Macroeconomic Uncertainty and Vector Autoregressions*

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Abstract

We estimate measures of macroeconomic uncertainty and compute the effects of uncertainty shocks by means of a new simple procedure based on standard VARs. Uncertainty and its effects are estimated using a single model so to ensure internal consistency. Under suitable assumptions, our procedure is equivalent to using the square of the VAR forecast error as an external instrument in a proxy SVAR. Our procedure allows to add orthogonality constraints to the standard proxy SVAR identification scheme. We apply our method to a US data set; we find that macroeconomic uncertainty is responsible of a large fraction of business-cycle fluctuations while financial uncertainty plays a modest role.

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1 Introduction

Uncertainty shocks have been in recent years at the heart of the business cycle debate. Since Bloom (2009), a vast literature studying the link between uncertainty and economic fluctuations has been growing. In particular, the focus has been paid in studying the effects on macroeconomic and financial variables of exogenous changes in uncertainty. [1] There are several theoretical channel through which increases in uncertainty can depress economic activity. A notable one is the real option channel. Higher uncertainty induces agents to postpone private expenditures and investment, thus producing a downturn in economic activity.

Two main econometric approaches have been used in the literature to measure the effects of uncertainty shocks. On the one side, Structural Vector Autoregressive (SVAR) models. Within this approach, the practice is to include in a VAR a measure of uncertainty, derived outside the model, as an additional endogenous variable and to identify the uncertainty shock in a standard way by means of some restrictions. Since Bloom (2009), it has become quite common to use a recursive ordering to identify the uncertainty shock, but, in principle,

[1] A partial list of contributions includes few contributions are Fernandez-Villaverde et al. (2011), Bachmann et al. (2013), Bekaert et al. (2013), Caggiano et al. (2014), Rossi and Sekhposyan (2015), Jurado et al. (2015), Scotti (2016), Baker et al. (2016), Caldara et al. (2016), Leduc and Liu (2016), Basu and Bundik (2017), Fajgelbaum et al. (2017), Piffer and Podstawsky (2017), Nakamura et al. (2017), Bloom et al. (2018), Shin and Zhong (2018), Jo and Sekkel (2019), Ludvigson et al. (2019), Angelini and Fanelli (2019), Dew-Baker and Giglio (2020). For more references, see the survey articles in Cascaldi-Garcia et al. (2020) and Fernandez-Villaverde and Guerron-Quintana (2020)

other restrictions can be used.[2]

On the other one side, researchers have relied on Stochastic Volatility VAR (SV-VAR) models, see for instance Carriero et al. (2018a, 2018b). In these models, an explicit dynamic process for uncertainty is specified, and uncertainty and its effects on economic variables are jointly estimated. The advantage of this second approach relative to the first, is that the estimates of uncertainty and its effects are internally consistent since they are obtained within a single framework. On the contrary, the first approach opens the door to a potential problem of inconsistency between the estimates of uncertainty and its effects. The cost of using SV-VAR is represented by a more complicated estimation procedure.

In this paper, we propose a new econometric procedure to estimate uncertainty and its effect based on a single standard homoschedastic VAR model. Throughout the paper, we focus on the definition of uncertainty adopted in JLN: uncertainty is the forecast error variance or, equivalently, the conditional expectation of the forecast error squared. The procedure unfolds in four steps: (i) estimating a VAR and the associated reduced form impulse response functions; (ii) computing the implied squared forecast error for the variable and horizon of interest; (iii) regressing the squared forecast error onto the current and past values of the

[2]Several papers have proposed proxies of uncertainty which are not model-based but exploit different sources of information, such as stock market volatility (Bloom, 2009, Bekaert et al., 2013, Caldara et al., 2016), forecast disagreement in survey data (Bachmann et al., 2013), the frequency of selected keywords in journal articles (Baker et al., 2016), the unconditional distribution of forecast errors (Jo and Sekkel, 2019). Other papers (e.g. Jurado et al. 2015, JLN henceforth, Ludvigson et al. 2019, LMN henceforth) start from a rigorous statistical definition of uncertainty as the conditional volatility of a forecast error and specify and estimate a stochastic volatility model by using sophisticated time series techniques.

