# Firm Dynamics, Job Turnover, and Wage Distributions in an Open Economy 

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#### Abstract

This paper explores the combined effects of reductions in trade frictions, tariffs, and firing costs on firm dynamics, job turnover, and wage distributions. It uses establishment-level data from Colombia to estimate an open economy dynamic model that links trade to job flows and wages. Counterfactual experiments imply that Colombia's integration with global product markets increased its national income at the expense of higher unemployment, greater wage inequality and increased firm-level volatility. In contrast, contemporaneous labor market reforms dampened the increase in unemployment and aggregate job turnover. The results speak more generally to the effects of globalization on labor markets.


Keywords: International trade, firm dynamics, size distribution, labor market frictions, inequality

JEL Codes: F12, F16, E24, J64, L11

[^0]
## 1 Introduction

During the late 1980s and early 1990s, as the forces of globalization gained momentum, many Latin American countries dismantled their trade barriers and implemented labor market reforms. ${ }^{1}$ Over the next two decades, these countries roughly doubled their trade-to-GDP ratios, and thereby reaped the well-known benefits of better access to foreign markets. But they also experienced increased wage inequality, higher unemployment rates, greater informal sector activity, and more rapid job turnover. ${ }^{2}$

These developments motivate the two basic questions we address in this paper. First, through what mechanisms and to what extent might the global integration of product markets have increased wage inequality, unemployment rates, and job insecurity in Latin America? Second, how might commercial policy reforms and changes in worker firing costs have conditioned the relationship between globalization and these labor market outcomes?

To answer these questions, we develop a dynamic general equilibrium model that links globalization and labor regulations to wage distributions, job flows, and unemployment. Then we fit our model to plant-level panel data from Colombia - a country that reduced firing costs, cut tariffs, and exhibited rapid growth in merchandise trade. Finally, by comparing simulated steady states under alternative regimes, we quantify the labor market consequences of global reductions in trade frictions (hereafter, "Globalization") and Colombia's policy reforms.

The estimated model closely replicates basic features of Colombian micro data in the decade preceding reforms, including the size distribution of firms, the rates of employment growth among firms of different sizes, producer entry and exit rates, exporting patterns, and the degree of persistence in firm-level employment levels. Also, although it is fit to prereform data, it nicely replicates many post-2000 features of the Colombian economy when it is evaluated at post-2000 tariff rates, firing costs, and global trade frictions. In particular, the quantified model successfully predicts the post-2000 plant size distribution.

The simulations indicate that, taken as a package, the policy reforms and the global reductions in trade frictions led to significant increases in wage inequality, lifetime income

[^1]inequality, and unemployment. On the other hand, while tariff reductions and globalization shocks would have increased job turnover by themselves, this impact of openness was offset by the labor market reforms. Finally, the globalization shock was in many respects the most potent. Hence, while improving per capita incomes through standard channels, the rapid expansion of global trade may also be reducing job security and increasing inequality in Latin America and elsewhere. ${ }^{3}$

While our model incorporates some well-known mechanisms, it also introduces several new channels through which openness is linked to labor market outcomes. First, by increasing the sensitivity of firms' revenues to their productivity and employment levels, openness makes firms more willing to incur the hiring and firing costs associated with adjusting their workforce. By itself, this sensitivity effect makes job turnover and unemployment higher when trade frictions are low. It also tends to create larger rents for the more successful firms and to thereby spread the cross-firm wage distribution. Second, however, openness concentrates workers at large firms, which are more stable than small firms and less likely to exit. ${ }^{4}$ This distribution effect works against the sensitivity effect, tending to reduce turnover and wage inequality as trade frictions fall.

Our formulation is related to several literatures. First, it shares some basic features with large-firm models in the labor-search literature. In particular, it can be viewed as an extension of Bertola and Caballero (1994), Bertola and Garibaldi (2001), and Koeniger and Prat (2007) to include fully articulated product markets, international trade, serially correlated productivity shocks, intermediate inputs, and endogenous firm entry and exit. ${ }^{5}$ And like these models, it explains firms' wage setting as reflecting their idiosyncratic demand or productivity shocks in the presence of convex hiring costs.

Second, our work contributes to the growing literature on the effects of international trade in the presence of labor market frictions. Like many papers therein, we link the crossfirm wage distribution to the cross-firm rent distribution, which we link in turn to trade costs through a Melitz (2003) mechanism (Egger and Kreickemeier 2009; Helpman, Itskhoki, and Redding 2010; Helpman et al. 2012; Felbermayr, Prat, and Schmerer 2011; Fajgelbaum 2013; Davis and Harrigan 2011; and Amiti and Davis 2012). ${ }^{6}$ Among these papers, our model

[^2]is relatively close to those that generate wage dispersion and unemployment by combining wage bargaining with Diamond-Mortensen-Pissarides search frictions (Helpman and Itskhoki 2010; Helpman, Itskhoki, and Redding 2010; Helpman et al. 2012; and Felbermayr, Prat and Schmerer 2011). The key distinction between these papers and ours is that we empirically characterize steady-states with ongoing productivity shocks, endogenous entry and exit, and job turnover.

Finally, because our paper offers a new explanation for size-dependent volatility, it contributes to the literature on firm dynamics (e.g., Hopenhayn 1992; Jovanovic 1982; Ericson and Pakes 1995; Klette and Kortum 2004; Luttmer 2007; Rossi-Hansberg and Wright 2007; and Arkolakis forthcoming). Unlike previous studies, we explain the relative stability of large firms as a consequence of nonlinear hiring costs, which make it relatively more expensive (per worker) for large firms to sustain any positive growth rate.

While we do not pretend to capture all of the channels through which openness and firing costs can affect labor market outcomes, our focus on firm-level entry, exit and idiosyncratic productivity shocks is supported by existing empirical evidence on the sources of job turnover and wage heterogeneity. Studies of job creation and job destruction invariably find that most reallocation is due to idiosyncratic (rather than industry-wide) adjustments (Davis, Haltiwanger, and Schuh 1998; Roberts 1996), even in Latin America's highly volatile macro environment (Chapter 2 of Inter-American Development Bank 2004). Further, as Goldberg and Pavcnik (2007) note, there is little evidence in support of trade-induced labor reallocation across sectors, so if openness has had a significant effect on job flows, it should have been through intra-sectoral effects. Finally, while observable worker characteristics do matter for wage differentials, much is attributable to labor market frictions and firm heterogeneity (Abowd, Kramarz, and Margolis 1999; Mortensen 2003; Helpman et al. 2012).

## 2 The Model

### 2.1 Preferences

We consider a small open economy populated by a unit measure of homogeneous, infinitelylived worker-consumers. Each period $t$, agents derive utility from the consumption of homogeneous, non-tradable services, $s_{t}$, and a composite industrial good, $c_{t}$, where

$$
\begin{equation*}
c_{t}=\left(\int_{0}^{N_{t}} c_{t}(n)^{\frac{\sigma-1}{\sigma}} d n\right)^{\frac{\sigma}{\sigma-1}} \tag{1}
\end{equation*}
$$

aggregates consumption of the differentiated goods varieties, $c_{t}(n), n \in\left[0, N_{t}\right]$, with a constant elasticity of substitution $\sigma>1$. Worker-consumers maximize the expected present value of their utility stream

$$
\mathcal{U}=\sum_{t=1}^{\infty} \frac{s_{t}^{1-\gamma} c_{t}^{\gamma}}{(1+r)^{t}}
$$

where $r$ is the discount rate and $\gamma \in(0,1)$ is the expenditure share of the industrial good. Being risk neutral, they do not save. In what follows, we suppress time subscripts $t$ for ease of notation.

### 2.2 Production technologies

Services are supplied by service sector firms and, less efficiently, by unemployed workers engaged in home production. Regardless of their source, services are produced with labor alone, homogeneous across suppliers, and sold in competitive product markets. Firms that supply services generate one unit of output per worker and face no hiring or firing costs. Unemployed workers who home-produce service goods each generate $b<1$ units of output. The economy-wide supply of services is thus

$$
\begin{equation*}
S=L_{s}+b L_{u} \tag{2}
\end{equation*}
$$

where $L_{s}$ is labor employed in the service sector and $L_{u}$ is unemployed labor.
Differentiated goods are supplied by industrial sector firms, each of which produces a unique product. These firms are created through sunk capital investments; thereafter their output levels are determined by their productivity levels, $z$, employment levels, $l$, and intermediate input usage, $m$, according to:

$$
q=\left\{\begin{array}{cc}
z l^{\alpha} m^{1-\alpha} & l \geq l_{e}  \tag{3}\\
0 & l<l_{e}
\end{array}\right.
$$

where $0<\alpha<1, l_{e}>0$ is the smallest viable firm size, and $m=\left(\int_{0}^{N} m(n)^{\frac{\sigma-1}{\sigma}} d n\right)^{\frac{\sigma}{\sigma-1}}$ aggregates differentiated goods used as intermediates in the same way the subutility function (1) aggregates differentiated goods used for final consumption. Note that, as in Melitz (2003), productivity variation can equally well be thought of as variation in product quality.

### 2.3 Price indices

Differentiated goods can be traded internationally. Measure $N_{F}$ of the measure $N$ differentiated goods are imported, and an endogenous set of domestically produced goods are
exported. Both exports and imports are subject to iceberg trade costs: for each $\tau_{c}>1$ units shipped, a single unit arrives at its destination. Moreover, imports are subject to an ad valorem tariff rate of $\tau_{a}-1>0$.

Let asterisks indicate that a variable is expressed in foreign currency, and define $p^{*}(n)$ to be the FOB price of imported variety $n \in\left[0, N_{F}\right]$. The exact home-currency price index for imported goods is then $P_{F}=\tau_{a} \tau_{c} k\left[\int_{0}^{N_{F}} p^{*}(n)^{1-\sigma} d n\right]^{1 /(1-\sigma)}$, where $k$ is the exchange rate. Similarly, letting $p(n)$ be the price of domestic variety $n \in\left(N_{F}, N\right]$ in the home market, the exact home price index for domestic goods is $P_{H}=\left(\int_{N_{F}}^{N} p(n)^{1-\sigma} d n\right)^{1 /(1-\sigma)}$. Finally, defining $p_{X}^{*}(n)$ to be the price of domestic variety $n$ in the foreign market, and letting $\mathcal{I}^{x}(n) \in\{0,1\}$ take a value of 1 if good $n$ is exported, $P_{X}^{*}=\left(\int_{N_{F}}^{N} \mathcal{I}^{x}(n) p_{X}^{*}(n)^{1-\sigma} d n\right)^{1 /(1-\sigma)}$ is the exact foreign market price index for exported goods.

Several normalizations simplify notation. First, since the measure of available foreign varieties and their FOB foreign-currency prices are exogenous to our model, we normalize $\left[\int_{0}^{N_{F}} p^{*}(n)^{1-\sigma} d n\right]^{1 /(1-\sigma)}$ to unity by choice of foreign currency units. This allows us to write the exact domestic price index for the composite industrial good as

$$
\begin{equation*}
P=\left[P_{H}^{1-\sigma}+\left(\tau_{a} \tau_{c} k\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} . \tag{4}
\end{equation*}
$$

Second, without loss of generality, we choose the price of services to be our numeraire. The real exchange rate $k$ endogenously adjusts so that in equilibrium, the two normalizations in domestic and foreign currency units are consistent.

### 2.4 Differentiated goods markets

Differentiated goods are sold in monopolistically competitive markets, where they are purchased by consumers as final goods and by producers as intermediate inputs. Utility maximization implies that worker $i$ with income $I_{i}$ demands $\frac{\gamma I_{i}}{P}\left(\frac{p(n)}{P}\right)^{-\sigma}$ units of domestic variety $n$ and $\frac{\gamma I_{i}}{P}\left(\frac{\tau_{a} \tau_{c} k p^{*}\left(n^{\prime}\right)}{P}\right)^{-\sigma}$ units of imported variety $n^{\prime}$. Similarly, firm $j$ with gross revenue $G_{j}$ optimally purchases $(1-\alpha) \frac{\sigma-1}{\sigma} \frac{G_{j}}{P}\left(\frac{p(n)}{P}\right)^{-\sigma}$ units of domestic variety $n$, and $(1-\alpha) \frac{\sigma-1}{\sigma} \frac{G_{j}}{P}\left(\frac{\tau_{a} \tau_{c} k p^{*}\left(n^{\prime}\right)}{P}\right)^{-\sigma}$ units of imported variety $n^{\prime}$.

Aggregating across domestic consumers and domestic producers yields total domestic demand for any domestic variety $n$ :

$$
\begin{equation*}
Q_{H}(n)=D_{H} p(n)^{-\sigma} \quad \text { for } n \in\left(N_{F}, N\right], \tag{5}
\end{equation*}
$$

where

$$
D_{H}=P^{\sigma-1}\left[\gamma \int_{0}^{1} I_{i} d i+(1-\alpha) \frac{\sigma-1}{\sigma} \int_{N_{F}}^{N} G_{j} d j\right]
$$

Note that the population of domestic worker-consumers is normalized to one, and domestic producers are indexed by $n \in\left(N_{F}, N\right]$. Likewise, total domestic demand for any imported variety $n$ is

$$
\begin{equation*}
Q_{H}(n)=D_{H}\left[\tau_{a} \tau_{c} k p^{*}(n)\right]^{-\sigma} \quad \text { for } \quad n \in\left[0, N_{F}\right] . \tag{6}
\end{equation*}
$$

Finally, assuming markets are internationally segmented, foreign demand for domestically produced good $n$ is given by

$$
\begin{equation*}
Q_{F}(n)=D_{F}^{*}\left[p_{X}^{*}(n)\right]^{-\sigma}, n \in\left(N_{F}, N\right], \tag{7}
\end{equation*}
$$

where $D_{F}^{*}$ measures aggregate expenditures abroad denominated in foreign currency, and is net of any effects of foreign commercial policy. Given our small country assumption, we take $D_{F}^{*}$ to be unaffected by the actions of domestic agents.

These expressions imply that, expressed in domestic currency, total domestic expenditures on domestic varieties amount to $D_{H} P_{H}^{1-\sigma}$, total domestic expenditures on imported varieties amount to $D_{H}\left(\tau_{a} \tau_{c} k\right)^{1-\sigma}$, and domestic firms' total export revenues amount to $k D_{F}^{*} P_{X}^{* 1-\sigma} / \tau_{c}$.

### 2.5 Producer dynamics

Industrial firms are subject to idiosyncratic productivity shocks. These are generated by the $A R(1)$ process

$$
\begin{equation*}
\ln z^{\prime}=\rho \ln z+\sigma_{z} \epsilon \tag{8}
\end{equation*}
$$

where $\rho \in(0,1)$ and $\sigma_{z}>0$ are parameters, primes indicate one-period leads, and $\epsilon \sim N(0,1)$ is a standard normal random variable independently and identically distributed across time and firms. Together with firms' employment policies and entry/exit decisions, (8) determines the steady state distribution of firms over the state space $(z, l)$ and the rates at which firms transit across pairs of states.

Producer dynamics in the industrial sector resemble those in Hopenhayn (1992) and Hopenhayn and Rogerson (1993) in that firms react to their productivity shocks by optimally hiring, firing, or exiting. Also, new firms enter whenever their expected future profit stream exceeds the entry costs they face. However, unlike these papers, we assume that hiring in the industrial sector is subject to search frictions captured by a standard matching function. We now describe the functioning of labor markets.

### 2.6 Labor markets and the matching technology

The service sector labor market is frictionless, so workers can obtain jobs there with certainty if they choose to do so. Since each service sector worker produces one unit of output, and the price of services is our numeraire, these jobs pay a wage of $w_{s}=1$.

The industrial sector labor market, in contrast, is subject to search frictions. These expose industrial job seekers to unemployment risk and create match-specific rents that workers and firms bargain over. The number of new matches between job seekers and vacancy posting firms each period is given by

$$
M(V, U)=\frac{V U}{\left(V^{\theta}+U^{\theta}\right)^{1 / \theta}}
$$

where $\theta>0$. Here, $U$ is the measure of workers searching for industrial sector jobs, and $V$ is the measure of industrial sector vacancies. The parameter $\theta$ governs the severity of matching frictions, since a higher value for $\theta$ results in a larger number of matches for given values of $U$ and $V .^{7}$ This matching function implies that industrial firms fill each vacancy with probability

$$
\phi(V, U)=\frac{M(V, U)}{V}=\frac{U}{\left(V^{\theta}+U^{\theta}\right)^{1 / \theta}}
$$

while workers searching for industrial jobs find matches with probability

$$
\widetilde{\phi}(V, U)=\frac{M(V, U)}{U}=\frac{V}{\left(V^{\theta}+U^{\theta}\right)^{1 / \theta}} .
$$

At the beginning of each period, workers who are not already employed in the industrial sector decide whether to accept a service sector job that pays wage $w_{s}=1$ with certainty, or to search for an industrial sector job. If they fail to match with an industrial sector producer, they subsist until the next period by home-producing services at the wage of $b<1 .{ }^{8}$ At the start of the matching process, among the unit measure of the worker population, $U$ are searching for an industrial job. At the end of the matching process, $L_{u}=(1-\widetilde{\phi}) U$ workers fail to find a job and stay unemployed while $L_{q}$ work in the industrial sector. As a result, a fraction $L_{u} /\left(L_{u}+L_{q}\right)$ of workers associated with the frictional labor market are unemployed. Workers employed in the service and industrial sectors together with the unemployed add up to the workforce: $L_{s}+L_{q}+L_{u}=1$.

[^3]Workers who begin a period employed in the industrial sector can continue with their current job unless their employer lays them off or shuts down entirely. In equilibrium, industrial sector workers are paid at least their reservation wage, so those who do not lose their jobs will never leave them voluntarily. Workers' job-seeking decisions and the bargaining game that determines industrial firms' wages will be described below in Sections 2.8 and 2.9, respectively. But before discussing either, we must characterize the firm's problem.

### 2.7 The firm's problem

At the beginning of each period, incumbent firms decide whether to continue operating and potential entrants decide whether to create new firms. Thereafter, active firms go on to choose their employment levels, intermediate input usage, and exporting policies. Entry, exit, hiring, and firing involve adjustment costs, so these decisions are solutions to forward-looking problems. In contrast, intermediate input purchases and exporting decisions involve frictionless static optimization after employment levels have been determined. We now characterize all firm decisions.

### 2.7.1 Export policy

Given the domestic demand function (5), any firm that sells some fraction $1-\eta$ of its output domestically will generate gross home sales amounting to $D_{H}^{\frac{1}{\sigma}}[(1-\eta) q]^{\left(\frac{\sigma-1}{\sigma}\right)}$. Similarly, given the foreign demand function (7), such a firm will generate gross foreign sales of $k\left(D_{F}^{*}\right)^{\frac{1}{\sigma}}\left[\frac{\eta}{\tau_{c}} q\right]^{\frac{\sigma-1}{\sigma}}$. Total gross revenue can thus be written as

$$
\begin{equation*}
G(q, \eta)=\exp \left[d_{H}+d_{F}(\eta)\right] q^{\frac{\sigma-1}{\sigma}} \tag{9}
\end{equation*}
$$

where $d_{H}=\ln \left(D_{H}^{\frac{1}{\sigma}}\right)$ and $d_{F}(\eta)=\ln \left[(1-\eta)^{\frac{\sigma-1}{\sigma}}+k\left(\frac{D_{F}^{*}}{D_{H}}\right)^{\frac{1}{\sigma}}\left(\eta / \tau_{c}\right)^{\frac{\sigma-1}{\sigma}}\right]$. While the term $d_{H}$ measures domestic demand, and is common to all firms, the term $d_{F}(\eta)$ captures the extra revenue generated by exporting, conditional on output.

Given output levels, firms choose their exporting levels each period to maximize their current sales revenues net of fixed exporting costs, $c_{x}$. Not all firms find it profitable to participate in foreign markets, but those that do share the same optimal level of $\eta$ :

$$
\begin{equation*}
\eta^{o}=\arg \max _{0 \leq \eta \leq 1} d_{F}(\eta)=\left(1+\frac{\tau_{c}^{\sigma-1} D_{H}}{k^{\sigma} D_{F}^{*}}\right)^{-1} \tag{10}
\end{equation*}
$$

The associated export market participation policy is

$$
\mathcal{I}^{x}(q)= \begin{cases}1 & \text { if }\left[\exp \left[d_{H}+d_{F}\left(\eta^{o}\right)\right]-\exp \left(d_{H}\right)\right] q^{\frac{\sigma-1}{\sigma}}>c_{x}  \tag{11}\\ 0 & \text { otherwise }\end{cases}
$$

so there is a threshold output level separating exporters from others.

### 2.7.2 Intermediates and the value-added function

Firms determine their output levels by choosing their intermediate input usage, $m$, given their current period $z$ and $l$ values. Optimizing over $m$ and suppressing market-wide variables (including $\eta^{o}$ ), we can thus use (3) and (9) to write value added net of exporting costs as a function of $z$ and $l$ alone:

$$
\begin{equation*}
R(z, l)=\max _{m}\left\{G\left(z l^{\alpha} m^{1-\alpha}\right)-P m-c_{x} \mathcal{I}^{x}\left(z l^{\alpha} m^{1-\alpha}\right)\right\} . \tag{12}
\end{equation*}
$$

As shown in online Appendix 1, the solution to this optimization problem is:

$$
\begin{equation*}
R(z, l)=\Delta(z, l)\left(z l^{\alpha}\right)^{\Lambda}-c_{x} \mathcal{I}^{x}(z, l) \tag{13}
\end{equation*}
$$

where

$$
\begin{equation*}
\Delta(z, l)=\Theta P^{-(1-\alpha) \Lambda} \exp \left[d_{H}+\mathcal{I}^{x}(z, l) d_{F}\left(\eta^{o}\right)\right]^{\frac{\sigma}{\sigma-1} \Lambda} \tag{14}
\end{equation*}
$$

$\Lambda=\frac{\sigma-1}{\sigma-(1-\alpha)(\sigma-1)}>0$, and $\Theta=\left(\frac{1}{(1-\alpha) \Lambda}\right)\left[\frac{(1-\alpha)(\sigma-1)}{\sigma}\right]^{\frac{\sigma}{\sigma-1} \Lambda}>0$.
The term $\Delta(z, l)$ is a firm-level market size index. It responds to anything that affects aggregate domestic demand $\left(D_{H}\right)$, trade $\operatorname{costs}\left(\tau_{c}\right)$, or the exchange rate $(k)$. But given these market-wide variables, the only source of cross-firm variation in $\Delta(z, l)$ is exporting status $\left(\mathcal{I}^{x}\right)$. Accordingly, below we suppress the arguments of $\Delta$ except where we wish to emphasize its dependence on these variables.

