Problem Set 1:

1. Show that estrict exogeneity is not satisfied on:
   $y_t = \beta y_{t-1} + \varepsilon_t$; even if $E(y_{t-1}\varepsilon_t) = 0$;

2. Show that P_X and M_X are both symmetric and idempotent.

3. Show that $P_X M_X = 0$.

4. Show that $\rho(P_X) = K$ and that $\rho(M_X) = n - K$.

5. Show that $\hat{\varepsilon} = M_X \varepsilon$.

6. Show graphically that if a column of ones is in the space spanned by X $(P_X \iota = \iota)$, then one can show that $0 \leq R_c^2 \leq 1$.

7. Prove that $X'\hat{\varepsilon} = 0$.

8. Find the distribution of $\widehat{\beta}$ using the property of linear combinations of random vectors distributed Normaly.

9. Consider the model $y = X\beta + \varepsilon$ subject to the restrictions $R\beta = r$.Show that $\hat{\beta}_R$ is unbiased if the restrictions are true.

10. Find the restriction on the parameters of the linearized model to test or impose that a Cobb-Douglas demand function is homogeneous of degree zero on prices and income.

11. Partition inverse matrices:

    Consider the following partitioned matrix: $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

    Let's define pivot elements the elements in the main diagonal:
    i) Construct the first tableau, pivot element is 1,1 (a):
    - element a becomes a^{-1}
    - in pivot row do:$(pivot-element)^{-1} * (pivot-row-element) : \begin{bmatrix} a^{-1} & a^{-1}b & a^{-1}c \end{bmatrix}$
    -in pivot column:$-(pivot-column-element) * (pivot-element)^{-1}$:
    $\begin{bmatrix} a^{-1} & a^{-1}b & a^{-1}c \\ -da^{-1} & & \\ -ga^{-1} & & \end{bmatrix}$
    -off elements: $(off-elements) - [(element-in-same-pivot-row) *$

$(pivot - element)^{-1}$
$*(element - same - pivot - col)]$:

$$\begin{bmatrix} a^{-1} & a^{-1}b & a^{-1}c \\ -da^{-1} & e - da^{-1}b & f - da^{-1c} \\ -ga^{-1} & h - ga^{-1b} & i - ga^{-1}c \end{bmatrix}$$

ii) Construct the next tableau with element 2,2 (e) as the pivot element.
iii) Construct the last tableau with element 3,3 (i) as the pivot element. This will be the final partitioned inverse.

Find the expression of $\widehat{\beta_R}$ and the lagrange multiplier $\widehat{\lambda}$ as the solution of the Lagrange optimization problem.

12. Show that $V(\hat{\beta}_R) - V(\widehat{\beta_U})$ is a negative semidefinite matrix

13. Justify this result using the adequate statistical propositions

$$\begin{aligned} \frac{\hat{\varepsilon}'\hat{\varepsilon}}{\sigma^2} &= \frac{\varepsilon' M_X \varepsilon}{\sigma^2} \\ &= \left(\frac{\varepsilon}{\sigma}\right)' M_X \left(\frac{\varepsilon}{\sigma}\right) \\ &\sim \chi^2(n-K) \end{aligned}$$

14. Show that in the case of testing the significance of an individual coefficient $t^2 = F$.

15. Given the model

$$y_t = \beta_1 + \beta_2 x_{t2} + \beta_3 x_{t3} + \beta x_{t4} + \varepsilon_t$$

OLS estimation results are

$$\begin{aligned} \hat{y}_t &= 2 + 1x_{t2} + 1x_{t3} + 4x_{t4} \\ \widehat{V}(\hat{\beta}) &= \begin{bmatrix} 0.25 & 0.01 & 0 & 0 \\ 0.01 & 0.04 & 0 & 0.01 \\ 0 & 0 & 1 & 0 \\ 0 & 0.01 & 0 & 1 \end{bmatrix} \\ \hat{\sigma}^2 &= 1 \end{aligned}$$

Find the value of the F statistic jointly to test the restrictions β_3=0 and $\beta_4 = 4$