VAR variables, the fitted values represent an estimate of uncertainty; (iv) using the coefficients of the regression in (ii) to combine the VAR impulse response functions obtained in (i), together with the desired restrictions to identify the uncertainty shock. This procedure, similarly to SV-VAR models, ensures consistency between the estimate of uncertainty and the estimate of effects of uncertainty shocks. Our method can be considered as an alternative to SV-VAR with advantages and disadvantages. The disadvantage advantage is that is much simpler.

Under suitable conditions, steps (iii-iv) are equivalent to using the squared forecast error as the instrument within a proxy SVAR (Mertens and Ravn (2013), Stock and Watson (2018), Plagborg-Møller and Wolf (forthcoming)). Hence, our method can be thought of as a proxy SVAR, where the proxy, instead of being an external variable, is a function of the estimated forecast error. The relevance condition of the instrument is clearly satisfied: the squared forecast error *is* correlated with the uncertainty shock by the very definition of uncertainty. However, in order for the exogeneity condition to hold, we need the additional assumption that uncertainty (or, more precisely, the squared prediction error) is not affected on impact by other structural shocks. This assumption is questionable.[3] To relax it, we impose orthogonality constraints with respect to other structural shocks within the standard proxy SVAR procedure. This represents a methodological innovation in the literature on Proxy SVAR where the effects are typically estimated without relying on any other additional restriction.

[3]Notice however that most papers in the uncertainty literature make precisely the same assumption, by adopting a Cholesky identification scheme with the external uncertainty measure ordered first.

Our method has a few noticeable advantages. First, it is extremely simple to implement. Second, there is a clear and rigorous definition of uncertainty for each variable and horizon in the VAR. Third, it avoids the problematic choice of an external uncertainty measure. Fourth, internal consistency between the estimate of uncertainty and its effects is ensured, as in SV-VAR, since they are both obtained with a single model. Fourth, making assumptions on the form of the conditional distribution of the shocks is not necessary.

We apply our procedure to a US macroeconomic data set and find that (a) our estimates of uncertainty are reliable, in that (a.1) the squared prediction errors are significantly predicted by a linear combination of the VAR variables, with sizable explained variances; (a.2) uncertainty estimates obtained with our linear approximation are strongly correlated with comparable estimates in the literature (notably, JLN and LMN measures); (a.3) price uncertainty and interest-rate uncertainty are related to recognizable economic events. As for the impulse response functions and variance decomposition, we find that (b) exogenous macroeconomic uncertainty shocks explain a large fraction of business-cycle fluctuations while financial uncertainty plays a modest role; (c) results are robust with respect to the choice of the uncertainty horizon and variable, the number of lags and the choice of the variables included in the VAR.

The remainder of the paper is organized as follows. Section 2 discusses the econometric approach. Section 3 presents the results. Section 4 concludes.

2 Econometric approach

This section discusses the econometric approach to estimate uncertainty and identify the effects of the uncertainty shock in a simple VAR model.

2.1 The VAR model

Our starting point is the assumption that the macroeconomic variables in the n-dimensional vector y_t follow the VAR model[4]

$$A(L)y_t = \mu + \varepsilon_t, \tag{1}$$

where ε_t is orthogonal to y_{t-k}, $k > 0$, and $A(L) = I - \sum_{k=1}^{p} A_k L^k$ is a matrix of degree-p polynomials in the lag operator L. By inverting the VAR, we get the VMA representation

$$y_t = \delta + B(L)\varepsilon_t, \tag{2}$$

where $B(L) = \sum_{k=0}^{\infty} B_k L^k = A(L)^{-1}$, with $B_0 = I_n$, is the matrix of reduced form impulse response functions and $\delta = B(1)\mu$. The implied h-step ahead prediction error is

$$e_{t+h} = \sum_{k=0}^{h-1} B_k \varepsilon_{t+h-k}. \tag{3}$$

2.2 VAR-based uncertainty

Following JLN, uncertainty is defined as the conditional volatility of the forecast error. We focus on a single. For variable i and horizon h we uncertainty is

$$U_{ht}^{i} = E_t e_{i,t+h}^{2}. \tag{4}$$

[4]Notice that our procedure could be easily extended to Factor Models or FAVARs.