### 2.7.3 Employment policy

We now turn to decisions that involve forward-looking behavior. When choosing employment levels, firms weigh the revenue stream implied by (13) against wage costs, the effects of $l$ on their continuation value, and current firing or hiring costs. To characterize the latter, let the cost of posting $v$ vacancies for a firm of size $l$ be

$$
C_{h}(l, v)=\left(\frac{c_{h}}{\lambda_{1}}\right)\left(\frac{v}{l^{\lambda_{2}}}\right)^{\lambda_{1}},
$$

where $c_{h}$ and $\lambda_{1}>1$ are positive parameters. ${ }^{9}$ The parameter $\lambda_{2} \in[0,1]$ determines the strength of scale economies in hiring. If $\lambda_{2}=0$, there are no economies of scale and the cost of posting $v$ vacancies is the same for all firms. On the other hand, if $\lambda_{2}=1$, the cost of a given employment growth rate is the same for all firms.

All firms in our model are large enough that cross-firm variation in realized worker arrival rates is ignorable. That is, all firms fill the same fraction $\phi$ of their posted vacancies. It follows that expansion from $l$ to $l^{\prime}$ simply requires the posting of $v=\left(l^{\prime}-l\right) / \phi$ vacancies, and we can write the cost of expanding from $l$ to $l^{\prime}$ workers as

$$
\begin{equation*}
C_{h}\left(l, l^{\prime}\right)=\left(\frac{c_{h}}{\lambda_{1}}\right) \phi^{-\lambda_{1}}\left(\frac{l^{\prime}-l}{l^{\lambda_{2}}}\right)^{\lambda_{1}} . \tag{15}
\end{equation*}
$$

Clearly, when labor markets are slack, hiring is less costly because each vacancy is more likely to be filled. Also, for $\lambda_{1}\left(1-\lambda_{2}\right)>1$, a given level of employment growth is more costly per worker for larger firms. ${ }^{10}$ So, other things equal, $\lambda_{1}\left(1-\lambda_{2}\right)>1$ means that larger firms expand relatively slowly in response to positive shocks, and they have stronger incentives to hoard labor in the face of transitory negative shocks. This feature of our model is the main reason it is able to replicate the well-known association between firm size and job stability.

Downward employment adjustments are also costly. When a firm reduces its workforce from $l^{\prime}$ to $l \geq l_{e}$, it incurs firing costs proportional to the number of workers shed: ${ }^{11}$

$$
\begin{equation*}
C_{f}\left(l, l^{\prime}\right)=c_{f}\left(l-l^{\prime}\right) . \tag{16}
\end{equation*}
$$

For convenience we assume hiring and firing costs are incurred in terms of service goods, and we describe both with the adjustment cost function:

$$
C\left(l, l^{\prime}\right)= \begin{cases}C_{h}\left(l, l^{\prime}\right) & \text { if } l^{\prime}>l \\ C_{f}\left(l, l^{\prime}\right) & \text { otherwise }\end{cases}
$$

Several observations concerning adjustment costs are in order. First, while convex hiring costs induce firms to expand gradually, firms cannot economize on firing costs by downsizing gradually. Second, when a firm exits, it is not liable for $c_{f}$. Finally, as will be discussed below, it is possible that a firm will find itself in a position where the marginal worker reduces operating profits, but it is more costly to fire her than retain her.

[^4]

Figure 1: Within-Period Sequencing of Events for Firms

Regardless of whether a firm expands, contracts, or remains at the same employment level, we assume it bargains with each of its workers individually and continuously. This implies that bargaining is over the marginal product of labor, and all workers at a firm in a particular state ( $z, l$ ) are paid the same wage (Stole and Zwiebel 1996; Cahuc and Wasmer 2001; Cahuc, Marque, and Wasmer 2008). Moreover, the marginal worker at an expanding firm generates rents, while the marginal worker at a contracting firm does not (Bertola and Caballero 1994; Bertola and Garibaldi 2001; Koeniger and Prat 2007). Hence expanding firms face different wage schedules than others. These schedules depend upon firms' states, so we denote the wage schedule paid by a hiring firm as $w_{h}(z, l)$ and the wage schedule paid by a non-hiring firm as $w_{f}(z, l)$. Details are deferred to Section 2.9 below.

We now elaborate firms' optimal employment policies within a period (see Figure 1). An incumbent firm enters the current period with the productivity level and work force ( $z, l$ ) determined in the previous period. Thereupon it may exit immediately, either because the expected present value of its profit stream is negative, or because it is hit with an exogenous death shock.

If a firm opts to stay active and is not hit with an exogenous exit shock, it proceeds to an interim stage in which it observes its current-period productivity realization $z^{\prime}$. Then, taking stock of its updated state, $\left(z^{\prime}, l\right)$, the relevant wage schedules, and adjustment costs $C\left(l, l^{\prime}\right)$, it chooses its current period work force, $l^{\prime}$. Both hiring and firing decisions take immediate effect and it enters the end of the period with $\left(z^{\prime}, l^{\prime}\right)$, making optimal intermediate usage and exporting decisions based on its new state. Profits are realized and wages are paid at this point. Depending on whether the firm is hiring or not, its profits are

$$
\pi\left(z^{\prime}, l, l^{\prime}\right)=\left\{\begin{array}{lr}
R\left(z^{\prime}, l^{\prime}\right)-w_{h}\left(z^{\prime}, l^{\prime}\right) l^{\prime}-C\left(l, l^{\prime}\right)-c_{p} \quad \text { if } l^{\prime}>l  \tag{17}\\
R\left(z^{\prime}, l^{\prime}\right)-w_{f}\left(z^{\prime}, l^{\prime}\right) l^{\prime}-C\left(l, l^{\prime}\right)-c_{p} & \text { otherwise }
\end{array}\right.
$$

where $c_{p}$, the per-period fixed cost of operation, is common to all firms.
Firms discount the future at the same rate $(1+r)$ as consumers. So the beginning-ofperiod value of a firm in state $(z, l)$ is

$$
\begin{equation*}
\mathcal{V}(z, l)=\max \left\{0, \quad \frac{1-\delta}{1+r} E_{z^{\prime} \mid z} \max _{l^{\prime}}\left[\pi\left(z^{\prime}, l, l^{\prime}\right)+\mathcal{V}\left(z^{\prime}, l^{\prime}\right)\right]\right\}, \tag{18}
\end{equation*}
$$

where $\delta$ is the probability of an exogenous death shock, and the maximum of the term in square brackets is the value of the firm in the interim state, after it has realized its productivity shock. All payoffs are discounted at the interim period.

The solution to (18) implies an employment policy function,

$$
\begin{equation*}
l^{\prime}=L\left(z^{\prime}, l\right) \tag{19}
\end{equation*}
$$

an indicator function $\mathcal{I}^{h}\left(z^{\prime}, l\right)$ that distinguishes hiring and firing firms, and an indicator function $\mathcal{I}^{c}(z, l)$ that characterizes firms' continuation and exit policy. $\mathcal{I}^{h}\left(z^{\prime}, l\right)$ and $\mathcal{I}^{c}(z, l)$ take the value one for firms that are hiring or continuing, respectively, and zero otherwise.

### 2.7.4 Entry

In the steady state, a constant fraction of firms exits the industry either endogenously or exogenously. These firms are replaced by an equal number of entrants, who find it optimal to pay a sunk entry cost of $c_{e}$ and create new firms. The cost of hiring the initial workforce, i.e., posting $l_{e} / \phi$ vacancies, is included in $c_{e}$, along with fixed capital costs. Entrants also draw their initial productivity level from the ergodic productivity distribution implied by (8), hereafter denoted as $\psi_{e}(z)$. Thereafter they behave exactly like incumbent firms, with interim state given by $\left(z, l_{e}\right)$ (see Figure 1). If they wish, they can further adjust their workforce to $l^{\prime}>l_{e}$ in accordance with their initial productivity by posting more vacancies subject to the hiring cost function (15). Free entry implies that

$$
\begin{equation*}
\mathcal{V}_{e}=\frac{1}{1+r} \int_{z} \max _{l^{\prime}}\left[\pi\left(z, l_{e}, l^{\prime}\right)+\mathcal{V}\left(z, l^{\prime}\right)\right] \psi_{e}(z) d z \leq c_{e} \tag{20}
\end{equation*}
$$

which holds with equality if there is a positive mass of entrants. We assume that each workerconsumer owns equal shares in a diversified fund that collects profits from firms, finances entry, and redistributes the residual as dividends to its owners.

### 2.8 The worker's problem

Figure 2 presents the intra-period timing of events for workers. Consider first a worker who is employed by an industrial firm in state $(z, l)$ at the beginning of the current period. This worker learns immediately whether her firm will continue operating. If it shuts down, she joins the pool of industrial job seekers (enters state $u$ ) in the interim stage. Otherwise, she enters the interim stage as an employee of the same firm she worked for in the previous period. Her firm then realizes its new productivity level $z^{\prime}$ and enters the interim state $\left(z^{\prime}, l\right)$. At this point her firm decides whether to hire workers. If it expands its workforce to $l^{\prime}>l$, she earns $w_{h}\left(z^{\prime}, l^{\prime}\right)$, and she is positioned to start the next period at a firm in state $\left(z^{\prime}, l^{\prime}\right)$. If the firm contracts or remains at the same employment level, she either loses her job and reverts to state $u$ or she retains her job, earns $w_{f}\left(z^{\prime}, l^{\prime}\right)$, and starts the next period at a firm in state $\left(z^{\prime}, l^{\prime}\right)$. All workers at contracting firms are equally likely to be laid off, so each loses her job with probability $\left(l-l^{\prime}\right) / l$.

Workers in state $u$ are searching for industrial jobs. They are hired by entering and expanding firms that post vacancies. If they are matched with a firm, they receive the same wage as those who were already employed by the firm. If they are not matched, they support themselves by home-producing $b<1$ units of the service good. At the start of the next period, they can choose to work in the service sector (enter state $s$ ) or search for a job in the industrial sector (remain in state $u$ ). Likewise, workers who start the current period in the service sector choose between continuing to work at the service wage $w_{s}=1$ and entering the pool of industrial job seekers. These workers are said to be in state $o$.

We now specify the value functions for the workers in the interim stage. Going into the service sector generates an end-of-period income of 1 and returns a worker to the $o$ state at the beginning of the next period. Accordingly, the interim value of this choice is

$$
\begin{equation*}
J^{s}=\frac{1}{1+r}\left(1+J^{o}\right) . \tag{21}
\end{equation*}
$$

Searching in the industrial sector exposes workers to the risk of spending the period unemployed, and supporting themselves by home-producing $b$ units of the service good. But it also opens the possibility of landing in a high-value industrial job. Since the probability of finding a match is $\widetilde{\phi}$, the interim value of searching for an industrial job is

$$
\begin{equation*}
J^{u}=\left[\widetilde{\phi} E J_{h}^{e}+\frac{(1-\widetilde{\phi})}{1+r}\left(b+J^{o}\right)\right], \tag{22}
\end{equation*}
$$

where $E J_{h}^{e}$ is the expected value of matching with a hiring firm, to be defined below.
The value of the sectorial choice is $J^{o}=\max \left\{J^{s}, J^{u}\right\}$. In an equilibrium with both sectors


Figure 2: Within-Period Sequencing of Events for Workers
in operation, workers must be indifferent between them, so $J^{o}=J^{s}=J^{u}$. Combined with (21), this condition implies that $J^{o}, J^{s}$, and $J^{u}$ are all equal to $1 / r$.

The expected value of matching with an industrial job, $E J_{h}^{e}$, depends on the distribution of hiring firms and the value of the jobs they offer. For workers who match with a hiring firm in the interim state $\left(z^{\prime}, l\right)$, the interim period value is given by

$$
\begin{equation*}
J_{h}^{e}\left(z^{\prime}, l\right)=\frac{1}{1+r}\left[w_{h}\left(z^{\prime}, l^{\prime}\right)+J^{e}\left(z^{\prime}, l^{\prime}\right)\right] \tag{23}
\end{equation*}
$$

where $l^{\prime}=L\left(z^{\prime}, l\right)$ and $J^{e}\left(z^{\prime}, l^{\prime}\right)$ is the value of being employed at an industrial firm in state $\left(z^{\prime}, l^{\prime}\right)$ at the start of the next period. Accordingly, the expected value of a match for a worker as perceived at the interim stage is

$$
\begin{equation*}
E J_{h}^{e}=\int_{z^{\prime}} \int_{l} J_{h}^{e}\left(z^{\prime}, l\right) g\left(z^{\prime}, l\right) d l d z^{\prime} \tag{24}
\end{equation*}
$$

where $g\left(z^{\prime}, l\right)=\tilde{g}\left(z^{\prime}, l\right) / \int_{z^{\prime \prime}} \int_{x} \tilde{g}\left(z^{\prime \prime}, x\right) d x d z^{\prime \prime}$ is the distribution of vacancies across hiring firms and

$$
\tilde{g}\left(z^{\prime}, l\right)=v\left(z^{\prime}, l\right) \tilde{\psi}\left(z^{\prime}, l\right)+\mathcal{I}_{l=l_{e}} \cdot \frac{N_{e}}{N-N_{F}} l_{e} / \phi
$$

Here $v\left(z^{\prime}, l\right)=I^{h}\left(z^{\prime}, l\right)\left[L\left(z^{\prime}, l\right)-l\right] / \phi$ is the number of vacancies posted by firms in state $\left(z^{\prime}, l\right), \tilde{\psi}\left(z^{\prime}, l\right)$ is the distribution of firms in interim state $\left(z^{\prime}, l\right)$ (see online Appendix 3 ), $I_{l=l_{e}}$
is an indicator variable for $l=l_{e}, l_{e} / \phi$ is the number of vacancies posted by entrants to hire their initial workforce, and $\frac{N_{e}}{N-N_{F}}$ is the fraction of firms that are new entrants.

It remains to specify the value of starting the period matched with an industrial firm, $J^{e}(z, l)$, which appears in (23) above. The value of being at a firm that exits immediately (exogenously or endogenously) is simply the value of being unemployed, $J^{u}$. This is also the value of being at a non-hiring firm, since workers at these firms are indifferent between being fired and retained. Hence $J^{e}(z, l)$ can be written as

$$
\begin{align*}
J^{e}(z, l)=[\delta & \left.+(1-\delta)\left(1-\mathcal{I}^{c}(z, l)\right)\right] J^{u} \\
& +(1-\delta) \mathcal{I}^{c}(z, l) \max \left\{J^{u}, E_{z^{\prime} \mid z}\left[\mathcal{I}^{h}\left(z^{\prime}, l\right) J_{h}^{e}\left(z^{\prime}, l\right)+\left(1-\mathcal{I}^{h}\left(z^{\prime}, l\right)\right) J^{u}\right]\right\} . \tag{25}
\end{align*}
$$

### 2.9 Wage schedules

We now characterize the wage schedules. Consider first a hiring firm. After vacancies have been posted and matching has taken place, the labor market closes. Firms then bargain with their workers simultaneously and on a one-to-one basis, treating each worker as the marginal one. At this point, vacancy posting costs are already sunk and workers who walk away from the bargaining table cannot be replaced in the current period. Similarly, if an agreement between the firm and the worker is not reached, the worker remains unemployed in the current period. These timing assumptions create rents to be split between the firm and the worker.

As detailed in online Appendix 2, it follows that the wage schedule for hiring firms with an end-of-period state $\left(z^{\prime}, l^{\prime}\right)$ is given by

$$
\begin{equation*}
w_{h}\left(z^{\prime}, l^{\prime}\right)=(1-\beta) b+\frac{\beta}{1-\beta+\alpha \beta \Lambda} \underbrace{\Delta\left(z^{\prime}, l^{\prime}\right) \alpha \Lambda\left(z^{\prime}\right)^{\Lambda}\left(l^{\prime}\right)^{\alpha \Lambda-1}}_{=\partial R\left(z^{\prime}, l^{\prime}\right) / \partial l^{\prime}} \tag{26}
\end{equation*}
$$

where $\beta \in[0,1]$ measures the bargaining power of the worker. Workers in expanding firms get their share of the marginal product of labor plus $(1-\beta)$ share of their outside option.

The marginal worker at a non-hiring firm generates no rents, so the firing wage just matches her reservation value (see online Appendix 2):

$$
\begin{equation*}
w_{f}\left(z^{\prime}, l^{\prime}\right)=r J^{u}-\left[J^{e}\left(z^{\prime}, l^{\prime}\right)-J^{u}\right] . \tag{27}
\end{equation*}
$$

Three assumptions lie behind this formulation. First, workers who quit do not trigger firing costs for their employers. Second, firms cannot use mixed strategies when bargaining with workers. Finally, fired workers are randomly chosen. The first assumption ensures that workers at contracting firms are paid no more than the reservation wage, and the remaining
assumptions prevent firms from avoiding firing costs by paying less than reservation wages to those workers they wish to shed. Importantly, $w_{f}(z, l)$ does vary across firing firms, since workers who continue with such firms may enjoy higher wages in the next period. This option to continue has a positive value, captured by the bracketed term in (27), so firing firms may pay their workers less than the flow value of being unemployed.

### 2.10 Equilibrium

Six basic conditions characterize the equilibrium. First, the distribution of firms over $(z, l)$ states in the interim and at the end of each period, denoted by $\widetilde{\psi}(z, l)$ and $\psi(z, l)$, respectively, reproduce themselves each period through the stochastic process on $z$, the policy functions, and the productivity draws that firms receive upon entry. Second, all markets clear: supply matches demand for services and for each differentiated good, where supplies are determined by employment and productivity levels in each firm. Third, the flow of workers into unemployment matches the flow of workers out of unemployment-that is, the Beveridge condition holds. Fourth, a positive mass of entrants replaces exiting firms every period so that free entry condition (20) holds with equality. Fifth, aggregate income matches aggregate expenditure, so trade is balanced. Finally, workers optimally choose the sector in which they are working or seeking work. Online Appendix 3 provides further details.

### 2.11 Discussion

Determinants of turnover and wages, and unemployment - Before moving on to quantitative analysis, it is instructive to summarize some key mechanisms in our model. We begin with the determinants of cross-firm variation in wages and job turnover, both of which are driven by interactions between productivity shocks and convex vacancy posting costs. To fix ideas, consider what would happen if, at some point in time, we were to set $\rho=1$ and $\sigma_{z}=0$, making the current set of productivity draws permanent. Thereafter, those firms with sufficiently high draws would gradually add workers, and they would exhibit relatively high wages as they expanded toward their long run desired size. Upon reaching this size, the rents from additional matches would disappear, and they would begin to pay reservation wages, along with all stationary or contracting firms. ${ }^{12}$ But during the transition, large firms would tend to be hiring and thus would tend to pay higher wages.

[^5]Now consider the unrestricted model with ongoing productivity shocks ( $\sigma_{z}>0$ ). These shocks introduce steady state job turnover by ensuring that some portion of the firm population is always in transition. Further, with sufficiently high persistence in productivity ( $\rho$ sufficiently close to 1 ), this turnover keeps enough large firms expanding to sustain a positive cross-sectional correlation between employment and wages (Bertola and Garibaldi 2001). Finally, since vacancy posting costs per worker vary with employment (refer to equation 15), the rates at which firms adjust their jobs and wages in response to shocks depend upon their size.

It remains to describe the determinants of the $Q$-sector unemployment rate, $L_{u} /\left(L_{u}+L_{q}\right)$. Given the wage distribution, this rate follows from the arbitrage condition $J^{s}=J^{u}$. That is, in order to keep each sector equally attractive to job seekers, increases in the expected value of a $Q$-sector job, $E J_{h}^{e}$, must be accompanied by offsetting reductions in the probability of finding one, just as in Harris and Todaro (1970).

Firing cost effects - We turn now to the effects of policy reforms and changes in openness, beginning with firing costs. Other things equal, lower firing costs induce firms to carry more workers by reducing the expected costs of shedding them later (Hopenhayn and Rogerson 1993). Further, low firing costs help large firms relatively more, since they are relatively unlikely to exit when they downsize in response to a negative productivity shock. (Exiting firms are unaffected by firing costs because they are not required to pay them.) For both reasons, reductions in $c_{f}$ shift the firm size distribution rightward.

By itself, this shift would reduce job turnover through the distribution effect mentioned in the introduction. But other forces are also in play. Most notably, holding the size distribution fixed, lower firings costs make firms' employment levels more volatile by encouraging larger adjustments to transitory shocks (e.g., Ljungqvist 2002; Mortensen and Pissarides 1999). Accordingly, the turnover effects of reductions in $c_{f}$ are ambiguous.

When a reduction in $c_{f}$ shifts the firm size distribution rightward, it induces a new distribution for the marginal revenue product of labor. This affects the wage distribution through (26) and (27), which in turn affect $E J_{h}^{e}$, the job filling rate, $\phi$, and the $Q$-sector unemployment rate. Finally, adjustments in $\phi$ feed back onto vacancy posting costs through equation (15), further confounding the net effect of firing cost reductions on job turnover.

Openness effects - What about reductions in $\tau_{c}$ or $\tau_{a}$ ? Increases in openness drive up the returns to exporting while intensifying import competition. As Melitz (2003) stresses, large firms benefit on net, since they find it worthwhile to pay the fixed costs of exporting, but other firms are hurt. Accordingly, the gap between wages at large and small firms increases, while the right-hand tail of the firm size distribution shifts outward.

The effects of openness on job turnover are more subtle. Partly, turnover adjustments reflect the sensitivity effect mentioned in the introduction: Greater openness increases the
market size index $\Delta$ for high-productivity firms, so these firms' value-added functions (13) become steeper with respect to employment levels, and they make larger workforce adjustments in response to $z$ shocks. ${ }^{13}$ So long as this heightened volatility among productive firms dominates the reductions in volatility that occur at unproductive firms (which experience $\Delta$ reductions with openness), the sensitivity effect tends to increase job turnover. A second linkage comes through adjustments in labor market tightness. To the extent that openness increases the expected value of industrial sector jobs, it induces offsetting increases in the unemployment rate, and these reduce the cost of posting vacancies (15) through $\phi$, creating further volatility. ${ }^{14}$ Finally, however, openness concentrates workers at larger firms, which tend to be relatively stable. This distribution effect works against the others, making turnover adjustments ambiguous.

