Econometrics, second part, 2022, questions for final exam.

1. In the context of GMM estimation with *overidentification*, let the moment conditions be $m_n(\theta)=\frac{1}{n}\sum_{t=1}^{n} m_t(\theta)$, and we assume that $E(m_t(\theta_0)) = 0$ (where the 0 is a vector of zeros). Assume that the covariance of $m_t(\theta_0)$ is Γ_0. The moment contributions $m_t(\theta)$ are covariance stationary. Assume that moment contributions are not autocorrelated over time: $E\left[m_t(\theta_0)m'_{t-s}(\theta_0)\right] = 0, s > 0$.

    (a) What is meant by "overidentification"?

    (b) What is meant by "covariance stationary"?

    (c) What is the form of Ω_n, the covariance matrix of $\sqrt{n}m_n(\theta_0)$? Give the steps used to find your answer.

    (d) For the purpose of obtaining a consistent GMM estimator, is it strictly necessary to estimate Ω_n? Explain why or why not.

    (e) Carefully explain how Ω_n may be estimated consistently, taking into account the specific information given in this problem.

    (f) Carefully explain how to implement the two step efficient GMM estimator.

2. Suppose that $\{y_i\}$ $i = 1,2,...,n$ is an i.i.d. sample of size n from the Poisson density with parameter λ_0. The Poisson density is $f_y(y;\lambda) = \frac{e^{-\lambda}\lambda^y}{y!}$.

    (a) Find the expression for the ML estimator, showing all steps.

    (b) Verify that the ML estimator is asymptotically distributed as $\sqrt{n}\left(\hat{\lambda} - \lambda_0\right) \xrightarrow{d} N(0,\lambda_0)$, where λ_0 is the true parameter value. Hint: compute the asymptotic variance using $-\mathcal{J}_\infty(\lambda_0)^{-1}$.

    (c) give a consistent estimator of the asymptotic variance of $\sqrt{n}\left(\hat{\lambda} - \lambda_0\right)$.

    (d) explain how to test $H_0 : \lambda_0 = 2$ versus $H_A : \lambda_0 \neq 2$, using the likelihood ratio test.

3. Suppose we have the model

$$
\begin{aligned}
y_t &= \alpha_1 + \alpha_2 x_t + \epsilon_t \\
\epsilon_t &= u_t + \phi u_{t-1}
\end{aligned}
$$

   where the shocks u_t are independent and identically distributed with mean zero and variance σ^2. Suppose that $x_t = 2w_t + 1z_t + u_t$. Suppose that $w_t$and z_t are uncorrelated with $u_s, \forall s$.

    (a) show that the regressor x_t is correlated with the error ϵ_t.

    (b) show that the regressor x_t is correlated with $w_t$and z_t.

    (c) explain in detail how to estimate the parameters α_1 and α_2 consistently and efficiently, using a GMM estimator that is based on the given information. Your answer should include a detailed explanation of:

        i. the specific moment conditions you propose

ii. a dscription of how to compute your proposed weight matrix

iii. a detailed description of the method of estimation

(d) explain in detail how one may test the hypothesis $H_0 : \alpha_2 = 0$ versus $H_A : \alpha_2 \neq 0$.

4. In the context of GMM estimation with *exact identification*, let the moment conditions be $m_n(\theta)=\frac{1}{n}\sum_{t=1}^{n} m_t(\theta)$, and we assume that $E(m_t(\theta_0)) = 0$. Consider the possibility that moment conditions contibutions may be correlated: $E(m_t m'_{t-s}) = \Gamma_s$.

   (a) for the purpose of simply computing the GMM estimator, does the potential correlation of moment function contributions matter? Explain why or why not.

   (b) for the purpose of testing hypotheses regarding θ_0, does the potential correlation of moment function contributions matter? Explain why or why not.

5. A particular way of parameterizing the Pareto density is

$$f_X(x) = \begin{cases} \frac{\alpha}{x^{\alpha+1}}, x \geqslant 1 \\ 0, x < 1 \end{cases}$$

Suppose we have an independently and identically distributed sample of size n, $\{x_i\}, i = 1,2,...,n$, where each x_i follows this Pareto distribution.

   (a) write the log likelihood function

   (b) compute the maximum likelihood estimator of the parameter α.

   (c) give a consistent estimator of the asymptotic variance of $\sqrt{n}\,(\hat{\alpha} - \alpha_0)$.

6. Are the following statements TRUE or FALSE?

   (a) the maximum likelihood estimator is unbiased

   (b) the maximum likelihood estimator is asymptotically unbiased

   (c) the maximum likelihood estimator is always more efficient than the GMM estimator

   (d) the parameters of a stationary GARCH(1,1) model must satisfy some restrictions

   (e) the parameters of a stationary ARCH(4) model must satisfy some restrictions

   (f) for a correctly specified model estimated by ML, the individual observations' contributions to the score vector (the first derivative of the log-likelihood function) may be correlated with one another

   (g) if we compute the Bayes information criterion (BIC) for a set of models, and the set includes the correctly specified model, the BIC will always be lowest for the correctly specified model

   (h) maximizing the likelihood function gives the same result as maximizing the log-likelihood function

(i) the Matlab function 'fminunc' will always find the global minimum of a function to be minimized

(j) given a particular restriction on a parameter, it is possible to consistently estimate the parameters of an ARCH(1,1) model by ordinary least squares

(k) For a linear regression model, if we estimate by instrumental variables estimation, in order for the estimator to be consistent, every instrument must be correlated with every regressor.

(l) For a linear regression model, if we estimate by instrumental variables estimation, in order for the estimator to be consistent, every instrument must be uncorrelated with the error term.