# Distortions, Endogenous Managerial Skills and Productivity Differences

Dhritiman Bhattacharya, Nezih Guner, and Gustavo Ventura[*,†]

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**Abstract**

We develop a span-of-control model where managerial skills are endogenous and the outcome of investments over the life cycle of managers. We calibrate this model to U.S plant-size data to quantify the effects of distortions that are correlated with the size of production units. These distortions lead to sharp reductions in plant productivity and the fraction of employment in large plants, with a quantitatively important role for managerial investments. We find that the model can account quite well for properties of Japanese size-distribution data, with a model-implied TFP of about 83% of the U.S. Distortions are critical in accounting for the differences in size distribution between the U.S. and Japan.

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[*]Bhattacharya, EQUIFAX (email: Dhritiman.Bhattacharya@ixicorp.com); Guner, ICREA-MOVE, Universitat Autonoma de Barcelona, Barcelona GSE, CEPR and IZA (email: nezih.guner@movebarcelona.eu); Ventura, Department of Economics, Arizona State University, USA (email: Gustavo.Ventura@asu.edu).

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# 1 Introduction

Why are some countries so much richer than others? From development accounting exercises, the answer depends heavily on Total Factor Productivity (TFP) differences across countries (Klenow and Rodriguez-Clare, 1997; Hall and Jones, 1999; Caselli, 2005). Consequently, much work in the last decade or so has been devoted to understanding the determinants of these measured TFP differences. As part of this effort, a growing body of recent literature (e.g. Banerjee and Duflo (2005), Bartelsman, Haltiwanger and Scarpetta (2008), Buera, Kabowski and Shin (2010), Caselli and Gennaioli (2005), Guner, Ventura and Xu (2008), Hsieh and Klenow (2009), Restuccia and Rogerson (2008)) studies the consequences of distortions that alter the efficient allocation of resources across production units, and show the extent to which these distortions can have large effects on aggregate productivity.

The exact nature of such distortions is the object of ongoing work. While some (e.g. Caselli and Gennaioli (2005) and Buera et al (2010)) focus on imperfections in contract design or financial frictions as sources of misallocation, others (e.g. Hsieh and Klenow (2009)) recover the underlying distortions from observed allocations. Hsieh and Klenow (2009), as well as Bertelsman et al (2008), follow Rogerson and Retuccia (2008) and model distortions as firm or plant-specific. Guner, Ventura and Xu (2008) focus on a particular form of these distortions: policies that only affect production units above or below a certain size (size-dependent policies), and study the implications of such distortions on aggregate outcomes.

There is little doubt that the distortions that alter the efficient allocation of resources across productive units are likely to lead to misallocation of managerial talent. Managerial talent, however, can arguably be endogenous, and as such, it is a form of human capital that agents can choose to invest on. In an environment with no distortions, intrinsically more able individuals are likely to spend resources to improve their managerial human capital in order to operate larger units. In a distorted environment, in contrast, there could be little incentives for managers to undertake costly investments. If the distortions are plant or firm specific, the returns to such investments would vary across managers and this is likely to generate a lower overall quality of managerial talent. If distortions are size dependent, relatively more able managers would reduce their skill investments, as the return to operate larger production units becomes lower. In sum, in order to evaluate how distortions and misallocation affect the aggregate economy, a natural next step is to understand and quantify the interplay between agents' incentives to accumulate managerial skills, distortions and the resulting effects on output and productivity. This is the central objective of the current paper.

We develop a span-of-control model with a life-cycle structure. Every period, a large number of finitely lived agents are born. These agents are heterogeneous in terms of their initial endowment of managerial skills. The objective of each agent is to maximize lifetime utility from consumption. In the first period of their lives, agents make an irreversible decision to be either workers or managers. If an agent chooses to be a worker, her managerial skills are of no use and she earns the market wage in every period until retirement. If an agent chooses to be a manager, she can use her managerial skills to operate a plant by employing labor and capital to produce output and collect the net proceeds (after paying labor and capital) as managerial income. Moreover, managers invest resources in skill formation, and as a result managerial skills grow over the life cycle. This implies that a manager can grow the size of her production operation and managerial income by investing a part of her current income each period in skill formation. As managers age and accumulate managerial skills, the distribution of skills (and productivity of production units) evolves endogenously. At the same time, the model delivers predictions on the growth of managerial earnings over the life-cycle as well as the share of GDP that is invested in improving skills.

In this model, the evolution of managerial skills and hence plant size will depend not only on initially endowed skills, but also on skill investment decisions. These investment decisions will reflect the costs (resources that have to be spent rather than being consumed) and the benefits (the future awards associated to being endowed with better managerial skills). A central assumption in our model is that there are complementarities between skills and investments: managers born with high skills find it optimal to invest more in skills over their lifetime than managers born with low skills. This model property amplifies initial heterogeneity in skills, and leads to increasing dispersion with age in the size of production plants that managers can operate.

We subsequently introduce *idiosyncratic* distortions in the model. At the start of life, all agents draw an output tax that applies if they become managers. They draw this tax from a distribution that is conditioned on their endowment of managerial ability: a higher initial managerial ability implies that a higher tax is more likely. This results in distortions that are positively *correlated* with the size of production units. In this context, we ask: What are the quantitative implications of these distortions for output, productivity and the size of establishments? How large should distortions be to generate given reductions in average plant size? What is the role of managerial investments in assessing the effects of distortions on resource allocation? What is the interplay between distortions and variation in exogenous, common-to-all establishments, productivity for output, productivity and plant size?

We calibrate the model to match macroeconomic statistics as well as cross sectional features of U.S. plant data. We assume for these purposes that the U.S. economy is relatively free of the distortions that we focus on. We find that the model can capture central features of the U.S. plant size distribution, including the upper and lower tails, something difficult to generate within a standard span-of-control model. This is critical; on one hand, the upper tail of the size distribution accounts for the bulk of the employment and output in the economy. On the other hand, the lower tail of the size distribution accounts for the bulk of the plants in the economy.

We subsequently find that size-dependent distortions lead to substantial effects on output and notions of productivity. Introducing distortions that reduce mean size by 30% (15%) relative to the benchmark economy lead to a reduction in aggregate output of about 8.8% (3.7%) and output per establishment by about 35.0% (16.6%). Such a reduction leads to a drastic drop in the share of employment in large plants (100 workers or more). The employment share of large plants drops from about 47.3% in the undistorted benchmark to about 31.1% (38.3%) in the distorted case. We find that the contribution of skill investments is substantial: in the absence of skill investments, the structure of distortions that lead to a reduction in mean size of 30% under the benchmark calibration, generates a reduction in mean size of only 19.4%. Our results also indicate non-trivial effects on the size distribution of establishments stemming from variation in exogenous aggregate productivity (common to all establishments). Reducing exogenous productivity by 50% (25%) leads to a reduction in mean establishment size of about 27.9% (14.2%), and a reduction in the share of employment at large units; from 47.3% in the benchmark case to 28.9% (39.1%).

