Graduate Program in Economics

IDEA

Econometrics II, second problem set.

Remember, "Matlab" can be any appropriate software of your choice that allows you to get the requested work done.

1. **Extremum estimation.** Suppose that $x_i \sim$ uniform(0,1), and $y_i = 1 - x_i^2 + \varepsilon_i$, where ε_i is iid(0,σ^2). Suppose we estimate the misspecified model $y_i = \alpha + \beta x_i + \eta_i$ by OLS.

   (a) Find, analytically, the numeric values of α^0 and β^0 that are the probability limits of $\hat{\alpha}$ and $\hat{\beta}$. Hint: the correct answers are 7/6 and -1.

   (b) Verify your results using "Matlab" by generating data that follows the above model, and calculating the OLS estimator. When the sample size is very large the estimator should be very close to the analytical results you obtained in the previous question.

2. **Numeric optimization.** Consider the data generating process $y_t = \beta_1 + \beta_2 x_t + u_t$, where $x_t \sim U(0,5)$ and $\epsilon_t \sim N(0,1)$. Suppose we have n observations on (y_t, x_t) . Suppose that the data satisfies the assumptions of the classical linear regression model. Suppose that we know that $\beta_1 = 1$ and that $0 < \beta_2 < 5$. Generate $n = 100$ observations from this model, and compute an estimator of β_2 *using the grid search method*. Your answer should include the program that you wrote, and the estimate of β_2.

3. **ML.** Estimate a Poisson model by ML using the 10 independent data points

y	0	0	0	1	1	5	8	16	20	30

   .
   For the Poisson model, the density $f_Y(y|\lambda) = \frac{\exp(-\lambda)\lambda^y}{y!}$, $y = 0, 1, 2, \ldots.$

   (a) create a data file that contains these observations

   (b) find the log-likelihood function

   (c) find the analytic expression for the ML estimator, and find an analytic expression for the asymptotic variance of $\sqrt{n}(\hat{\lambda} - \lambda^0)$.

   (d) write a Matlab function that computes the log-likelihood function, using the form obj=loglik(theta, data) and use fminunc to find the ML estimator. You need to use an anonymous function for this.

   (e) compute the ML estimator using your analytic expression. It should be very close to what you got using fminunc. Is it? If not, revise your code to make it work better.

   (f) compute the estimated standard deviation of $\hat{\lambda}$ and report an asymptotic 95% confidence interval for λ^0.

   (g) now, reparameterize the model as $f_Y(y|\alpha) = \frac{\exp(-\lambda)\lambda^y}{y!}$ where $\lambda = \exp\alpha$. The advantage of this is that α is unrestricted in sign, while the original λ must be positive. This doesn't matter much at present, but it will when you allow the conditional mean to depend on other variables. Verify the invariance property of ML by estimating α, and then showing that $\hat{\lambda}$ from part (d) is equal to $\exp\hat{\alpha}$.

4. **ML.**

   (a) Estimate the Nerlove model $\ln C = \beta_1 + \beta_Q \ln Q + \beta_L \ln P_L + \beta_F \ln P_F + \beta_K \ln P_K + \epsilon$ by ML, assuming that the errors are i.i.d. $N(0, \sigma^2)$. The data is available here.

   (b) Estimate subject to the restriction that the cost function satisfies homogeneity of degree one in factor prices), $\beta_L + \beta_F + \beta_K = 1$.

   (c) Test this restriction using the likelihood ratio test.

   (d) Test the restriction that $\beta_Q = 1$ (the model exhibits constant returns to scale) using the LR test.

   (e) Test homogeneity of degree 1 and constant returns to scale, jointly, using the LR test.

5. **ML.** The exponential density is

$$f_Y(y) = \begin{cases} \frac{e^{-\frac{y}{\lambda_0}}}{\lambda_0}, \; y \geqslant 0 \\ 0, \; y < 0 \end{cases}$$

   Suppose we have an independently and identically distributed sample of size n, $\{y_i\}, i = 1, 2, ..., n$, where each y_i follows this exponential distribution.

   (a) write the log likelihood function

   (b) find an analytic expression for the maximum likelihood estimator of the parameter λ.

   (c) explain how to estimate the asymptotic variance of the ML estimator. That is, if $\sqrt{n}\left(\hat{\lambda} - \lambda_0\right) \to^d N(0, V_\infty)$, give a consistent estimator of V_∞.

   (d) explain how to compute an estimator of the standard error of $\hat{\lambda}$.