Exporters - We conclude this section with a brief discussion of exporters' characteristics and their determinants. Firms expanding into export markets experience a discrete jump in demand, which causes their prices to rise and increases the marginal revenue product of their workers. Thus, our model is consistent with the common finding that exporters tend to pay their workers better (e.g., Bernard and Jensen 1995), and it falls into the "wider class of models in which wages are increasing in firm revenue and there is selection into export markets" (Helpman, Itskhoki, and Redding 2010, p. 1240). ${ }^{15}$ However, unlike others in this class, our formulation implies exporters will eventually drive down the marginal revenue product of their workers as they expand. It thus delivers a distinctive explanation for the finding that long term exporters do not routinely pay higher wages than non-exporters

[^6](Bernard and Jensen 1999).
Implications for exporters' mark-ups and productivity also obtain. First, the higher average prices charged by new exporters explain de Loecker and Warzynski's (2012) finding of higher mark-ups among Slovenian exporting firms. Frictionless trade models with CES preferences cannot explain this result because firms in these models freely expand or contract until their mark-ups are the same. Second, since exporters' higher prices translate into higher revenue per unit input bundle, our model provides a pricing-based explanation for the common finding that exporters enjoy higher revenue productivity.

## 3 Quantitative Analysis

### 3.1 Pre- and post-reform conditions in Colombia

To explore the quantitative implications of our model, we fit it to Colombian data. This country suits our purposes for several reasons. First, Colombia underwent a significant trade liberalization during the late 1980s and early 1990s, reducing its average nominal tariff rate from 21 percent to 11 percent (Goldberg and Pavcnik 2004). Second, Colombia also implemented labor market reforms in 1991 that substantially reduced firing costs. According to Heckman and Pages (2000), the average cost of dismissing a worker fell from an equivalent of 6-7 months' wages in 1990 to 3 months' wages in 1999. Finally, major changes in Colombian trade volumes and labor markets followed these reforms, suggesting that they and/or external reductions in trade frictions may well have been important.

Key features of the Colombian economy during the pre- and post-reform period are summarized in Figure 3. The first panel shows the fraction of manufacturing establishments that were exporters, as well as the aggregate revenue share of exports. Before 1991, about 12 percent of all plants were exporters on average, and total exports accounted for 9 percent of aggregate manufacturing revenues. Reflecting the globalization of the Colombian economy, both ratios increased by about 250 percent from the 1980s to the 2000s. The second panel shows manufacturing job turnover due to entry, exit, and changing employment levels among continuing producers. This series went from an average of 18.1 percent during the pre-reform period 1981-1990 to 23 percent during the post-reform period 1993-1998. (Unfortunately, post-2000 turnover figures are unavailable.)

The third panel of Figure 3 shows the evolution of the urban unemployment rate. During the post-reform years 1991-1998, this series hovered around its 1981-90 average of 10.8 percent. During 2000-2006, its average was a somewhat higher 13 percent, but this increase mainly reflected a financial crisis at the end of the 1990s. The fourth panel shows that after reforms, the manufacturing share of urban employment dropped from 24 percent in

Figure 3: Colombian Aggregates


Notes: In all panels, 1991 is marked as the reform year. See text for details about variables. Missing data points were unavailable. Data sources for each panel in clockwise order: 1) DANE Annual Survey of Manufacturers. Pre-1991 series are based on own calculations from the micro-data, post-1991 series has been obtained from DANE; 2) Inter-American Development Bank (2004). See footnote 24 for the definition of job turnover; 3) International Monetary Fund (2011); 4) Blue line: urban employment share of Manufacturing (ISIC-Rev.2 code 3), black line: combined employment shares of Wholesale and Retail Trade and Restaurants and Hotels, and Community, Social and Personal Services (ISIC-Rev. 2 codes 6 and 9), ILOSTAT Database; 5) Approximate numbers based on Figure 3.1 in Mondragón-Vélez and Pena (2010), Colombian Household Survey (Encuesta Nacional de Hogares, ENH); 6) World Bank (2013).

1991 to roughly 20 percent in 2000. The corresponding increase in the service sector was largely driven by two sub-sectors, both of which exhibit a high level of self-employment: wholesale-retail trade and personal services (Mondragón-Vélez and Pena 2010). So the sustained increase in the share of self-employed urban workers (fifth panel) can be taken as a sign of weakening demand in formal labor markets. Finally, the sixth panel shows that over the same time period, the Gini coefficient for Colombia rose from roughly 53 percent to roughly 58 percent.

These aggregate trends were accompanied by a dramatic shift in the plant size distribution. Figure 4 shows the size distribution of manufacturing plants in the 1980s (black bars) and 2000s (white bars). The average size increased from 45 to 60 workers, and the proportion of plants with more than 100 workers increased from 15 percent to 22 percent.

In sum, Colombia experienced a significant shift in its manufacturing plant size distribution and an overall decline in manufacturing employment. Manufacturing jobs also became more unstable as job turnover rates increased. Unemployment, self-employment, and wage inequality also increased. We now investigate how, in the context of our model, these changes might be linked to the changes in tariffs, firing costs and foreign market conditions that Colombian firms experienced during the late 1980s and early 1990s.

### 3.2 Fitting the model to the data

Model Period - Assuming that Colombia was in a steady state prior to reforms, we fit our model to annual data from 1981-1990. In doing so, we treat all plants as single-plant firms. Also, in order to exploit "control function" techniques (discussed below), we match the periodicity of our model to the periodicity of our data. This means imposing that unemployment spells occur in one-year increments, which is longer than some calibrated labor search models for the U.S. economy have presumed (e.g., Elsby and Michaels 2013; Cooper, Haltiwanger and Willis 2007). Nonetheless, since the average unemployment spell in urban Colombian labor markets is around 11 months (page 16 in Medina, Nunez, and Tamayo 2013), this drawback does not strike us as critical. We will return to the issue of periodicity when we discuss the robustness of our findings.

Parameters Not Estimated - Several parameters are not identified by the model; these we take from external sources. The real borrowing rate in Colombia fluctuated around 15 percent between the late 1980s and early 2000s, so we set $r=0.15$ (Bond, Utar, and Tybout, forthcoming). The average share of services in Colombian GDP during the sample period was 0.49 , so this is our estimate for $\gamma{ }^{16}$ Heckman and Pages (2000) estimate that dismissal costs amounted to $6-7$ months' wages in 1990 (their Figure 1), so we fix firings costs at $c_{f}=0.6$ in the benchmark economy. ${ }^{17}$ Eaton and Kortum (2002) estimate that the tariff equivalent of iceberg costs falls between 123 percent and 174 percent, so we choose our prereform value of $\tau_{c}-1$ to be 1.50. Finally, we take our estimate of the pre-reform nominal tariff rate, $\tau_{a}-1=0.21$, from Goldberg and Pavcnik (2004).

The Estimator - This leaves us with 16 parameters to estimate, collected in the vector

$$
\boldsymbol{\Omega}=\left(\sigma, \alpha, \rho, \sigma_{z}, \beta, \theta, \delta, \lambda_{1}, \lambda_{2}, b, l_{e}, c_{h}, c_{p}, c_{x}, c_{e}, D_{F}^{*}\right)
$$

These we estimate using the method of simulated moments (Gouriéroux and Monfort 1996). Specifically, let $\overline{\mathbf{m}}$ be a vector of sample statistics that our model is designed to explain and define $\mathbf{m}(\boldsymbol{\Omega})$ as the vector of model-based counterparts to these sample statistics. Our estimator is then given by

$$
\widehat{\boldsymbol{\Omega}}=\arg \min (\overline{\mathbf{m}}-\mathbf{m}(\boldsymbol{\Omega}))^{\prime} \widehat{\mathbf{W}}(\overline{\mathbf{m}}-\mathbf{m}(\boldsymbol{\Omega})),
$$

[^7]Table 1: Data-based versus Simulated Statistics

|  | Data | Model | Data | Model |  |
| :--- | :---: | :---: | :--- | :---: | :---: |
| First and second moments |  |  | Size distribution |  |  |
| $E\left(\ln G_{t}\right)$ | 5.442 | 5.274 | 20th percentile cutoff | 14.617 | 15.087 |
| $E\left(\ln l_{t}\right)$ | 3.622 | 3.638 | 40th percentile cutoff | 24.010 | 24.736 |
| $E\left(\mathcal{I}_{t}^{x}\right)$ | 0.118 | 0.108 | 60th percentile cutoff | 41.502 | 42.559 |
| $\operatorname{var}\left(\ln G_{t}\right)$ | 2.807 | 3.334 | 80th percentile cutoff | 90.108 | 87.137 |
| $\operatorname{cov}\left(\ln G_{t}, \ln l_{t}\right)$ | 1.573 | 1.888 | Firm growth rates |  |  |
| $\operatorname{var}\left(\ln l_{t}\right)$ | 1.271 | 1.326 | $<20$ th percentile | 1.425 | 1.287 |
| $\operatorname{cov}\left(\ln G_{t}, \mathcal{I}_{t}^{x}\right)$ | 0.230 | 0.264 | 20th-40th percentile | 0.255 | 0.251 |
| $\operatorname{cov}\left(\ln l_{t}, \mathcal{I}_{t}^{x}\right)$ | 0.153 | 0.175 | 40th-60th percentile | 0.209 | 0.191 |
| $\operatorname{cov}\left(\ln G_{t}, \ln G_{t+1}\right)$ | 2.702 | 2.119 | 60th-80th percentile | 0.184 | 0.155 |
| $\operatorname{cov}\left(\ln G_{t}, \ln l_{t+1}\right)$ | 1.538 | 1.534 | Aggregate turnover/ |  |  |
| $\operatorname{cov}\left(\ln G_{t}, \mathcal{I}_{t+1}^{x}\right)$ | 0.225 | 0.283 | Wage dispersion |  |  |
| $\operatorname{cov}\left(\ln l_{t}, \ln G_{t+1}\right)$ | 1.543 | 1.409 | Firm exit rate | 0.108 | 0.104 |
| $\operatorname{cov}\left(\ln l_{t}, \ln l_{t+1}\right)$ | 1.214 | 1.192 | Job turnover | 0.197 | 0.222 |
| $\operatorname{cov}\left(\ln l_{t}, \mathcal{I}_{t+1}^{x}\right)$ | 0.152 | 0.195 | Std. dev. of log wages | 0.461 | 0.380 |
| $\operatorname{cov}\left(\mathcal{I}_{t}^{x}, \ln G_{t+1}\right)$ | 0.220 | 0.273 | Olley-Pakes statistics |  |  |
| $\operatorname{cov}\left(\mathcal{I}_{t}^{x}, \ln l_{t+1}\right)$ | 0.149 | 0.200 | (1- $\alpha)\left(\frac{\sigma-1}{\sigma}\right)$ | 0.685 | 0.685 |
| $\operatorname{cov}\left(\mathcal{I}_{t}^{x}, \mathcal{I}_{t+1}^{x}\right)$ | 0.090 | 0.075 | $d_{F}$ | 0.090 | 0.094 |

Notes: All data-based statistics are calculated using Colombian plant-level panel data for the pre-liberalization period, 198190. These data were collected by the Colombian National Administrative Department of Statistics (DANE) in its Annual Manufacturer Survey (EAM), which covers all establishments with at least 10 workers.
where $\widehat{\mathbf{W}}$ is a bootstrapped estimate of $[\operatorname{var}(\overline{\mathbf{m}})]^{-1}$ with off-diagonal elements set to zero. ${ }^{18}$
The Sample Statistics - The vector $\overline{\mathbf{m}}$ and the associated weighting matrix are based on plant-level panel data from Colombia. These data are annual observations on all manufacturing plants with at least 10 workers, covering the 1981-1990 period. ${ }^{19}$

Table 1 lists the elements of $\overline{\mathbf{m}}$, grouped according to the type of information they convey. The first group consists of means, variances, and covariances for the vector $\left(\ln l_{t}, \ln G_{t}, \mathcal{I}_{t}^{x}, \ln l_{t+1}\right.$, $\left.\ln G_{t+1}, \mathcal{I}_{t+1}^{x}\right) \cdot{ }^{20}$ Gross revenues $G$ are gross sales, expressed in thousands of 1977 pesos. The indicator $\mathcal{I}^{x}$ takes a value of unity for those plant-year observations with positive exports. Finally, since workers are all identical in the model economy, we control for the effects of worker heterogeneity on output by measuring the labor input $l$ in terms of "effective worker" units (see online Appendix 5.1 for details).

The second and third groups of moments in $\overline{\mathbf{m}}$ include quintiles of the plant size dis-

[^8]tribution and the average rate of employment growth among expanding plants within each size category, respectively. We target employment growth among expanding plants because, given linear firing costs, contractions are not gradual in the model. Quintiles are based on effective employment levels, $l$, and constructed using the pooled panel of plants. ${ }^{21}$ Employment growth rates for quintile $j$ are constructed as cross-plant averages of $\left(l_{t+1}-l_{t}\right) /\left[\frac{1}{2}\left(l_{t+1}+l_{t}\right)\right]$, including only expanding plants that were in quintile $j$ at the beginning of the period. New plants are included in these growth rates, and are treated as having an initial employment of zero.

The fourth group in $\overline{\mathbf{m}}$ contains aggregate statistics for the pooled sample of plants. These include the job turnover rate, the plant exit rate, and the standard deviation in effective wages. Job turnover is a cross-year average of the annual turnover rate, net of aggregate employment growth or contraction. ${ }^{22}$ The plant exit rate is the fraction of plants that exit the panel in year $t$, averaged over the 10-year sample period. Finally, the standard deviation in effective wages is constructed as the cross-plant standard deviation of the log of real payments to labor (wages and benefits) per effective worker. Given that our measure of effective workers has been adjusted for workforce composition, this measure of wage dispersion controls for observable worker characteristics to the extent possible. Unavoidably, it partly reflects variation in unobservable worker characteristics. But this latter source of noise is averaged across individual workers within a firm, and thus is hopefully relatively modest. ${ }^{23}$

The last two elements of $\overline{\mathbf{m}}$ are not simple descriptive statistics. Rather, they are samplebased estimates of $\left(\frac{\sigma-1}{\sigma}\right)(1-\alpha)$ and $d_{F}$ obtained by applying the logic of Olley and Pakes (1996) to the gross revenue function. By including these statistics in the moment vector rather than treating them as fixed parameters when estimating $\boldsymbol{\Omega}$, we recognize the effects

[^9]of their sampling error on $\widehat{\boldsymbol{\Omega}} .{ }^{24}$
Our approach to estimating these two statistics merits further explanation. By (3) and (9), gross revenues before fixed exporting costs can be written as
\[

$$
\begin{equation*}
\ln G_{i t}=d_{H}+\mathcal{I}_{i t}^{x} d_{F}\left(\eta_{0}\right)+\left(\frac{\sigma-1}{\sigma}\right)\left[\ln z_{i t}+\alpha \ln l_{i t}+(1-\alpha) \ln m_{i t}\right] \tag{28}
\end{equation*}
$$

\]

Also, among firms that adjust their employment levels, the policy function $l^{\prime}=L\left(z^{\prime}, l\right)$ can be inverted to express $z^{\prime}$ as a monotonic function of $l^{\prime}: \ln z^{\prime}=g\left(\ln l, \ln l^{\prime}\right)$. This "control function" allows us to eliminate $z$ from (28):

$$
\begin{equation*}
\ln G_{i t}=\tilde{d}_{H}+\mathcal{I}_{i t}^{x} d_{F}\left(\eta_{0}\right)+\left[\frac{\sigma-1}{\sigma}(1-\alpha)\right] \ln \left(P m_{i t}\right)+\varphi\left(\ln l_{i t-1}, \ln l_{i t}\right)+\xi_{i t} . \tag{29}
\end{equation*}
$$

Here $\varphi\left(\ln l_{i t-1}, \ln l_{i t}\right)=\frac{\sigma-1}{\sigma}\left[\alpha \ln l_{i t}+g\left(\ln l_{i t-1}, \ln l_{i t}\right)\right]$ is treated as a flexible function of its arguments, and the intercept $\tilde{d}_{H}=d_{H}-(1-\alpha) \frac{\sigma-1}{\sigma} \ln P$ reflects the fact that we have replaced the unobservable $m_{i t}$ with observable input expenditures, $P m_{i t}$. The error term $\xi_{i t}$ captures measurement error in $\ln G_{i t}$ and any productivity shocks that are unobserved at the time variable inputs and exporting decisions are made. Because $\xi_{i t}$ is orthogonal to $P m_{i t}$ and $\mathcal{I}_{i t}^{x}$, we obtain our estimates of $\frac{\sigma-1}{\sigma}(1-\alpha)$ and $d_{F}\left(\eta_{0}\right)$ by applying least squares to equation (29). Just as Olley and Pakes (1996) excluded observations with zero investment to keep their policy function invertible, we exclude observations for which $l_{i t}=l_{i t-1 .}{ }^{25}$

Identification - While it is not possible to associate individual parameters in $\boldsymbol{\Omega}$ with individual statistics in $\overline{\mathbf{m}}$, particular statistics play relatively key roles in identifying particular parameters. We devote this section to a discussion of these relationships.

To begin, the sample-based estimates of $\frac{\sigma-1}{\sigma}(1-\alpha)$ and $d_{F}\left(\eta_{0}\right)$ provide a basis for inference regarding $\alpha$ and $\sigma$. This is because, for any given value of $\left(\frac{\sigma-1}{\sigma}\right)(1-\alpha)$, the elasticity of revenue with respect to labor, $\alpha \Lambda$, increases monotonically in $\sigma$. Thus, loosely speaking, the regression of revenue on employment-which is implied by the sample moments $\operatorname{cov}\left(\ln l_{t}, \ln G_{t}\right), \operatorname{var}\left(\ln G_{t}\right)$, and $\operatorname{var}\left(\ln l_{t}\right)$ —pins down $\sigma .{ }^{26}$ And once $\sigma$ and $\frac{\sigma-1}{\sigma}(1-\alpha)$ are determined, $\alpha$ and $\Lambda$ are also implied. These moments also discipline $\theta$. As we mentioned above, greater job turnover increases the pool of unemployed workers, and increases the

[^10]vacancy fill rate, $\phi(V, U)$. Hence $\theta$ also helps determine how responsive firms' employment levels are to $z$.

Next note that by inverting the revenue function, we can express $\ln z_{t}$ as a function of the data $\left(\ln l_{t}, \ln G_{t}, \mathcal{I}_{t}^{x}\right)$ and several parameters discussed above $\left(d_{F}\left(\eta_{0}\right), \alpha, \Lambda\right)$. Thus, given these parameters, the data vector $\left(\ln l_{t}, \ln G_{t}, \mathcal{I}_{t}^{x}, \ln l_{t+1}, \ln G_{t+1}, \mathcal{I}_{t+1}^{x}\right)$ determines $\ln z_{t+1}$ and $\ln z_{t}$, and the second moments of this vector imply the parameters of the autoregressive process that generates $\ln z_{t}$, i.e., $\rho$ and $\sigma_{z}$.

The average level of gross revenues - proxied by $E\left(\ln G_{t}\right)$-helps to identify the fixed cost of operating a firm, $c_{p}$. That is, larger fixed costs force low-revenue firms to exit, and thereby increase $E\left(\ln G_{t}\right)$ among survivors. The mean exporting rate $E\left(\mathcal{I}^{x}\right)$ is informative about the fixed costs of exporting, $c_{x}$. Also, since the cost of creating a firm, $c_{e}$, must match the equilibrium value of entry, the estimated intercept $\widetilde{d}_{H}$ from (29) helps us to pin down the price level $P$ in the estimation. In turn, this pins down the value of entry $\mathcal{V}_{e}$ from (20). Further details are provided in online Appendix 4.

The job turnover rate among continuing firms is informative about the general magnitude of hiring costs, which scale with $c_{h}$. Similarly, the firm-size-specific job add rates are informative about frictions faced by firms in different states. More precisely, in the absence of labor market frictions, the job turnover rate, the firm size distribution, and the quintile-specific add rates would simply be determined by the productivity process. Deviations from these patterns require adjustment frictions, and quintile-specific patterns require different frictions for firms of different sizes. The parameter $\lambda_{1}$, which governs the convexity of hiring costs, determines the overall level of adjustment frictions. And $\lambda_{2}$, which governs scale economies in hiring costs, determines the relative stability of large versus small firms, as discussed above in Section 2.7. Other things equal, the smaller $\lambda_{2}$ is, the more rapidly the employment growth rate declines with firm size.

Finally, in combination with information on job turnover and hiring rates, the share of employment in the non-traded services sector and the cross-firm dispersion in log wages help to identify the matching function parameter $(\theta)$, workers' bargaining power $(\beta)$, and the value of being unemployed $(b)$. These parameters determine how rents are shared between the workers and the employers in hiring firms, and, as a result, the wage dispersion across firms.

Estimates and Model Fit - Table 2 reports our estimates of $\boldsymbol{\Omega}$. Standard errors are constructed using the standard asymptotic variance expression, with $v \hat{a} r(\overline{\mathbf{m}})$ bootstrapped from the sample data. ${ }^{27}$ Since our data-based moments are calculated from a large survey of plants, sample variation in the moments is small. Almost by construction, this leads to small

[^11]Table 2: Parameters Estimated with Simulated Method of Moments

| Parameter | Description | Estimate | Std. Err. |
| :---: | :--- | :---: | :---: |
| $\sigma$ | Elasticity of substitution | 6.667 | 0.0127 |
| $\alpha$ | Elasticity of output with respect to labor | 0.195 | 0.0009 |
| $\rho$ | Persistence of the $z$ process | 0.962 | 0.0001 |
| $\sigma_{z}$ | Standard deviation of the $z$ process | 0.137 | 0.0003 |
| $\beta$ | Bargaining power of workers | 0.441 | 0.0011 |
| $\theta$ | Elasticity of the matching function | 1.838 | 0.0086 |
| $\delta$ | Exogenous exit hazard | 0.064 | 0.0005 |
| $c_{h}$ | Scalar, vacancy cost function | 0.448 | 0.0035 |
| $\lambda_{1}$ | Convexity, vacancy cost function | 3.101 | 0.0110 |
| $\lambda_{2}$ | Scale effect, vacancy cost function | 0.385 | 0.00012 |
| $b$ | Value of home production | 0.433 | 0.0014 |
| $l_{e}$ | Initial size of entering firms | 5.906 | 0.0359 |
| $c_{p}$ | Fixed cost of operating | 7.839 | 0.0245 |
| $c_{x}$ | Fixed exporting cost | 112.943 | 1.2062 |
| $c_{e}$ | Entry cost for new firms | 15.794 | 0.1194 |
| $D_{F}^{*}$ | Foreign market size | $2,379.900$ | 150.6402 |

diagonal elements of $\operatorname{var} r(\overline{\mathbf{m}})$. Our solution algorithm is summarized in online Appendix 4.
Overall, the model fits the data quite well. ${ }^{28}$ In particular, it captures the size distribution of firms (Figure 4, first two bars in each bin), the exit rate, the persistence in employment levels, and the variation in growth rates across the plant size distribution. The model underestimates wage dispersion a bit, but this is to be expected, since our data-based measure of wage dispersion controls for only five types of workers, and thus reflects some unobserved worker heterogeneity. In contrast, our model-based dispersion measure is based on the assumption of homogenous effective labor units.