We finally use the model to assess the combined effects of distortions and exogenous variation in economy-wide productivity. For these purposes, we force the model economy to reproduce Japanese data via idiosyncratic distortions and exogenous productivity variation. We find that our model can account for properties of the Japanese size distribution very well. Our model implies an endogenous level of TFP for Japan that is only 83% of the U.S. level. We find that variation in exogenous productivity alone accounts for only 49%, 60% and 11% of the differences in output, TFP and mean size, respectively; the reminder is due to distortions and the interaction between distortions and productivity differences.

The paper is organized as follows. Section 2 presents the model and the modeling of idiosyncratic distortions. Section 3 discusses the calibration of the benchmark model. Section 4 presents the findings associated to the introduction of distortions and exogenous changes in economy-wide productivity. Section 5 provides a quantification of the importance of skill investments. Section 6 illustrates the quantitative implications of the model when applied to Japan. Finally, section 7 concludes.

# 2 Model

Consider the following life-cycle version of Lucas (1978) span-of-control model. Each period, an overlapping generation of heterogeneous agents are born into economy that lives for $J$ periods. The objective of each agent is to maximize the present value of lifetime utility from consumption

$$\sum_{j=1}^{J} \beta^{j-1} \log(c_j), \tag{1}$$

where $\beta \in (0,1)$ and $c_j$ is the consumption of an age-$j$ agent.

Each agent is born with an initial endowment of managerial ability. We denote managerial ability by $z$, and assume that initial (age-1) abilities are drawn from an exogenous distribution with cdf $F(z)$ and density $f(z)$. Until retirement age $J_R$, each agent is also endowed with one unit of time which she supplies in-elastically as a manager or as a worker. In the very first period of their lives, agents must choose either to be a *worker* or a *manager*. This decision is irreversible. A worker inelastically supplies her endowed labor time to earn the market wage every period until retirement. The decision problem of a worker is to choose how much to consume and save every period.

A manager's problem, however, is more complicated. A manager has access to a technology to produce output, which requires managerial ability plus capital and labor services. Hence, given factor prices, she decides how much labor and capital to employ every period. In addition, in every period, a manager decides how much of his net income to allocate towards current consumption, savings and investments in improving her/his managerial skills.

We assume that each cohort is $1 + n$ bigger than the previous one. These demographic patterns are stationary so that age $j$ agents are a fraction $\mu_j$ of the population at any point in time. The weights are normalized to add up to one, and obey the recursion, $\mu_{j+1} = \mu_j/(1+n)$.

There is a government that taxes labor income of workers and incomes (profits) of managers at a proportional rate $\tau$. We assume that tax revenue is used to finance government consumption.

**Technology** Each manager has access to a Lucas (1978) span-of-control technology. A plant comprises of a manager with ability $z$ along with labor and capital,

$$y = Az^{1-\gamma} \left(k^{\alpha} n^{1-\alpha}\right)^{\gamma},$$

where $\gamma$ is the span-of-control parameter and $\alpha\gamma$ is the share of capital. The term $A$ is productivity term that is common to all establishments.[1] Every manager can enhance her future skills by investing current income in skill accumulation. The law of motion for managerial skills is given by

$$z' = z + g(z, x) = z + z^{\theta_1} x^{\theta_2},$$

where $z'$ is next period's ability and $x$ denotes investment in skill accumulation. The skill accumulation technology described above satisfies three important properties, of which the first two follow from the functional form and the last one is an assumption. First, the technology shows *complementarities* between current ability and investments in next period's ability; i.e. $g_{zx} > 0$. Second, $g(z, 0) = 0$. That is, investments are essential to increase the stock of managerial skills. Finally, there are diminishing returns to skill investments: $g_{xx} < 0$. This naturally requires $\theta_2 < 1$.

## 2.1 Decisions

Consider a stationary environment with a constant factor prices $R$ and $w$. Let $a$ denote assets that pay the risk-free rate of return $r = R - \delta$.

[1] In referring to production units, we use the terms *establishment* and *plant* interchangeably.

**Managers** The problem of a manager of age $j$ is given by

$$V_j(z,a) = \max_{x,a'}\{\log(c) + \beta V_{j+1}(z',a')\} \tag{2}$$

subject to

$$c + x + a' = (1-\tau)\pi(z;r,w) + (1+r)a \qquad \forall 1 \leq j < J_R - 1, \tag{3}$$

$$c + a' = (1+r)a \qquad \forall j \geq J_R, \tag{4}$$

and

$$z' = z + g\left(z, x\right) \qquad \forall j < J_R - 1, \tag{5}$$

with

$$V_{J+1}(z,a) = \begin{cases} 0 \text{ if } a \geq 0 \\ -\infty, \text{ otherwise} \end{cases}.$$

Given her state $(z,a)$, a $j$-years old manager decides how much to save, $a'$, and how much to invest to enhance her skills. Up to the retirement age $J_R$, a manager's income consists of her after-tax managerial profits and her assets, while after age $J_R$ her only source of income is from her assets. We assume that agents (managers as well as workers) can lend or borrow at the interest rate $r$ as long as they do not die in debt.

Since there are no borrowing constraints, factor demands and per-period profits of a manager only depend on her ability $z$. Managerial income for a manager with ability $z$ is given by

$$\pi(z;r,w) \equiv \max_{n,k} \ \{Az^{1-\gamma}\left(k^{\alpha}n^{1-\alpha}\right)^{\gamma} - wn - (r+\delta)k\}.$$

Factor demands are given by

$$k(z;r,w) = (A(1-\alpha)\gamma)^{\frac{1}{1-\gamma}}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1-\gamma(1-\alpha)}{1-\gamma}}\left(\frac{1}{r+\delta}\right)^{\frac{1-\gamma(1-\alpha)}{1-\gamma}}\left(\frac{1}{w}\right)^{\frac{\gamma(1-\alpha)}{1-\gamma}} z, \tag{6}$$

and

$$n(z;r,w) = (A(1-\alpha)\gamma)^{\frac{1}{1-\gamma}}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{\alpha\gamma}{1-\gamma}}\left(\frac{1}{r+\delta}\right)^{\frac{\alpha\gamma}{1-\gamma}}\left(\frac{1}{w}\right)^{\frac{1-\alpha\gamma}{1-\gamma}} z. \tag{7}$$

