Problem Set 2

1. The F test is valid in small samples only if the errors are normally distributed. One can adjust the F test to obtain an asymptotically valid test with nonnormal errors. Find the asymptotic distribution of q times the F test statistic, under nonnormal errors.

2. Given the model subject to a set of linear restrictions

$$\begin{aligned} y &= X\beta + \epsilon \\ R\beta &= r \\ \mathcal{E}(\epsilon) &= 0 \\ V(\epsilon) &= \Sigma \end{aligned}$$

   find the limiting distribution of the restricted OLS estimator. That is, find the limiting distribution of $\sqrt{n}(\hat{\beta}_R - \beta)$ where $\hat{\beta}_R$ is the OLS estimator subject to the restriction $R\beta = r$.

3. Under the classical assumptions, the ϵ_t are uncorrelated. Are the OLS residuals $\hat{\epsilon}_t$ also uncorrelated?

4. The Cobb-Douglas model

$$z = A w_2^{\beta_2} w_3^{\beta_3} \exp(\epsilon)$$

   can be transformed logarithmically to obtain

$$\ln z = \ln A + \beta_2 \ln w_2 + \beta_3 \ln w_3 + \epsilon.$$

   If we estimate this last by OLS, explain how to calculate an estimator $\hat{A}$ of A, and give an explicit formula for how to calculate an asymptotically valid estimated standard error for $\hat{A}$.

5. Suppose we have two models:

$$\begin{aligned} \text{Model 1: } y_t &= \beta_1 + \beta_2 x_t + \beta_3 z_t + \epsilon_t \\ \text{Model 2: } y_t &= \alpha_1 + \alpha_2 x_t + \eta_t \end{aligned}$$

   (a) Discuss the properties of the OLS estimator of the parameters of the first model, assuming that the second model is correctly specified and satisfies the classical assumptions.

   (b) Discuss the properties of the OLS estimator of the parameters of the second model, assuming that the first model is correctly specified and satisfies the classical assumptions.

6. Obtain the Cramer-rao lower bound for: $y = X\beta + \epsilon$
   $\epsilon \sim N(0, \sum)$.

7. Consider the model $y = X\beta + \epsilon$; $\epsilon \sim N(0, \sum)$.
Obtain the formula for the estimator of β that :
$Min \epsilon' \sum^{-1} \epsilon$.

8. Show that the GLS estimator is a CAN estimator.

9. Show that in the context of a model with a lagged dependent variable the consistency or the inconsistency of OLS in the following assumptions for the error term:
a)$u_t = \epsilon_t + \lambda\epsilon_{t-1}$; $u_t \sim MA(1)$
b)$u_t = (1-\lambda)\epsilon_t$
c)$u_t = \epsilon_t + \lambda u_{t-1}$; $u_t \sim AR(1)$.

10. Give an intuition for the shape of the objective function, that is the sum of the squares of the resiuals, when we have problems of multicollinearity.

11. Proof formally that when missing observations in the dependent variable, substituting $\widehat{y_2} = \overline{y_1}$ leads to a OLS inconsistent estimator.

12. Show that in the context of collinearity $(X'X)^{-1}_{1,1} = (SSR_{x/w})^{-1}$ (following the notation in the notes).

13. Show that in the context of panel data when we assume heteroskedasticity across units, contemporaneous correlation between units and autocorrelation (AR(1) within units:

$$\sum = \begin{bmatrix} \sigma_{11}P_{11} & \sigma_{12}P_{12} & . & . & \sigma_{1G}P_{1G} \\ . & \sigma_{22}P_{22} & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ \sigma_{1G}P_{1G} & . & . & . & \sigma_{GG}P_{G} \end{bmatrix}; P_{ij} = \begin{bmatrix} 1 & \rho_j & \rho_j^2 & . & . & \rho_j^{T-1} \\ \rho_i & 1 & . & . & . & . \\ \rho_i^2 & . & 1 & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \rho_i^{T-1} & . & . & . & . & 1 \end{bmatrix}$$