Estimates of $b, c_{p}, c_{x}$ and $c_{e}$ are measured in terms of our numeraire - the price of the service good, or equivalently, the average annual service sector wage. We calculate this to be $w_{s}=\$ 3$, 461 in 2012 US dollars during the sample period so, expressed in dollars, the sunk cost of creating a new firm is $15.794 \times \$ 3,461=\$ 54,663$, the annual fixed cost of operating a business amounts to $7.839 \times \$ 3,461=\$ 27,131$, and the fixed cost of exporting is $112.943 \times \$ 3,461=\$ 390,896 .{ }^{29}$ (The magnitude of the latter figure reflects the large gap in our sample between the average revenues of exporters and non-exporters.) To put these numbers in context, the mean and median annual sales of a Colombian manufacturing firm during the sample period were $\$ 4,418,360$ and $\$ 508,970$, respectively.

Several other features of our results on preferences and technology merit comment. First,

[^12]our estimate of the elasticity of substitution among differentiated industrial goods, $\sigma=6.67$, is very much in line with the literature. ${ }^{30}$ Second, given our estimates of $\alpha$ and $\sigma$, the elasticity of value added with respect to labor is $\alpha \Lambda=0.52$. (Recall that $\Lambda=\frac{\sigma-1}{\sigma-(1-\alpha)(\sigma-1)}$.) This figure falls a bit below the range typically estimated for value-added production functions. ${ }^{31}$ Third, we find substantial persistence in the $z$ process $(\rho=0.96)$. This relatively high estimate reflects the fact that, unlike most estimates of productivity processes, we treat capital stocks as fixed upon entry and common across firms. This effectively bundles persistence in employment due to capital stocks into the $z$ process. Fourth, we estimate that about 40 percent of the 10.4 percent firm exit rate (Table1) is due to adverse productivity shocks, and 60 percent is due to factors outside our model $(\delta=0.064)$. Finally, our model allows us to infer the typical size at which firms enter, recognizing that they do not actually appear in the database until they have acquired 10 workers. This entry size amounts to $l_{e}=5.91$ workers.

The remaining parameter estimates in Table 2 concern labor markets. These too are plausible. The returns to home production by unemployed workers is 43 percent of the the secure wage they could have earned if they had committed to work in the service sector. The matching function parameter, $\theta=1.84$, is close to the value of 2.16 that Coşar (2013) calibrates using aggregate labor market statistics from Brazil, and not far from the value of 1.27 that den Haan, Ramey, and Watson (2000) obtain in calibrating their model to the US economy. The bargaining parameter, $\beta=0.44$, implies workers have a bit less bargaining power than firms when dividing the rents from a match. Finally, the parameters of the vacancy cost function imply both short-run convexities $\left(\lambda_{1}=3.10\right)$ and substantial scale economies $\left(\lambda_{2}=0.39\right) .{ }^{32}$

[^13]Table 3: Model Implications for Aditional Statistics

|  | Data | Model |
| :---: | :---: | :---: |
| Aggregates |  |  |
| Revenue share of exports | 0.090 | 0.156 |
| Relative market size (COL/RoW) | 0.006 | 0.007 |
| Manufacturing share of employment | 0.226 | 0.335 |
| Unemployment rate | 0.108 | 0.049 |
| Exporters versus Non-exporters |  |  |
| ${\overline{\ln }{ }_{\mathcal{I}^{x}=1}-{\overline{\ln } \underline{\mathcal{I}}^{x}=0}(\text { size premium) }}^{\text {a }}$ | 1.402 | 1.977 |
|  | 0.420 | 0.481 |
| Aggregate employment share of exporters | 0.360 | 0.457 |
| Aggregate revenue share of exporters | 0.518 | 0.612 |
| Wage-Size Relationship $\operatorname{corr}(w, l)$ | 0.402 | 0.065 |
| $\ln w=\alpha+\beta_{l} \ln l+\beta_{x} I^{x}+\varepsilon$ |  |  |
| $\ln l$ coefficient $\left(\beta_{l}\right)$ | 0.202 | -0.086 |
|  | (0.001) | (0.001) |
| $I^{x}$ coefficient ( $\beta_{x}$ ) | 0.137 | 0.651 |
|  | (0.005) | (0.004) |
| $R^{2}$ | 0.295 | 0.183 |

Notes: Data-based statistics are constructed using the same panel of establishment used for Table 1. Numbers in parentheses are OLS standard errors.

Drawing on our discussion in Section 2.7 above, we infer that per-worker vacancy posting costs rise substantially with firm size, holding the rate of employment growth constant: $\lambda_{1}\left(1-\lambda_{2}\right)=1.91>1$. Hence large firms in our model will tend to grow relatively slowly in response to productivity shocks. Since $z$ innovations are similar at large and small firmsthat is, $\rho=0.96$ implies that mean reversion in $z$ is very slow-this feature of the vacancy cost function plays a dominant role in shaping employment dynamics.

Non-Targeted Statistics and Out-of-Sample Fit - Before discussing policy implications of these estimates, we ask how well the model replicates features of the data that we did not use as a basis for identification. To address this question we construct several additional statistics in Table 3.

We start with the aggregates in the first panel. The pre-reform revenue share of exports, plotted in Figure 3, is 9 percent. In our estimation, we targeted the fraction of firms that export, and the revenue increment due to exporting $d_{F}$, but did not explicitly target the revenue share of exports. The model generates a 15.6 percent share, reflecting the fact that the model does not allow new exporters to start with a lower revenue share of foreign sales (Eaton et al. 2014). In the model, $D_{H} /\left(k^{\sigma} D_{F}^{*}\right)$ measures the size of domestic expenditures on tradable goods relative to total foreign demand for tradables. We estimate this ratio as 0.007. While it is hard to find an exact empirical counterpart to this statistic, we calculate

Colombia's average GDP relative to the sum of its trade partners' GDP over 1981-1990 and find a value of 0.006 . Another relevant statistic is the employment share of manufacturing, which averaged 0.23 in the pre-reform period. Our model predicts an employment share of $L_{q} /\left(L_{q}+L_{s}\right)=0.34$ for the industrial sector, so strictly speaking, it somewhat overstates the role of manufacturing in the Colombian economy. On the other hand, some tradable nonmanufactured goods like coffee and cut flowers might be considered to be monopolistically competitive, so this discrepancy does not strike us as problematic. Finally, the modelgenerated unemployment rate among workers affiliated with the industrial sector, $L_{u} /\left(L_{u}+\right.$ $L_{q}$ ), is 0.049 with a job finding rate of $\widetilde{\phi}=0.71$. It is difficult to compare this rate to aggregate unemployment figures from Colombia, since our model abstracts both from labor market frictions in the service sector and observed or unobserved worker heterogeneity. While these extensions are beyond the scope of the current paper, we nonetheless note that the Colombian urban unemployment rate averaged 11 percent during the pre-reform period, an a back-of-the-envelope calculation implies that a service sector unemployment rate of 14 percent would reconcile our simulated industrial unemployment rate with pre-reform conditions. ${ }^{33}$ The implication that unemployment is relatively high in the service sector is consistent with sector-specific job separation patterns observed in Colombia (See online Appendix 5.2).

As the second panel of Table 3 shows, Colombian exporters are larger (size premium) and pay higher wages (wage premium) than non-exporters. Also, as a group exporters account for more than a third of industrial employment and slightly more than half of total revenues. The model generates all of these patterns, although it overstates the gap between exporters and non-exporters. This tendency to overstate exporter premia while matching other moments reflects the fact that in our model, all firms above a threshold output level are exporters (see equation 11). The contrast between exporters and others could be weakened without sacrificing model fit by adding another source of firm heterogeneity-for example random fixed exporting costs. But the workings of the model that we wish to focus upon would be unaffected, so we opt for simplicity here.

This same deterministic relationship between output and exporting status makes it difficult for our model to generate the observed positive association between size and wages, conditional on exporting status. The third panel of Table 3 reports the wage-size relation-

[^14]ship. While our model generates a positive unconditional correlation, adding an exporter dummy to the model-based regression of log wages on log employment turns the coefficient on $\log$ employment slightly negative. ${ }^{34}$ With the exporter dummy absorbing much of the cross-firm rent variation, two remaining forces are at work in our model. On the one hand, holding productivity and exporting status constant, the marginal revenue product of labor falls with employment, putting downward pressure on wages at large firms. On the other hand, holding exporting status constant, productivity shocks tend to induce a positive correlation between firm size and wages, as discussed in section I.K. In the model, the marginal revenue product effect dominates. But in the data, the relation between employment and exports is noisier and additional forces are at work, including unobserved worker heterogeneity and perhaps greater monopoly power among larger firms. ${ }^{35}$

Finally, we ask how well our model does in capturing cross-worker (as opposed to crossfirm) residual wage inequality. Since our establishment survey data do not provide information on individual workers, we are unable to construct our own data-based version of this concept. However, we note that the pre-reform average Gini coefficient was around 0.53 (World Bank, 2013, and figure 3), while our model generates a Gini of 0.224 . As an alternative statistic, Attanasio, Goldberg, and Pavcnik (2004) report the 1984-1990 average of unconditional standard deviation of log worker wages as 0.80 using the Colombian Household Survey (their Table 2a). The model counterpart is 0.380 . Since both data-based measures incorporate observable and unobservable characteristics of workers and firms, and observable worker characteristics typically explain around a third of the wage variation in Mincer regressions (Mortensen 2005), it seems reasonable that our model generates around half of total dispersion by these measures.

## 4 Simulated Effects of Reforms and Globalization

### 4.1 The experiments

We are now prepared to examine the effects of reforms and global reductions in trade frictions in our estimated model. Our aim is to determine the extent to which these simulated effects capture the long-term changes in labor market outcomes documented in Figure 3. ${ }^{36}$ We

[^15]begin by exploring the effects of reducing firing costs $\left(c_{f}\right)$, tariffs $\left(\tau_{a}\right)$, and iceberg costs $\left(\tau_{c}\right)$ one by one, holding all other parameters at their baseline values. Then we consider two combinations of changes in these friction parameters. The first experiment ("Reforms") corresponds to Colombia's reform package, which we approximate by cutting $\tau_{a}$ from 1.21 to 1.11 and cutting $c_{f}$ from 0.6 to 0.3 . The second experiment ("Reforms and Globalization") keeps these two changes, but also reduces $\tau_{c}$ from 2.50 to 2.19.

This 12.3 percent drop in $\tau_{c}$ is chosen to match the observed increase in the aggregate revenue share of exports, given the reform-induced reductions in $\tau_{a}$ and $c_{f}$. It can be interpreted as capturing additional forces of globalization during the period under study, including the increased income and openness of Colombia's trading partners, improvements in global communications, and general reductions in shipping costs (Hummels 2007). It also captures the integration of rapidly growing emerging markets into the global economy. We view these shocks as originating beyond Colombia's borders, inasmuch as Latin America in general experienced a surge in trade that roughly matched Colombia's.

### 4.2 Findings

Table 4 reports our baseline steady state equilibrium in column 1 and our counterfactual equilibria in columns 2 through 6 . The firing $\operatorname{costs} c_{f}$, tariff rates $\tau_{a}$, and iceberg $\operatorname{costs} \tau_{c}$ that correspond to each equilibrium are reported in the top panel of the table, and the remaining panels reports characteristics of the associated steady states. To keep orders of magnitude in a similar range, statistics reported in the bottom panel are normalized relative to their baseline values. Also, in order to isolate firm-size-specific turnover effects from the effects of shifts in the size distribution, we hold the size class cutoffs constant at their baseline values when calculating the average firm growth rates reported in the third panel. Further details on adjustments to the reforms can be found in online Appendix 5.4.

Firing Cost Reductions - First consider the effects of Colombia's firing cost $\left(c_{f}\right)$ reductions in isolation, holding tariffs $\left(\tau_{a}\right)$ and external trade frictions $\left(\tau_{c}\right)$ constant at their pre-reform levels (column 2). As discussed in section I.K, this reform favors large firms, inducing a rightward shift in the size distribution. And the concentration of workers at large, stable firms puts downward pressure on the job turnover rate through the distribution effect.

The rightward shift in the size distribution also concentrates job vacancies at firms that pay lower hiring wages, driving down the expected value of new jobs. As $E J_{h}^{e}$ falls, the unemployment rate falls too: Without such a tightening of the labor market, workers would cease searching for $Q$-sector jobs. Further, as tighter labor markets push down the job filling rate $\phi$, they increase the cost of hiring (15) and compound the downward pressure on job turnover. For this reason, employment growth rates decline 1-2 percent for all but

Table 4: The Effects of Reforms and Globalization

|  | Baseline | Labor | Tariff | Iceberg | Reforms | Reforms and <br> Globalization |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $c_{f}$ (firing cost) | 0.60 | 0.30 | 0.60 | 0.60 | 0.30 | 0.30 |
| $\tau_{a}$ (ad valorem tariff rate) | 1.21 | 1.21 | 1.11 | 1.21 | 1.11 | 1.11 |
| $\tau_{c}$ (iceberg trade cost) | 2.50 | 2.50 | 2.50 | 2.19 | 2.50 | 2.19 |
| Size Distribution |  |  |  |  |  |  |
| 20th percentile |  |  |  |  |  |  |
| 40th percentile | 15.09 | 16.84 | 15.24 | 14.64 | 18.05 | 17.87 |
| 60th percentile | 25.22 | 27.54 | 24.74 | 23.55 | 29.77 | 30.06 |
| 80th percentile | 42.97 | 47.30 | 42.97 | 42.15 | 51.57 | 54.09 |
| Average firm size | 87.14 | 98.59 | 90.51 | 99.53 | 107.38 | 129.79 |
|  | 52.69 | 58.73 | 55.89 | 61.99 | 64.08 | 77.91 |
| Firm Growth Rates |  |  |  |  |  |  |
| (at the baseline size quantiles) |  |  |  |  |  |  |
| <20th percentile | 1.28 | 1.36 | 1.28 | 1.26 | 1.43 | 1.49 |
| 20th-40th percentile | 0.25 | 0.23 | 0.26 | 0.28 | 0.26 | 0.29 |
| 40th-60th percentile | 0.19 | 0.17 | 0.20 | 0.22 | 0.18 | 0.21 |
| 60th-80th percentile | 0.15 | 0.14 | 0.16 | 0.18 | 0.15 | 0.17 |
| Aggregates |  |  |  |  |  |  |
| Rev. share of exports | 1.00 | 1.02 | 1.36 | 2.01 | 1.39 | 2.50 |
| Exit rate | 1.00 | 0.96 | 1.02 | 1.13 | 0.96 | 1.03 |
| Job turnover | 1.00 | 0.90 | 1.01 | 1.03 | 0.92 | 0.94 |
| Mass of firms | 1.00 | 0.96 | 0.95 | 0.74 | 0.88 | 0.66 |
| Share of labor, $Q$ sector | 1.00 | 1.06 | 1.01 | 0.88 | 1.07 | 0.98 |
| Vacancy filling rate $(\phi$ ) | 1.00 | 0.93 | 1.04 | 1.11 | 0.99 | 1.09 |
| Unemp. rate, $Q$ sector | 1.00 | 0.73 | 1.11 | 1.38 | 0.88 | 1.19 |
| Std. wages (firms) | 1.00 | 1.09 | 1.01 | 1.03 | 1.12 | 1.18 |
| Std. wages (workers) | 1.00 | 1.10 | 1.02 | 1.04 | 1.11 | 1.14 |
| Std. J (firms) | 1.00 | 1.05 | 1.03 | 1.06 | 1.09 | 1.18 |
| Std. J (workers) | 1.00 | 1.04 | 1.04 | 1.06 | 1.07 | 1.21 |
| Exchange rate | 1.00 | 1.04 | 0.99 | 0.89 | 1.02 | 0.84 |
| Real income | 1.00 | 0.95 | 1.00 | 1.14 | 0.96 | 1.12 |
|  |  |  |  |  |  |  |

Note: Aggregate statistics in the bottom panel are normalized by their baseline levels. Since there is no wage heterogeneity within firms, but firms are heterogeneous in size, wage and value ( $J$ ) dispersion measures are reported at both the firm- and worker-level.
the smallest producers. Combined, the tightness effect and the distribution effect discussed above reduce the aggregate job turnover rate by 10 percent.

Despite inducing tighter labor markets, lower firing costs $c_{f}$ in isolation actually pull down real GDP $\left(I / P^{\gamma}\right)$ by 4 percent. (See online Appendix 3 for the definition of aggregate income $I$.) This result is partly due to the reduction in the number of varieties $(N)$ available to consumers, as production becomes concentrated at fewer firms. It is also due to an increase in the severity of the overhiring distortion that appears in this class of models (Stole and

Zwiebel 1996). ${ }^{37}$
Finally, reductions in firing costs increase $Q$-sector income inequality by concentrating more workers at the low end of the wage distribution. This shift reflects the aforementioned reallocation of jobs toward large, stable firms, where the marginal revenue product of labor is relatively low. It also reflects the lingering presence of some low-productivity firms that would have exited to avoid paying firing costs under the baseline regime. By sticking around, these marginal firms increase the share of the workforce that is paid reservation wages. (Note that our inequality measures only characterize the population of employed $Q$-sector workers.)

Trade Liberalization - Consider next the simulated effects of Colombia's tariff reductions, holding $c_{f}$ and $\tau_{c}$ constant at their pre-reform levels. This experiment is summarized in column 3 of Table 4. As with the firing cost reduction, this reform induces a rightward shift in the upper tail of the plant size distribution. Now, however, the shift occurs because larger producers take advantage of cheaper imported intermediates to expand their exports. By itself, this concentration of workers at large firms would reduce job turnover through the distribution effect. But with increased foreign demand, the sensitivity effect is also in play, causing firm and job turnover to rise with tariff reductions. Further, small unproductive firms tend to exit relatively frequently after tariffs fall, since they face greater import competition without the benefit of more sales in export markets. Overall, the net effect is a small increase in turnover rates, an 11 percent increase in $Q$-sector unemployment, and a 4 percent increase in job filling rates $(\phi)$ as the expected value of $Q$-sector jobs rises and each vacancy attracts more searching workers.

Inequality also tends to rise with Colombia's trade liberalization, especially when measured in terms of lifetime welfare $(J)$. The reason is that, like firing cost reductions, tariff reductions favor large firms. As these firms expand their exports, they generate additional rents to share with their workers, who were already relatively well-paid, and whose jobs were already relatively secure. Finally, by itself, Colombia's trade liberalization would have left real income unchanged, as the standard Melitzian (2003) selection effects favoring productive firms would have been counteracted by the loss of variety when the number of firms in the industrial sector declines.

Reduced Iceberg Costs - Reductions in iceberg costs and reductions in tariffs both encourage trade, although reductions in $\tau_{c}$ have a direct effect on exporting costs that is not present when we reduce $\tau_{a}$. This similarity of the two experiments explains why qualitatively, the direction of movement for all variables is the same in column 4 as it is column 3 . Note, however, that the effects of the reduction in iceberg costs are much stronger across the

[^16]board. This is partly because the reduction is larger- 17 percent for $\tau_{c}$ versus 10 percent for $\tau_{a}$-and partly because of the direct effect of iceberg costs on the returns to exporting. The reduction in the mass of firms, the increase in real income, and the increase in the unemployment rate are particularly dramatic.

The Reform Package - The combined effects of Colombia's trade and labor market reforms are summarized in column 5 of Table 4. When studied in isolation, these two reforms had opposing effects on some variables, like exit, unemployment, and job turnover. So it is unsurprising that the net effect of the reform package on these variables tends to be muted. Other variables, including the number of firms, firm size quantiles, and inequality measures responded in qualitatively similar ways to tariff reductions and to lower firing costs. Among this latter group of variables, the effects of the two reforms tend to compound each other. Note in particular that the responses of the size distribution and the percentage of firms exporting suggest that there were some complementaries between the two policies. That is, reductions in firing costs appear to have encouraged relatively productive firms to respond more dramatically to trade liberalization, participating in the foreign market at a higher rate and becoming larger than they would have otherwise. So just as policy makers hoped, greater labor market flexibility appears to have increased the competitiveness of Colombian producers (refer to footnote 1).

Reforms and Globalization - Our final experiment examines the combined effects of the three shocks discussed above. We begin our discussion of findings by examining the associated shifts in the plant size distribution. Figure 4 juxtaposes the simulated distribution with the distribution observed in the data during the pre- and post-reform periods. Since the post-reform data are only available in terms of the number of workers, we report numbers of workers (not effective workers as we did in Table 1), both for the model and for the data (see online Appendix 5.1 for details). Note that our post-reform simulation matches the actual movement in the Colombian plant size distribution quite closely, not only in terms of average size, but also in terms of shape. (Refer to the third and fourth bar in each cluster.) The firing cost effects discussed earlier and the trade-related selection effects emphasized by Melitz (2003) both play a role in inducing this shift.

Interestingly, however, our simulations do not explain the increase in aggregate job turnover documented in Panel 2 of Figure 3. In fact, they imply a small decline (column 6 of Table 4). This reflects the importance of the induced rightward shift in the size distribution of firms. The left-hand panel of Figure 5 shows that without any distribution effect, the reforms and globalization would have increased employment growth rates by several percentage points for firms in all states. However, the tendency for all of the shocks to concentrate workers at relatively stable firms was more than sufficient to offset the tendency for openness to reduce job security. It is not clear how much of the gap between simulated

Figure 4: Firm Size Distribution, Model vs. Data


Notes: 1981-1990 average is calculated from plant-level data. 2000-2006 data is obtained from DANE. Since labor in the model is in effective labor units, while the post-reform data is only available in terms of number of employees, we convert model-generated firm size into number of employees using the fit of the two units in the pre-reform plant level data. The details of this procedure are described in online Appendix 5.1.
and actual turnover rates reflects lack of transitional dynamics in the model economy. Our Figure 3 series on post-reform turnover rates is particularly short, and it appears to trend downward until the 1999 financial crisis.