Substituting these into the profit function, one can show that profits are a linear function of managerial ability, $z$

$$\pi(z;r,w) = A^{\frac{1}{1-\gamma}}\ \Omega\ \left(\frac{1}{r+\delta}\right)^{\frac{\alpha\gamma}{1-\gamma}} \left(\frac{1}{w}\right)^{\frac{\gamma(1-\alpha)}{1-\gamma}}\ z, \tag{8}$$

where $\Omega$ is a constant, given by

$$\Omega \equiv (1-\alpha)^{\frac{\gamma(1-\alpha)}{(1-\gamma)}}\ \alpha^{\frac{\gamma\alpha}{(1-\gamma)}}\ (1-\gamma)^{\frac{1}{1-\gamma}}. \tag{9}$$

The solution to the dynamic programming problem of a manager is characterized by two conditions. First, the solution for next-period assets, $a'$, is characterized by the standard Euler equation for asset accumulation

$$\frac{1}{C_j} = \beta(1+r)\frac{1}{C_{j+1}}. \tag{10}$$

Second, the optimality condition for $x$ and (10) imply the following no-arbitrage condition for investing in physical capital and skills

$$(1+r) = (1-\tau)\pi_z\left(z_j;r,w\right) g_x(z_j,x_j). \tag{11}$$

The left-hand side of the above equation is next period's gain in income from one unit of current savings. The manager can also use this one unit as an investment on her skills. Hence, the term $g_x(z_j,x_j)$ on the right-hand side is the additional skills generated next period from an additional unit of investment in the current period. The term $(1-\tau)\pi_z\left(r,w\right)$ is the additional profit generated from an additional unit of managerial skills, which is decreasing in $\tau$. Therefore, the right-hand side is the gain in utility by the $j$-period old manager from investing one unit of the current consumption good in skill accumulation. To get a unique interior optimum $g_{xx}$ must be negative, as assumed earlier. This implies that the marginal benefit of investing in skill accumulation is monotonically decreasing in the level of skill investment while the marginal cost, given by $(1+r)$, is constant.

Figure 1 illustrates the optimal decision for skill investments $x$ at a given age $j$. As the figure illustrates, a higher level of current (age $j$) managerial ability leads to higher skill investments as the result of complementarities built into the production of new managerial skills. Since this occurs at all ages, given prices, initial heterogeneity in skills is *magnified* by investments in skill acquisition.

The manager's problem generates decision rules for savings $a' = a_j^m(z, a)$, investment in managerial skills, $x = x_j\ (z, a)$, as well as the associated factor demands given by $k = k(z; r, w)$ and $n = n(z; r, w)$.

**Workers** The problem of an age-$j$ worker is simpler and is given by

$$W_j(a) = \max_{a'}\{\log(c) + \beta W_{j+1}(a')\}$$

subject to

$$c + a' = (1 - \tau)w + (1 + r)a \qquad \forall 1 \leq j < J_R - 1 \tag{12}$$

and

$$c + a' = (1 + r)a \qquad \forall j \geq J_R, \tag{13}$$

with

$$W_{J+1}(a) = \begin{cases} 0 \text{ if } a \geq 0 \\ -\infty, \text{ otherwise} \end{cases}.$$

Let the associated savings decision of a worker be $a' = a_j^w(a)$. Like managers, workers can borrow and lend without any constraint as long as they do not die with negative assets.

The objective of each agent born every period is to maximize lifetime utility by choosing to be a worker or a manager. Let $z^*$ be the ability level at which a 1-year old agent is indifferent between being a manager and a worker. This threshold level of $z$ is given by (as agents are born with no assets)

$$V_1(z^*, 0) = W_1(0). \tag{14}$$

Given all the assumptions made, $V_1$ is a continuous and a strictly increasing function of $z$. Therefore, (14) has a well-defined solution, $z^*$. Figure 2 depicts the solution.

## 2.2 Equilibrium

As we mentioned above, member of each new generation is endowed with managerial ability levels distributed with cdf $F(z)$ and density $f(z)$ on $[\underline{z}, z^{\max}]$. After the age-1, the distribution of managerial abilities is endogenous since it depends on investment decisions of managers over their life-cycle.

Let managerial abilities take values in set $Z = [\underline{z}, \overline{z}]$ with the endogenous upper bound $\overline{z}$. Similarly, let $A = [0, \overline{a}]$ denote the possible asset levels. Let $\psi_j(a,z)$ be the mass of age-$j$ agents with assets $a$ and skill level $z$. Given $\psi_j(a,z)$, let

$$\widetilde{f}_j(z) = \int \psi_j(a,z)da,$$

be the skill distribution for age-$j$ agents. Note that $\widetilde{f}_1(z) = f(z)$ by construction.

Since the decision to become a manager is irreversible, each period those agents whose ability is above $z^*$ work as managers, whereas the rest are workers. Then, in a stationary equilibrium with given prices, $(r,w)$, labor, capital and goods market must clear. The labor market equilibrium condition can be written as

$$\sum_{j=1}^{J_R-1} \mu_j \int_{z^*}^{\overline{z}} n(z;r,w)\widetilde{f}_j(z)dz = F(z^*) \sum_{i=1}^{J_R-1} \mu_j \tag{15}$$

where $\mu_j$ is the total mass of cohorts of age $j$. The left-hand side is the labor demand from $J_R - 1$ different cohorts of managers. A manager with ability level $z$ demands $n(z,r,w)$ units of labor and there are $\widetilde{f}_j(z)$ of these agents. The right-hand side is the fraction of each cohort employed as workers times the total mass of all non-retired cohorts in the economy.

In the capital market, the demand for savings is not only generated by managers renting physical capital. There is an additional demand for savings from managers borrowing funds from the capital market to invest in skill accumulation. The capital market equilibrium condition can be written as

$$\begin{aligned}\sum_{j=1}^{J_R-1} \mu_j \int_{z^*}^{\overline{z}} k(z;r,w)\widetilde{f}_j(z)dz &= \sum_{j=1}^{J-1} \mu_j \int_{z}^{z^*} a_j^w(a)\psi_j(z,a)dzda \\ &+ \sum_{j=1}^{J-1} \mu_j \int_{z^*}^{\overline{z}} \int_A a_j^m(z,a)\psi_j(z,a)dzda \\ &- \sum_{j=1}^{J_R-2} \mu_j \int_{z^*}^{\overline{z}} \int_A x_j(z,a)\psi_j(z,a)dzda \end{aligned} \tag{16}$$

The left-hand-side of the equation (16) above is the capital demand from $J_R - 1$ different cohorts of managers. The first two terms on the right-hand-side are the supply of savings from $J - 1$ different cohorts of managers and workers. The third term is the demand for skills investments from $J_R - 2$ different cohorts of managers (a manger will stop investing in his skills the period right before his retirement).

The goods market equilibrium condition requires that the sum of undepreciated capital stock and aggregate output produced in all plants in the economy is equal to the sum of aggregate consumption and savings across all cohorts, and skill investments by all managers across all cohorts. Finally, total tax collections must cover government consumption:

$$G = w\tau \ F(z^*) \sum_{j=1}^{J_R-1} \mu_j + \tau \sum_{j=1}^{J_R-1} \mu_j \int_{z^*}^{\overline{z}} \pi(z, r, w) \widetilde{f}_j(z) dz$$

where the first term is tax collections from workers, and the second term is tax collection from managers.