The conditional expectation cannot be computed without introducing additional assumptions about the conditional distribution of the VAR residuals, for instance a stochastic volatility model. However, it can be approximated by means of linear projections. More precisely, we approximate the logarithm of uncertainty by taking the orthogonal projection of the log of the squared prediction error onto the linear space spanned by the constant and the present and past values of the y's:[5]

$$\log(U^i_{ht}) \approx P^i_t = Proj\left(\log(e^2_{i,t+h})|y_{i,t-k}, i = 1,\dots,n; k = 0,\dots,q\right) \tag{5}$$

$$\tag{6}$$

where

$$\begin{aligned} Proj\left(\log(e^2_{i,t+h})|y_{i,t-k}, i = 1,\dots,n; k = 0,\dots,q\right) &= \theta + c(L)'y_t \\ &= \theta + c_0'y_t + \cdots + c_q'y_{t-q} \end{aligned}$$

where c_j is an n-dimensional column vector of coefficients and θ the constant term. Notice that, if the VAR residuals were serially independent (and therefore independent of lagged y's), then $\log(e^2_{i,t+h})$ would be orthogonal to the predictors, implying $c(L) = 0$. CHANGE THIS Hence our procedure requires that the VAR residuals, while being serially uncorrelated, are not serially independent.

Using the estimated (in-sample) forecast errors, the parameters of the projection above can be estimated from the regression

$$\log(e^2_{i,t+h}) = \theta + c(L)'y_t + \nu_t = \theta + c_0'y_t + \cdots + c_q'y_{t-q} + \nu_t, \tag{7}$$

[5] We approximate the log uncertainty rather than uncertainty itself to avoid negative estimates of uncertainty. However by approximating directly uncertainty very similar results are obtained.

where the error ν_t is orthogonal to y_t and its past history. In the empirical section we document that the estimated coefficients are significantly different from zero thus rejecting serial independence. Uncertainty can then be estimated as the exponential of the fitted values $\hat{P}_t^i = \hat{\theta} + \hat{c}(L)' y_t$.

2.3 Identifying uncertainty shocks

Here we discuss the identification of uncertainty shocks. The standard VAR approach at this point would be to include the measure of uncertainty derived in the previous subsection as an additional endogenous variable in a VAR, re-estimate the model and impose some restrictions, like a recursive ordering, to identify the exogenous uncertainty shock. It is important to stress that we do not pursue this route here. Instead, we directly combine equations (2) and (7) as discussed next.

To understand shock identification in this context, notice, first of all, that log uncertainty is a linear combination of the VAR variables, see equation (7), and therefore a combination of VAR residuals. Precisely,

$$
\begin{aligned}
P_t^i &= \theta + c(L)' y_t \\
&= \theta + c(L)' B(L) \varepsilon_t \\
&= \theta + g(L)' \varepsilon_t. \qquad (8)
\end{aligned}
$$

where $g(L) = \sum_{j=0}^{\infty} g_j L^j$. Given that uncertainty is just a combination of the vector y_t, and therefore of the VAR residuals ε_t, we can write the full model as

$$\begin{pmatrix} P_t^i \\ \\ y_t \end{pmatrix} = \begin{pmatrix} \theta \\ \\ \delta \end{pmatrix} + \begin{pmatrix} g(L)' \\ \\ B(L) \end{pmatrix} \varepsilon_t. \tag{9}$$

In other words, uncertainty becomes an additional variable in the VAR driven by the same VAR shocks driving y_t. The goal is to identify the uncertainty shock as a combination of the VAR residuals ε_t and study its impulse response functions. We use two identification schemes. The first identification simply postulates that the uncertainty shock is the innovation in uncertainty. In the second, we use zero short-run and zero long-run restrictions to account for potential endogeneity of uncertainty.