By increasing rents at large, productive firms, globalization increases expected industrial wages. Hence, as discussed in section I.K, the industrial unemployment rate must rise to keep the expected payoff the same for workers in each sector. Largely for this reason, our globalization and reform experiment implies a one percentage point increase in the unemployment rate - about half of the 2.2 percentage point increase observed in the data between 1981-1990 and 2000-2006 (panel 3 of Figure 3). However, as with turnover rates, the effects of Colombia's financial crisis and recession at the end of the 1990s make it difficult to infer reform-induced changes in unemployment from the data.

As we argued in Section 3, unemployment alone is an insufficient measure of labor market conditions in developing countries. The declining employment share of manufacturing and the corresponding rise in self-employment, mostly associated with personal services, point to a further deterioration of stable employment opportunities for workers (Panels 4 and 5 of Figure 3). As shown in the last column of Table 4, our model implies a 6 percent decline in industry's share of employment, thereby accounting for about a third of the observed 17

Figure 5: Employment Policy and Wage Effects


Notes: $z$ is firm productivity, ln(l) is log labor. Both panels display changes from the baseline to the "Reforms and Globalization" scenario reported in the sixth column of table 4, plotted as decile averages over the equilibrium productivity and labor distribution of firms in baseline. The left panel plots the percentage change in firm growth rates. The right panel plots the change in wages.
percent contraction observed in the data (see Section 3.1). ${ }^{38}$
Next consider wage inequality. While the average real wage $\left(\bar{w} / P^{\gamma}\right)$ increases with reforms and globalization, the shift in the wage schedule depends very much upon employer states $(z, l)$. The right-hand panel of Figure 5 shows changes in firm-level wages from their baseline values. Wages become more polarized as relatively productive firms benefit from additional export sales and pay higher wages, while smaller, less productive firms suffer from increased import competition and lower their wages. This rent polarization is reflected in cross-firm wage dispersion, cross-worker wage dispersion, and lifetime earnings $(J)$ dispersion (Table 4, column 6). Our model therefore provides a lens through which to interpret the substantial increase in overall inequality observed in Colombia (Panel 6 of Figure 3).

Finally, our "Reforms and Globalization" experiment predicts sizeable aggregate income gains from globalization through increased selection, market share reallocations, and cheaper intermediates. These effects dominate the upward pressure on our exact price index $(P)$ that results from a fall in the measure of varieties $(N)$, generating a 12 percent increase in real income with respect to the baseline. The net welfare implications of these income gains would ideally be calculated by weighing them against the negative effects of greater wage dispersion and higher unemployment rates. But to do so properly would require introducing risk aversion into the model, substantially complicating the analysis.

Robustness - As we noted earlier, our estimation strategy dictated that we set the unit of time in our model equal to one year. To explore the implications of this choice,

[^17]we have calibrated a quarterly version of our model (see online Appendix 6 for details). Since this exercise is based on a number of approximations, we view it as only suggestive. Nonetheless, it indicates how our results might have been affected by our choice of periodicity. In particular, other things equal, allowing workers to search more frequently increases their reservation wages. This reduces wage dispersion and tightens the labor market. So, in order to still match the data in terms of wage inequality and job turnover, the quarterly version of our model requires a lower self-employment income (b), a lower matching function elasticity $(\theta)$, and a higher elasticity of demand $(\sigma)$.

These parameter adjustments do not affect the model's qualitative and quantitative predictions regarding labor market responses to tariffs, iceberg costs and firing costs ( $\tau_{a}, \tau_{c}$, and $c_{f}$ ). Job turnover decreases while wage dispersion and industrial sector unemployment increase in response to the "Reforms and Globalization" experiment. However, the relative magnitudes of some adjustments do depend on the unit of time in the model. Specifically, in the quarterly model, increased inequality shows up more through the dispersion of worker welfare and less through wage dispersion. Also, the responses in job turnover and unemployment are slightly less dramatic in the quarterly model (see Tables 4 and A4).

## 5 Summary

In Latin America and elsewhere, globalization and labor market reforms have been associated with greater wage inequality, higher unemployment rates, and more job turnover. We formulate and estimate a dynamic structural model that links these developments. Our formulation combines ongoing firm-level productivity shocks and Melitz-type (2003) trade effects with labor market search frictions, firing costs, and worker-firm wage bargaining.

Fit to micro data from Colombia, the model delivers several basic messages. First, this country's tariff reductions and labor market reforms in the early 1990s explain a significant fraction of the heightened inequality it experienced during the following decade, but they are unlikely to have been the reason that job turnover and unemployment increased. Second, global reductions in trade frictions compounded the inequality effects of reforms, and go some way toward explaining higher unemployment rates as well. Finally, had tariff reductions and global reductions in trade frictions not been accompanied by labor market reforms, their negative effects on unemployment would have been larger, and they would also have tended to increase job turnover.

Many other countries registered growth rates in merchandise trade similar to Colombia's over the past two decades, even without major commercial policy reforms. To the extent that these surges were caused by the international integration of product markets, globalization may have contributed to similar labor market outcomes in these countries as well.

In principle, our analysis could be extended in several directions. First, incorporating worker heterogeneity would permit us to link openness with wage effects among workers with different skills and/or at different stages in their careers. Second, a more fully-articulated representation of the service sector would allow us to better characterize economy-wide patterns of unemployment and perhaps also explicitly deal with informal jobs. Finally, introducing risk aversion would permit us to formally link job turnover rates to welfare, and to examine the trade-off between static gains from trade and losses from heightened risks of job loss. We see these extensions as interesting directions for future work.

## References

[1] Abowd, John, Francis Kramarz and David Margolis. 1999. "High Wage Workers and High Wage Firms." Econometrica 67 (2): 251-333.
[2] Ackerberg, Daniel, Kevin Caves and Garth Frazer. 2006. "Structural Identification of Production Functions." Unpublished.
[3] Amiti, Mary and Donald R. Davis. 2012. "Trade, Firms, and Wages: Theory and Evidence." Review of Economic Studies 79 (1): 1-36.
[4] Arkolakis, Costas. forthcoming. "A Unified Theory of Firm Selection and Growth." Quarterly Journal of Economics.
[5] Artuc, Erhan, Shubham Chaudhuri and John McLaren. 2010. "Trade Shocks and Labor Adjustment: A Structural Empirical Approach." American Economic Review 100 (3):1008-1045.
[6] Attanasio, Orazio, Pinelopi K. Goldberg and Nina Pavcnik. 2004. "Trade Reforms and Wage Inequality in Colombia." Journal of Development Economics 74 (2): 331-366
[7] Baier, Scott L. and Jeffrey H. Bergstrand. 2001. "The Growth of World Trade: Tariffs, Transport Costs, and Income Similarity." Journal of International Economics 53 (1): 1-27.
[8] Bernard, Andrew B. and J. Bradford Jensen. 1995. "Exporters, Jobs and Wages in U.S. Manufacturing, 1976-87." Brookings Papers on Economic Activity: Microeconomics, volume 1995, 67-112.
[9] Bernard, Andrew B. and J. Bradford Jensen. 1999. "Exceptional exporter performance: Cause, Effect, or Both?" Journal of International Economics 47 (1): 1-25.
[10] Bertola, Giuseppe and Ricardo Caballero. 1994. "Cross-sectional Efficiency and

Labour Hoarding in a Matching Model of Unemployment." Review of Economic Studies 61 (3): 435-47.
[11] Bertola, Giuseppe and Pietro Garibaldi. 2001. "Wages and the Size of Firms in Dynamic Matching Models." Review of Economic Dynamics 4 (2): 335-368.
[12] Bond, Eric, James Tybout and Hale Utar. Forthcoming. "Credit Rationing, Risk Aversion and Industrial Evolution in Developing Countries." International Economic Review.
[13] Broda, Christian and David Weinstein. 2006. "Globalization and the Gains from Variety." Quarterly Journal of Economics 121 (2): 541-585
[14] Cahuc, Pierre and Etienne Wasmer. 2001. "Does Intrafirm Bargaining Matter in Large Firm Matching Model?" Macroeconomic Dynamics 5 (5): 742-747.
[15] Cahuc, Pierre, Francois Marque and Etienne Wasmer. 2008. "Intrafirm Wage Bargaining in Matching Models: Macroeconomic Implications and Resolution Methods with Multiple Labor Inputs." International Economic Review 49 (3): 943-972.
[16] Cooper, Russell, John Haltiwanger and Jonathan Willis. 2007. "Search Frictions: Matching Aggregate and Establishment Observations." Journal of Monetary Economics 54 (1): 56-78.
[17] Coşar, Kerem. 2013. "Adjusting to Trade Liberalization: Reallocation and Labor Market Policies." Unpublished.
[18] Davidson, Carl, Steven Matusz and Andrei Shevchenko. 2008. "Globalization and Firm-Level Adjustment with Imperfect Labor Markets." Journal of International Economics 75 (2): 295-309.
[19] Davidson, Carl, Lawrence Martin and Steven Matusz. 1999. "Trade and Search Generated Unemployment." Journal of International Economics 48 (2): 271-299.
[20] Davis, Donald R. and James Harrigan. 2011. "Good Jobs, Bad Jobs, and Trade Liberalization." Journal of International Economics 84 (1): 26-36.
[21] Davis, Steven, John Haltiwanger and Scott Schuh. 1998. Job Creation and Destruction. Cambridge, MA: MIT Press.
[22] den Haan, Wouter J., Garey Ramey and Joel Watson. 2000. "Job Destruction and the Propagation of Shocks." American Economic Review 90 (3): 482-98.
[23] De Loecker, Jan. 2011. "Product Differentiation, Multiproduct Firms, and Estimating the Impact of Trade Liberalization on Productivity." Econometrica 79 (5): 1407-1451.
[24] De Loecker, Jan, Pinelopi Goldberg, Amit Khandelwal and Nina Pavcnik. forthcoming. "Prices, Mark-ups and Trade Reforms." Econometrica.
[25] De Loecker, Jan and Frederick Warzynski. 2012. "Markups and Firm-Level Export Status." American Economic Review 102 (6): 2437-2471.
[26] Dix-Carneiro, Rafael. 2014. "Trade Liberalization and Labor Market Dynamics." Econometrica 82 (3).
[27] Eaton, Jonathan, Marcela Eslava, David Jinkins, C. J. Krizan, and James Tybout. 2014. "A Search and Learning Model of Export Dynamics." Penn State University WP.
[28] Eaton, Jonathan and Samuel Kortum. 2002. "Technology, Geography, and Trade." Econometrica 70 (5): 1741-1779.
[29] Egger, Hartmut and Udo Kreickemeier. 2009. "Firm Heterogeneity and the Labour Market Effects of Trade Liberalisation." International Economic Review 50 (1): 187216.
[30] Elsby, Michael W. L., and Ryan Michaels. 2013. "Marginal Jobs, Heterogeneous Firms, and Unemployment Flows." American Economic Journal: Macroeconomics 5(1): 1-48.
[31] Ericson, Richard and Ariel Pakes. 1995. "Markov Perfect Industry Dynamics: A Framework for Empirical Work." Review of Economic Studies 62 (1): 53-82.
[32] Fajgelbaum, Pablo. 2013. "Labor Market Frictions, Firm Growth, and International Trade." Unpublished.
[33] Feenstra, Robert C., Robert Inklaar and Marcel P. Timmer. 2013. "The Next Generation of the Penn World Table." National Bureau of Economic Research Working Paper 19255.
[34] Felbermayr, Gabriel, Giammario Impullitti and Julien Prat. 2014. "Firm Dynamics and Residual Inequality in Open Economies." IZA Discussion Paper 7960.
[35] Felbermayr, Gabriel, Julien Prat and Hans-Jörg Schmerer. 2011. "Globalization and Labor Market Outcomes: Wage Bargaining, Search Frictions, and Firm Heterogeneity." Journal of Economic Theory 146 (1): 39-73.
[36] Goldberg, Pinelopi K. and Nina Pavcnik. 2004. "Trade Reforms and Wage Inequality in Colombia." Journal of Development Economics 74 (2): 331-366.
[37] Goldberg, Pinelopi K. and Nina Pavcnik. 2007. "Distributional Effects of Glob-
alization in Developing Countries." Journal of Economic Literature 45 (1): 39-82.
[38] Gouriéroux and Monfort. 1996. Simulation-Based Econometric Methods. Oxford, UK: Oxford University Press.
[39] Haltiwanger, John, Adriana Kugler, Maurice Kugler, Alejandro Micco, and Carmen Pages. 2004. "Effects of Tariffs and Real Exchange Rates on Job Reallocation: Evidence from Latin America." Journal of Policy Reform 7 (4): 191-208.
[40] Haltiwanger, John, Ron S. Jarmin and Javier Miranda. 2013. "Who Creates Jobs? Small versus Large versus Young." Review of Economics and Statistics 95 (2): 347-361.
[41] Harris, John R. and Michael P. Todaro. 1970. "Migration, Unemployment, and Development: A Two-Sector Analysis." American Economic Review 60 (1): 126-142.
[42] Heckman, James and Carmen Pages. 2000. "The Cost of Job Security Regulation: Evidence from Latin American Labor Markets." National Bureau of Economic Research Working Paper 7773.
[43] Heckman, James and Carmen Pages. 2004. "Law and Enforcement: Lessons from Latin America and the Caribbean: An Introduction." In Law and Employment: Lessons from Latin America and the Caribbean, edited by James Heckman and Carmen Pages, 1-107. Chicago, IL: University of Chicago Press.
[44] Helpman, Elhanan and Oleg Itskhoki. 2010. "Labor Market Rigidities, Trade and Unemployment." Review of Economic Studies 77 (3): 1100-1137.
[45] Helpman, Elhanan, Oleg Itskhoki and Stephen Redding. 2010. "Inequality and Unemployment in a Global Economy." Econometrica 78 (4): 1239-1283.
[46] Helpman, Elhanan, Marc Muendler, Oleg Itskhoki and Stephen Redding. 2012. "Trade and Inequality: From Theory to Estimation." National Bureau of Economic Research Working Paper 17991.
[47] Hobijn, Bart and Aysegul Sahin. 2013. "Firms and Flexibility." Economic Inquiry 51 (1): 922-940.
[48] Hopenhayn, Hugo. 1992. "Entry, Exit, and Firm Dynamics in Long Run Equilibrium." Econometrica 60 (5): 1127-1150.
[49] Hopenhayn, Hugo and Richard Rogerson. 1993. "Job Turnover and Policy Evaluation: A General Equilibrium Analysis." Journal of Political Economy 101 (5): 915-938.
[50] Hummels, David. 2007. "Transportation costs and international trade in the second
era of globalization." The Journal of Economic Perspectives 21 (3): 131-154.
[51] Inter-American Development Bank. 2004. Good Job Wanted: Labor Markets in Latin America. Economic and Social Progress Report. Washington, DC: Inter-American Development Bank.
[52] International Monetary Fund (IMF). 2011. International Financial Statistics. Washington, D.C: International Monetary Fund.
[53] Jovanovic, Boyan. 1982. "Selection and the Evolution of Industry." Econometrica 50 (3): 649-70.
[54] Kambourov, Gueorgui. 2009. "Labor Market Restrictions and the Sectoral Reallocation of Workers: The Case of Trade Liberalizations." Review of Economic Studies 76 (4): 1321-1358.
[55] Klette, Tor Jakob and Samuel Kortum. 2004. "Innovating Firms and Aggregate Innovation." Journal of Political Economy 112 (5): 986-1018.
[56] Koeniger, Winfried and Julien Prat. 2007. "Employment Protection, Product Market Regulation and Firm Selection." The Economic Journal 117 (521): F302-F332.
[57] Kugler, Adriana. 1999. "The Impact of Firing Costs on Turnover and Unemployment: Evidence from the Colombian Labour Market Reform." International Tax and Public Finance 6 (3): 389-410.
[58] Lee, Donghoon and Kenneth Wolpin. 2006. "Intersectoral Labor Mobility and the Growth of the Service Sector." Econometrica 74 (1): 1-46.
[59] Ljungqvist, Lars. 2002. "How Do Lay-Off Costs Affect Employment." Economic Journal 112 (482): 829-853.
[60] Lentz, Rasmus and Mortensen, Dale T. 2012. "Labor Market Frictions, Firm Heterogeneity, and Aggregate Employment and Productivity." Unpublished.
[61] Luttmer, Erzo. 2007. "Selection, Growth, and the Size Distribution of Firms." Quarterly Journal of Economics 122 |(3): 1103-1144.
[62] Medina, Carlos, Jairo Nunez and Jorge Andres Tamayo. 2013. "The Unemployment Subsidy Program in Colombia: An Assessment." Inter-American Development Bank Research Department Working Paper 369.
[63] Melitz, Marc. 2003. "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." Econometrica 71 (6): 1695-1725.
[64] Merz, Monika and Eran Yashiv. 2007. "Labor and the Market Value of the Firm."

American Economic Review 97 (4): 1419-1431.
[65] Mondragón-Vélez, Camilo, and Ximena Peña. 2010. "Business Ownership and Self-Employment in Developing Economies: The Colombian Case." In International Differences in Entrepreneurship, edited Josh Lerner and Antoinette Schoar, 89-127. Chicago, IL: University of Chicago Press.
[66] Mortensen, Dale T. 2005. Wage Dispersion: Why Are Similar Workers Paid Differently? (Zeuthen Lectures). Cambridge, MA: MIT Press.
[67] Mortensen, Dale T. and Christopher A. Pissarides. 1999. "New Developments in Models of Search in the Labour Market." In Handbook of Labour Economics. Vol. 3B, edited by Orley Ashenfelter and David Card. 2567-2627. Amsterdam: Elsevier Science, North-Holland.
[68] Nilsen, Øivind A., Kjell G. Salvanesa, and Fabio Schiantarelli. 2007. "Employment Changes, the Structure of Adjustment Costs, and Plant Size." European Economic Review 51 (3):577-98.
[69] Olley, G. Steven and Ariel Pakes. 1996. "The Dynamics of Productivity in the Telecommunications Equipment Industry." Econometrica 64 (6): 1263-98.
[70] Pissarides, Christopher A. 2000. Equilibrium Unemployment Theory. Cambridge, MA: MIT Press.
[71] Roberts, Mark. 1996. "Employment Flows and Producer Turnover." In Industrial Evolution in Developing Countries, edited by Mark Roberts and James Tybout, 18-42. Oxford, UK: Oxford University Press.
[72] Rodrik, Dani. 1997. Has Globalization Gone too Far? Washington, D.C.: Institute for International Economics.
[73] Rossi-Hansberg, Esteban and Mark L. J. Wright. 2007. "Establishment Dynamics in the Aggregate Economy." American Economic Review 97 (5): 1639-1665.
[74] Stole, Lars A. and Jeffrey Zwiebel. 1996. "Intra-Firm Bargaining under NonBinding Contracts." The Review of Economic Studies 63 (3): 375-410.
[75] Utar, Hale. 2008. "Import Competition and Employment Dynamics." Unpublished.
[76] World Bank. 2013. World Development Indicators. Washington D.C.: World Bank.
[77] Yashiv, Eran. 2006. "Evaluating the Performance of the Search and Matching Model." European Economic Review 50 (4): 909-936.