## 2.3 Idiosyncratic Distortions

Consider now the environment in which managers face distortions to operate production plants. We model these distortions as output taxes that are dependent on the ability level of the manager. As the size of production that a manager can operate is (strictly) increasing in her ability, on average, relatively larger plants will be more distorted than smaller ones. In this sense, distortions will be *correlated* with size.

At age 1, each potential manager with ability levels $z$ makes a draw of an idiosyncratic tax $\tau_d$ from a distribution $D(\tau_d)$, with support $[0,\tau_d^{\max}(z)]$. We assume in particular that $\tau_d^{\max}(z) = \kappa \frac{z}{z^{\max}}$, with $\kappa > 0$. Once a manager is attached to a particular tax (distortion) $\tau_d$, he/she obtains managerial rents $\pi(z; r, w, \tau_d)$ that obey

$$\pi(z; r, w, \tau_d) \equiv \max_{n,k} \ \{(1 - \tau_d) A z^{1-\gamma} \left(k^{\alpha} n^{1-\alpha}\right)^{\gamma} - wn - (r + \delta)k\}.$$

We assume further that $D(\tau_d)$ is a uniform distribution. Hence, the mean value of output taxes for a manager of type $z$ is given by

$$E(\tau_d|z) = \frac{\kappa}{2} \frac{z}{z^{\max}},$$

which is increasing in $z$.

We note that our formulation delivers plant-specific, correlated distortions in a simple, parsimonious way. If $z$ is low, $\kappa \frac{z}{z^{\max}}$ is small (for a given $\kappa$), and managers are more likely to draw a low tax (distortion) level. As $z$ increases and approaches $z^{\max}$, a manager is more likely to draw high levels of taxes. Note also that as taxes are stochastically assigned to managers and managers invest at different rates, our formulation allows for the coexistence of managers of similar managerial skills who face different distortions.

Our formulation implies that production taxes distort the choice of capital and labor hired, and thus reduce optimal size measured in either capital or labor used, but leave the capital-labor ratio unaltered. Since distortions affect managerial rents, they matter for skill accumulation. The key condition for skill accumulation is now

$$(1+r) = (1-\tau)\pi_z(z_j; r, w, \tau_d) g_x(z_j, x_j). \tag{17}$$

As managerial rents are decreasing in $\tau_d$, the marginal benefit from skill investment declines as $\tau_d$ increases. Therefore, all the same, higher levels of distortions lead to lower levels of managerial skills.

# 3 Parameter Values

We assume that the U.S. economy to be distortion free and calibrate the benchmark model parameters to match central aggregate and cross sectional features of the U.S. plant data. Before discussing the calibration strategy, it is worthwhile to emphasize important features of the U.S. plant size data collected from the 2004 U.S. Economic Census. The average size of a plant in the U.S. was about 17.9. The distribution of employment across plants is quite skewed. As many as 72.5% of plants in the economy employed less than 10 workers, but accounted for only 15% of the total employment. On the other hand, less than 2.7% of plants employed more than 100 employees but accounted for about 46% of total employment. These are key features of the data for our analysis of distortions that are correlated with the size of production units.

The exogenous skill distribution of new born agents in the model is assumed to follow a log normal distribution, with parameters $\mu_z$ and $\sigma_z$. We let the model period correspond to 10 years. Each cohort of agents enter the model at age 20 and live until they are 80 years old. Agents retire at age 60. Hence, in the model agents live for 6 model periods; 4 as workers or managers and 2 as retirees. There is a total of 9 parameters to calibrate, in conjunction with a tax rate, as listed in Table 1. The product of two of these parameters, importance of capital $(\alpha)$ and returns to scale $(\gamma)$, determine the share of capital in output. We determine the values of capital share in output and the depreciation rate from the data. A measure of capital consistent with the current model on business plants should include capital accounted for by the business sector. Similarly a measure of output consistent with our definition of capital should only include output accounted for by the business sector. The measure of capital and output discussed in Guner et al (2008) is consistent with the current plant size distribution model. Hence we adopt the value of capital output ratio, capital share and depreciation rate reported in that paper. These values are 2.325, 0.317, and 0.04 respectively. We choose the population growth rate in the model such that the annual population growth rate is 1.1%.

After calibrating the depreciation rate and the population growth rate, we have 7 more parameters to calibrate: importance of capital, returns to scale, discount factor, two parameters of the skill accumulation technology and the mean and variance of the skill distribution. Note that the capital share in the model is given by $\gamma\alpha$, and since this value has to be equal to 0.317, a calibrated value for $\gamma$ determines $\alpha$ as well. Hence we have indeed 6 parameters to determine: $\gamma, \beta, \theta_1, \theta_2$, $\mu_z$ and $\sigma_z$.

At the aggregate level, we want the benchmark model to replicate the capital output ratio in the U.S. economy. At the cross sectional level, the model implied distribution of plants should capture some of the important features of the U.S. plant size distribution discussed in the beginning of this section. We normalize the mean of the skill distribution to zero and jointly calibrate the 5 remaining parameters to match the following 5 moments of the U.S. plant size distribution: mean plant size, fraction of plants with less than 10 workers, fraction of plants with more than 100 workers, fraction of the labor force employed in plants with 100 or more employees, and the aggregate capital output ratio. These moments together with their model counterparts are given in Table 2. We also impose a tax rate $\tau$ equal to 26%, which results in tax collections as a fraction of output of about 18% in the benchmark economy.

The benchmark model is successful in replicating multiple features of the U.S. plant size distribution. The coefficient of variation of the plant size distribution implied by the skill accumulation model is 4.05 which is close to the corresponding value (3.98) in the data. Indeed, the model is able to replicate properties of the entire plant size distribution fairly well as illustrated in Figures 3 and 4. The success of the skill accumulation model in accounting for the tail of the plant-size distribution is important; as we argued earlier, the bulk of employment is *there.* Here, it is worth noting that the ability of the model to account for the top tail of the distribution of employment by size, is closely connected to the skill accumulation mechanism that we concentrate in this paper. As shown in Guner et al (2008), the standard span-of-control setup with a log-normal distribution of managerial ability cannot reproduce the concentration of employment in the upper tail.

**Skill Investments** In our calibration, the fraction of resources that are invested in skill accumulation is of about 1.8% of GDP in the benchmark economy. Viewed as an *intangible* investment, this is a relatively small fraction of available estimates for these type of investments. McGrattan and Prescott (2010) calculations, for instance, yield an investment rate in a broad notion of intangibles of about 10.8% of output.