Model (8) makes clear why, when uncertainty is proxied by the forecast error variance, there is no need of relying on external measures. Uncertainty is directly derived from the forecast errors obtained from the data generating process for y_t. This, in turn, ensures the internal consistency between the model generating the economic time series and the implied estimates of uncertainty.

2.3.1 Innovation

To begin with, we consider the simple case in which the uncertainty shock is simply the innovation of log uncertainty, normalized to have unit variance. Although quite common, this is a strong assumption and will be relaxed later on. From equation (8) the innovation is

$$g_0'\varepsilon_t = c_0'B_0\varepsilon_t = c_0'\varepsilon_t$$

(recall that $B_0 = I_n$). Then the normalized innovation u_t^* is

$$u_t^* = \frac{c_0'}{\sqrt{c_0' \Sigma_\varepsilon c_0}} \varepsilon_t = v' \varepsilon_t, \tag{10}$$

where Σ_ε is the variance-covariance matrix of ε_t. The corresponding vector of impulse response functions for the variables included in the VAR to an innovation in uncertainty is

$$d^*(L) = B(L) \Sigma_\varepsilon v, \tag{11}$$

with contemporaneous effects equal to $d^*(0) = \Sigma_\varepsilon v$, being $B(0) = I_n$ (see Appendix A for details on the derivation of the impulse response functions). So the model can be written in terms of the uncertainty innovation as

$$\begin{pmatrix} P_t^i \\ \\ y_t \end{pmatrix} = \begin{pmatrix} \theta \\ \\ \delta \end{pmatrix} + \begin{pmatrix} c(L)' B(L) \Sigma_\varepsilon v \\ \\ B(L) \Sigma_\varepsilon v \end{pmatrix} u_t^* + \Psi(L) w_t. \tag{12}$$

where $\Psi(L) w_t$ is the term containing the $n-1$ remaining unidentified shocks times their impulse response functions.

2.3.2 Short-run and long-run zero restrictions

The identification procedure in the previous section imposes that on impact only the uncertainty shock affects uncertainty since the shock is just the innovation. While quite common in the literature, this assumption is questionable, see for instance Bachmann et al. (2013) since there could be other shocks which might affect uncertainty contemporaneously. Here we show how to relax this assumption and impose other restrictions. More specifically in this subsection we discuss, from a theoretical point of view, how to impose both short-run and long-run restrictions to zero. We postpone to section 3.3, after having discussed model specification, the discussion of the specific restrictions used to identify the uncertainty shock.

Suppose we want to impose that the uncertainty shock has a zero impact effect on variable y_{1t}. To impose the restriction it suffices to impose orthogonality of the uncertainty shock with respect to $\varepsilon_{1t} = D_1\varepsilon_t$, where $D_1 = [1\ 0\ \cdots\ 0]$, i.e. the innovation in the first variable. This amounts at considering the shock as the residual in the projection of the uncertainty innovation onto $D_1\varepsilon_t$. The non-normalized uncertainty shock orthogonal to the shock $D_1\varepsilon_t$, is $u_t = [c_0' - c_0'\Sigma_\varepsilon D_1' D_1]\varepsilon_t$. To impose that the shock has no long run effect on some variable, for instance GDP, fist a long run shock, $D_1\varepsilon_t$, has to be identify assuming that is the only one shock affecting GDP in the long run. The vector D_1 is the identifying vector ensuring that the long run restriction is satisfied. Second, the orthogonality of the uncertainty innovation with respect to the long-run shock $D_1\varepsilon_t$ has to be imposed. This ensures that the uncertainty shock has zero long run effect. As before the non-normalized uncertainty shock orthogonal to the shock $D_1\varepsilon_t$, is $u_t = [c_0' - c_0'\Sigma_\varepsilon D_1' D_1]\varepsilon_t$.