# Appendices <br> for 

Firm Dynamics, Job Turnover, and Wage Distributions in an Open Economy
by

## A. Kerem Coşar, Nezih Guner and James Tybout

## 1 The Revenue Function

Value added net of exporting costs as a function of $z$ and $l$ (equation 12 in the paper) is given by

$$
\begin{equation*}
R(z, l)=\max _{m}\left\{G\left(z l^{\alpha} m^{1-\alpha}\right)-P m-c_{x} \mathcal{I}^{x}\left(z l^{\alpha} m^{1-\alpha}\right)\right\} . \tag{1}
\end{equation*}
$$

Given $G(z, l, m)=\exp \left[d_{H}+d_{F}\left(\eta^{0}\right)\right]\left(z l^{\alpha} m^{(1-\alpha)}\right)^{\frac{\sigma-1}{\sigma}}$, the first order condition for $m$ reads as

$$
P m=(1-\alpha) \frac{(\sigma-1)}{\sigma} \exp \left[d_{H}+\mathcal{I}^{x} d_{F}\left(\eta^{0}\right)\right]\left(z l^{\alpha} m^{(1-\alpha)}\right)^{\frac{\sigma-1}{\sigma}},
$$

or rearranging,

$$
\begin{equation*}
m=\left(\frac{(1-\alpha)}{P} \frac{\sigma-1}{\sigma} \exp \left[d_{H}+\mathcal{I}^{x} d_{F}\left(\eta^{0}\right)\right]\right)^{\frac{\sigma}{\sigma-1} \Lambda}\left(z \alpha^{\alpha}\right)^{\Lambda}, \tag{2}
\end{equation*}
$$

where $\Lambda=\frac{\sigma-1}{\sigma-(1-\alpha)(\sigma-1)}>0$.
Using this expression to eliminate $m$ from $G(z, l, m)$ yields:

$$
\begin{aligned}
G(z, l) & =\exp \left[d_{H}+\mathcal{I}^{x} d_{F}\left(\eta^{0}\right)\right]\left(z l^{\alpha}\right)^{\frac{\sigma-1}{\sigma}} \\
& \left\{\left(\frac{(1-\alpha)}{P} \frac{\sigma-1}{\sigma} \exp \left[d_{H}+\mathcal{I}^{x} d_{F}\left(\eta^{0}\right)\right]\right)^{\frac{\sigma}{\sigma-1} \Lambda}\left(z l^{\alpha}\right)^{\Lambda}\right\}^{\frac{(1-\alpha)(\sigma-1)}{\sigma}}, \\
& =\exp \left[d_{H}+\mathcal{I}^{x} d_{F}\left(\eta^{0}\right)\right]\left(z l^{\alpha}\right)^{\frac{\sigma-1}{\sigma}} \\
& \left\{\left(\frac{1-\alpha}{P}\right)\left(\frac{\sigma-1}{\sigma}\right) \exp \left[d_{H}+\mathcal{I}^{x} d_{F}\left(\eta^{0}\right)\right]\right\}^{(1-\alpha) \Lambda}\left(z l^{\alpha}\right)^{\Lambda \frac{(1-\alpha)(\sigma-1)}{\sigma}}, \\
& =P^{-(1-\alpha) \Lambda}\left[(1-\alpha)\left(\frac{\sigma-1}{\sigma}\right)\right]^{(1-\alpha) \Lambda} \exp \left[d_{H}+\mathcal{I}^{x} d_{F}\left(\eta^{0}\right)\right]^{\frac{\sigma}{\sigma-1} \Lambda}\left(z l^{\alpha}\right)^{\Lambda},
\end{aligned}
$$

where the derivation uses the fact that

$$
\frac{\sigma-1}{\sigma}+\Lambda \frac{(1-\alpha)(\sigma-1)}{\sigma}=\frac{(\sigma-1)}{\sigma}[1+(1-\alpha) \Lambda]=\Lambda .
$$

We can now derive a parameterized version of the net revenue function (equation 13 in the paper). From (2), optimal expenditures on intermediate inputs are:

$$
P m=P^{-(1-\alpha) \Lambda}\left[\left((1-\alpha) \frac{\sigma-1}{\sigma}\right) \exp \left[d_{H}+\mathcal{I}^{x} d_{F}\left(\eta^{0}\right)\right]\right]^{\frac{\sigma}{\sigma-1} \Lambda}\left(z l^{\alpha}\right)^{\Lambda} .
$$

Subtracting this expression and fixed exporting costs from gross revenues yields:

$$
\begin{aligned}
& R(z, l)=G(z, l)-P m-c_{x} \mathcal{I}^{x}(z, l) \\
& =\left[1-(1-\alpha) \frac{\sigma-1}{\sigma}\right] P^{-(1-\alpha) \Lambda}\left((1-\alpha) \frac{\sigma-1}{\sigma}\right)^{(1-\alpha) \Lambda} \\
& \exp \left[d_{H}+\mathcal{I}^{x} d_{F}\left(\eta^{0}\right)\right]^{\frac{\sigma}{\sigma-1}} \Lambda\left(z l^{\alpha}\right)^{\Lambda}-c_{x} \mathcal{I}^{x}(z, l) \\
& =\left[\frac{\sigma-(1-\alpha)(\sigma-1)}{\sigma}\right] P^{-(1-\alpha) \Lambda}\left((1-\alpha) \frac{\sigma-1}{\sigma}\right)^{(1-\alpha) \Lambda} \\
& \exp \left[d_{H}+\mathcal{I}^{x} d_{F}\left(\eta^{0}\right)\right]^{\frac{\sigma}{\sigma-1}}\left(z l^{\alpha}\right)^{\Lambda}-c_{x} \mathcal{I}^{x}(z, l) \\
& =P^{-(1-\alpha) \Lambda}\left(\frac{\sigma-1}{\sigma \Lambda}\right)\left((1-\alpha) \frac{\sigma-1}{\sigma}\right)^{(1-\alpha) \Lambda} \\
& \exp \left[d_{H}+\mathcal{I}^{x} d_{F}\left(\eta^{0}\right)\right]^{\frac{\sigma}{\sigma-1} \Lambda}\left(z l^{\alpha}\right)^{\Lambda}-c_{x} \mathcal{I}^{x}(z, l) \\
& =\underbrace{\Theta P^{-(1-\alpha) \Lambda} \exp \left[d_{H}+\mathcal{I}^{x} d_{F}\left(\eta^{0}\right)\right]^{\frac{\sigma}{\sigma-1}}}_{=\Delta(z, l)}\left(z l^{\alpha}\right)^{\Lambda}-c_{x} \mathcal{I}^{x}(z, l),
\end{aligned}
$$

where $\Theta=\left(\frac{1}{(1-\alpha) \Lambda}\right)\left[\frac{(1-\alpha)(\sigma-1)}{\sigma}\right]^{\frac{\sigma}{\sigma-1} \Lambda}$. Note that $\alpha \Lambda<1$, so the net revenue function displays diminishing marginal returns to labor.

## 2 The Wage Functions

### 2.1 Hiring Wages

For a hiring firm in state $\left(z^{\prime}, l^{\prime}\right)$, the marginal worker generates surplus

$$
\begin{equation*}
\Pi^{f i r m}=\frac{1}{1+r}\left[\frac{\partial \tilde{\pi}\left(z^{\prime}, l^{\prime}\right)}{\partial l^{\prime}}+\frac{\partial V\left(z^{\prime}, l^{\prime}\right)}{\partial l^{\prime}}\right] \tag{3}
\end{equation*}
$$

where $\tilde{\pi}\left(z^{\prime}, l^{\prime}\right)=R\left(z^{\prime}, l^{\prime}\right)-w\left(z^{\prime}, l^{\prime}\right) l^{\prime}$ measures current period profits and $\frac{\partial V\left(z^{\prime}, l^{\prime}\right)}{\partial l^{\prime}}$ measures the effect of the marginal worker on next period's expected hiring or firing costs. ${ }^{1}$ Note that it is possible that a firm hires in the current period and exits at the beginning of the next period. In this case, the effect of the marginal worker on the firm's continuation value is $\frac{\partial V\left(z^{\prime}, l^{\prime}\right)}{\partial l^{\prime}}=0$.

Given the wage schedule $w\left(z^{\prime}, l^{\prime}\right)$, the marginal worker at this hiring firm enjoys surplus:

$$
\begin{equation*}
\Pi^{w o r k}=\frac{1}{1+r}\left[w\left(z^{\prime}, l^{\prime}\right)+J^{e}\left(z^{\prime}, l^{\prime}\right)-\left(b+J^{0}\right)\right] . \tag{4}
\end{equation*}
$$

Combined with the Nash bargaining condition,

$$
\begin{equation*}
(1-\beta) \Pi^{w o r k}=\beta \Pi^{f i r m} \tag{5}
\end{equation*}
$$

these expressions allow us to derive the hiring wage schedule. The formulation of the bargaining problem follows Bertola and Caballero (1994), Bertola and Garibaldi (2001), and Koeniger and Prat (2007). If one assumes that a firm has to pay the firing costs in case the bargaining fails, then these costs should be subtracted from its threat point. Ljungqvist (2002) discusses alternative assumptions for the case of one-worker one-firm matching models. We assume that separations are costless when bargaining fails (which don't happen in equilibrium), so firing costs do not figure into (3).

The derivation first uses (3), (4) and (5) to express total worker surplus in terms of total firm surplus:

$$
\begin{equation*}
\Pi^{\text {work }}=\frac{\beta}{1-\beta} \frac{1}{1+r}\left[\frac{\partial \tilde{\pi}\left(z^{\prime}, l^{\prime}\right)}{\partial l^{\prime}}+\frac{\partial V\left(z^{\prime}, l^{\prime}\right)}{\partial l^{\prime}}\right] . \tag{6}
\end{equation*}
$$

Assuming that continuation values are shared the same way as current flows, we can write: ${ }^{2}$

$$
\begin{equation*}
\beta \frac{\partial V\left(z^{\prime}, l^{\prime}\right)}{\partial l^{\prime}}=(1-\beta)\left[J^{e}\left(z^{\prime}, l^{\prime}\right)-J^{0}\right] . \tag{7}
\end{equation*}
$$

[^18]Substituting these sharing rules into the worker's surplus equation (4) and the total surplus sharing rule (6), the wage function must solve:

$$
\begin{equation*}
\frac{\beta}{1-\beta}\left[\frac{\partial \tilde{\pi}\left(z^{\prime}, l^{\prime}\right)}{\partial l^{\prime}}+\frac{\partial V\left(z^{\prime}, l^{\prime}\right)}{\partial l^{\prime}}\right]=w\left(z^{\prime}, l^{\prime}\right)-b+\frac{\beta}{(1-\beta)} \frac{\partial V\left(z^{\prime}, l^{\prime}\right)}{\partial l^{\prime}} . \tag{8}
\end{equation*}
$$

Cancelling terms, re-arranging, and using $\frac{\partial \tilde{\pi}\left(z^{\prime}, l^{\prime}\right)}{\partial l^{\prime}}=\frac{\partial R\left(z^{\prime}, l^{\prime}\right)}{\partial l^{\prime}}-w\left(z^{\prime}, l^{\prime}\right)-\frac{\partial w\left(z^{\prime}, l^{\prime}\right)}{\partial l^{\prime}} l^{\prime}$, equation (8) implies:

$$
\begin{equation*}
w\left(z^{\prime}, l^{\prime}\right)=(1-\beta) b+\beta\left[\frac{\partial R\left(z^{\prime}, l^{\prime}\right)}{\partial l^{\prime}}-\frac{\partial w\left(z^{\prime}, l^{\prime}\right)}{\partial l^{\prime}} l^{\prime}\right] . \tag{9}
\end{equation*}
$$

Finally, following footnote 10 in Koeninger and Prat (2007), which in turn is based on Bertola and Garibaldi (2001), one can solve this differential equation. First re-write (9) in the form:

$$
\begin{equation*}
\frac{d x\left(l^{\prime}\right)}{d l^{\prime}}+x\left(l^{\prime}\right) p\left(l^{\prime}\right)+q\left(l^{\prime}\right)=0, \tag{10}
\end{equation*}
$$

where

$$
\begin{aligned}
& x\left(l^{\prime}\right)=w\left(z^{\prime}, l^{\prime}\right), \\
& p\left(l^{\prime}\right)=1 /\left(\beta l^{\prime}\right),
\end{aligned}
$$

and

$$
q\left(l^{\prime}\right)=-\left[\frac{\partial R\left(z^{\prime}, l^{\prime}\right)}{\partial l^{\prime}}+\frac{(1-\beta)}{\beta} b\right] / l^{\prime} .
$$

Plugging these expressions for $x\left(l^{\prime}\right), p\left(l^{\prime}\right)$, and $q\left(l^{\prime}\right)$ into the solution to equation (10), and recalling $\frac{\partial R(z, l)}{\partial l}=\Delta \alpha \Lambda z^{\Lambda} l^{\alpha \Lambda-1}$, we can then express wages as ${ }^{3}$

$$
w\left(z^{\prime}, l^{\prime}\right)=\left(l^{\prime}\right)^{-1 / \beta} \int_{0}^{l^{\prime}} \Delta \alpha \Lambda\left(z^{\prime}\right)^{\Lambda} x^{\frac{1-\beta}{\beta}} x^{\alpha \Lambda-1} d x+(1-\beta) b .
$$

Integration yields the wage expression in the text:

$$
\begin{equation*}
w_{h}\left(z^{\prime}, l^{\prime}\right)=(1-\beta) b+\frac{\beta}{1-\beta+\alpha \beta \Lambda} \underbrace{\Delta \alpha \Lambda\left(z^{\prime}\right)^{\Lambda}\left(l^{\prime}\right)^{\alpha \Lambda-1}}_{=\partial R\left(z^{\prime}, l^{\prime}\right) / \partial l^{\prime}} . \tag{11}
\end{equation*}
$$

### 2.2 Firing Wages

To derive the firing wage schedule, we begin by writing the value of employment at a firing firm in the interim stage as

[^19]$$
J_{f}^{e}\left(z^{\prime}, l\right)=\frac{1}{1+r}\left[p_{f}\left(z^{\prime}, l\right)(1+r) J^{u}+\left(1-p_{f}\left(z^{\prime}, l\right)\right)\left(w_{f}\left(z^{\prime}, l^{\prime}\right)+J^{e}\left(z^{\prime}, l^{\prime}\right)\right)\right]
$$
where $l^{\prime}=L\left(z^{\prime}, l\right)$ and $p_{f}\left(z^{\prime}, l\right)=\left(l-l^{\prime}\right) / l^{\prime}$ is the probability of being fired. Workers randomly laid off in the interim stage go back to the unemployment pool and can search for a job within the same period. Those who are retained do so for a wage level $w_{f}\left(z^{\prime}, l^{\prime}\right)$ that leaves them indifferent between staying and leaving. For this to be the case, the two terms inside the bracket must be equal, i.e.,
$$
w_{f}\left(z^{\prime}, l^{\prime}\right)+J^{e}\left(z^{\prime}, l^{\prime}\right)=(1+r) J^{u},
$$
which yields the wage schedule according to which workers in firing firms are paid:
$$
w_{f}\left(z^{\prime}, l^{\prime}\right)=r J^{u}-\left[J^{e}\left(z^{\prime}, l^{\prime}\right)-J^{u}\right] .
$$

## 3 Steady State Equilibrium

Let the transition density of the Markov process on $z$ be denoted by $\Omega\left(z^{\prime} \mid z\right)$. Given a measure of aggregate expenditure abroad denominated in foreign currency, $D_{F}^{*}$, a steady state equilibrium for a small open economy consists of a measure of domestic differentiated goods $N_{H}$; an exact price index for the composite good $P$; an aggregate domestic demand index for industrial goods $D_{H}$; aggregate income $I$; a measure of workforce in services $L_{s}$; a measure workers in differentiated goods sector $L_{q}$; a measure of workers searching for jobs in the industrial sector $U$; a measure of unemployed workers $L_{u}$; the job finding rate $\widetilde{\phi}$; the vacancy filling rate $\phi$; the exit rate $\mu_{e x i t}$; the fraction of firms exporting $\mu_{x}$; the measure of entrants $N_{e}$; the value and associated policy functions $\mathcal{V}(z, l), L(z, l), \mathcal{I}^{h}(z, l), \mathcal{I}^{c}(z, l)$, $\mathcal{I}^{x}(z, l), J^{o}, J^{u}, J^{s}$, and $J^{e}$; the wage schedules $w_{h}(z, l)$ and $w_{f}(z, l)$; the exchange rate $k$; and end-of period and interim distributions $\psi(z, l)$ and $\widetilde{\psi}(z, l)$ satisfying the following conditions:

1. Steady state distributions: In equilibrium, $\psi(z, l)$ and $\widetilde{\psi}\left(z^{\prime}, l\right)$ reproduce themselves through the Markov processes on $z$, the policy functions, and the productivity draws upon entry. The interim distribution is composed of incumbent firms that did not exit at the beginning of the period and entering firms. Since all entering firms start the interim period with $l_{e}$ workers and a productivity draw from $\psi_{e}\left(z^{\prime}\right)$, we can measure the firms in state $\left(z^{\prime}, l\right)$ as:

$$
\widetilde{\psi}\left(z^{\prime}, l\right)= \begin{cases}(1-\delta) \int_{z} \Omega\left(z^{\prime} \mid z\right) \psi(z, l) \mathcal{I}^{c}(z, l) d z & \text { if } l \neq l_{e} \\ \frac{N_{e}}{N_{H}} \psi_{e}\left(z^{\prime}\right)+(1-\delta) \int_{z} \Omega\left(z^{\prime} \mid z\right) \psi(z, l) \mathcal{I}^{c}(z, l) d z & \text { if } l=l_{e}\end{cases}
$$

where $\frac{N_{e}}{N_{H}}$ is the fraction of firms that turn over every period for exogenous or endogenous reasons:

$$
\frac{N_{e}}{N_{H}}=1-(1-\delta) \int_{l} \int_{z} \psi(z, l) \mathcal{I}^{c}(z, l) d z d l .
$$

Applying the policy function for employment choices to $\widetilde{\psi}\left(z^{\prime}, l\right)$, we obtain the end-of period distribution of firms across states:

$$
\psi\left(z^{\prime}, l^{\prime}\right)=\frac{\int_{l} \widetilde{\psi}\left(z^{\prime}, l\right) \mathcal{I}_{L\left(z^{\prime}, l\right)=l^{\prime}} d l}{\int_{z^{\prime \prime}} \int_{l} \widetilde{\psi}\left(z^{\prime \prime}, l\right) \mathcal{I}_{L\left(z^{\prime \prime}, l\right)=l^{\prime}} d z^{\prime \prime} d l},
$$

where $\mathcal{I}_{L\left(z^{\prime}, l\right)=l^{\prime}}=1$ if $L\left(z^{\prime}, l\right)=l^{\prime}$ and $\mathcal{I}_{L\left(z^{\prime}, l\right)=l^{\prime}}=0$ otherwise.
2. Market clearance in the service sector: Demand for services comes from two sources: consumers spend a $(1-\gamma)$ fraction of aggregate income $I$ on it, and firms demand it to pay their fixed operation and exporting costs, as well as labor adjustment and market entry costs. Aggregate income $I$ itself is the sum of wage income earned by service and industrial sector workers, market services supplied by unemployed workers, tariff revenues rebated to worker-consumers, and aggregate profits in the industrial sector distributed to worker-consumers who own the firms, given by

$$
\begin{aligned}
I & =\underbrace{\int_{l} \int_{z} \psi(z, l)\left[\mathcal{I}^{h}(z, l) w_{h}(z, l)+\left(1-\mathcal{I}^{h}(z, l) w_{f}(z,) l\right] l d z d l+L_{s}+L_{u} b\right.}_{\text {total wage income }} \\
& +\underbrace{D_{H} \tau_{a}^{-\sigma}\left(\tau_{c} k\right)^{1-\sigma}\left(\tau_{a}-1\right)}_{\text {tariff revenue }} \\
& +\underbrace{N_{H} \int_{l} \int_{z} \psi(z, l)\left[\pi(z, l, L(z, l))-\mu_{e x i t} c_{e}\right] d z d l}_{\text {aggregate profits }} .
\end{aligned}
$$

The average labor adjustment cost for firms in the interim with $l$ is given by

$$
\bar{c}=\int_{z} \int_{l} C(l, L(z, l)) \widetilde{\psi}(z, l) d l d z .
$$

The market clearance condition is then given by

$$
L_{s}+b L_{u}=(1-\gamma) I+N_{H}\left(\bar{c}+c_{p}+\mu_{x} c_{x}\right)+N_{e} c_{e} .
$$

3. Labor market clearing: At the beginning of each period, the total number of industrial production jobs is

$$
L_{q}=N_{H} \bar{l},
$$

where

$$
\begin{equation*}
\bar{l}=\int_{z} \int_{l} l \psi(z, l) d l d z, \tag{12}
\end{equation*}
$$

is the sector's average employment. Some of these jobs are destroyed as firms exit-for exogenous or endogenous reasons - or downsize. Summing these sources of job destruction, we obtain our measure of industrial workers who are thrown into unemployment before the interim period:

$$
\begin{aligned}
\tilde{U} & =\delta N_{H} \int_{z} \int_{l} l \psi(z, l) d l d z \\
& +(1-\delta) N_{H} \int_{z} \int_{l}\left[1-\mathcal{I}^{c}(z, l)\right] l \psi(z, l) d l d z \\
& +N_{H} \int_{z^{\prime}} \int_{l} \mathcal{I}^{f}\left(z^{\prime}, l\right)\left[l-L\left(z^{\prime}, l\right)\right] \tilde{\psi}\left(z^{\prime}, l\right) d l d z^{\prime}
\end{aligned}
$$

Recall that $\mathcal{I}^{f}\left(z^{\prime}, l\right)=0$ for all entering firms, which must hire at least $l_{e}$ workers. The associated rate of job destruction is:

$$
\begin{equation*}
\mu_{l}=\frac{\tilde{U}}{L_{q}} . \tag{13}
\end{equation*}
$$

In the steady state equilibrium there are no net flows of workers out of the service sector. Accordingly, the total number of industrial job seekers each period includes those who just lost their jobs $(\tilde{U})$, and those who searched unsuccessfully for industrial jobs last period $\left(L_{u}\right)$ :

$$
\begin{equation*}
U=\tilde{U}+L_{u} \tag{14}
\end{equation*}
$$

Since $L_{u}=(1-\widetilde{\phi}) U$, equations (13) and (14) imply:

$$
U \widetilde{\phi}=L_{q} \mu_{l} .
$$

That is, the number of workers flowing into industrial jobs, $U \widetilde{\phi}$, must match the number of industrial jobs that are turning over. Finally, at the end of each period, workers either must have jobs in one of the sectors or be unsuccessful industrial job seekers:

$$
1=L_{s}+L_{q}+L_{u} .
$$

On the vacancies side, the aggregate number of vacancies posted is given by

$$
V=N_{H} \int_{z^{\prime}} \int_{l} v\left(z^{\prime}, l\right) \widetilde{\psi}\left(z^{\prime}, l\right) d l d z^{\prime}+N_{e} l_{e} / \phi
$$

which includes the $l_{e} / \phi$ vacancies that entrants post to hire their initial workforce. The total number of vacancies, $V$, together with $U$, determines matching probabilities $\phi(V, U)$ and $\widetilde{\phi}(V, U)$ that firms and workers take as given.
4. Firm turnover: In equilibrium, there is a positive mass of entry $N_{e}$ every period so that the free entry condition

$$
\begin{equation*}
\mathcal{V}_{e}=\frac{1}{1+r} \int_{z} \max _{l^{\prime}}\left[\pi\left(z, l_{e}, l^{\prime}\right)+\mathcal{V}\left(z, l^{\prime}\right)\right] \psi_{e}(z) d z \leq c_{e} \tag{15}
\end{equation*}
$$

holds with equality. The fraction of firms exiting is implied by the steady state distribution and the exit policy function,

$$
\mu_{e x i t}=(1-\delta) \int_{z} \int_{l}\left[1-\mathcal{I}^{c}(z, l)\right] \psi(z, l) d l d z+\delta,
$$

and measure of exits equals that of entrants,

$$
N_{e}=\mu_{e x i t} N_{H} .
$$

5. Trade balance: Adding up final and intermediate demand, total domestic expenditures on imported varieties equals $D_{H}\left(\tau_{a} \tau_{c} k\right)^{1-\sigma}$. Taking the import tariff into account, domestic demand for foreign currency (expressed in domestic currency) is thus $\frac{D_{H}\left(\tau_{a} \tau_{c} k\right)^{1-\sigma}}{\tau_{a}}=D_{H} \tau_{a}^{-\sigma}\left(\tau_{c} k\right)^{1-\sigma}$. Tariff revenue is given by $D_{H} \tau_{a}^{-\sigma}\left(\tau_{c} k\right)^{1-\sigma}\left(\tau_{a}-1\right)$, and is returned to worker-consumers in the form of lump-sum transfers. Total export revenues are $\frac{k D_{F}^{*} P_{1}^{* 1-\sigma}}{\tau_{c}}$ with the foreign market price index for exported goods $P_{X}^{*}$ as defined in Section I.C in the paper. Trade is balance given by

$$
\underbrace{\frac{D_{H}\left(\tau_{a} \tau_{c} k\right)^{1-\sigma}}{\tau_{a}}}_{\text {domestic demand for foreign currency }}=\underbrace{\frac{k D_{F}^{*} P_{X}^{* 1-\sigma}}{\tau_{c}}}_{\text {export revenue }} .
$$

The exchange rate $k$ moves to ensure that this condition holds. Balanced trade ensures that national income matches national expenditure.
6. Workers are indifferent between taking a certain job in the undifferentiated sector and searching for a job in the industrial sector: $J^{o}=J^{s}=J^{u}$.

