Fertility by female earnings in 1994 survey data

Female earnings quintile	Statistic	USA		Canada	
		Age interval		Age interval	
		25–35	36–45	25–35	36–45
1	Mean	1.95	1.68	1.6	1.53
	std	52.45	53.31	19.12	20.76
	median	2	2	2	2
	<i>N</i>	5315	5976	4543	4716
2	Mean	1.34	1.28	1	1.16
	std	47.53	49.5	17.06	17.31
	median	1	1	1	1
	<i>N</i>	1096	1179	716	808
3	Mean	1.04	1.08	0.87	1.15
	std	44.17	42.73	17.69	17.76
	median	1	1	1	1
	<i>N</i>	1088	1159	636	745
4	Mean	0.99	1.09	0.78	1.01
	std	44.51	45.72	17.46	17.74
	median	1	1	0	1
	<i>N</i>	1062	1108	606	742
5	Mean	0.84	1.13	0.78	1.08
	std	41.37	44.46	18.06	18.77
	median	1	1	0	1
	<i>N</i>	1055	1126	620	690

SOURCE: Luxembourg Income Study, 1994

Table 3 explores the relationship between female earnings and fertility in the data. In both countries, the number of children at home declines with a woman's earnings, and the size of this effect is similar in both countries, although the number of children is higher in the U.S. than in Canada.

### 3. The model

The basic structure of the model is based on Greenwood, Guner, and Knowles (2003). The economy is populated by overlapping generations that live two periods as children and two periods as adults. We refer to the first and second periods of adulthood as young and old. The mass of each of these age groups is equally divided between a continuum of males and one of females, distinguished by their productivity levels (types). Let the productivity of agents be denoted by  $x$  for females, and by  $z$  for males and assume that they are contained in the finite sets  $\mathcal{X} = \{x_1, x_2, \dots, x_N\}$  and  $\mathcal{Z} = \{z_1, z_2, \dots, z_N\}$ . Each adult is endowed with one unit of time.

On becoming young adults (after two periods with their parents), agents learn their productivity levels and meet potential spouses from the same

cohort. Potential couples then draw a random match quality, denoted by  $\gamma \in \mathcal{G} = \{\gamma_1, \gamma_2, \dots, \gamma_M\}$ . At this point, the productivity of the two potential partners is common knowledge, as is the quality of their match. If both parties agree, a marriage ensues; otherwise both remain single. At the start of the second period, each agent learns her next-period productivity and, if married, that of her spouse as well as the future match quality. Married agents then decide whether to stay together or divorce. There is no remarriage for divorced agents. At this time, agents who remained unmarried in the first period meet new potential partners (among those who also remained single in the first period) and can choose to marry.

A newly matched couple, young or old, draws its match quality from the following distribution:

$$\Pr[\gamma = \gamma_i] = \Gamma(\gamma_i).$$

For a married young couple, the match quality in the second period,  $\gamma'$ , depends on the initial draw and represented by

$$\Pr[\gamma' = \gamma_j \mid \gamma = \gamma_i] = \Lambda(\gamma_j \mid \gamma_i).$$

After the matching decisions of the first period, young married couples and young single females decide how many children to have, how much to work, and how much of the mother's time and family income should be spent on educating the children. Young males simply decide how much to work. Hence, whether married or single, males allocate their time between leisure and labour, while that of females is allocated across labour, leisure, and the nurture of the children.<sup>9</sup> Children are not differentiated by sex until they become adults. Let  $k$  denote the number of children; we assume that  $k \in \mathcal{K} = \{0, 1, \dots, K\}$ . Similarly, after the matching decisions of the second period, households decide how to allocate their time and income. We assume that females can have children only when they are young, and if their parents get divorced, children stay with their mothers. There is a child support payment system in effect. A divorced male has to pay  $\pi\%$  of his second-period income per child that he has to support.

Education per child in a family with  $k$  children is an increasing, deterministic function of parental spending on education, denoted by  $d$ , and the nurture time of the mother, denoted by  $t$ , and is represented by

$$e = Q(t, d, k).$$

9 Including father's time allocation to children's education would have been too burdensome computationally. Empirical studies suggest mothers spend much more time with children than fathers do in the U.S.; see, for example, Juster and Stafford (1985).

Consumption per capita of a household with income level  $Y$  that has  $a$  adults and  $k$  children is given by

$$c = \frac{1}{\Psi(a, k)} Y,$$

where  $\Psi(a, k)$  is the adult-equivalent size of a household with  $a$  adults and  $k$  children.

Agents' per period utility function depends on  $c$ ,  $k$ ,  $e$ , and  $\gamma$  (if married) and are given by

$$F(c, e, k, 1-l-t, \gamma) = \begin{cases} v^c(c) + v^e(e, k) + v^\ell(1-l-t-\phi_f k) - \gamma, & \text{if married} \\ v^c(c) + v^e(e, k) + v^\ell(1-l-t-\phi_f k), & \text{if single} \end{cases}$$

for females. Females put  $l$  units of their time to market work and  $t$  units of their time to child care. There is also a fixed time cost of having  $k$  children, denoted by  $\phi_f k$ . Note that both married and single females can have children, so both  $e$  and  $k$  enter into their utility function. Similarly, the utility function for males is given by

$$M(c, e, k, 1-n, \gamma) = \begin{cases} u^c(c) + u^e(e, k) + u^\ell(1-n-\phi_m k) - \gamma, & \text{if married} \\ u^c(c) + u^\ell(1-n), & \text{if single} \end{cases}.$$

Note that a single (or divorced) male does not care about the human capital investment of children. Males simply allocate  $n$  units of their time to market work and face (if married) a fixed time cost of having  $k$  children. We assume that the household decisions of married couples are determined by the Nash solution to the fixed-threat bargaining game in which the threat point is the value of being single.<sup>10</sup>

In the first period of adult life, the probability of different productivity realizations depends on the education received during childhood and is denoted by

$$\Pr[x = x_i | e] = \Pi^x(x_i | e) \text{ and } \Pr[z = z_i | e] = \Pi^z(z_i | e),$$

where  $e = e_{-1} + e_{-2}$  is the total human capital investment that a child receives during his/her childhood (which depends on the marital history of his/her mother). The probability distributions  $\Pi^x(x_i | e)$  and  $\Pi^z(z_i | e)$  are stochastically increasing in  $e$  in the sense of first-order stochastic dominance.<sup>11</sup>

10 There is a large literature on different approaches to households' decision making. See Del-Boca and Flinn (2005) for a recent review and empirical evidence in favour of the Nash bargaining solution.

11 We do not differentiate between early and late education, in order to reduce computational burden. See Restuccia and Uruttia (2004) and Caucutt and Lochner (2005) for models of human capital accumulation in which this distinction is explicitly modelled.

The productivity in the second period of adult life does not depend directly on childhood education, but rather depends on the initial productivity draw and is given by

$$\Pr[x' = x_j | x = x_i] = \Delta^x(x_j | x_i) \text{ and } \Pr[z' = z_j | z = z_i] = \Delta^z(z_j | z_i),$$

where  $x'$  and  $z'$  denote next period's productivity levels.

Finally, each household can receive welfare payments, which depend on the family type, number of children, and family income. For households with no labour income, we denote by  $w_g(k)$ ,  $w_b$ , and  $w_m(k)$  the guaranteed income level for a single female, a single male, and a married couple, respectively. As labour income increases, however, welfare payments are reduced at rate  $r$ . Welfare payments are financed by a lump-sum tax  $\tau$  on households. We assume that households who are on welfare do not pay this lump-sum tax. We also assume that divorced males' welfare payments are subject to child-support payments. Given these assumptions, the income of a young single female of type  $x$  who has  $k$  children and works  $l$  units is given by  $w_g(k) + xl(1 - r)$  if she is on welfare and by  $xl - \tau$  if she is out of welfare. Similarly, for a divorced female who has  $k$  children and her ex-husband has  $zn$  units of income, income is given by  $xl + \pi knz - \tau$  if she is out of welfare and by  $w_g(k) + (1 - r)xl + \pi knz$  if she is on welfare.