More generally, let D be the $m \times n$ matrix having on the rows the vectors D_1, $D_2, \ldots$, D_m, with $m < n$. To impose orthogonality with respect to the corresponding m shocks $D_1\varepsilon_t, D_2\varepsilon_t \ldots, D_m\varepsilon_t$, we have to take the residual of the orthogonal projection of the uncertainty innovation u_t^* onto $D\varepsilon_t$, normalized to have unit variance. The corresponding uncertainty shock, call it u_t, can be computed from the VAR coefficients by applying the formulas

$$\begin{aligned} u_t &= \gamma \varepsilon_t \\ \gamma &= \frac{\beta}{\sqrt{\beta' \Sigma_\varepsilon \beta}} \\ \beta &= c_0' - c_0' \Sigma_\varepsilon D' (D \Sigma_\varepsilon D')^{-1} D. \end{aligned} \tag{13}$$

The impulse-response functions for the variables included in the VAR corresponding to the shock $u_t = \gamma \varepsilon_t$ are

$$d(L) = B(L) \Sigma_\varepsilon \gamma. \tag{14}$$

The full model becomes

$$\begin{pmatrix} P_t^i \\ \\ y_t \end{pmatrix} = \begin{pmatrix} \theta \\ \\ \delta \end{pmatrix} + \begin{pmatrix} c(L)' B(L) \Sigma_\varepsilon \gamma \\ \\ B(L) \Sigma_\varepsilon \gamma \end{pmatrix} u_t + \Psi(L) w_t. \tag{15}$$

where $\Psi(L) w_t$ is again the term containing the $n-1$ remaining unidentified shocks times their impulse response functions.

Notice that the term $d_u(L) = c(L) d(L)'$ identifies the effect of the uncertainty shock on uncertainty as $d_u(L) u_t = d_u(L) \gamma \varepsilon_t$, let us call it the *exogenous component*. The part of uncertainty not driven by the uncertainty shock, i.e. the *endogenous component*, is therefore $c(L) y_t - d_u(L) u_t = [c(L) B(L) - d_u(L) \gamma] \varepsilon_t$. Since the two components are orthogonal, we can compute a variance decomposition both for the total variance and for the prediction errors at all horizons.

2.4 Equivalence with proxy SVAR

Our procedure is equivalent in population to estimating a proxy SVAR using $z_t = \log(e^2_{i,t+h})$ as the external instrument for the uncertainty shock.[6] When the number of lags in equation (7) is the same as the number of lags in the VAR, the results of the two procedures are identical even in sample.

For the instrument to be valid, the standard assumptions of relevance and exogeneity have to hold. The intuition of why the squared forecast error is a good candidate is the following. Consider the orthogonal decomposition

$$e^2_{i,t+h} = E_t e^2_{i,t+h} + v_{it} = U^i_{ht} + v_{it}.$$

Since v_{it} is independent of uncertainty, $e^2_{i,t+h}$ must be correlated with the uncertainty shock and so will be the log, which is the instrument we use. Hence relevance is ensured by the very definition of uncertainty. If the other shocks have zero impact effect on uncertainty, as assumed in Section 2.3, then the exogeneity assumption is also fulfilled, so that $\log(e^2_{i,t+h})$ is a valid proxy to identify the uncertainty shock.