## 4 Numerical Solution Algorithm

To compute the value functions, we discretize the state space on a log scale using 550 grid points for employment and 60 grid points for productivity. We set the maximum firm size as 2000 workers and numerically check that this is not restrictive. In the steady state, a negligible fraction of firms reaches this size, which is also the case in the data. The algorithm works as follows:

1. Formulate guesses for $D_{H}, w_{f}(z, l), w_{h}(z, l), d_{F}$ and $\phi$. Given $\phi$, calculate $\widetilde{\phi}=\left(1-\phi^{\theta}\right)^{1 / \theta}$.
2. Given $D_{H}, w_{f}(z, l), d_{F}, \phi$ and $w_{h}(z, l)$, calculate the value function for the firm, $\mathcal{V}(z, l)$ as

$$
\begin{equation*}
\mathcal{V}(z, l)=\max \left\{0, \quad \frac{1-\delta}{1+r} E_{z^{\prime} \mid z} \max _{l^{\prime}}\left[\pi\left(z^{\prime}, l, l^{\prime}\right)+\mathcal{V}\left(z^{\prime}, l^{\prime}\right)\right]\right\}, \tag{16}
\end{equation*}
$$

and find the associated decision rules for exiting, hiring, and exporting. Calculate the expected value of entry, $\mathcal{V}_{e}$, using equation (15). Compare $\mathcal{V}_{e}$ with $c_{e}$. If $\mathcal{V}_{e}>c_{e}$, decrease $D_{H}$ (to make entry less valuable) and if $\mathcal{V}_{e}<c_{e}$, increase $D_{H}$ (to make entry more valuable). Go back to Step 1 with the updated value of $D_{H}$ and repeat until $D_{H}$ converges.
3. Given $w_{f}(z, l), d_{F}, \phi$ and the converged value of $D_{H}$ from Step 2, update $w_{f}(z, l)$. To do this, first calculate $J^{e}\left(z^{\prime}, l^{\prime}\right)$ using

$$
\begin{equation*}
J_{h}^{e}\left(z^{\prime}, l\right)=\frac{1}{1+r}\left[w_{h}\left(z^{\prime}, l^{\prime}\right)+J^{e}\left(z^{\prime}, l^{\prime}\right)\right] \tag{17}
\end{equation*}
$$

and

$$
\begin{align*}
J^{e}(z, l)= & {\left[\delta+(1-\delta)\left(1-\mathcal{I}^{c}(z, l)\right)\right] J^{u} } \\
& +(1-\delta) \mathcal{I}^{c}(z, l) \max \left\{J^{u}, E_{z^{\prime} \mid z}\left[\mathcal{I}^{h}\left(z^{\prime}, l\right) J_{h}^{e}\left(z^{\prime}, l\right)+\left(1-\mathcal{I}^{h}\left(z^{\prime}, l\right)\right) J^{u}\right]\right\} \tag{18}
\end{align*}
$$

and imposing the equilibrium condition $J^{u}=J^{o}$. Given $J^{e}(z, l)$, update firing wage schedule using

$$
\begin{equation*}
w_{f}\left(z^{\prime}, l^{\prime}\right)=r J^{u}-\left[J^{e}\left(z^{\prime}, l^{\prime}\right)-J^{u}\right] . \tag{19}
\end{equation*}
$$

Compare the updated firing wage schedule with the initial guess. If they are not close enough go back to Step 1 with the new firing wage schedule and repeat Steps 1 to 3 until $w_{f}$ converges. Note that if firing wages are too high, then $J^{e}(z, l)$-the value of being in a firm at the start of a period- is high, since the firm is less likely to fire workers. A high value of $J^{e}(z, l)$, however, lowers firing wages. Similarly, if firing wages are too low, then $J^{e}$ is low, which pushes firing wages up.
4. Given $d_{F}$ and $\phi$, the converged value of $D_{H}$ from step 2 , and the converged value of $w_{f}(z, l)$ from Step 3, update $w_{h}(z, l)$ using

$$
\begin{equation*}
w_{h}\left(z^{\prime}, l^{\prime}\right)=(1-\beta) b+\frac{\beta}{1-\beta+\alpha \beta \Lambda} \Delta\left(z^{\prime}, l^{\prime}\right) \alpha \Lambda\left(z^{\prime}\right)^{\Lambda}\left(l^{\prime}\right)^{\alpha \Lambda-1} . \tag{20}
\end{equation*}
$$

5. Given $\phi$, the converged value of $D_{H}$ from Step 2, the converged value of $w_{f}(z, l)$ from Step 3, and the converged value of $w_{h}(z, l)$ from step 4, calculate the trade balance. To do this:
(a) Given firms' decisions, calculate $\psi(z, l)$ and $\widetilde{\psi}(z, l)$, the stationary probability distributions over $(z, l)$ at the end and interim states, respectively.
(b) Given $\widetilde{\psi}(z, l)$, calculate the average number of vacancies $\bar{v}=\int_{z^{\prime}} \int_{l} v\left(z^{\prime}, l\right) \widetilde{\psi}\left(z^{\prime}, l\right) d l d z^{\prime}$ and the average employment in the industrial sector using equation 12 .
(c) Take a guess for $N_{H}$. Firms' decisions and the steady state distribution $\psi(z, l)$ pin down exit rate $\mu_{\text {exit }}$ as defined above, which implies a mass of entrants $N_{e}=$ $\mu_{e x i t} N_{H}$. Given $N_{H}, N_{e}, \bar{v}$ and $v_{e}=l_{e} / \phi$, calculate the unique mass of unemployed $U$ in the industrial sector solving

$$
\phi(V, U)=\frac{M(V, U)}{V}=\frac{U}{\left[\left(N_{H} \bar{v}+N_{e} v_{e}\right)^{\theta}+U^{\theta}\right]^{1 / \theta}}
$$

Given $U$, calculate $L_{u}=(1-\widetilde{\phi}) U$. Then, given $\bar{l}$, the size employment in the service sector is given by $L_{s}=1-L_{u}-N_{H} \bar{l}$. With $N_{H}, L_{s}, L_{u}, N_{e}$, and I (aggregate income) at hand, check if supply and demand are equal in the service sector:

$$
\underbrace{L_{s}+b L_{u}}_{\text {supply }}=\underbrace{(1-\gamma) I+N_{H}\left(\bar{c}+c_{p}+\mu_{x} c_{x}\right)+N_{e} c_{e}}_{\text {demand }} .
$$

Update $N_{H}$ until this market clearance condition holds.
(d) Given the value of $N_{H}$ from Step 5c, calculate exports and imports. If exports are larger than imports, lower $d_{F}$; if exports are less than imports, increase $d_{F}$. Go back to Step 1 with the updated value of $d_{F}$, and repeat until convergence.
6. Given the converged value of $D_{H}$ from Step 2, the converged value of $w_{f}(z, l)$ from Step 3, the converged value of $w_{h}(z, l)$ from Step 4, and the converged value of $d_{F}$ from Step 5, update $\phi$. In order to do that, first calculate $E J_{h}^{e}$ using (17). Given $E J_{h}^{e}$ and $\widetilde{\phi}$, calculate $J^{u}$ using

$$
\begin{equation*}
J^{u}=\left[\widetilde{\phi} E J_{h}^{e}+\frac{(1-\widetilde{\phi})}{1+r}\left(b+J^{o}\right)\right] . \tag{21}
\end{equation*}
$$

If $J^{o}>J^{u}$, increase $\phi$ (to attract workers to the differentiated goods sector) and if $J^{o}<J^{u}$, we lower $\phi$ (to make the differentiated goods sector less attractive). Go back to Step 2, and repeat until $\phi$ converges.

### 4.1 Estimation Procedure

In the estimation of the model, we set $D_{H}$ and $d_{F}$ to their data counterparts, which allows us to skip Steps 2 and $5 d$ above and considerably reduces the computation time. In order to compute $D_{H}$, we use the Olley-Pakes intercept $\tilde{d}_{H}$ estimated from

$$
\begin{equation*}
\ln G_{i t}=\tilde{d}_{H}+\mathcal{I}_{i t}^{x} d_{F}\left(\eta_{0}\right)+\left[\frac{\sigma-1}{\sigma}(1-\alpha)\right] \ln \left(P m_{i t}\right)+\varphi\left(\ln l_{i t-1}, \ln l_{i t}\right)+\xi_{i t} \tag{22}
\end{equation*}
$$

to calculate firms' net revenue schedule $R(\cdot)$.
Furthermore, in the estimation, we treat foreign market size $D_{F}^{*}$ as a parameter to be estimated and $d_{F}$ as a moment to be matched. Given $\tilde{d}_{H}$ and the estimated value of the foreign market size parameter $D_{F}^{*}$, we calculate $\eta$ using

$$
\begin{equation*}
\eta^{o}=\arg \max _{0 \leq \eta \leq 1} d_{F}(\eta)=\left(1+\frac{\tau_{c}^{\sigma-1} D_{H}}{k^{\sigma} D_{F}^{*}}\right)^{-1} \tag{23}
\end{equation*}
$$

which allows to use the implied $d_{F}$ directly in our solution algorithm. The price level $P$ and exchange rate $k$ can easily be solved in equilibrium so that trade balance holds and $\tilde{d}_{H}$ is consistent with $D_{H}$. Having used the empirical $\tilde{d}_{H}$ to construct the gross revenue function and solve for firms' problem in the estimation, we can calculate the value of entry $\mathcal{V}_{e}$. Assuming that the economy is in a steady state with positive entry, we back out $c_{e}$ by $c_{e}=\mathcal{V}_{e}$. This approach to discipline the cost of entry $c_{e}$ is in line with the quantitative literature (Hopenhayn and Rogerson 1993).

In our policy experiments, however, we use the complete algorithm to compute equilibrium outcomes for given a set of parameters, including the cost of entry $c_{e}$. In these experiments, both $d_{F}$ and $D_{H}$ are equilibrium objects that respond to changes in $\tau_{a}, \tau_{c}$ and $c_{f}$.

## 5 Further Results and Data Sources

### 5.1 Labor Units

Since workers are all identical in the model economy, we measure the labor input $l$ in terms of "effective worker" units in our estimation. This allows us to control for the effects of
worker heterogeneity on output. In the plant-level data we use to estimate our model for the pre-reform period, we observe five categories of workers: managerial, technical, skilled, unskilled, and apprentice. For a given plant-year, effective labor $l$ is the sum of all workers in the plant, each weighted by the average wage (including fringe benefits) for workers in its category. For each category of worker, the average wage is based on the mean real wage in the entire 10-year panel and expressed as a ratio to the average real wage for unskilled workers during the same period. Thus wage weights are constant across plants and time, and the only source of variation in $l$ is variation in the employment level of at least one category of worker.

After fitting the model to the pre-reform data, we simulate Colombian reforms and decreased trade costs in Section II of the paper. To evaluate the success of the model in explaining post-reform outcomes, we wish to compare the firm size distribution as predicted by the model to its empirical counterpart. Since we do not have access to the plant-level data from the post-reform period, we don't observe the above-described variables used to construct effective labor. While The Colombian Statistical Agency DANE publishes summary statistics on the size distribution of plants for the 2000-2006 period (http://www. dane.gov. co/index.php/industria/encuesta-anual-manufacturera-eam), these are based on the number of total employees.

The following procedure facilitates the comparison of model based and empirical size distributions in both periods (see Figure 4 in the paper). Using the pre-reform plant level, we first fit total number of workers to a polynomial of effective labor $l$. We then use the coefficients from this regression to convert model-generated effective labor $l$ units to worker count for both the estimated pre-reform and simulated post-reform periods. The bars with patterns in Figure 4 in the paper-representing the model-based size distributions - are generated using this transformation. The bars with solid colors representing the empirical size distributions are generated directly from the data using total number of employees.

### 5.2 Sectoral Labor Flows in Colombia

The Colombian Statistical Agency DANE publishes monthly labor market indicators. We accessed the following link on August 11, 2015:
https://www.dane.gov.co/files/investigaciones/boletines/ech/ech/anexo_ech_may12.xls.
The file is in Spanish but variable names can be easily translated using online translators. In this file, the worksheets titled " ocup ramas trim tnal" indicates sectoral employment levels. The worksheet titled "cesantes ramas trim tnal" reports the sector of previous employment for separations. The ratio of the latter over the former gives sectoral separation rates. We exclude agriculture and mining, and find employment-weighted average for service industries.

For the 2001-2008 period, average separation rates are 0.122 for manufacturing and 0.14 for services.

### 5.3 Size-Wage Relationship

Table A1 shows the effect of size (measured by the number of workers) and productivity on wages for hiring and non-hiring firms in the Colombian establishment survey data and the model.

Table A1: Wage-Size Relationship Controlling for Productivity

| $\ln w=\alpha+\beta_{l} \ln l+\beta_{z} \ln z+\varepsilon$ | Non-expanding firms |  | Expanding firms |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Data | Model | Data | Model |
| $\beta_{l}$ | 0.075 | -0.232 | 0.064 | -0.231 |
|  | $(0.002)$ | $(0.003)$ | $(0.002)$ | $(0.004)$ |
| $\beta_{z}$ | 0.586 | 1.381 | 0.502 | 1.374 |
|  | $(0.005)$ | $(0.009)$ | $(0.006)$ | $(0.015)$ |
| $R^{2}$ | 0.46 | 0.55 | 0.51 | 0.54 |

Notes: l is effective workers defined in online Appendix 5.1, $w$ is average wage per effective worker, and $z$ is firm-level productivity implied by equation (29) in the paper given the data and estimated parameter values.

### 5.4 Additional Figures



Figure A1: Baseline Outcomes


Figure A2: Change in Wage Schedules


Figure A3: Change in Worker Value Functions


Figure A4: Change in Average Wages by Firm Type


Figure A5: Change in Average Worker Value by Firm Type


Figure A6: Shifts in the Size Distribution


Figure A7: Employment Policy Changes

Figure A1 shows wage schedules (equation 26 or 27 in the main text depending on whether a firm is hiring or not), percentage employment adjustments $\left(L\left(z^{\prime}, l\right)-l\right) / l$, worker value functions $J$ and the distribution $\psi(z, l)$.

In order to shed light on the mechanisms described in sections I.K and III.B of the paper, Figures A2-3 show the change with respect to the baseline in wage schedules and worker value functions for any point in the state space $(z, \ln (l))$, regardless of whether it is populated by firms. In this and related figures, the first three panels feature the isolated policy changes (corresponding to columns 2-4 in table 4) while the fourth panel (bottom, right) features the case of reforms and globalization (column 6 in table 4).

Figures A4-5 are based on 100 grid points, each of which is the mid-point of a size and productivity decile in the baseline $(z, l)$ distribution. Holding these grid points fixed, we plot the changes in average wages and worker value functions. These graphs show how wages and employment policies shift in the populated portions of the state space.

Figure A6 shows the size distribution of firms above $l>10$ (mimicking the empirical sampling in the Colombian firm data), fit with a spline using a smoothing parameter of 0.6.

Figure A7 shows the change in employment policies, i.e., the percentage change in $\left(L\left(z^{\prime}, l\right)-l\right) / l$ with respect to the baseline.

## 6 Robustness to the Choice of Model Period

To isolate the role of model period in driving our results, we hold the estimation strategy fixed by using our estimated revenue function and productivity process to approximate their quarterly counterparts. ${ }^{4}$ Then we re-estimated remaining parameters using the same moment vector as in the annual baseline, aggregating simulated quarterly outcomes on flow variables to their annual equivalents, and taking simulated fourth quarter realizations on stock variables to be representative of their annual counterparts (as is done in the annual manufacturing surveys).

Specifically, we kept our estimate of the elasticity of value added with respect to labor $(\alpha \Lambda)$ based on annual data, and we chose the root of the quarterly productivity process to replicate our estimate of persistence in the annual process: $\rho_{q}=\rho_{a}^{1 / 4}$. Likewise, we adjusted the discount rate to $r_{q}=\left(1+r_{a}\right)^{1 / 4}-1$, and we divide the log revenue function intercept $\widetilde{d}_{H}$ by four to put revenue flows on a quarterly basis. Finally, since we saw no good way to approximate the relationship between the variance of the innovations in the annual data $\left(\sigma_{z, a}^{2}\right)$ and the variance of the innovations in the quarterly data $\left(\sigma_{z, q}^{2}\right)$, we included $\sigma_{z, q}^{2}$ in

[^20]the set of parameters to be estimated.
Tables A2 and A3 present the resulting parameter estimates and the fit of the model. The quarterly version doesn't fit as well as the annual baseline, perhaps because of the way we have constrained our revenue function estimates. Nonetheless, the quarterly results do give us some insight into the effects of model period choice on parameter estimates and model performance.

The major differences in parameter estimates are in the elasticity of substitution $\sigma$, the elasticity of the matching function $\theta$, and the value of home production $b$. The change in $b$ can be explained by the effect of model frequency on wage inequality. Allowing workers to search more frequently increases their reservation wages, which in turn affects the entire wage schedule. Other things equal, this would lower wage dispersion in the model. So, in order to still match the dispersion of $\log$ wages, the quarterly calibration lowers the constant term $(1-\beta) b$ in the hiring wage schedule (20). It does so by reducing $b$ from 0.433 to $0.302 .{ }^{5}$ The other major change in parameter values is the decrease in matching function elasticity $\theta$ from 1.838 to 1.154. This compensates for the fact that, other things equal, switching to a quarterly frequency would have increased labor market tightness as workers enjoyed more opportunities to match with firms. In turn, this would have made it more difficult for firms hire, and thus shifted the simulated firm size distribution leftward. Dropping $\theta$ improves the ability of firms to meet workers over the relevant range of $(U, V)$ values, and thus prevents this from occurring. Other parameter values such as exogenous exit rate $\delta$ and the initial firm size $l_{e}$ drop due to the increase in model frequency.

Table A4 addresses the main question of interest: how robust are the policy experiments in the paper to the change in model period? That is, it redoes Table 4 using the quarterly version of our model. Note that in order to facilitate comparison, we use the same $\tau_{c}=2.19$. This number, calibrated to replicate the 150 percent increase in the revenue share of exports in the baseline model, generates a similar (143 percent) increase in the quarterly model.

The results in Table A4 show that the effects of policy experiments are robust. First, while "Reforms" and "Reforms and Globalization" experiments generate higher firm growth rates at the baseline size quantiles, aggregate job turnover declines in both experiments. The decline in job turnover is, however, slightly smaller with the quarterly model. This reflects the shifts in parameter estimates described above. On the one hand, with a smaller home production payoff, $b$, wages are more sensitive (percentagewise) to firm characteristics $(z, l)$. On the other hand, a lower matching function elasticity makes job finding and fill rates less responsive to changes in aggregate labor market conditions. Second, "Reforms"

[^21]and "Reforms and Globalization" experiments result in similar levels of inequality in worker values $(J)$, measured either at the firm or worker levels. Finally, "Reforms and Globalization" generates a smaller increase in $Q$-sector unemployment compared to the baseline model (9.1 percent versus 19 percent) and a higher increase in real income (41.4 percent versus 12 percent).