#### 4. Equilibrium

Since agents live for two periods, second-period decisions are rather straightforward. We start by characterizing old agents' problems and then, given the values assigned to the second-period outcomes, define the value functions for the first period.

##### 4.1. Single old

A single old female can be never-married or divorced. For a never-married female, the individual state is given by her type,  $x$ , and the number of children she has,  $k$ . Divorced agents receive child-support payments from their ex-husbands, so the current productivity of their ex-husbands,  $z$ , is also a state variable. The problem solved by a divorced female is given by

$$G_2(x, k, z) = \max_{l,t,d} F(c, e, k, 1 - l - t, 0), \tag{1}$$

subject to

$$c = \Psi(1, k) \max\{xl + \pi z N_2^s(z, k)k - d - \tau, w_g(k) + (1 - r)xl + \pi z N_2^s(z, k)k - d\}$$

and

$$e = Q(t, d, k),$$

where the function  $N_2^s(z, k)$  denotes the labour supply of a single male who has  $k$  children from his first-period marriage. For a never-married old female, the problem is simply given by setting  $z = 0$ .

The value of being a single old male is given by the following problem:

$$B_2(z, k) = \max_n M(c, 0, 0, 1 - n, 0), \quad (2)$$

subject to

$$c = \max\{zn(1 - \pi k) - \tau, (w_b + (1 - r)zn)(1 - \pi k)\},$$

where  $k$  denotes the number of children for whom he has to pay child support. Note that for a never-married old male,  $k = 0$ . Let  $N_2^s(z, k)$  be the optimal labour-support decision associated with this problem.

#### 4.2. Married old

Consider a couple of type  $(x, z, \gamma, k)$  that is married at the start of the second period and has been married in the first period as well. Its problem is given by

$$\begin{aligned} & \max_{l, t, n, d} [F(c, e, k, 1 - l - t, \gamma) - G_2(x, k, z)] \\ & \quad \times [M(c, e, k, 1 - n, \gamma) - B_2(z, k)], \end{aligned} \quad (3)$$

subject to

$$c = \Psi(2, k) \max\{xl + zn - d - \tau, w_m(k) + (1 - r)(xl + zn) - d\}$$

and

$$e = Q(t, d, k).$$

Here,  $B_2(z, k)$  and  $G_2(x, k, z)$  are the threat points for the husband and wife. They are the values of being single in the second period and are given by the solutions to the old single agent problems, (1) and (2). Let the resulting utility levels for an old husband and wife in a  $(x, z, \gamma, k)$ -marriage or the values for  $M$  and  $F$  in (3) evaluated at the optimal choices be represented by  $H_2(x, z, \gamma, k)$  and  $W_2(x, z, \gamma, k)$ .

Each party faces a decision: should s/he choose married or divorced life for the period. A married female will remain married if and only if  $W_2(x, z, \gamma, k) \geq G_2(x, k, z)$ . Similarly, a married male will remain so if and only if  $H_2(x, z, \gamma, k) \geq B_2(z, k)$ . The marital decision of an age-2 couple who is considering divorce is

then given by the following indicator function:

$$I_2^m(x, z, \gamma, k) = \begin{cases} 1, & \text{if } W_2(x, z, \gamma, k) \geq G_2(x, k, z) \text{ and } H_2(x, z, \gamma, k) \geq B_2(z, k) \\ 0, & \text{otherwise} \end{cases} \quad (I2)$$

The problem of a couple who has just matched at the start of the second period is identical to (3), with  $k = 0$  in  $B_2(z, k)$  and  $z = 0$  in  $G_2(x, k, z)$ . Let  $I_2^s(x, z, \gamma, k)$  be the indicator function for a newly matched couple in the second period.

### 4.3. Young

Consider first a young female of type  $x$  who meets a young male of type  $z$  in the marriage market and that their match quality is  $\gamma$ . Suppose that the expected lifetime utility of single life for the female is  $G_1(x)$ , while the expected lifetime utility from marriage is  $W_1(x, z, \gamma)$ . She will choose to marry if  $W_1(x, z, \gamma) \geq G_1(x)$  and to remain single otherwise. Let  $B_1(z)$  and  $H_1(x, z, \gamma)$  denote the corresponding first-period values for males. Since her partner faces the same decision, the marriage will occur if and only if  $W_1(x, z, \gamma) \geq G_1(x)$  and  $H_1(x, z, \gamma) \geq B_1(z)$ .

How is  $G_1(x)$  determined? The value of being a young single female of type  $x$ ,  $G_1(x)$ , is the sum of current utility and expected future utility, which in turn depends on the values of single life and married life in the second period. It is given by

$$G_1(x) = \max_{c,e,d,l,t,k} \{F(c, e, k, 1 - l - t, 0) + \beta E\{W_2(x', z', \gamma', k)I_2^s(x', z', \gamma', k) + G_2(x', k)[1 - I_2^s(x', z', \gamma', k)]\}, \quad (4)$$

subject to

$$c = \frac{1}{\Psi(a, k)} \max\{w_g(k) + (1 - r)xl - d, xl - d - \tau\}$$

and

$$e = Q(t, d, k).$$

The term  $E\{W_2(x', z', \gamma', k)I_2^s(x', z', \gamma', k) + G_2(x', k)[1 - I_2^s(x', z', \gamma', k)]\}$  represents the expected value of entering into the second period as a single female. A single female of type  $x$  will have a new productivity draw  $x'$ , meet a single male of type  $z'$ , and draw a match quality  $\gamma'$ . A similar problem determines the value

of being a young single male,  $B_1(z)$ , as

$$B_1(z) = \max_{c,n} \{M(c, 0, 0, 1 - n, 0) + \beta E\{H_2(x', z', \gamma', k)I_2^s(x', z', \gamma', k) + B_2(z')[1 - I_2^s(x', z', \gamma', k)]\}, \quad (5)$$

subject to

$$c = \max\{w_b + (1 - r)zn, zn - \tau\},$$

where the term  $E\{H_2(x', z', \gamma', k)I_2^s(x', z', \gamma', k) + B_2(z')[1 - I_2^s(x', z', \gamma', k)]\}$  now captures the probability of meeting a type- $x'$  female with  $k$  children.

The decision problem facing a young married couple indexed by  $(x, z, \gamma)$  is

$$\begin{aligned} \max_{c,e,k,l,t,n} \{ & F(c, e, k, 1 - l - t, \gamma) + \beta E[W_2(x', z', \gamma', k)I_2^m(x', z', \gamma', k) \\ & + G_2(x', k)[1 - I_2^m(x', z', \gamma', k)] - G_1(x)] \\ & \times \{M(c, e, k, 1 - n, \gamma) + \beta E[H_2(x', z', \gamma', k)I_2^m(x', z', \gamma', k) \\ & + B_2(z', k)I_2^m(x', z', \gamma', k)] - B_1(z)\}, \end{aligned} \quad (6)$$

subject to

$$c = \frac{1}{\Psi(a, k)} \max\{w_m(k) + (xl + zn)(1 - r) - d, xl + zn - \tau - d\}$$

and

$$e = Q(t, d, k).$$

Here,  $G_1(x)$  and  $B_1(z)$  represent the female's and male's threat points defined in problems (4) and (5).