Despite the relatively small fraction of resources devoted to the improvement of managerial skills, the incomes of managers grow significantly with age. A manager who is in his 40s (age 3) earns about 2.7 times as much as a manager who is in his 20s (age 1). How does this compare to U.S. data? For comparison purposes, we calculate the growth of managerial incomes over the life-cycle using the U.S. Census data, which provides large number of obsrevations even for narrowly-defined occupational categories. In particular, we use the 1980 U.S. Census to construct managerial incomes for ages 20-29, the 1990 Census for ages 30-39, and the 2000 Census for ages 40-49. The set of occupational categories for managers is a narrow one, in the spirit of the model (*one plant per manager*). It delivers an overall fraction of managers in the labor force of about 4.8-4.9%, which is close to the fraction of managers in the model economy (5.3%). From this data, we conclude that the model is in line with evidence. The data reveals that manager's incomes indeed grow significantly with age, as they grow by a factor of about 3.2, from ages 20-29 to ages 40-49.[2]

[2] Incomes are wage and salary income, and are deflated by the Consumer Price Index. The sample is restricted to those whose income is above half of minimum wage income, with at least 2000 yearly working hours. We used OCC 1990 classification with occupation codes 4 to 21 counted as managers. The set of occupations that are classified as managers include `Managers of service organizations`, `Managers and administrators`, `Financial managers`, `Human resources and labor relations managers`, `Managers and specialists in marketing and advertising, and public relations`, `Managers in education and related fields`, `Managers of medicine and health occupations`, `Postmasters and mail superintendents`, `Managers of food-serving and lodging establishments`, `Managers of properties and real estate`, and `Funeral directors`.

# 4 Results

In this section, we present and discuss the central quantitative findings of the paper. We first introduce distortions as described in section 2.3. Subsequently, we explore the implied responses of our model economy to variations in economy-wide productivity.

## 4.1 Effects of Idiosyncratic Distortions

We evaluate the effects of idiosyncratic distortions by changing the parameter $\kappa$, which governs the distribution of distortions. We vary $\kappa$ across steady states, so that mean plant size is reduced by 15% and 30% relative to the (undistorted) benchmark economy. Our exercises are *neutral* in terms of revenues: as $\kappa$ increases, we reduce the tax rate $\tau$ in order to keep the magnitude of resources extracted either by regular or idiosyncratic taxes the same.

Table 3 and 4 show the main findings. As Table 3 demonstrates, reducing mean size by 15% (30%) leads to a reduction in aggregate output of about 2.7% (8.8%), and to a substantial increase in the number of production establishments of about 16.6% (40.2%). As a result of these changes, output per establishment drops by much more than the reduction in aggregate output, 16.6% (35.0%). This occurs as with the introduction of distortions that are correlated with size, relatively large, distorted establishments reduce their demand for capital and labor services, leading to a reduction in the wage rate across steady states. This prompts the emergence of smaller production units, as individuals with low initial managerial ability become managers.

The effects outlined above are also present in the analysis in Guner et al (2008), in the context of a standard span-of-control model with capital accumulation. In the current context with skill investments, the consequences of distortions that affect the size of production establishments are more severe. Distortions have detrimental consequences on skill investments, and on average, managers with higher initial ability are more severely distorted. This contributes to the decline in average managerial ability, which declines by 16.6% with a reduction in mean size of 15% and by 37.0% with a reduction of 30%.

Indeed, mean ability of managers declines due to two reasons. The first reason is the reallocation of resources across establishments that leads to lower demand for labor and lower wages and results in the emergence of small establishments. As managers of these establishments have initial skills that are below the cutoff level, they reduce average managerial skills. The second reason is due to the forces that we highlight in this paper. Distorted individuals, who tend to be the initially most able ones, reduce their investments in managerial skills, leading in turn to a further reduction in the economy-wide level of managerial ability. Both forces contribute to the decline in aggregate output, and the degree of reallocation of resources from large establishments to small ones. Indeed, as Table 4 shows, idiosyncratic distortions lead to substantial reallocation of production across establishments. A reduction in mean size of 30% leads to reduction in the share of employment accounted for by large establishments (100 and more workers) from about 47.3% to 31.1%, and an increase in the share of small ones (less than 10 workers) from 17.7% to 26.7%.

**The Magnitude of Distortions** How large are the implicit tax distortions that lead to the reductions in mean size discussed above? As Table 4 shows, the average tax rates are relatively small, as an average rate of 3.3% (1.3%) is needed to reduce mean size by 30% (15%). Not surprisingly, median distortion rates are lower than average ones. Note, however, that the relatively low values of means masks somewhat the magnitude of distortions faced by some managers. Table 4 indicates that while the average distortion when mean size is reduced by 30% is 3.3%, the distortions at the top 10% (1%) amount to 16.9% (43.5%).

Distortions, not surprisingly, are much larger on average at the top of the distribution of establishment size. For establishments with more than 100 workers, reducing mean size by 15% (30%) leads to distortions averaging 5.7% (6.8%). It is worth noting here that doubling the reduction in mean size requires a disproportionately low increase in the rate on establishments with more than 100 workers. This occurs as $\kappa$ is increased, fewer severely distorted establishments remain with a size of 100 workers or more. In addition, large but lightly distorted establishments become larger, and their managers invest more in skills as distortions become more severe. Hence, both forces contribute to the relatively low average value of distortions at large establishments.

**The Importance of Distortions Correlated with Productivity** Previous work has shown that for distortions to matter for aggregates and productivity, they have to be heavier for more productive establishments than for less productive ones (Guner et al (2008), Restuccia and Rogerson (2008)). Does this result still hold in the current environment? Note that distortions as considered here, even if the same for all, affect investment in skills and thus may matter.

To assess the importance of the connection between productivity and distortions in the current environment, we conduct the following experiment: we apply the average rate that is required to reduce mean size by 30% to *all* managers and reduce the tax rate $\tau$ in order to maintain revenue neutrality. We find that in this case, the drop in aggregate output is quite small (about 0.8%), while leaving mean establishment size effectively unchanged. We conclude from this exercise that the positive association between size and distortions is a central force in the current environment as well.

## 4.2 Variation in Economy-wide Productivity

We now consider the effects of changes in economy-wide productivity levels; the term $A$ that is common to all establishments. We do this for multiple reasons. First, there is substantial variation in the size of establishments across countries that is correlated to the level of development. As it is well documented, productivity differences are a central factor in accounting for the large observed disparities in income across countries.[3] Second, differences in productivity across countries can have effects in human capital decisions, and thus their role in development can be therefore be amplified. If productivity differences affect the accumulation of managerial skills, variation in productivity can contribute to account for cross-country differences in establishment size.

We consider two exogenous reductions in productivity ($A$) relative to the benchmark case: 25% and 50%. Results are presented in Table 5. Not surprisingly, reducing productivity by 25% (50%) leads to a reduction in output across steady states of 38.7% (68.6%). Changes in the level of productivity also affect the distribution of plant size: reducing productivity by 25% (50%) leads to an increase in the number of establishments by about 15.5% (36.0%) with a corresponding decline in the mean size of establishments.