Table A2: Parameters Estimated with SMM - annual vs quarterly

| Parameter | Description | Annual | Quarterly |
| :---: | :--- | :---: | :---: |
| $\sigma$ | Elasticity of substitution | 6.667 | 10.358 |
| $\alpha$ | Elasticity of output with respect to labor | 0.195 | 0.240 |
| $\beta$ | Bargaining power of workers | 0.441 | 0.411 |
| $\theta$ | Elasticity of the matching function | 1.838 | 1.154 |
| $\delta$ | Exogenous exit hazard | 0.064 | 0.019 |
| $c_{h}$ | Scalar, vacancy cost function | 0.448 | 2.549 |
| $\lambda_{1}$ | Convexity, vacancy cost function | 3.101 | 3.699 |
| $\lambda_{2}$ | Scale effect, vacancy cost function | 0.385 | 0.332 |
| $b$ | Value of home production | 0.433 | 0.302 |
| $l_{e}$ | Initial size of entering firms | 5.906 | 3.338 |
| $c_{p}$ | Fixed cost of operating | 7.839 | 12.291 |
| $c_{x}$ | Fixed exporting cost | 112.943 | 71.378 |
| $c_{e}$ | Entry cost for new firms | 15.794 | 146.816 |
| $\sigma_{z}$ | Standard deviation of the $z$ process | 0.137 | 0.061 |

Table A3: Data-based versus Simulated Statistics - annual vs quarterly

| Moment | Data | Annual | Quarterly | Size Distribution | Data | Annual | Quarterly |
| :--- | :---: | :---: | :---: | :--- | :---: | :---: | :---: |
| $E\left(\ln G_{t}\right)$ | 5.442 | 5.274 | 5.919 | 20th percentile cutoff | 14.617 | 15.087 | 35.043 |
| $E\left(\ln l_{t}\right)$ | 3.622 | 3.638 | 4.632 | 40th percentile cutoff | 24.010 | 24.736 | 74.043 |
| $E\left(\mathcal{I}_{t}^{x}\right)$ | 0.118 | 0.108 | 0.320 | 60th percentile cutoff | 41.502 | 42.559 | 148.92 |
| $\operatorname{var}\left(\ln G_{t}\right)$ | 2.807 | 3.334 | 67.94 | 80th percentile cutoff | 90.108 | 87.137 | 334.54 |
| $\operatorname{cov}\left(\ln G_{t}, \ln l_{t}\right)$ | 1.573 | 1.888 | 11.535 | Firm Growth Rates |  |  |  |
| $\operatorname{var}\left(\ln l_{t}\right)$ | 1.271 | 1.326 | 2.172 | <20th percentile | 1.425 | 1.287 | 1.321 |
| $\operatorname{cov}\left(\ln G_{t}, \mathcal{I}_{t}^{x}\right)$ | 0.230 | 0.264 | 2.788 | 20th-40th percentile | 0.255 | 0.251 | 0.403 |
| $\operatorname{cov}\left(\ln l_{t}, \mathcal{I}_{t}^{x}\right)$ | 0.153 | 0.175 | 0.480 | 40th-60th percentile | 0.209 | 0.191 | 0.242 |
| $\operatorname{cov}\left(\ln G_{t}, \ln G_{t+1}\right)$ | 2.702 | 2.119 | 43.48 |  | 60th-80th percentile | 0.184 | 0.155 |
| $\operatorname{cov}\left(\ln G_{t}, \ln l_{t+1}\right)$ | 1.538 | 1.534 | 9.091 | Aggregate Turnover |  |  | 0.168 |
| $\operatorname{cov}\left(\ln G_{t}, \mathcal{I}_{t+1}^{x}\right)$ | 0.225 | 0.283 | 2.784 | Wage Dispersion |  |  |  |
| $\operatorname{cov}\left(\ln l_{t}, \ln G_{t+1}\right)$ | 1.543 | 1.409 | -17.45 | Firm exit rate | 0.108 | 0.104 | 0.031 |
| $\operatorname{cov}\left(\ln l_{t}, \ln l_{t+1}\right)$ | 1.214 | 1.192 | 2.696 | Job turnover | 0.198 | 0.222 | 0.237 |
| $\operatorname{cov}\left(\ln l_{t}, \mathcal{I}_{t+1}^{x}\right)$ | 0.152 | 0.195 | 0.292 | Std. dev. of log wages | 0.461 | 0.380 | 0.449 |
| $\operatorname{cov}\left(\mathcal{I}_{t}^{x}, \ln G_{t+1}\right)$ | 0.220 | 0.273 | 2.292 | Olley-Pakes Statistics |  |  |  |
| $\operatorname{cov}\left(\mathcal{I}_{t}^{x}, \ln l_{t+1}\right)$ | 0.149 | 0.200 | 0.426 | (1-a)( $\left.\frac{\sigma-1}{\sigma}\right)$ | 0.685 | 0.685 | 0.686 |
| $\operatorname{cov}\left(\mathcal{I}_{t}^{x}, \mathcal{I}_{t+1}^{x}\right)$ | 0.090 | 0.075 | 0.193 |  | $d_{F}$ | 0.090 | 0.094 |

Table A4: Simulation Results

|  | Baseline | Reforms | Reforms and <br> Globalization |
| :--- | :---: | :---: | :---: |
| $c_{f}$ (firing cost) | 0.60 | 0.30 | 0.30 |
| $\tau_{a}$ (ad valorem tariff rate) | 1.21 | 1.11 | 1.11 |
| $\tau_{c}$ (iceberg trade cost) | 2.50 | 2.50 | 2.19 |
| Firm Growth Rates |  |  |  |
| (at the baseline size quantiles) |  |  |  |
| <20th percentile |  |  |  |
| 20th-40th percentile | 1.321 | 1.323 | 1.439 |
| 40th-60th percentile | 0.403 | 0.426 | 0.503 |
| 60th-80th percentile | 0.242 | 0.246 | 0.313 |
|  | 0.168 | 0.170 | 0.207 |
| Aggregates |  |  |  |
| Revenue share of exports | 1 |  |  |
| Exit rate | 1 | 1.404 | 2.434 |
| Job turnover | 1 | 0.974 | 1.106 |
| Mass of firms | 1 | 0.952 | 0.963 |
| Share of labor in $Q$ sector | 1 | 0.924 | 0.730 |
| Vacancy filling rate ( $\phi$ ) | 1.040 | 0.937 |  |
| Unemployment rate in $Q$ sector | 1 | 1 | 1.077 |
| Std. wages (firms) | 1 | 0.951 | 1.091 |
| Std. wages (workers) | 1 | 1.054 | 1.147 |
| Std. J (firms) | 1.037 | 1.019 |  |
| Std. J (workers) | 1 | 1.080 | 1.210 |
| Exchange rate | 1 | 1.079 | 1.215 |
| Real income | 1 | 0.950 | 0.610 |


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[^1]:    ${ }^{1}$ Global trade expansion stalled in the early 1980s, but resumed growing much more rapidly than production thereafter (Word Trade Organization Statistics Database, Time Series on international trade, accessed on 12/7/2014, http://stat.wto.org/StatisticalProgram/WSDBStatProgramHome.aspx?Language=E). Haltiwanger et al. (2004) document the general reduction in Latin American trade barriers during this period. Heckman and Pages (2004) survey labor market regulations and reforms in Latin America, noting that openness to international trade increased the demand for labor market flexibility.
    ${ }^{2}$ Trade data are taken from Feenstra, Inklaar, and Timmer (2013). The Inter-American Development Bank (2004) summarizes the deterioration in Latin American labor market conditions. Goldberg and Pavcnik (2007) survey the evidence linking openness to wage inequality and informality in Latin America and other developing countries. Haltiwanger et al. (2004) document the association between job turnover and openness in Latin America.

[^2]:    ${ }^{3}$ Rodrik (1996) makes a related argument, though he points to different mechanisms.
    ${ }^{4}$ This feature of our model captures a well-known empirical regularity. Haltiwanger, Jarmin, and Miranda (2013) provide recent evidence from the U.S.
    ${ }^{5}$ Other recent papers that study firm dynamics and labor market frictions in a closed economy context include Cooper, Haltiwanger, and Willis (2007), Lentz and Mortensen (2012), Hobijn and Sahin (2013), and Elsby and Michaels (2014). Utar (2008) studies firm dynamics and labor market frictions in an importcompeting industry that takes the wage rate as given.
    ${ }^{6}$ Many other recent papers examine the effects of trade on imperfect labor markets, but presume competitive product markets and homogenous firms. Examples of theoretical and empirical work in this vein include Davisdon, Martin, and Matusz (1999), Davidson, Matusz and Shevchenko (2008), Kambourov (2009), Artuc, Chaudhuri, and McLaren (2010), Coşar (2013), and Dix-Carneiro (2014).

[^3]:    ${ }^{7}$ The functional form of the matching function follows den Haan, Ramey, and Watson (2000). It is subject to constant returns to scale, and increasing in both arguments. In contrast to the standard Cobb-Douglas form, it has no scale parameter and the implied matching rates are bounded between zero and one. Note that for $V=U$, as $\theta$ approaches infinity, job finding and filling probabilities approach to one.
    ${ }^{8}$ The notion that workers trade job security in a low wage sector for the opportunity to search in a higher wage sector traces back at least to the Harris and Todaro (1970) model.

[^4]:    ${ }^{9}$ This specification generalizes Nilsen, Salvanesa, and Schiantarelli (2007), who set $\lambda_{2}=1-1 / \lambda_{1}$. See also Merz and Yashiv (2007), and Yashiv (2006).
    ${ }^{10} \mathrm{By}$ (15), labor force expansion at any given rate $g=\frac{l^{\prime}-l}{l}$ involves vacancy posting costs $\left(\frac{c_{h}}{\lambda_{1}}\right) \phi^{-\lambda_{1}} g^{\lambda_{1}} l^{\lambda_{1}\left(1-\lambda_{2}\right)}$. So for $\lambda_{1}\left(1-\lambda_{2}\right)>1, C_{h}\left(l, l^{\prime}\right) / l$ is increasing in $l$.
    ${ }^{11}$ As it is standard in the literature (see Ljungqvist 2002 for a review), we assume that firing costs take the form of a resource cost and are not pure transfers from firms to workers.

[^5]:    ${ }^{12}$ This feature of the model is not specific to the random search framework: Felbermayr, Impullitti, and Prat (2014) show that convex vacancy posting costs play a similar role in generating residual wage inequality in a directed search model. This is in contrast to Felbermayr, Prat, and Schmerer (2011) where vacancy posting costs are linear and independent of size. Firms then immediately expand to equalize the marginal revenue product of labor to the expected marginal recruitment cost. The latter being equal across all firms, there is no wage inequality.

[^6]:    ${ }^{13}$ Convex vacancy posting costs are necessary for this result. To better understand this feature of our model, suppose the marginal value of an additional worker is simply her marginal revenue product, $\alpha \Lambda \Delta z\left(l^{\prime}\right)^{\alpha \Lambda-1}$, and assume the entire cost of hiring $l^{\prime}$ workers is captured by the vacancy posting cost, $\left(\frac{c_{h}}{\lambda_{1}}\right) \phi^{-\lambda_{1}} \frac{\left(l^{\prime}-l\right)^{\lambda_{1}}}{l^{\lambda_{2}}}$. Then the first order condition for employment implies a positive relationship between $l^{\prime}$ and $\Delta$ among all firms in states where hiring occurs: $l^{\prime}=f(\Delta \mid z, l), f_{\Delta}>0$. Further, the elasticity of $l^{\prime}$ with respect to $z$ increases with $\Delta$ :

    $$
    \frac{d \ln l^{\prime}}{d \ln z}=\left[\frac{\left(\lambda_{1}-1\right) f(\Delta \mid z, l)}{f(\Delta \mid z, l)-l}+1-\alpha \Lambda\right]^{-1}
    $$

    Of course, other properties of our model complicate this relationship, including wage schedules, firing costs, and the distinction between the value of a worker and her marginal revenue product.
    ${ }^{14}$ This labor market tightness effect is known as a "congestion exernality" in the labor-search literatrure (Pissarides 2000).
    ${ }^{15}$ Within this class of model, size and profitability are generally associated with exporting for standard Melitz-type (2003) reasons. But wages are linked to these firm characteristics through a variety of mechanisms. For example, in Helpman, Itskhoki, and Redding (2010), firms screen workers to improve their average productivity, and incentives to do so increase with firms' sales volume. The exporter wage premium then follows from the fact that highly screened workers command the highest wages and are concentrated at exporters. In Fajgelbaum's (2013) on-the-job search model, relatively productive firms expand into export markets by poaching workers from other firms, and they pay relatively high wages in order to do so. In the "fair wage" models of Amiti and Davis (2008) and Egger and Kreickmeier (2009), firms must share their rents with their workers to keep them from shirking.

[^7]:    ${ }^{16}$ Calculations are based on ICP Table 8 downloaded from http://www.eclac.cl/deype/PCI_resultados/eng/index.htm.
    ${ }^{17}$ In the benchmark economy, average wage in the industrial sector is about 1 . Since the model period is a year, $c_{f}=0.6$ corresponds to about 7 months' wages. In order to save computational time, $c_{f}$ was calibrated by a simple trial and error procedure, i.e. given a $c_{f}$ value, we compute average wage in the industrial sector and verify that $c_{f}$ amounts to 6-7 months' wages.

[^8]:    ${ }^{18}$ Setting off-diagonal terms to zero improves the stability of our estimator while maintaining consistency and keeping it independent of units of measurement. Examples of other studies employing the same strategy include Lee and Wolpin (2006) and Dix-Carneiro (2014).
    ${ }^{19}$ The data were collected by Colombia's National Statistics Department (DANE) and cleaned as described in Roberts (1996). They cover 88,815 plant-observations during the sample period. Estimates of $v \widehat{a} r(\overline{\mathbf{m}})$ are generated by bootstrapping the sample.
    ${ }^{20}$ In a stationary equilibrium, $E\left(\ln l_{t+1}, \ln G_{t+1}, \mathcal{I}_{t+1}^{x}\right)=E\left(\ln l_{t}, \ln G_{t}, \mathcal{I}_{t}^{x}\right)$ and $\operatorname{cov}\left(\ln l_{t+1}, \ln G_{t+1}, \mathcal{I}_{t+1}^{x}\right)=$ $\operatorname{cov}\left(\ln l_{t}, \ln G_{t}, \mathcal{I}_{t}^{x}\right)$. We therefore exclude $E\left(\ln l_{t+1}, \ln G_{t+1}, \mathcal{I}_{t+1}^{x}\right)$ and $\operatorname{cov}\left(\ln l_{t+1}, \ln G_{t+1}, \mathcal{I}_{t+1}^{x}\right)$ from our moment vector. We also drop $\operatorname{var}\left(I_{t}^{x}\right)$ because it is redundant: the variance of a Bernoulli random variable depends solely on its mean. This leaves 3 means, 2 variances, and 12 covariances.

[^9]:    ${ }^{21}$ While our estimation allows $l_{e}$ (the size of entering plants) to be arbitrarily small, our database does not cover plants with less than 10 workers. This means that plants appearing in the database for the first time can either be plants crossing the 10 -worker threshold from below, or plants in their first year of operation. We apply the same truncation to our simulated moments. This means, for example, that statistics describing the smallest quintile characterize the smallest quintile among observed producers.
    ${ }^{22}$ Let $c, e$, and $d$ be the set of continuing, entering, and exiting plants, respectively. Also, let $i$ index plants. Our year $t$ job turnover measure is then:

    $$
    X_{t}=\left(\Sigma_{i \in c}\left|l_{i t}-l_{i t-1}\right|+\Sigma_{i \in e} l_{t}+\Sigma_{i \in d} l_{t-1}-\left|\Sigma_{i} l_{i t}-\Sigma_{i} l_{i t-1}\right|\right) / \Sigma_{i} l_{i t-1}
    $$

    and our turnover statistic is $\frac{1}{10} \sum_{t=1981}^{1990} X_{t}$. The job turnover numbers in Table 2 are slightly higher than those depicted in Figure 3 for two reasons. First, Figure 3 is based on worker head counts, while our moment is based on effective workers. Second, the turnover rates in Figure 3 are taken from a study limited to establishments with at least 15 workers, while our moment is based on establishments with at least 10 workers. It was not possible to construct Figure 3 using effective workers and a 10 worker cutoff because we did not have access to establishment level data more recent than 1991.
    ${ }^{23}$ The cross-plant distribution of average wages provides a very natural measure of wage dispersion in a model with homogenous workers. See also Lentz and Mortensen (2012).

[^10]:    ${ }^{24}$ The alternative approach, commonly used, is to pre-estimate technology and taste parameters that can be identified without solving the dynamic problem, then treat them as parameters at the computationally intensive stage when parameters identified by the dynamic problem are estimated.
    ${ }^{25}$ While a standard application of Olley-Pakes would involve correcting for selection bias, this is not appropriate in the present context. The reason is that our timing assumptions in Section 2.7 imply entry and exit decisions are made before the current productivity shock is realized.
    ${ }^{26}$ In this regression, the error term is a function of $z$ and thus is correlated with labor. But the dependence of $l$ on $z$ is built into our model, so under the maintained hypothesis that the model is correctly specified, there is no simultaneity bias. Put differently, by exploiting our model's structure and assuming constant returns to scale, we avoid the need for a second stage Olley-Pakes step.

[^11]:    ${ }^{27}$ Specifically, the variance covariance matrix is $\left(\mathbf{J}^{\prime} \mathbf{W J}\right)^{-1}\left(\mathbf{J}^{\prime} \mathbf{W}\right) \widehat{\mathbf{Q}}(\mathbf{W J})\left(\mathbf{J}^{\prime} \mathbf{W} \mathbf{J}\right)^{-1}$, where $\mathbf{J}=\partial \boldsymbol{\Omega}^{\prime} / \partial \mathbf{m}$, $\mathbf{W}$ is the weighting matrix, and $\widehat{\mathbf{Q}}=\widehat{\operatorname{cov}}(\overline{\mathbf{m}}-\mathbf{m}(\boldsymbol{\Omega}))$.

[^12]:    ${ }^{28}$ At fitted values, the average percentage deviation between data- and model-based moments is 11.1 percent.
    ${ }^{29}$ The data are expressed in thousands of 1977 pesos. In 1977 , there were 46.11 pesos per dollar, and based on the US producer price index, a dollar in 1977 was worth 3.116 dollars in 2012. We therefore convert the average industrial wage per effective worker into 2012 US dollars as: $\bar{w} \times 3.116 / 46.11=\$ 4,153$. Then using the ratio of service sector wages to average industrial wages, we compute the service sector wage $w_{s}=$ $4,153 / 1.2=3,461$.

[^13]:    ${ }^{30}$ Estimates of the elasticity of substitution vary widely; our figure falls somewhere in the middle. For example, using establishment data from Slovenia, De Loecker and Warzynski (2012, Tables 2 and 3) estimate mark-ups ranging from 0.13 to 0.28 , implying demand elasticities that range from 2.27 to 8.3. Similarly, using firm-level Indian data, De Loecker et al. (forthcoming) estimate a median mark-up of 1.10, implying a demand elasticity of 11 , although they find the distribution of mark-ups is spread over a wide range of values. Using trade data, Baier and Bergstrand (2001) estimate a demand elasticity of 6.43 , while Broda and Weinstein (2006) get estimates around 12 for their most disaggregated (10 digit HTS) data.
    ${ }^{31}$ Direct comparisons with other recent studies are difficult because most control for capital stocks, and most estimate gross production functions rather than value added functions. One well-known study that does estimate a value-added function is Olley and Pakes (1996). Their preferred estimate of the elasticity of valueadded with respect to labor is $0.61-\mathrm{a}$ bit higher than our 0.56 . Another well-known study, Ackerberg, Caves, and Frazer (2006), reports estimates between 0.75 and 1.0. When comparing to studies that estimate gross physical production functions (correcting for price variation), it is perhaps best to focus on the ratio of the labor elasticity $(\alpha)$ to the materials elasticity. In our model this figure is $\alpha /(1-\alpha)=0.205 /(1-0.205)=0.26$. In other studies it is either $\alpha_{\text {labor }} / \alpha_{\text {materials }}$ or $\alpha_{\text {labor }} /\left(\alpha_{\text {materials }}+\alpha_{\text {capital }}\right)$, depending upon whether one treats capital as a material input. Several recent studies of selected industries find the first measure falls around 0.33 while they find the latter falls around 0.25 (e.g., De Loecker 2011; De Loecker et al. forthcoming).
    ${ }^{32}$ Our estimate of $\lambda_{1}$ is consistent with the available evidence on hiring cost convexities (e.g., Merz and Yashiv 2007, and Yashiv 2006). We also come close to satisfying the relationship $\lambda_{2}=1-1 / \lambda_{1}$ implied by Nilsen, Salvanesa, and Schiantarelli's (2007) specification.

[^14]:    ${ }^{33}$ Assuming that workers must commit to one sector before searching, we can define sector-specific unemployment rates. Call the service sector unemployment rate $u_{s}$ and the industrial sector unemployment rate $u_{q}$. Then, with approximately one-third of the work force employed or searching for jobs in the industrial sector (i.e., $\left(L_{u}+L_{q}\right) / L=1 / 3$ ), the economy-wide unemployment rate is related to the sector-specific unemployment rate by $\frac{1}{3} u_{q}+\frac{2}{3} u_{s}=0.11$. While a higher unemployment rate would imply, contrary to what we observe in the data higher wages in the current model without heterogeneity, a model with worker heterogeneity and labor market frictions in the service sector can generate higher unemployment and lower wages in the service sector if lower skilled workers sort themselves into the service sector.

[^15]:    ${ }^{34}$ Despite the differences in their wage setting, this result holds equally for both expanding and nonexpanding firms.
    ${ }^{35}$ Similarly, since exporting is strongly correlated with productivity, a negative size-wage relationship obtains when we condition on productivity (see Table A1 in online Appendix 5.3).
    ${ }^{36}$ Whenever possible, we focus on the post-2000 period because the early 1990s were too close to the reform years to plausibly approximate a new steady state and the late 1990s were characterized by a financial crisis and recession. However, some series such as job turnover and wage inequality are only available up to 2000 .

[^16]:    ${ }^{37}$ While workers bargain individually, each is treated marginally and paid the same wage. As the net revenue function (13) is subject to diminishing marginal returns to labor, wages decrease as firms expand. This creates an externality that firms exploit by overhiring.

[^17]:    ${ }^{38}$ Given our simple characterization of service sector production technology, we cannot generate predictions on the share of service sector output coming from own-account (self-employed) producers. In the data, however, around 90 percent of self-employed workers (who are not employers) are in the service sector (Figure 3.4 in Mondragón-Vélez and Peña 2010). Therefore, we find it plausible to associate an expansion of the service sector with increased self-employment.

[^18]:    ${ }^{1}$ To see the latter, let $I^{f}\left(z^{\prime \prime}, l^{\prime}\right)$ be an indicator function for whether a firm in interim state $\left(z^{\prime \prime}, l^{\prime}\right)$ will fire workers. Also, let $I^{c}\left(z^{\prime}, l^{\prime}\right)$ be an indicator function for whether a firm beginning next period in state $\left(z^{\prime}, l^{\prime}\right)$ will continue to operate. Then the envelope theorem implies the derivative of the continuation value of the marginal worker can be written as the probability-weighted average of the expected savings in hiring costs next period, given expansion, and the extra firing costs incurred, given contraction:

    $$
    \begin{aligned}
    \frac{\partial V\left(z^{\prime}, l^{\prime}\right)}{\partial l^{\prime}} & =\frac{\partial E_{z^{\prime \prime} \mid z^{\prime}} \max _{l^{\prime \prime}}\left[\pi\left(z^{\prime \prime}, l^{\prime}, l^{\prime \prime}\right)+V\left(z^{\prime \prime}, l^{\prime \prime}\right)\right]}{\partial l^{\prime}}, \\
    & =E_{z^{\prime \prime} \mid z^{\prime}} \frac{\partial \pi\left(z^{\prime \prime}, l^{\prime}, l^{\prime \prime}\right)}{\partial l^{\prime}}, \\
    & =E_{z^{\prime \prime} \left\lvert\, z^{\prime} I^{c}\left(z^{\prime}, l^{\prime}\right)\left[\left[1-I^{f}\left(z^{\prime \prime}, l^{\prime}\right)\right] \frac{\partial C_{h}\left(l^{\prime}, l^{\prime \prime}\right)}{\partial l^{\prime}}-I^{f}\left(z^{\prime \prime}, l^{\prime}\right) c_{f}\right] .\right.} .\left\{\begin{array}{l}
    \end{array}\right)
    \end{aligned}
    $$

    ${ }^{2}$ An alternative assumption would be to share the continuation values according to the bargaining rule only when they are positive, with firms absorbing all firing costs when they are incurred. An earlier version of our paper used this sharing rule.

[^19]:    ${ }^{3}$ Here, we suppressed the dependence of $\Delta(\cdot)$ on $l^{\prime}$ since $\partial \Delta / \partial l^{\prime}=0$ if the firm's exporting decision does not depend on the marginal worker. Since workers bargain individually and simultaneously with the firm, no single worker will be taken as the marginal worker for the export decision.

[^20]:    ${ }^{4}$ We emphasize "approximate" here because there is no analytical relationship linking the parameters of the annual objects to their quarterly counterparts. The reason is that our revenue function characterizes logs of flows, and thus annual variables are not linear combinations of quarterly variables.

[^21]:    ${ }^{5}$ Note that the unit of account is the service sector wage per period, so $b=0.302$ from the quarterly estimation is directly comparable to the $b$ from the annual baseline.

