

Restrictions

They are linear equations on the true coefficients of the model:

1. Testing:
 - a) from the unrestricted model only.
 - b) from both models the restricted and the unrestricted.
2. Estimating:
 - a) from the unrestricted model only.
 - b) from the restricted model by substitution.

1) a) from the unrestricted model only:

The F test allows testing multiple restrictions jointly. We have learned in class to derive the F statistic for this case:

$$F = \frac{\left(R\hat{\beta}_{UR} - r\right)' \left(R(X'X)^{-1}R'\right)^{-1} \left(R\hat{\beta}_{UR} - r\right)}{q\hat{\sigma}^2} \sim F(q, n - K).$$

1)b) Comparing the sum of the squares of the residuals of both models

$$\frac{(RSS_R - RSS_{UR})/q}{RSS_{UR}/(n - K)} \sim F(q, n - K).$$

In order to estimate the RSS_R we will use the technic that corresponds to section

2)b) named by substitution, example:

$$\begin{aligned} y_i &= \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \beta_5 x_{i5} + \varepsilon_i \\ \beta_2 + \beta_3 &= 1 \Rightarrow \beta_2 = 1 - \beta_3 \\ \beta_4 &= 0 \end{aligned}$$

We substitute the restrictions in the unrestricted model:

$$\begin{aligned} y_i &= \beta_1 + (1 - \beta_3)x_{i2} + \beta_3 x_{i3} + 0x_{i4} + \beta_5 x_{i5} + \varepsilon_i \\ y_i &= \beta_1 + x_{i2} - \beta_3 x_{i2} + \beta_3 x_{i3} + \beta_5 x_{i5} + \varepsilon_i \\ y_i - x_{i2} &= \beta_1 + \beta_3(x_{i3} - x_{i2}) + \beta_5 x_{i5} + \varepsilon_i \end{aligned}$$

Refine the vector of the dependent variable and the matrix of the regressors in the restricted model as:

$$y_{iR} = [y_i - x_{i2}]; X_{iR} = [1 \ x_{i3} - x_{i2} \ x_{i5}]$$

and y_R and X_R are the correspondent vector and matrix, then $\hat{\beta}_R^* = (X_R' X_R)^{-1} (X_R' y_R)$. You can obtain the

$$\begin{aligned} RSS_R &= \hat{\epsilon}_R' \hat{\epsilon}_R \text{ where} \\ \hat{\epsilon}_R &= y_R - X_R \hat{\beta}_R^* \text{ or} \\ \hat{\epsilon}_R &= y - X \hat{\beta}_R \end{aligned}$$

where $\hat{\beta}_R$ is the vector of the k coefficient estimates by OLS that satisfy the restrictions, $\hat{\beta}_R^*$ is the $k - q$ vector of coefficients estimates from estimating the restricted model by substitution. Once you obtain the RSS_R , you can perform the F test.

2)a) You can estimate the restricted vector of coefficients from the unrestricted vector of estimated coefficients, just substitute in the following formula:

$$\hat{\beta}_R = \hat{\beta}_{UR} - (X'X)^{-1} R' [R(X'X)^{-1} R']^{-1} [R \hat{\beta}_{UR} - r]$$

check that the dimensions of all these matrices are conformable and note that with this formula you would be able to obtain the vector of estimated restricted coefficients from having estimated the unrestricted vector of coefficients only.