The maximized value of the first term in braces gives the value of being in a  $(x, z, \gamma)$  marriage for the female,  $W_1(x, z, \gamma)$ , while the second term yields  $H_1(x, z, \gamma)$ . Once we have these first-period values, we can define a marriage indicator for the first period as  $I_1(x, z, \gamma) = 1$  if and only if  $W_1(x, z, \gamma) \geq G_1(x)$  and  $H_1(x, z, \gamma) \geq B_1(z)$ .

Since we assume that  $x$  and  $z$  take values from finite sets, let  $\Phi_1(x_i)$  and  $\Omega_1(z_i)$  be the distribution of female and male agents who participate in the first period's marriage market and  $\Phi_2(x_i, k)$  and  $\Omega_2(z_i)$  be the distribution of female and male agents who participate in the second period's marriage market. First, note that the second-period distributions,  $\Phi_2(x_i, k)$  and  $\Omega_2(z_i)$ , together with transition functions for  $x$  and  $z$ , that is,  $\Delta^x$  and  $\Delta^z$ , define the expectations in problems (4), (5), and (6). Therefore, given  $\Phi_2(x_i, k)$  and  $\Omega_2(z_i)$ , the values of being married and single in the first period, that is,  $G_1(x)$ ,  $B_1(z)$ ,  $W_1(x, z, \gamma)$ , and  $H_1(x, z, \gamma)$ , can be calculated. Second, the value functions  $G_2$ ,  $B_2$ ,  $W_2$ , and  $H_2$  are

TABLE 4  
Log hourly wage distributions

	Men	Women
Mean	2.65	2.29
Std.	0.63	0.67

SOURCE: Luxembourg Income Study, 1994

determined trivially, since agents live for only two periods. Third, given these value functions, marriage indicators,  $I_1(x, z, \gamma)$ ,  $I_2^s(x, x, \gamma, k)$ , and  $I_2^m(x, x, \gamma, k)$  can be constructed. Finally, given the marriage indicator functions and the education decisions associated with first- and second-period value functions, we can update  $\Phi_1(x_i)$ ,  $\Omega_1(z_i)$ ,  $\Phi_2(x_i, k)$  and  $\Omega_2(z_i)$ .

This updating involves two parts. The first part is trivial. The distribution of agents in the first-period marriage market, that is,  $\Phi_1(x_i)$  and  $\Omega_1(z_i)$ , first-period marriage indicators, and fertility decisions are used to determine  $\Phi_2(x_i, k)$  and  $\Omega_2(z_i)$ . The second-period distributions consist simply of agents who decided not to or could not marry in the first period. The second part involves updating  $\Phi_1(x_i)$  and  $\Omega_1(z_i)$ . Given marital histories, we can use fertility and education decisions to update  $\Phi_1(x_i)$  and  $\Omega_1(z_i)$  in line with transition functions  $\Pi^x(x_i | e)$  and  $\Pi^z(z_i | e)$ . This updating procedure is characterized formally in the appendix.

A steady-state equilibrium for this economy consists of a fixed point between household decisions about marriage, fertility, and education, and the distribution of agents in the first- and second-period marriage markets. We solve this fixed-point problem numerically.

## 5. Computational analysis

The first step in the computational analysis is to select functional forms and parameterize the model to be able to generate a set of observables regarding the distribution of children by parent's marital status and income distribution by family structure. To this end, we first set  $N = 13$  and distribute the grid points (values of  $x_i$ s and  $z_i$ s) over a range of two standard deviations around the mean of log wages, as reported in table 4.

In the benchmark calibration, people can choose to have children from the following set,  $k \in \{0, 1, 2, 3, 4, 5\}$ , which is the smallest non-binding set for  $k$ . We restrict the number of match-quality shocks to two, that is,  $\gamma \in \mathcal{G} = \{\gamma_1, \gamma_2\}$ , and set the stochastic structure to be *iid*; that is,

$$\begin{aligned} \Pr[\gamma = \gamma_1] &= \Pr[\gamma = \gamma_2] = 0.5 \text{ and } \Pr[\gamma' = \gamma_1 | \gamma = \gamma_1] \\ &= \Pr[\gamma' = \gamma_2 | \gamma = \gamma_2] = 0.5. \end{aligned}$$

Second, there are a few parameters that can be chosen on the basis of previous research. We interpret a model period as 10 years and set  $\beta = 0.67$ , which corresponds to a 4% yearly interest rate, as is standard in the macroeconomics literature. Economies of scale in household consumption is given by

$$c = \frac{1}{\Psi(a, k)} Y = \frac{1}{(a + bk)^\theta} Y = \frac{1}{(a + 0.4k)^{0.5}} Y.$$

There is a large empirical literature on household economies of scale. We take the functional form for  $\Psi(a, k)$  from Cutler and Katz (1992) and set  $b = 0.4$  and  $\theta = 0.5$ , which they consider as consensus and intermediate values in the literature.

Third, we set  $\pi = 0.0607$ , so that a divorced male pays about 6.1% of his income for child-support payments per child he is supporting. According to Bertrand et al. (2003), based on more than 33,000 divorce cases between 1998 and 2002, the mean monthly child-support payment was \$544 and the mean annual income of paying parents was \$43,532. Hence, a divorced father paid about 15% of his income in child-support payments.<sup>12</sup> Of course, unlike our model economy, some parents do not pay the child-support payments that are due. According to Bertrand et al. (2003), about 10% of divorce cases and associated child-support payments were contested. Taking this as a measure of potential non-compliance by ex-husbands, and assuming two children per divorce (as in our benchmark economy), effective payment per child is about 6.07%.

Finally, we borrow the functional form and parameters of the child-quality production function from Greenwood, Guner, and Knowles (2003):

$$e = Q(t, d, k) = \left(\frac{t}{k^{\chi_1}}\right)^\alpha \left(\frac{d}{k^{\chi_2}}\right)^{1-\alpha} = \left(\frac{t}{k^{0.4}}\right)^{0.5} \left(\frac{d}{k^{0.5}}\right)^{1-0.5} = \frac{1}{k^{0.45}} t^{0.5} d^{0.5}.$$

We provide a more detailed discussion of the child-quality production function in section 5.1.

The rest of the parameter values are calibrated to match a set of targets from the data. We assume that momentary utility functions are given by

$$F(c, e, k, 1 - l - t, \gamma) \equiv \frac{c^{\sigma_1}}{\sigma_1} + \frac{k^{\sigma_2} e^{\sigma_3 f}}{\sigma_2 \sigma_3 f} + \delta \frac{(1 - l - t - \phi_f k)^{\sigma_4}}{\sigma_4} - \gamma$$

for females and by

$$M(c, e, k, 1 - n, \gamma) \equiv \frac{c^{\sigma_1}}{\sigma_1} + \frac{k^{\sigma_2} e^{\sigma_3 m}}{\sigma_2 \sigma_3 m} + \delta \frac{(1 - l - t - \phi_m k)^{\sigma_4}}{\sigma_4} - \gamma$$

for males.

We assume that  $\Delta^x(x_j | x_i)$  and  $\Delta^z(z_j | z_i)$  are discrete approximations to log-normal distributions. These distributions map first-period productivity levels into

12 Fathers had custody in only about 8% of cases.

second-period productivity levels and determine the volatility of earnings from the first to the second model period. We assume that, for a female with first-period productivity level  $x$ , her next-period productivity  $x'$  is a draw from a log normal distribution with mean  $2.29(1 - \rho) + \rho \ln x$  and standard deviation  $s$ . Similarly, a male's productivity evolves to  $z' \sim \ln N(2.65[1 - \rho] + \rho \ln z, s)$ . Hence, both males and females have on average a  $\rho\%$  chance of keeping their current productivity and a  $(1 - \rho)\%$  chance of moving to the mean productivity.