Changes in exogenous productivity, as modeled here, do *not* generate size differences in a growth model with a Lucas (1978) span-of-control technology, as changing $A$ has no effect

[3]While the mean size of establishments in the U.S. is about 17.9 employees in the U.S. and 15.0 in Norway, it amounts to about 9.7 in Japan. The size of typical establishments is substantially lower in poorer countries. For instance, mean establishment size is 7.5 employees in Turkey, 4.3 in Jordan and 4.4 in India. See Bhattacharya (2010) for a documentation of differences in establishment size across countries.

on occupational decisions.[4] The consequences of changing aggregate productivity, however, are different in the current setup. As productivity drops, both wage rates and managerial rents drop as in the standard span-of-control model. But a productivity drop also reduces the marginal benefit associated to an extra unit of income invested in skill accumulation (see equation 11). As a result, managerial ability drops across steady states as well, which translates into further reductions in labor demand and therefore, on the wage rate. The net result is a reduction in the value of becoming a worker relative to a manager at the start of life, which leads in turn to an increase in the number of managers (establishments).

Quantitatively, the size-distribution effects associated to a reduction in productivity are not trivial. As Table 5 demonstrates, a 25% (50%) reduction in productivity reduces mean managerial ability by 28.9% (51.1%). It leads to a reduction in the share of employment accounted for by establishments with 100 workers or more from 47.2% to 39.1% (28.9%), and to an increase in the share of establishments of less than 10 workers from 17.7% to 20.8% (24.7%).

Overall, changes in aggregate productivity concentrate employment at smaller production units as idiosyncratic distortions do. As a result, they can matter in quantitatively accounting for the observed cross-country differences. In Section 6, we explore in detail the interplay between distortions and exogenous productivity differences for the case of Japan.

# 5 The Importance of Skill Investments

In this section we quantify the importance of the mechanism that we highlight in the paper. We ask the extent to which the effects on output, productivity and the size of plants depend on the presence of skill investments by managers.

The importance of skill investments is intimately connected with the magnitude of the parameter $\theta_2$ in the skill accumulation technology. If $\theta_2 \to 0$, managerial investments approach zero, managerial skills become exogenous and invariant to changes in the environment. We start by noting that if $\theta_2$ is halved in the benchmark economy (keeping all other parameters fixed at their benchmark values), the effects on size distribution statistics are non trivial. Mean establishment size drops from about 17.9 employees to 13.4. These changes are largely accounted for by the behavior of the upper tail of the distribution of employment: the share

[4]This requires a Cobb-Douglas specification as modeled here.

of establishments with 100 workers or more drops from the benchmark value of of 47.2% to 24.5%. This is not surprising: changes in $\theta_2$ affect the growth rate of managerial skills as individuals age, which in turn are positively associated with the level of initial skills. Hence, a reduction in $\theta_2$, for a given distribution of initial managerial ability, affects disproportionately the capacity of the model to account for the employment share at the top of the distribution.

To evaluate the importance of skill investments in the effects of idiosyncratic distortions, we conduct the following experiments. We compare the effects of distortions that reduce mean size by 30% with those emerging under lower values of $\theta_2$. We consider two alternative values of $\theta_2$: an intermediate value ($\theta_2' = \theta_2/2$) and a limiting one ($\theta_2'' = 0$). To highlight the changes under lower values of $\theta_2$, in both alternative cases we use the same parameter values and the same distortions at the start of the life cycle as in the benchmark results in section 4.

Table 6 shows the main results. The distortions considered in the previous sections lead now to lower effects on the average values of managerial ability and therefore, on output. Under less investment in managerial skills, output drops by 4.3% in the intermediate case and by about 3.2% in the absence of skill investments. Hence, the model with no skill investments captures about 36% of the output changes implied by the distortions. Table 6 also shows the changes in establishment size driven by the introduction of idiosyncratic distortions. For instance, the distortions that lead to a reduction in mean size of 30% in the benchmark results, lead to a reduction in mean size of 19.4% in the absence of skill investments. Similarly, while the number establishments rises by 40.2% under the benchmark results, it rises by only 11.6% in the case of no investments.

Summing up, our findings imply that the effects of the introduction of idiosyncratic distortions on output, productivity and the size of establishments is substantially amplified by managerial skill investments.[5] Henceforth, a central implication of our findings is that to generate given effects on establishment size, output and productivity, significantly *smaller* distortions are required. For instance, while reducing mean size by 30% requires an average distortionary tax rate of 3.3% under benchmark parameters, the average tax rate must be

[5]This result is similar to the amplification effects of productivity differences driven by human capital accumulation in the development literature, e.g. Manuelli and Seshadri (2010) and Erosa, Koreshkova and Restuccia (2010).

around 4.9% in order to achieve the same reduction in the absence of skill investments.

# 6 Distortions and their Interplay with Productivity Differences

In previous sections, we showed the quantitative implications of the model in terms of output, productivity measures and the size distribution of establishments. In this section, we use data from the Japanese economy to evaluate the performance of the model in several dimensions, and provide quantitative estimates of economy-wide productivity differences between Japan and the United States.

Japan is quite a relevant case to consider from the perspective of this paper. First, while a relatively rich country, its level of output per worker is only about 70% of the U.S. level and it has been at that level for roughly twenty years. Second, there are substantial differences in the size distribution of establishments in relation to the United States. Using data from the Japanese Establishment and Enterprise Census, we calculate that mean establishment size in Japan is substantially below the U.S. level: 9.7 versus 17.9 employees. Not surprisingly, production in Japan is effectively concentrated in small units. As we documented earlier, the fraction of small establishments (less than 10 workers) in the U.S. is 72.5%, the fraction of large establishments ( 100 workers or more) is about 2.6% and large establishments account for about 46.2% of employment. The corresponding values for Japan differ non trivially: 79.1%, 1.0% and 26.0%, respectively. Finally, Japan is a case that fits well with the case of distortions that are correlated with the size of establishments. As documented in Guner, Ventura and Yi (2006, 2008), Japan regulates severely size of the retail sector at the national level, with policies that go back to the pre-World War II era.

Using the calibrated parameters of the benchmark economy, we proceed to find the levels of economy-wide productivity ($A$) and distortions as modeled earlier, in order to reproduce, in a stationary equilibrium, two targets: (i) the level of output of Japan relative to the United States; (ii) mean size in Japan. We then contrast the model implications for other properties of the size distribution with Japanese data, and quantify the importance of aggregate productivity differences vis-a-vis idiosyncratic distortions.

