Statistics 1: Problem Set 5

Abbr.: r.v. = random variable, pmf = probability mass function, pdf = probability density function

1. Consider an experiment of tossing a fair coin twice. Let (X, Y) be a bivariate r.v., where X is the number of heads that occurs in the two tosses and Y is the number of tails that occurs in the two tosses. (a) What is the range R_x of X? (b) What is the range R_y of Y? (c) Find and sketch the range R_{xy} of (X, Y). (d) Find $P(X = 2, Y = 0)$, $P(X = 0, Y = 2)$, and $P(X = 1, Y = 1)$.

2. Two fair dice are thrown. Consider a bivariate r.v. (X, Y). Let $X = 0$ or 1 according to whether the first die shows an even number or an odd number of dots. Similarly, let $Y =$ 0 or 1 according to the second die. (a) Find the range R_{xy} of (X, Y). (b) Find the joint pmf's of (X, Y).

3. The joint pmf of a bivariate r.v. (X, Y) is given by

$$p_{XY}(x_i, y_j) = \begin{cases} k(2x_i + y_j) & x_i = 1, 2; y_j = 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant. (a) Find the value of k. (b) Find the marginal pmf's of X and Y. (c) Are X and Y independent?

4. The joint pmf of a bivariate r.v. (X, Y) is given by

$$p_{XY}(x_i, y_j) = \begin{cases} kx_i^2 y_j & x_i = 1, 2; y_j = 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant. (a) Find the value of k. (b) Find the marginal pmf's of X and Y. (c) Are X and Y independent?

5. The joint pdf of a bivariate r.v. (X, Y) is given by

$$f_{XY}(x, y) = \begin{cases} k(x + y) & 0 < x < 2; 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant. (a) Find the value of k. (b) Find the marginal pdf's of X and Y. (c) Are X and Y independent?

6. The joint pdf of a bivariate r.v. (X, Y) is given by

$$f_{XY}(x, y) = \begin{cases} kxy & 0 < x < 1; 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant. (a) Find the value of k. (b) Are X and Y independent? (c) Find $P(X+Y<1)$.

7. The joint pdf of a bivariate r.v. (X,Y) is given by

$$f_{XY}(x,y) = \begin{cases} k & 0 < y \leq x < 1 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant. (a) Determine the value of k. (b) Find the marginal pdf's of X and Y. (c) Find $P(0<X<1/2, 0<Y<1/2)$.

8. Consider the bivariate r.v. (X,Y) of Prob. 3. (a) Find the conditional pmf's $p_{Y|X}(y_j|x_i)$ and $p_{X|Y}(x_i|y_j)$. (b) Find $P(Y=2|X=2)$ and $P(X=2|Y=2)$.

9. Find the conditional pmf's $p_{Y|X}(y_j|x_i)$ and $p_{X|Y}(x_i|y_j)$ for the bivariate r.v. (X, Y)of Prob. 4.

10. Consider the bivariate r.v. (X,Y) of Prob. 5. (a) Find the conditional pdf's $f_{Y|X}(y|x)$ and $f_{X|Y}(x|y)$. (b) Find $P(0<Y<1/2)|X=1)$.

11. Find the conditional pdf's $f_{Y|X}(y|x)$ and $f_{X|Y}(x|y)$ for the bivariate r.v. (X,Y) of Prob. 6.

12. Find the conditional pdf's $f_{Y|X}(y|x)$ and $f_{X|Y}(x|y)$ for the bivariate r.v. (X,Y) of Prob. 7.

13. Consider the bivariate r.v. (X,Y) of Prob. 3 (or Prob. 8). Compute the conditional mean and the conditional variance of Y given $x_i = 2$.

14. Let (X,Y) be the bivariate r.v. of Prob. 7 (or Prob. 12). Compute the conditional means $E(Y|x)$ and $E(X|y)$.

a) Analyze whether the random variables X and Y are independent. b) Compute $Var(X+Y)$. c) Compute $E(Y|x)$ and $Var(Y|x)$.

15. Let (X,Y,Z) be a trivariate r.v., where X, Y, and Z are independent uniform r.v.'s over (0, 1). Compute $P(Z \geq XY)$.