Let us come now to the equivalence. The proxy SVAR approach consists in projecting the VAR residuals ε_t onto the proxy z_t. The population parameters are $\phi = E z_t \varepsilon_t / E z_t^2$ (see Mertens and Ravn, 2013). The impact effects ϕ are therefore proportional to $E z_t \varepsilon_t$. It is easily seen that our population impact effects are also proportional to $E z_t \varepsilon_t$, so that they are equal to those of the proxy SVAR when the same normalization is imposed. If the proxy

[6] On the proxy SVAR approach see Mertens and Ravn (2013), Stock and Watson (2018) and Plagborg-Møller and Wolf (forthcoming).

z_t is $\log(e^2_{i,t+h})$, from equations (7) and (2) we get

$$z_t = \omega + c(L)'B(L)\varepsilon_t + \nu_t, \tag{16}$$

where $\omega = \theta + c_0'\delta$ and ν_t is orthogonal to y_{t-k}, $k \geq 0$ and therefore to ε_{t-k}, $k \geq 0$. Post-multiplying by ε_t' and taking expected values we get $Ez_t\varepsilon_t' = c_0'\Sigma_\varepsilon$, since $B(0) = I$. But we have already seen that our impact effects are $\Sigma_\varepsilon v = \Sigma_\varepsilon c_0/\alpha$ with $\alpha = \sqrt{c_0'\Sigma_\varepsilon c_0}$ (see equations (10) and (11)). Hence our impact effects are $Ez_t\varepsilon_t/\alpha$.

In Appedix B we also show that the OLS estimates are equal to those of Mertens and Ravn (2013) if $q = p$, i.e. when the number of lags of y_t included in the regression of z_t is equal to the number of lags of the VAR. Hence, as far as the estimation of the effects of uncertainty are concerned, our approach and the standard proxy SVAR approach produce the same results.

Te advantage of our method is that it allows us to get an estimate of uncertainty itself, besides the uncertainty shock and its impulse-response functions. On the other hand, the above discussion clarifies that, for the identification of the uncertainty shock, the linear approximation of uncertainty in equation (7) is not needed: we just need the standard assumptions of relevance and exogeneity.

2.5 Summary of the procedure

Summing up, our procedure is the following.

1. Estimate by OLS the VAR in equation (1) to get $\hat{B}(L) = \hat{A}(L)^{-1}$, the vector of residuals $\hat{\varepsilon}_t$ and its sample variance-covariance matrix $\hat{\Sigma}_\varepsilon$. Compute $\hat{e}_{t+h}$ according to equation

(3).

2. Compute $\hat{z}_t = \log(\hat{e}^2_{i,t+h})$. Estimate by OLS equation (7) to get $\hat{\theta}$ and $\hat{c}(L)$ and compute $\hat{U}^i_{ht}$ according to equation (7) as $\hat{U}^i_{ht} = \exp(\hat{\theta} + \hat{c}(L)'y_t)$.

3. Compute $\hat{u}^*_t$ and $\hat{d}^*(L)$ according to equations (10) and (11) by replacing c_0 and Σ_ε with the corresponding estimates. Alternatively:

3′. Specify the relevant orthogonality restrictions by choosing the matrix D. Compute the estimates $\hat{u}_t$ and $\hat{d}(L)$ according to equations (13) and (14) by replacing c_0 and Σ_ε with the corresponding estimates.

4. Get the estimate of the IRFs of uncertainty, either $d^*_u(L)$ or $d_u(L)$, according to (**??**).

In Appendix C we describe in detail our bootstrap procedure to construct confidence bands.

If the goal is to exclusively estimate the effects of uncertainty shocks, an alternative and equivalent procedure is the following.

a. Estimate by OLS the VAR in equation (1) to get $\hat{B}(L) = \hat{A}(L)^{-1}$, the vector of residuals $\hat{\varepsilon}_t$ and its sample variance-covariance matrix $\hat{\Sigma}_\varepsilon$. Compute $\hat{e}_{t+h}$ according to equation (3).

b. Compute $\hat{z}_t = \log(\hat{e}^2_{i,t+h})$ and use it as the external instrument in a proxy SVAR to obtain the effects of the uncertainty shock.

3 Empirics

In this section, we present the main results of our empirical analysis.