We also assume that  $\Pi^x(x_i | e)$  and  $\Pi^z(z_i | e)$  are discrete approximations to log normal distributions. These functions map accumulated education during childhood,  $e = e_{-2} + e_{-1}$ , into first-period productivity levels. We assume that  $\Pi^x(x_i | e)$  is a discrete approximation to log normal distribution with mean  $m_x e^\eta$  and a standard deviation of  $s_e$ ; similarly,  $\Pi^z(z_i | e)$  is a discrete approximation to a log normal distribution with mean  $m_z e^\eta$  and a standard deviation of  $s_e$ .

The parameters we need to determine are then as follows: eight utility parameters,  $\{\sigma_1, \sigma_2, \sigma_{3f}, \sigma_{3m}, \phi_m, \phi_f, \delta, \sigma_4\}$ , two match-quality levels,  $\{\gamma_1, \gamma_2\}$ , welfare policy parameters,  $\{w_m(k), w_g(k), w_b, r\}$ , parameters determining stochastic structure of productivity levels between periods 1 and 2,  $\{\rho, s\}$ , and parameters that map childhood histories in education into first-period productivity levels,  $\{m_z, m_x, \eta, s_e\}$ . We calibrate these parameters to match an equal number of targets from the data.

1. In the data, about 17% of younger children and about 15% of older children live with single parents. The match-quality levels,  $\gamma_1 = 0$  and  $\gamma_2 = 1.439$ , are chosen to generate these statistics.
2. In the data, incomes differ significantly by marital status. On average, a single mother earns about 24% of the income of a married couple when she is young and about 32% of the income of a married couple when she is old. The curvature of the utility from consumption,  $\sigma_1 = 0.48$ , is picked to generate this match.
3. In the data, single females use about 12% of their time for the market work when they are young and about 16% of it when they are old (assuming a weekly time endowment of 122 hours). The parameter of the utility from leisure,  $\sigma_4 = 0.255$ , is picked to match these statistics.
4. In the data, transfer incomes amount to about 10.64% of married couples' income in the first period. The labour supply parameter  $\delta = 2.7$  is picked to generate the right amount of welfare dependence for married couples in the first model period.
5. The overall fertility level is 2 per female in the model (which generates a stationary population structure). Low-income families tend to have more children than high-income families; the dependence of fertility on income generated by the model is shown in figure 1. Figure 1 shows the relation between female earnings and fertility in the model and in the data (for age interval 25–35, as documented in table 3).<sup>13</sup> We set  $\sigma_2 = 0.302$  to get the overall fertility level of

13 The data source for figure 1, as it was for table 3, is the Luxembourg Income Study.

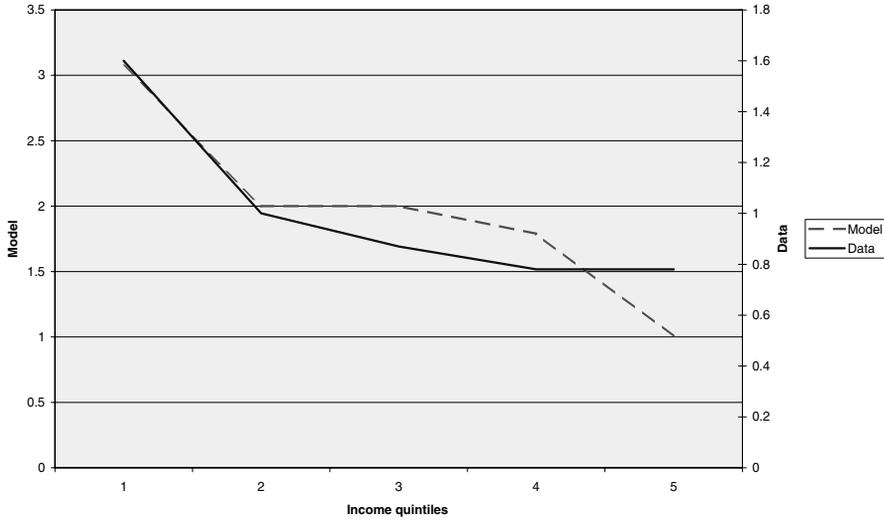


FIGURE 1 Income and fertility

2 and picked the remaining parameters that determine fertility,  $\sigma_{3m} = 0.325$ ,  $\sigma_{3f} = 0.22$ ,  $\phi_m = 0.025$  and  $\phi_f = 0.05$ , to generate a relation between income and fertility similar to what we observe in the data.

6. To calibrate  $\Delta^x$  and  $\Delta^z$ , we set  $\rho = 0.69$ ; that is, both males and females have on average a 69% chance of keeping their current productivity and a 31% chance of moving to the mean productivity level. In the data, transfer incomes amount to about 9.49.% of married couples' income in the second period. Parameter  $\rho$  was chosen to generate the same level of welfare dependence in the model economy. Note that, given welfare dependence for married couples in the first period, welfare dependence in the second period is determined by the mass of households at the low end of the productivity distribution. This allows us to pin down the level of income mobility between two periods. Given  $\rho$ , the parameter  $s$  is then selected so that the standard deviation of second-period distribution of female and male types in the steady state are consistent with the data in table 4; that is, the standard deviation of log earnings is about 0.63 for males and 0.67 for females.
7. To calibrate  $\Pi^x$  and  $\Pi^z$ , we set  $m_z = 14$ ,  $m_x = 9.86$ ,  $s_e = 0.4$ , and  $\eta = 0.525$ . These four parameters were chosen so that the initial (period 1) distributions of female and male types in the steady state, that is,  $\Phi_1(x_i)$  and  $\Omega_1(z_j)$ , are consistent with the data; that is, the four moments of these distributions match the ones reported in table 4.
8. Finally, in order to calibrate our welfare parameters, we use Canadian data on welfare payments. According to the National Council of Welfare (2000), in British Columbia, Ontario, and Quebec, welfare incomes constitute about 48%

of the single mothers' average income. We set  $w_g(k) = w_g = 1.3$ , which is about 48% of the average income for single females in the benchmark economy. Then we used the ratio of welfare payments for single mothers to that for single males and married in the data to set  $w_b = 0.65$  and  $w_m(k) = w_m = 1.8$ .<sup>14</sup> We set  $r = 0.66$ , based on the estimate by Charette and Meng (1994). This variable captures how welfare payments are reduced with household income and therefore is a summary measure of quite complex rules.<sup>15</sup>

Table 5 lists the parameter values (except those that are selected to match the statistics in table 4) and the corresponding targets. Table 6 compares statistics from table 1 with their analogues from the model economy for married and single agents. Overall, the model generates a good fit with the data on the moments that are most directly linked to family structure: the share of children by age in single-parent families, the relative earnings of young married and single mothers, and the relation between female earnings and fertility. The main divergences between the model results and the data are quantitative rather than qualitative. Single mothers (especially when young) are less dependent on welfare in the model than they are in the data. A single mother gets about 47.8% of her income from welfare when young and 51.75% of it from welfare when old. In contrast, the welfare dependence of young and old single mothers is 83.7% and 59.7% in the data, respectively. The model also generates a higher fertility differential between single and married parents than in the data.