**Findings** The findings from the experiment are shown in Table 7. As the table demonstrates, the model is successful in generating the quantitative properties of the Japanese size-distribution data. The model implies a fraction of small (large) establishments of about 81.6% (0.7%) and a share of employment at large establishments of 24.9%. As mentioned earlier, the corresponding values from the data are 79.1% (1.0%) and 26.0%, respectively. We view these findings as important since they illustrate the capacity of the framework to account for size observations; they give us confidence to take the quantitative findings of the model seriously.

The model generates the Japanese facts with about an aggregate, economy-wide productivity level of about 0.91 (benchmark value 1.0), and distortions that amount to an average of 5%. Given the endogeneity of managerial skills, the results can be used to provide a measure of *Total Factor Productivity* (TFP). Note that since the production structure of the model allows for aggregation, the model implies an aggregate production technology

$$Y = \underbrace{AZ}_{\equiv \text{TFP}} K^{\nu\gamma} L^{\nu(1-\gamma)},$$

where $K$ is aggregate capital, $L$ is the aggregate fraction of the population engaged in regular work (see equation 15) and the term $Z$ is the level of endogenous managerial skills raised to the power $1 - \gamma$, i.e.,

$$Z \equiv \left( \sum_{j=1}^{J_R - 1} \mu_j \int_{z^*}^{\overline{z}} z_j \psi_j(a, z) da dz \right)^{1-\gamma}.$$

Hence, TFP amounts to $AZ$. Changes in the level of exogenous productivity affect TFP in two ways. First, the reduction in $A$ reduce TFP directly. Second, since managerial skills are endogenous, the introduction of distortions and the reduction in the exogenous productivity level $A$ reduce aggregate managerial skills, and induce a further reduction in TFP. Table 7 shows that TFP measured in this way drops by about 17% across steady states, with a significant contribution stemming from the combined effects on managerial skills across steady states.

How large are contribution of exogenous productivity differences vis-a-vis distortions in order to generate the results in Table 7? The last column in the Table answers this question. Using the previously found values of $A$, we compute the effects of changes in exogenous productivity *only*. The results reveal that differences in $\bar{A}$ play a non-trivial role: they capture nearly half of the changes in output, about 60% of the changes in TFP and about 11% and of the changes in mean size relative to the benchmark economy. On the other hand, the share of employment at large establishments drops only marginally to 45% due to the reduction in exogenous productivity.

From these findings, we conclude that distortions as modeled here matter: exogenous differences in aggregate productivity alone cannot account for the differences between Japan and the United States. Indeed, as the results demonstrate, distortions appear to be the main driver in accounting for differences in the size of establishments between the U.S. and Japan.

# 7 Conclusion

We developed a span-of-control model where managers invest in the quality of their skills, and used it to quantify the significance of idiosyncratic, correlated distortions and their interplay with aggregate productivity levels. We found that these distortions, when selected to generate given reductions in the mean size of establishments, can lead to substantial effects on output, productivity and the size distribution. These effects are non-trivially magnified by the endogeneity of managerial skills.

A central implication of our findings is that relatively small distortions on average can lead to substantial effects in the long run. As we elaborate in the text, the endogeneity of managerial skills is key for theses result; as the importance of managerial investments is reduced, larger distortions are needed to generate given effects on the size distribution of establishments, and thus on output and productivity.

Another important implication of our analysis is that exogenous variation in economy-wide productivity is *not* neutral for occupational choice and thus, the size distribution of establishments. This implies that the model is flexible enough to capture and account for the large differences in size across countries by differences in aggregate productivity, distortions and the interplay between them. For the specific case of Japan, we find that when the model is forced to account for the output gap and the large differences in mean size with the U.S., that differences in productivity alone can capture about 49% of the output gap and 11% of the differences in mean size.

We close the paper with two comments. First, we note that we introduce idiosyncratic, correlated distortions by assigning them stochastically to agents at the start of the life cycle. Managers who are distorted cannot avoid the burden imposed on them, even when no agent is forced to operate a distorted establishment if he/she chooses not to. Perhaps a more realistic approach to the problem would involve the random introduction of distortions in each period that are correlated with managerial productivity. Such an approach, however, would have the drawback of introducing uncertainty in individual decisions, making the comparison between different distortion levels somewhat more involved. Nevertheless, we conjecture that our main quantitative findings would remain unaltered under this extension.

The second comment pertains to the use in detail of observations on the life cycle of managers in the parameterization of the model. We are unaware of empirical work that has focused at the life cycle of managers in detail. The discussion in section 3 indicates that our model is broadly consistent with the growth of managerial income over the life cycle, providing support for our parameterization. In any case, a deeper study that connects an occupational choice model with data on multiple aspects of the life cycle of managers seems warranted. We leave this and other extensions for future work.

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Table 1: Parameter Values (annualized)

| Parameter | Value |
|---|---|
| Population Growth Rate ($n$) | 0.011 |
| Depreciation Rate ($\delta$) | 0.04 |
| Importance of Capital ($\nu$) | 0.428 |
| Returns to Scale ($\gamma$) | 0.760 |
| Mean Log-managerial Ability ($\mu_z$) | 0 |
| Dispersion in Log-managerial Ability ($\sigma_z$) | 2.293 |
| Discount Factor ($\beta$) | 0.945 |
| Skill accumulation technology ($\theta_1$) | 0.953 |
| Skill accumulation technology ($\theta_2$) | 0.405 |
| Tax rate ($\tau$) | 0.26 |

Table 2: Empirical Targets: Model and Data

| Statistic | Data | Model |
|---|---|---|
| Mean Size | 17.9 | 17.7 |
| Capital Output Ratio | 2.325 | 2.304 |
| Fraction of Small (0-9 workers) establishments | 0.725 | 0.747 |
| Fraction of Large (100+ workers) establishments | 0.026 | 0.027 |
| Employment Share of Large establishments | 0.462 | 0.473 |

Table 3: Effects of Idiosyncratic Distortions

| Statistic | Benchmark | 15% Reduction | 30% Reduction |
|---|---|---|---|
| Aggregate Output | 100.00 | 97.3 | 91.2 |
| Number of Establishments | 100.00 | 116.6 | 140.2 |
| Output per Establishment | 100.00 | 83.4 | 65.0 |
| Mean Ability | 100.0 | 83.3 | 63.0 |
| Investment in Skills (% of GDP) | 1.8 | 1.7 | 1.6 |
| Employment Share 0-9 workers (%) | 17.7 | 21.0 | 26.7 |
| Employment Share 100+ workers (%) | 47.2 | 38.3 | 31.1 |

Note: Entries show the effects on displayed variables associated to the introduction of idiosyncratic distortions. The distortions are selected in order to generate reductions in mean establishment size of 15% and 30% relative to the benchmark economy.

