Problem set # 3

1. In the context of non-nested testing, think of how you would designe a Monte Carlo experiment where you show that the tests proposed by Davidson and Mackinon, say test number (1) in the notes, reject too often a correctly specified model.

2. Determine the identification status, of the following macro model (this is the widely known Klein's Model 1)

$$\begin{aligned}
\text{(Consumption)}\ C_t &= \alpha_0 + \alpha_1 P_t + \alpha_2 P_{t-1} + \alpha_3 (W_t^p + W_t^g) + \epsilon_{1t} \\
\text{(Investment)}\ I_t &= \beta_0 + \beta_1 P_t + \beta_2 P_{t-1} + \beta_3 K_{t-1} + \epsilon_{2t} \\
\text{(Private Wages)}\ W_t^p &= \gamma_0 + \gamma_1 X_t + \gamma_2 X_{t-1} + \gamma_3 A_t + \epsilon_{3t} \\
\text{(Output)}\ X_t &= C_t + I_t + G_t \\
\text{(Profits)}\ P_t &= X_t - T_t - W_t^p \\
\text{(Capital Stock)}\ K_t &= K_{t-1} + I_t
\end{aligned}$$

The other variables are the government wage bill, W_t^g, taxes, T_t, government nonwage spending, G_t,and a time trend, A_t. The endogenous variables are the lhs variables,

$$Y_t' = \left[\begin{array}{cccccc} C_t & I_t & W_t^p & X_t & P_t & K_t \end{array}\right]$$

and the predetermined variables are all others:

$$X_t' = \left[\begin{array}{cccccccc} 1 & W_t^g & G_t & T_t & A_t & P_{t-1} & K_{t-1} & X_{t-1} \end{array}\right].$$

The model written as $Y\Gamma = XB + E$ gives

$$\Gamma = \begin{bmatrix}
1 & 0 & 0 & -1 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & -1 \\
-\alpha_3 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & -\gamma_1 & 1 & -1 & 0 \\
-\alpha_1 & -\beta_1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

$$B = \begin{bmatrix}
\alpha_0 & \beta_0 & \gamma_0 & 0 & 0 & 0 \\
\alpha_3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & \gamma_3 & 0 & 0 & 0 \\
\alpha_2 & \beta_2 & 0 & 0 & 0 & 0 \\
0 & \beta_3 & 0 & 0 & 0 & 1 \\
0 & 0 & \gamma_2 & 0 & 0 & 0
\end{bmatrix}$$

3. Show that the generalized instrumental variable estimator is a CAN estimator.

4. Show that the objective function of the IV estimator is zero in the case of exact identification.

5. In the context of simultaneous equations show that:

$A'P_WA$ is idempotent. Where $A = I - X(X'P_WX)^{-1}X'P_W$; and $P_W = W(W'W)^{-1}W'$.

6. Also show that $\rho(A'P_WA) = L - K_X$.