### 5.1. Discussion

Table 7 reports additional statistics from the benchmark economy. These statistics highlight two aspects of our calibration strategy that we did not discuss in detail above. First, in the simulations we assume that the match quality shocks are *iid* with  $\Pr[\gamma = \gamma_1] = \Pr[\gamma = \gamma_2] = 0.5$  and  $\Pr[\gamma' = \gamma_1 | \gamma = \gamma_1] = \Pr[\gamma' = \gamma_2 | \gamma = \gamma_2] = 0.5$ . This is undoubtedly restrictive. However, given that  $\gamma_1$  and  $\gamma_2$  are selected to generate the fraction of children living with single mothers, the simple life-cycle structure of the model restricts our ability to generate additional statistics that can be used for calibrating the stochastic structure of match qualities. After all, the fraction of children living with single mothers is simply determined by the fertility behaviour and the marital status of the population. The only additional statistic that the model generates is the fraction of population that is divorced in the second period. In the model (see table 7), about 10% of second period population is divorced. This compares surprisingly well with the data.

14 According to Cragg (1996) and Barrett and Cragg (1998), most welfare spells are shorter than 10 years (a model period), although for single mothers with children spells can be quite long (more than two years). However, Barrett and Cragg (1998) show that more than half of single mothers who exit welfare return to it after a year.

15 The available estimates for the U.S. welfare system are quite varied and range from low values of around 30% by McKinnish, Sanders, and Smith (1999) to much higher rates of 70–90% by Hoynes (1997) and Keane and Wolpin (2002).

TABLE 5  
Parameter values

Parameter	Value	Target	Model	Data
$\gamma_1$	0	Fraction of Kids with Single Mothers (period 1)	17.10%	17.31%
$\gamma_2$	1.439	Fraction of Kids with Single Mothers (period 2)	15.23%	15.24%
$\sigma_1$	0.480	Single females' Income/Married Couples' Incomes (period 1)	0.22	0.24
$\sigma_2$	0.302	Aggregate Fertility		2
$\alpha_{3f}$	0.220	Income Fertility Relation		Figure 1
$\alpha_{3m}$	0.325	Income Fertility Relation		Figure 1
$\alpha_4$	0.255	Single females' labor supply (period 1)	0.12	0.12
$\phi_f$	0.05	Income Fertility Relation		Figure 1
$\phi_m$	0.025	Income Fertility Relation		Figure 1
$\delta$	2.70	Transfer Income/Family Income (married, period 1)	10.89	10.64
$\rho$	0.690	Transfer Income/Family Income (married, period 2)	9.93	9.49
$w_g(k)$	1.30	Welfare income/average income of single females	0.48	0.48
$w_m(k)$	1.80	welfare payments to married/welfare payments single mothers	1.38	1.38
$w_b$	0.65	welfare payments to single males/welfare payments to single mothers	0.5	0.5

TABLE 6  
Calibration

	Married parents				Single parents			
	Period 1		Period 2		Period 1		Period 2	
	Model	Canada	Model	Canada	Model	Canada	Model	Canada
Children	82.90	82.69	84.77	84.76	17.10	17.31	15.23	15.24
Fam. earnings*	1.00	1.00	1.00	1.00	0.22	0.24	0.27	0.32
Transfers/Income (%)	10.89	10.64	9.93	9.49	47.80	83.70	51.75	59.65
Fertility	1.88	2.03	NA	2.64	2.66	1.91	NA	2.32
Labor supply**								
Mother	0.33	0.17	0.25	0.20	0.12	0.12	0.13	0.16

\*Canadian data for fam. earnings are based on table 1. The numbers are normalized to total family earnings of married couples

\*\*Canadian data for labour supply are based on table 1. Weekly hours are normalized by 112 hours.

TABLE 7  
Benchmark economy

	Marital status (%)		Spending on children (%)*		Human capital investment**	
	Period 1	Period 2	Period 1	Period 2	Period 1	Period 2
Married	87.31	86.71	18.26	18.37	1.00	1.00
Single	12.69	13.29	33.88	33.00	0.34	0.39
Never Married		2.69		34.72		0.33
Divorced		10.6		27.67		0.40

\*As a fraction of household income

\*\*As a fraction of young married couples' investment

According to the 1996 Census, about 10% of females between ages 35 and 59 were divorced (available at <http://www.statcan.ca/english/census96/oct14/law.htm>).

Second, we borrowed the parameters for  $Q(k, d, t)$  from Greenwood, Guner, and Knowles (2003). Their calibration is based on the fraction of income that single and married parents spend on their children. In the benchmark economy, married couples spend about 18% of their income on children, while the fraction for single parents is about 33%. A simple calculation from Statistics Canada (1999) shows that the model is not far from the data. In 1997, married couples with children spent about 22% of their total consumption expenditure on children, while the same fraction for single households was about 28%.<sup>16</sup> Given these differences in spending and the number of children (table 6), the model

16 In order to calculate spending on children, we take from Statistics Canada (1999, table 3 total consumption spending as well as spending on food, shelter, household operations, furnishing, clothing, transportation, health, personal care, recreation, reading materials, and education. We consider education as totally spent on children. For the rest of the items, we allocate 20% of

generates wide disparities in human-capital investment per child; in married couple households human-capital investment per child is about three times as high as in single-mother families.

## 6. Policy experiments

Given that the model can reproduce the basic features of the Canadian data discussed above, we now conduct policy simulations in order to compute the impact on family structure and the inequality of moving from the Canadian-style welfare policy of our benchmark model to the more targeted type of welfare policy that was in effect in the U.S. The basic policy differences we consider are (1) eligibility of married women and single men, (2) dependence of transfers on the number of children, and (3) average level of transfers. In this section, we modify the benchmark model by introducing these differences sequentially. Our objective is to find out to what extent such differences could explain the higher proportion of single-parent children in the U.S., which of these differences is most important for our explanation, and what type of policy is most effective in making the poor better off in the long run.

The results of these policy experiments are reported in table 8a together with the data for the U.S. economy. In Experiment 1, we simply assume that Canada stops providing welfare payments to married people and single males. Instead, only single mothers are eligible for welfare. As table 8a demonstrates, the first experiment does not affect the number of children with single mothers. The income inequality measures also remain the same.

In Experiment 2, we make welfare payments for single mothers dependent on the number of children; in particular, we assume that  $w_g(k) = a + bk$ . In order to determine the parameters  $a$  and  $b$ , first note that in the U.S. (table 2), having children increases the income of a single mother (with no additional earnings) by about 25%. Hence,  $(a + b)/a = 1.25$ . We also require that under this policy, a single female with three children (the average number of children that welfare mothers have in the benchmark economy) receives the same welfare payments as she did in Experiment 1; that is,  $a + 3b = 1.3$ . The policy parameters that satisfy these two restrictions give us  $w_g(k) = 0.748 + 0.184k$ . The effects of this policy are dramatic: the number of children with single mothers and the income gap between single mothers and married couples widen significantly. Indeed, the average number of children with young single mothers jumps to about 31% in the model, in

spending in married-couple households and 28% of spending in single-mother households to children. We arrive at these allocation rules from our economies-of-scale parameters. Married couples have 1.8 children in the data; hence, weighting children as 0.4 adults, children consume  $(1.8)(0.4)/(2 + (1.8)(0.4)) = 0.20$  of household consumption. For single mothers, who have 1.6 children in the data, we get  $(1.6)(0.4)/(1 + (1.6)(0.4)) = 0.28$ . By dividing total consumption of children by total consumption (which includes additional items such as spending on alcohol and tobacco), we find the fraction of total spending on children.

