Final Exam , Econometrics, English group 04
10 Juny 2003
Profs. M. Creel and M. Farell.

I. Consider the true model of a demand funtion where the quantity demanded is a function of the price of the good. If y_i is the logarithm of the quantities and $x_{i2}$is the logarithm of the correspondent prices:

$$y_i = \beta_1 + \beta_2 x_{i2} + \varepsilon_i$$

where $i = 1, 2, ...n$; and ε_i satisfies the classical assumptions.

1. (1 point) Give an economic interpretation of the parameters.
2. (0.5 points) Make a picture where one could see clearly the distance from one observation (x_i, y_i) to the estimated unconditional mean of y, $\overline{y}$ and the distance to the estimated mean of y conditional on x_i.
3. (0.5 points) If the values of the estimated parameters are 100 and -0.7 what would be the prediction of the logarithm of the quantity demanded if the logarithm of price was 5.

II. Given the model

$$y_i = \beta_1 + \beta_2 x_i + \beta_3 w_i + \varepsilon_i,$$

which satisfies all the classical assumptions, we have the following results from OLS estimation with $n = 50$ observations

$$\begin{aligned} \hat{y}_i &= 10 + 1x_i + 4w_i \\ \hat{V}(\hat{\beta}) &= \begin{pmatrix} 9 & 0 & 1 \\ 0 & 4 & 0 \\ 1 & 0 & 4 \end{pmatrix} \end{aligned}$$

1. (2 points) Which coefficients are significatively different from zero? Which statistical test will you use, its distribution, and the degrees of freedom? Use 2 as the critical value.
2. (2 points) Do the following hypothesis test $H_0 : \beta_1 = 0 \ and \ \beta_2 = 0$ using the estimation of the non-restricted model. Which is the statistic used in the test, its distribution, and the degrees of freedom? Use 4 as a critical value. If you don't remember the statistic recall that you can derive it applying the following statistical propositions to $R\hat{\beta}$. If x is a $(nx1)$ vector of random variables such that $x \sim N(\mu, \Sigma)$ then $a + Ax \sim N(a + A\mu, A\Sigma A')$, the other proposition you need is, if x is a $nx1$ vector of random variables such that $x \sim N(0, V)$, then $x'V^{-1}x \sim \chi^2(n)$(this proposition is true under some assumptions on V which are satisfied in this case).

III. Consider the model:

$$y_i = \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + \varepsilon_i$$

where $i = 1, 2, ...100$;i ε_i satisfies the classical assumptions. The model represents individual salary as a function of level of studies of an individual and level of experience in the labor force. We assume these two variables are continuous since they can be meassured in minutes of studying during the whole life of an individual and minutes of work realized correspondently.

You have also data on the sex of the individuals. You have to test if there is discrimination (of any kind) between men and women. That means, imagine you have data on individuals from 8 years old to 65, you have to test if there is discrimination at the entrance of the labor market and/or there is discrimination in their promotion (the increase of salary you would get as you get more level of studies and/or more experience).

1. (1 point) Give the expression of the model you would have to estimate.

2. (0.5 points) Indicate the contents of the matrix of regressors.

3. (2 points) Explain in detail how would you perform the test.

4. (0.5 points) Suppose there is no discrimination in the true model, which would be the properties of the estimator of the model you proposed in 1). Justify your answer with arguments.

IV. Multicolinearity

1. (1 point) Define multicolinearity formally.

2. (1 point) Ennumerate and give a short explanation of the main consequences of multicolinearity.

3. (1 point) Define each of the techniques you know to detect multicolinearity.

V. Generalized least squares

1. (0.5 points) Discuss under which assumptions we would prefer GLS estimation versus OLS estimation. Define both estimators.

2. (1.5 points) Derive the variance-covariance matrix of the GLS estimator.

3. (2 points) Discuss the properties of the following estimators when we are under the assumptions you made in 1):
   -OLS with var-cov wrongly assumed $\sigma^2(X'X)^{-1}$.
   - OLS with the right var-cov.
   -GLS

VI. (3 points) Explain any method you know to detect heteroscedasticiy, techniques and/or tests.