

HOME-WORK I NOV. 2007

The objective of this exercise is to simulate empirically the theoretical distribution of probability of  $\hat{\beta} \sim N(\beta, \sigma^2(x'x)^{-1})$ . You should report the results, if you can, printing the GRETTL outputs and commenting them.

You should do the following:

1) Generate a population  $(x_2, y)$  linear, with parameters  $\beta_1 = a$   $\beta_2 = b$ ; keep the values of  $x_2$  fixed like we did in the computer room, keeping the generation of  $x_2$  outside the loop; and for the error  $\varepsilon \sim N(0, 1)$ .

2) For a certain size of the sample  $n = 75$  find the empirical distribution of  $\hat{\beta}_1$  or  $\hat{\beta}_2$ , do  $m = 1000$  repetitions and afterwards do a histogram of frequencies.

3) When you obtain the results, the gretl output reports the average results of the experiment. Interpret those results and tell me which formula the GRETTL routine must have used to obtain that mean and that standard deviation.

4) Do steps 1) and 2) now with  $\varepsilon \sim N(0, 4)$ .

5) Do steps 1) and 2) now with  $\varepsilon \sim N(0, 8)$ .

6) Compare the results of the histograms obtained in 2), 4) and 5), which relationship do you see between them and why? Think carefully about your answers.

ANSWER KEY:

3) Just the regular formulas for the sample mean and the standard deviation you know, applied to the 1000 realizations of the random variables, you have obtained.

6) As you know by now, in this exercise you are illustrating empirically that the probability distribution of  $\hat{\beta} \sim N(\sigma^2(x'x)^{-1})$ , therefore the variance of  $\hat{\beta}_2 = \sigma^2(x'x)^{-1}_{2,2}$ , therefore when we increase the variance of the error, the variance of the distribution of probability of  $\hat{\beta}_2$  has to increase also. This is what you should observe empirically when you compare 2), 4) and 5).