TABLE 8a  
Welfare experiments

	Married parents				Single parents			
	Period 0		Period 1		Period 0		Period 1	
	Model	US	Model	U.S.	Model	U.S.	Model	U.S.
<i>Benchmark Economy</i>								
% of Kids	82.90	76.88	84.77	74.68	17.10	23.12	15.23	25.32
Fam. Earnings*	1.00	1.00	1.00	1.15	0.22	0.24	0.27	0.37
<i>Expt. 1: restriction to single mothers</i>								
% of Kids	82.97	76.88	85.40	74.68	17.03	23.12	14.60	25.32
Fam. Earnings*	1.00	1.00	1.00	1.15	0.22	0.24	0.27	0.37
<i>Expt. 2: Expt. 1 + fertility bonus</i>								
% of Kids	69.03	76.88	76.65	74.68	30.97	23.12	23.35	25.32
Fam. Earnings*	1.00	1.00	0.98	1.15	0.19	0.24	0.23	0.37
<i>Expt. 3: Expt. 2 + lower base payment</i>								
% of Kids	77.26	76.88	83.48	74.68	22.74	23.12	16.52	25.32
Fam. Earnings*	1.00	1.00	0.99	1.15	0.21	0.24	0.28	0.37

\*Earnings data are based on table 1. The numbers are normalized to total family earnings of married couples

contrast to 23% for the U.S. economy. Income inequality also worsens. A young single mother now has about 19% of the income of a married couple (instead of 22%). The same statistic for an old single mother is 23% (instead of 27%).

The U.S. welfare payments, however, are not as generous as the Canadian ones. In Experiment 3, we reduce the welfare payments to reflect the average AFDC and food stamps payments in the U.S. In particular, we set  $w_g(k) = 0.575 + 0.141k$ . With this policy rule, a single mother with three children receives about 10% of the average income in the economy as welfare payments. Furthermore, this welfare rule still delivers the fact that having children increases the income of a single mother (with no additional earnings) by about 25% that is,  $(a + b)/a = 1.25$ . In this final experiment the average number of children with young single mothers is about 22.7%, a number very close to 23.12% for the U.S. economy. However, the model creates fewer single mothers for the second period than the U.S. economy. The fraction of children with an old single mother is about 16.52% in the model, whereas it is 25.32% in the U.S. The results suggest that welfare system plays a significant role in accounting for the differences in single motherhood between the U.S. and Canada.

In order to illuminate the role of incentives on marital decisions, table 8b repeats the analysis of table 8a while forcing all marital decisions to be the same as in the benchmark economy.<sup>17</sup> A comparison between the tables shows that

17 For this experiment we change the parameter values but assume that marriages are determined exogenously according to indicator functions,  $I_1$ ,  $I_2^*$ , and  $I_2^*$  from the benchmark economy. Given these marriage rules, agents make all other decisions optimally.

TABLE 8b  
Welfare experiments (with benchmark marriage decisions)

	Married parents				Single parents			
	Period 0		Period 1		Period 0		Period 1	
	Model	U.S.	Model	U.S.	Model	U.S.	Model	U.S.
<i>Benchmark Economy</i>								
% of Kids	82.90	76.88	84.77	74.68	17.10	23.12	15.23	25.32
Fam. Earnings*	1.00	1.00	1.00	1.15	0.22	0.24	0.27	0.37
<i>Expt. 1: restriction to single mothers</i>								
% of Kids	83.32	76.88	85.42	74.68	16.68	23.12	14.58	25.32
Fam. Earnings*	1.00	1.00	1.00	1.15	0.22	0.24	0.27	0.37
<i>Expt. 2: Expt. 1 + fertility bonus</i>								
% of Kids	73.31	76.88	77.66	74.68	26.69	30.97	22.34	23.35
Fam. Earnings*	1.00	1.00	0.98	1.15	0.22	0.24	0.24	0.37
<i>Expt. 3: Expt. 2 + lower base payment</i>								
% of Kids	78.99	76.88	80.02	74.68	21.01	23.12	19.98	25.32
Fam. Earnings*	1.00	1.00	0.99	1.15	0.21	0.24	0.24	0.37

\*Earnings data are based on table 1. The numbers are normalized to total family earnings of married couples

Experiment 1 did not affect the fraction of children with single mothers, because there were two opposite forces in play: non-marital fertility declines slightly, but this decline is compensated for by higher degree of single motherhood. As a result, in table 8b, with the benchmark marital decisions, the fertility effect dominates and we end up with a smaller fraction of children living with single mothers.

For Experiments 2 and 3, table 8b shows that both marital and fertility effects move in the same direction. Non-marital fertility increases from 2.66 to 3.7 with Experiment 2 and from 2.66 to 3.4 with Experiment 3. Table 8b also shows that the fertility decision plays the major role in these experiments. Even with marriage decision fixed, Experiment 2 in table 8b still increases the fraction of children with young single mothers by about 70% of the total increase in table 8a. Similarly, Experiment 3 in table 8b results in about 21.01% of children living with young single mothers compared with 22.7% in table 8a; again, about 70% of the total rise takes place even if we hold the marriage decisions fixed. In Experiments 2 and 3, per child human capital in single-mother households increases. With experiment 2, for example, single mothers on average make about 5% more human-capital investment in their children than they do in the benchmark economy. This positive effect is, however, more than compensated for by the rise in the number of children living in single-mother households, an issue we address in the next section.

These results imply that the distortions induced by targeting are largely due to the effect on the fertility margin, while the marriage margin accounts for about 30%. Experiments 2 and 3 have a direct effect on fertility, as welfare payments depend directly on the number of children. The effect on marriages, however, is less

TABLE 9  
Human capital investment in children

	Mean	Household income quantile				
		1st	2nd	3d	4th	5th
Benchmark	100.00	100	100	100	100	100
Expt. 1: restriction to single mothers	100.84	101.94	100.09	100.23	100.59	100.52
Expt. 2: Expt. 1 + fertility bonus	94.15	75.80	87.95	94.67	95.09	95.15
Expt. 3: Expt. 2 + lower base payment	98.12	92.02	98.47	99.24	99.00	98.62

TABLE 10  
Income inequality

	Married/ Single	5th quantile/ 1st quantile
Benchmark	3.787	3.873
Expt. 1: restriction to single mothers	3.755	3.798
Expt. 2: Expt. 1 + fertility bonus	4.088	5.312
Expt. 3: Expt. 2 + lower base payment	3.763	4.194

pronounced, since, while being a single female is more attractive with Experiments 2 and 3, males (who lose their welfare benefits with these experiments) are more eager to enter marriages.

### 6.1. Effectiveness of social policy

In this section we revisit the economies studied in the previous two sections, in order to find out which social policies are most effective in making poor children better off and reducing inequality. In table 9, we show the average education level of children by their percentile rank in the income distribution (with benchmark values normalized to 100). What is striking in these results is that the Canadian policy is much more effective than Experiment 3, which is the policy that most closely resembles AFDC. Parents in all five income quantiles invest more in children under the Canadian policy than under the AFDC-like policy (those in the lowest income quantile invest about 8% more, while the ones in the highest quantile invest about 2% more). Furthermore, most of the disadvantage of AFDC comes from the subsidy to fertility (Experiment 2). The restriction of welfare to unmarried women does not have much impact on children's education.

Table 10 shows that the implications for income inequality are also in line with the results in table 8a. Experiment 1, the restriction of transfers to the unmarried, minimizes the ratio of mean income in the highest-income quantile to the mean income in lowest-income quantile. It also minimizes the income gap between married households and single-mother households. The policy that maximizes

TABLE 11a  
Utility distribution – females

Economy	Household income quantile				
	1st	2nd	3d	4th	5th
Benchmark	100	100	100	100	100
Expt. 1: restriction to single mothers	99.959	100.099	100.108	100.114	100.082
Expt. 2: Expt. 1 + fertility bonus	99.068	99.053	98.825	98.871	98.993
Expt. 3: Expt. 2 + lower base payment	99.074	99.313	99.398	99.501	99.644

TABLE 11b  
Utility distribution – males

Economy	Household income quantile				
	1st	2nd	3d	4th	5th
Benchmark	100	100	100	100	100
Expt. 1: restriction to single mothers	99.975	100.232	100.193	100.210	100.173
Expt. 2: Expt. 1 + fertility bonus	94.553	97.012	97.979	98.216	98.235
Expt. 3: Expt. 2 + lower base payment	98.437	99.619	99.660	99.570	99.571

inequality is Experiment 2, which is a generous version of the AFDC policy, with rewards for extra fertility.

