Statistics II Exam, Juny 12

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You have to answer all the questions

(2 points) 1) Statistical concepts:

i) Define unbiasedness

ii) Define efficiency

(2 points) 2) Experiments: Supose you are generating data from a random variable $x \sim N(5,2)$, you obtain every time $n = 5000$ observations in the sample, then you calculate the mean $\overline{x} = \sum x_i/n$, you do this calculations 1000 times. What would you do to ilustrate that when doing hypothesis testing at a certain critical level α, say $\alpha = 0.05$, this α, is precisely the probability of making a type I error?

(2 points) 3) Linear combinations of random variables expressed in matrix notation: Assume we have three different populations, X, Y *and* Z, we have n observations of each population, and we are interested on testing if the sum of the population mean of X, μ_x, and the mean of Y, μ_Y, equals the mean of Z, μ_Z, we use the correspondent sample means to estimate each of the population means, and they are distributed as follows:

$$\overline{X} \sim N(\mu_x, \frac{\sigma_X^2}{n})$$
$$\overline{Y} \sim N(\mu_Y, \frac{\sigma_Y^2}{n})$$
$$\overline{Z} \sim N(\mu_Z, \frac{\sigma_Z^2}{n})$$
$$\sigma_{\overline{XY}} = 0$$
$$\sigma_{\overline{XZ}} \neq 0$$
$$\sigma_{\overline{YZ}} = 0$$

Using *matrix notation,* and the statistical property that if a vector $nx1$, X, of random variables $X \sim N(\mu, \Sigma)$ then a linear combination $a + AX \sim N(a + A\mu, A\Sigma A')$, and assuming the variances and covariances are known, explain all steps of the hypotheis testing proposed in this question, that is:1) setting the hypothesis, 2) the statistic we would use, 3) decision rules.

(4 points) 4) Exercise

i) You have the following data set, the sample: 5, 6.5, 7, 7.8, 7.2, 8, 8.7, 6.2, 6.5, 8.5, 9, 10, 5.8, 6. Coming from a random variable which distribution of probability is unknown but has mean μ and $\sigma^2 = 9$, Give a confidence interval for the population mean at an α of 0.1. Interpret the CI.

ii) Can we believe that the true mean is 8 against that the true mean is greater that 8 ? Answer the question doing all the steps of a hypothesis testing: 1) set both hypothesis, 2) justify the statistic you will use for the test, YOU WILL NOT GET FULL CREDIT FOR THIS PART IF YOU DO NOT JUSTIFY THE STATISTIC YOU ARE USING, 3) make a decision.

iii) Redo parts i) and ii) assuming that the variance is unknown.