Table 4: Idiosyncratic Distortions (%)

| Statistic | Benchmark | 15% Reduction | 30% Reduction |
|---|---|---|---|
| Mean Distortion | 0.0 | 1.3 | 3.3 |
| Median Distortion | 0.0 | 0.6 | 1.6 |
| Mean Distortion (100+ workers) | 0.0 | 5.7 | 6.8 |
| Mean Distortion (Top 10%) | 0.0 | 6.4 | 16.9 |
| Mean Distortion (Top 1%) | 0.0 | 15.1 | 43.5 |

Note: Entries show statistics on the magnitude of idiosyncratic distortions. The distortions are selected in order to generate reductions in mean establishment size of 15% and 30% relative to the benchmark economy.

Table 5: Role of Aggregate Productivity

| Statistic | Benchmark | A=0.75 | A=0.5 |
|---|---|---|---|
| Aggregate Output | 100 | 61.3 | 31.4 |
| Number Establishments | 100.0 | 115.5 | 136.0 |
| Output per Establishment | 100.0 | 53.0 | 23.1 |
| Mean Ability | 100 | 71.1 | 48.9 |
| Investment in Skills (% of GDP) | 1.76 | 1.44 | 1.03 |
| Mean Size | 100 | 85.8 | 72.1 |
| Employment Share 0-9 workers (%) | 17.7 | 20.8 | 24.7 |
| Employment Share 100+ workers (%) | 47.2 | 39.1 | 28.9 |

Note: Entries show the effects on displayed variables associated to exogenous reductions in the level of economy-wide productivity.

Table 6: Importance of Skill Investments

| Statistic | No Distortions | Distortions (benchmark $\theta_2$) | Distortions ($\theta' = \theta_2/2$) | Distortions ($\theta_2'' = 0$) |
|---|---|---|---|---|
| Aggregate Output | 100.0 | 91.2 | 95.7 | 96.9 |
| Number of Establishments | 100.0 | 140.2 | 114.7 | 111.6 |
| Mean Ability | 100.0 | 63.0 | 86.0 | 91.8 |
| Mean Size | 100.0 | 70.0 | 76.7 | 80.6 |
| Employment Share 0-9 workers | 100.0 | 150.5 | 119.7 | 116.0 |
| Employment Share 100 + workers | 100.0 | 65.8 | 62.9 | 58.8 |

Note: Entries show the effects on displayed variables associated to the introduction of idiosyncratic distortions in different cases. The fourth (fifth) column shows the effects driven by the same distortions in the benchmark case, but with a value of the elasticity parameter $\theta_2$ equal to half the value in the benchmark case (zero).

Table 7: Japan

| | Benchmark (U.S.) | Japan Data | Japan Model | Japan Model (No Distortions) |
|---|---|---|---|---|
| Output per Worker | 100 | 70.0 | 70.0 | 85.4 |
| Mean Size | 17.7 | 9.7 | 9.7 | 16.8 |
| Exogenous Productivity ($\bar{A}$) | 100 | - | 91.25 | 91.25 |
| Mean Ability | 100 | - | 38.6 | 89.1 |
| Total Factor Productivity | 100 | - | 83.0 | 89.8 |
| Mean Distortion (%) | 0.0 | - | 5.0 | 0.0 |
| Number of Establishments (%) | 100 | | 174.4 | 104.9 |
| Fraction of 0-9 workers establishments | 0.747 | 0.791 | 0.816 | 0.752 |
| Fraction of 100+ establishments | 0.027 | 0.010 | 0.007 | 0.026 |
| Employment Share of 100+ establishments | 0.473 | 0.260 | 0.249 | 0.448 |

Note: Entries show the effects on displayed variables when the model is applied to the case of Japan. The fourth column shows the effects when the distortions and exogenous productivity are varied to match Japan's output relative to the U.S. and Japan's mean establishment size. The last column shows the effects for the exogenous productivity level previously found.

Figure 1: Determination of Skill Investments

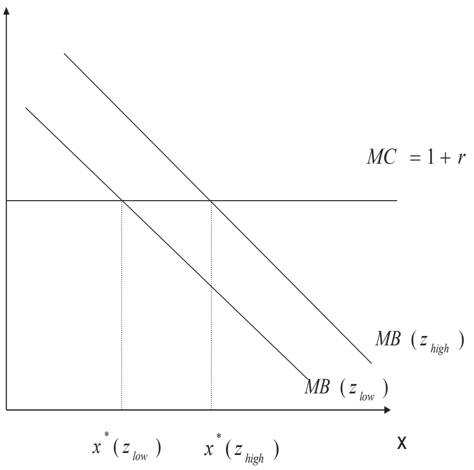


Figure 2: Occupational Choice

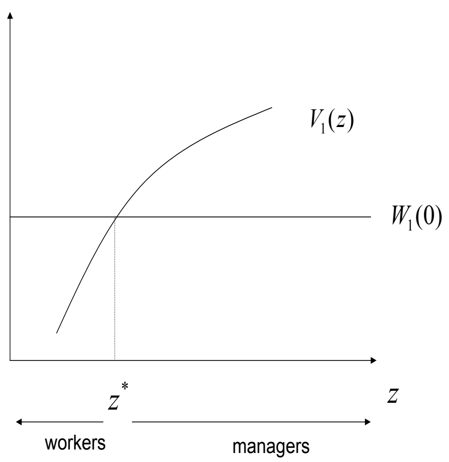

Figure 3: Size Distribution: Model and Data

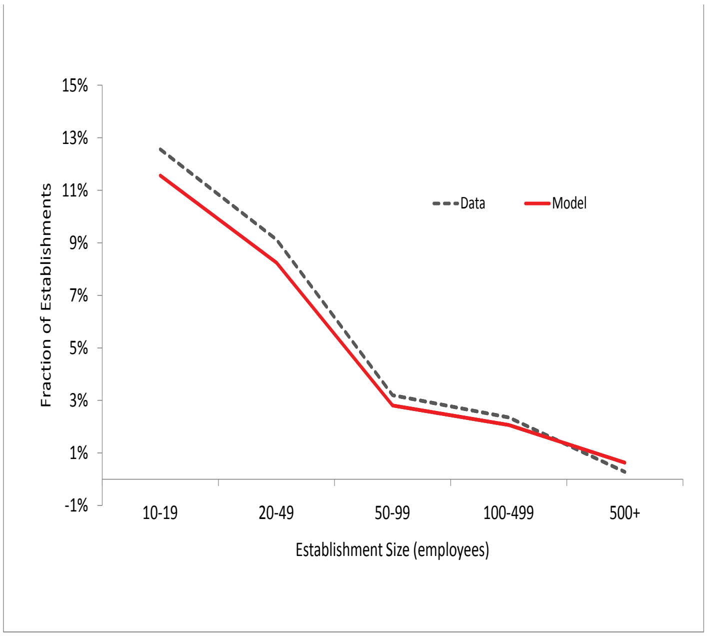


Figure 4: Employment Shares: Model and Data