### 6.2. Preferred policy

The tax rate implied by the Canadian policy is about 28% higher than the tax required to pay for Experiment 3. Thus, if inequality or children's education is the predominant concern of social policy, then it is clear that the Canadian policy is better suited than the U.S. policy to address this. However, it may be that the average income under the U.S. policy is sufficiently higher to outweigh this advantage.

In tables 11a and 11b, we show the relation between the percentile rank of the household and expected utilities in the steady-state economies under different policies.<sup>18</sup> These rankings (again with benchmark values normalized to 100) are the same for men and women and show that poorest households are best off under the Canadian (benchmark) policy, while richer quantiles have the highest utility under the policy that excludes married people from welfare (Experiment 1). The other policies are never the second choice of these households; in

<sup>18</sup> The numbers in table 11 are calculated as the expected lifetime utility of an agent who is randomly thrown into the model economy.

particular, Experiment 3, which most resembles the former U.S. policy, is ranked third by all households. The fact that the majority of people prefer a welfare policy that targets single females reflects the cost effectiveness of these policies in helping children raised in single-mother families. Agents in this economy are better off when these children receive better education, since educational investments determines the steady-state productivity distributions. While agents prefer to be in an economy where single mothers receive welfare, they do not want to make these welfare payments dependent on the number of children, since this results in higher fertility and makes the welfare payments per child much less effective.

## **7. Conclusion**

In this paper we asked to what extent the higher rate of single-parent children and larger income inequality in the U.S. were long-run responses to the differences in the social-transfer programs in the two countries. Our basic hypothesis is that single-motherhood and long-run poverty are connected by human capital investment in children and that both are affected by the incentives implicit in the welfare policy.

We constructed an equilibrium model of the interaction between family structure and social policy. The basic premise is not only that family structure decisions are dependent on the human capital of the parents, but that in turn they help to determine the human capital of the children. In the model, marriage and divorce decisions depend on the outside options of both partners, which in turn depend on the decisions of all other adults, because these determine the probability distribution of potential spouses.

We calibrated the steady-state equilibrium of this model to the Canadian economy in the mid-1990s. The parameters of the calibrated model were chosen so as to match the following features of the data: the distribution of children across dual and single-parent households, the earnings differential between single and dual parents, average fertility, and a pattern of lower fertility for higher-income households. The social policy was set to resemble an 'average' Canadian welfare policy, according to empirical models of transfer income estimated on household survey data for 1994. A similar procedure was used for U.S. data to define a 'U.S.-style' welfare policy.

The main result of this paper is that when the social policy in our benchmark economy is replaced by the U.S.-style policy, almost all of the difference between the two countries in the fraction of children 0–8 years old in single-parent families is explained by this change in social policy alone.

Our results also suggest that the Canadian policy is more effective than the AFDC-style policy in helping poor children and in reducing the level of income inequality among households. The U.S. policy, on the other hand, is less costly and results in higher average income. Nevertheless, in terms of ex ante utility,

all households in our model economy prefer to be born into an economy with the Canadian policy than the AFDC-style policy. Interestingly, while for the poorest households the Canadian policy is the most preferred one, the majority of households prefer an in-between policy: one that targets single mothers but does not provides fertility bonuses.<sup>19</sup>

### Appendix: Stationary equilibrium

Let us denote an old single mother's level of human capital investment in her children in problem (Pg2) by  $e = E_2^s(x, k, z)$  and an old married couple's level of human capital investment per child in problem (Pm2) by  $e = E_2^m(x, z, \gamma, k)$ . Similarly, let  $k = K^s(x)$  be the fertility decision and  $e = E_1^s(x)$  be the education decision of a young single female in problem (P1g), and let  $k = K^m(x)$  be the fertility decision and  $e = E_1^m(x, z, \gamma)$  be the education decision of young married couple in problem (P1m). Then, the average number of children per female in this economy is given by

$$\mathbf{k} = \sum_{i=1}^N \sum_{j=1}^N \sum_{h=1}^M \Phi_1(x_i) \Omega_1(z_j) \Gamma(\gamma_h) I_1^s(x_i, z_j, \gamma_h) K^m(x_i, z_j, \gamma_h) \\ + \sum_{i=1}^N \Phi_1(x_i) \left[ 1 - \sum_{j=1}^N \sum_{h=1}^M \Omega_1(z_j) \Gamma(\gamma_h) I_1^s(x_i, z_j, \gamma_h) \right] K^s(x_i).$$

To understand this formula, note that the probability of a type- $(x_i, z_j, \gamma_h)$  marriage between young adults is  $\Phi_1(x_i) \Omega_1(z_j) \Gamma(\gamma_h) I_1^s(x_i, z_j, \gamma_h)$ . This match will generate  $K^m(x_i, z_j, \gamma_h)$  children. The odds that a woman will be type  $x_i$  and remain single are  $\Phi_1(x_i) [1 - \sum_{j=1}^N \sum_{h=1}^M \Omega_1(z_j) \Gamma(\gamma_h) I_1^s(x_i, z_j, \gamma_h)]$ . This woman will have  $K^s(x_i)$  children. In a stationary equilibrium the growth rate of the population,  $g$ , will therefore be  $g = \sqrt{k/2}$ .

#### A.1. Steady-state matching probabilities

*Young Adults:* The probabilities of meeting a young female and male of a given type in the marriage market are  $\Phi_1(x)$  and  $\Omega_1(z)$ . To determine these probabilities, let  $\Upsilon^{mm}(x_i, z_j, \gamma_h, x_k, z_l, \gamma_n)$  represent the fraction of females who were married in both periods and transited from state  $(x_i, z_j, \gamma_h)$  to  $(x_k, z_l, \gamma_n)$ . Likewise, let  $\Upsilon^{ss}(x_i, x_k)$  denote the fraction of females who were single in both periods and transited from  $x_i$  to  $x_k$ , and  $\Upsilon^{ms}(x_i, z_j, \gamma_h, x_k, z_l)$  denote the fraction of females

19 Although the emphasis of the analysis has been on differences in welfare policy, it is worth noting that the model is also amenable to the analysis of other types of policy that affect or respond to family structure, such as alimony, child-support and other divorce-contingent transfers.

who suffered a marriage breakup, and so forth. Hence,

$$\begin{aligned}
 \Upsilon^{mm}(x_i, z_j, \gamma_h, x_k, z_l, \gamma_n) &\equiv \Phi_1(x_i) \Omega_1(z_j) \Gamma(\gamma_h) I_1^s(x_i, z_j, \gamma_h) \\
 &\quad \times I_2^m(x_k, z_l, \gamma_n, k^m) \Lambda(\gamma_n | \gamma_h) \Delta^x(x_k | x_i) \Delta^z(z_l | z_j), \\
 \Upsilon^{ss}(x_i, x_k) &\equiv \Phi_1(x_i) \left[ 1 - \sum_{j=1}^N \sum_{h=1}^M \Gamma(\gamma_h) \Omega_1(z_j) I_1^s(x_i, z_j, \gamma_h) \right] \\
 &\quad \times \Delta^x(x_k | x_i) \left[ 1 - \sum_{l=1}^N \sum_{n=1}^M \Gamma(\gamma_n) I_2^s(x_k, z_l, \gamma_n, k^s) \Omega_2(z_l) \right], \\
 \Upsilon^{ms}(x_i, z_j, \gamma_h, x_k, z_l) &\equiv \Phi_1(x_i) \Omega_1(z_j) \Gamma(\gamma_h) I_1^s(x_i, z_j, \gamma_h) \Delta^x(x_k | x_i) \Delta^z(z_l | z_j) \\
 &\quad \times \left\{ \sum_{n=1}^m \Lambda(\gamma_n | \gamma_h) [1 - I_2^m(x_k, z_l, \gamma_n, k^m)] \right\}, \\
 \Upsilon^{sm}(x_i, x_k, z_l, \gamma_n) &\equiv \Phi_1(x_i) \left[ 1 - \sum_{j=1}^S \sum_{h=1}^m \Gamma(\gamma_h) \Omega_1(z_j) I_1^s(x_i, z_j, \gamma_h) \right] \\
 &\quad \times I_2^s(x_k, z_l, \gamma_n, k^s) \Gamma(\gamma_n) \Delta^x(x_k | x_i) \Omega_2(z_l), \tag{A1}
 \end{aligned}$$

where  $k^m \equiv K^m(x_i, z_j, \gamma_h)$  and  $k^s \equiv K^s(x_i)$ .

Then, it is easy to see that the odds of meeting a young woman of type  $x_r$  in the marriage market are given by

$$\begin{aligned}
 \Phi_1(x_r) &= \left\{ \sum_{i,j,k,l,h,n} \Pi^x(x_r | E_1^m(x_i, z_j, \gamma_h) + E_2^m(x_k, z_l, \gamma_n, K^m(x_i, z_j, \gamma_h))) \right. \\
 &\quad \times \Upsilon^{mm}(x_i, z_j, \gamma_h, x_k, z_l, \gamma_n) K^m(x_i, z_j, \gamma_h) \\
 &\quad + \sum_{i,k} \Pi^x(x_r | E_1^s(x_i) + E_2^s(x_k, K^s(x_i), 0)) \Upsilon^{ss}(x_i, x_k) K^s(x_i) \\
 &\quad + \sum_{i,j,k,l,h} \Pi^x(x_r | E_1^m(x_i, z_j, \gamma_h) + E_2^s(x_k, K^m(x_i, z_j, \gamma_h), z_l)) \\
 &\quad \times \Upsilon^{ms}(x_i, z_j, \gamma_h, x_k, z_l) K^m(x_i, z_j, \gamma_h) \\
 &\quad + \sum_{i,k,l,n} \Pi^x(x_r | E_1^s(x_i) + E_2^m(x_k, z_l, \gamma_n, K^s(x_i))) \\
 &\quad \left. \times \Upsilon^{sm}(x_i, x_k, z_l, \gamma_n) K^s(x_i) \right\} / \mathbf{k}. \tag{A2}
 \end{aligned}$$

The probability of meeting a type- $z_r$  young man is determined analogously:

$$\begin{aligned} \Omega_1(z_r) = & \left\{ \sum_{i,j,k,l,h,n} \Pi^z(z_r | E_1^m(x_i, z_j, \gamma_h) + E_2^m(x_k, z_l, \gamma_n, K^m(x_i, z_j, \gamma_h))) \right. \\ & \times \Upsilon^{mm}(x_i, z_j, \gamma_h, x_k, z_l, \gamma_n) K^m(x_i, z_j, \gamma_h) \\ & + \sum_{i,k} \Pi^x(z_r | E_1^s(x_i) + E_2^s(x_k, K^s(x_i), 0)) \Upsilon^{ss}(x_i, x_k) K^s(x_i) \\ & + \sum_{i,j,k,l,h} \Pi^z(z_r | E_1^m(x_i, z_j, \gamma_h) + E_2^s(x_k, K^m(x_i, z_j, \gamma_h), z_l)) \\ & \times \Upsilon^{ms}(x_i, z_j, \gamma_h, x_k, z_l) K^m(x_i, z_j, \gamma_h) \\ & + \sum_{i,k,l,n} \Pi^z(z_r | E_1^s(x_i) + E_2^m(x_k, z_l, \gamma_n, K^s(x_i))) \\ & \left. \times \Upsilon^{sm}(x_i, x_k, z_l, \gamma_n) K^s(x_i) \right\} / \mathbf{k}. \end{aligned}$$

*Old Adults:* Next, how are the odds of meeting a single age-2 type- $x$  female with  $k$  children,  $\Phi_2(x, k)$ , or of a single age-2 type- $z$  male,  $\Omega_2(z)$  determined in stationary equilibrium? This depends upon the number of single agents who remain unmarried from the previous period. So, how many are there? Again, the number of married and single one-period-old type- $x_i$  females is given by  $\Phi_1(x_i) \sum_{j=1}^N \sum_{h=1}^M \Omega_1(z_j) \Gamma(\gamma_h) I_1^s(x_i, z_j, \gamma_h)$  and  $\Phi_1(x_i) [1 - \sum_{j=1}^N \sum_{h=1}^M \Omega_1(z_j) \Gamma(\gamma_h) I_1^s(x_i, z_j, \gamma_h)]$ . Given this supply of one-period-old single females, the quantity of two-period-old type  $x_k$  single females will be  $\sum_{i=1}^N \Delta^x(x_k | x_i) \Phi_1(x_i) [1 - \sum_{j=1}^N \sum_{h=1}^M \Omega_1(z_j) \Gamma(\gamma_h) I_1^s(x_i, z_j, \gamma_h)]$ .

Let

$$\mathfrak{N}(x_i, k) = \begin{cases} 1, & \text{if } K^s(x_i) = k \\ 0, & \text{otherwise} \end{cases}$$

be an indicator function representing the number of children that a single one-period-old female of type  $x_i$  has. Then, the odds of drawing a single two-periods-old type- $x_k$  female with  $k$  children in the marriage market, or  $\Phi_2(x_k, k)$ , will be given by

$$\begin{aligned} \Phi_2(x_k, k) &= \frac{\sum_{i=1}^N \mathfrak{N}(x_i, k) \Delta^x(x_k | x_i) \Phi_1(x_i) \left[ 1 - \sum_{j=1}^N \sum_{h=1}^M \Gamma(\gamma_h) \Omega_1(z_j) I_1^s(x_i, z_j, \gamma_h) \right]}{\sum_{k=1}^N \sum_{i=1}^S \Delta^x(x_k | x_i) \Phi_1(x_i) \left[ 1 - \sum_{j=1}^N \sum_{h=1}^M \Gamma(\gamma_h) \Omega_1(z_j) I_1^s(x_i, z_j, \gamma_h) \right]}. \end{aligned}$$

The same formula for the odds of meeting a single two-period-old male of type  $z_l$ , or for  $\Omega_2(z_l)$ , reads

$$\Omega_2(z_l) = \frac{\sum_{j=1}^N Z(z_l | z_j) \Omega_1(z_j) \left[ 1 - \sum_{i=1}^N \sum_{h=1}^M \Gamma(\gamma_h) \Phi_1(x_i) I_1^s(x_i, z_j, \gamma_h) \right]}{\sum_{i=1}^N \sum_{j=1}^N Z(z_l | z_j) \Omega_1(z_j) \left[ 1 - \sum_{i=1}^N \sum_{h=1}^M \Gamma(\gamma_h) \Phi_1(x_i) I_1^s(x_i, z_j, \gamma_h) \right]}. \quad (\text{A3})$$

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