3.1 Specification

We use US quarterly data spanning the period from 1960:Q1 to 2019:Q3. Our benchmark VAR includes seven variables: the log of real per-capita GDP, the unemployment rate, CPI inflation, the federal funds rate, the log of the S&P500 stock price index, a component of the Michigan Consumer Confidence Index, i.e. expected business conditions for the next 12 months (E1Y), and the spread between BAA corporate bond yield and GS10 (BAA-GS10).[7] The last four variables are included essentially because they are supposed to quickly react to shocks and therefore are hopefully able to better capture the information necessary to reveal uncertainty. In the robustness section, we replace stock prices and the spread BAA-GS10 with a different set of forward-looking variables.

We include just one lag in the VAR, as suggested by the BIC criterion. In the robustness section we show results for 2 and 4 lags.

We estimate equation (7) for all the variables included in the model and considering 1, 4 and 8 quarters ahead forecast horizons. In all cases, following the BIC criterion, we include y_t without further lags on the right-hand side (i.e. $q = 0$ and $c(L) = c_0$). In the robustness section we include also y_{t-1}, so that $p = q$ and our method produce exactly the same result as the proxy-SVAR method discussed above.

[7]GDP and stock prices are taken in log levels to take into account potential cointegration relations.

3.2 Estimated uncertainty

We start documenting the overall significance of the regressors in equation (7). Table 1 shows the R^2 statistic along with the F-test for the overall significance of the regression, for all variables and horizons, when using just the contemporaneous VAR variables as regressors ($q = 0$). All regressions but the one for stock price uncertainty at horizon 8 are significant at the 5% level, and 16 regressions out of 21 are significant at the 1% level. The VAR variables predict the squared prediction errors implied by the VAR itself. This result, to our knowledge, was not found before and, as already observed, implies that the VAR residuals are not serially independent. This preliminary step lends support to the validity of our approximation procedure and the idea that shocks, although white noise, are far from being independent. The finding call into question the normality assumption typically made in Stcchastic volatility models and in Bayesian settings in general.[8]

Figure 1 plots the estimated uncertainty indexes for 1- and 4-quarter ahead. Real economic activity uncertainty, GDP and unemployment uncertainty, behave as already largely documented in the literature. The indexes tend to lead recessions, they begin to increase right before the onset of the recessions and to reduce right before the end of the recessions. The results are quite similar across horizons except that the 4-quarter ahead uncertainty is much more volatile.

Inflation uncertainty, 4-quarters, and federal funds rate uncertainty, 1-quarter, present

[8]The R^2 might appear small for several equations; notice however that $R^2 = 0.15$, corresponding to unemployment uncertainty at the one-year horizon (which is the uncertainty used in our baseline VAR below) roughly corresponds to the R^2 of a univariate AR(1) model with the sizable coefficient 0.4.

some differences compared to real economic activity uncertainty. Interestingly they do not exhibit a peak corresponding to the Great Recession. Inflation uncertainty is large during periods of high inflation, with peaks corresponding roughly to oil shocks. Federal funds rate uncertainty is high when the federal funds rate is high, i.e. during the so-called "stop and go" monetary policy period and during the Volcker era; it is very low at the end of the sample, when interest rates are close to zero.

Stock prices uncertainty, 1-quarter, behaves very similarly to real economic activity uncertainty, with correlations around 0.7, see Table 2 . The 4-quarter uncertainty display an opposite behavior, especially since the early 80s. Uncertainty steadily increases during expansionary periods and suddenly drops right before the recession remaining relatively low during recession. Stock prices become hard to forecast at long horizon compared to short horizons in periods of booms while in recession the forecast error variances are very similar at both horizons. The result seems to suggest that the longer is the period of economic expansion, the larger is the probability assigned to a fall in prices. As prices drop, uncertainty suddenly reduces because good outcomes are no longer expected. The result show that protracted period of increasing uncertainty are followed by economic downturns. It would be interesting to understand whether long-run stock prices uncertainty is able to predict in real time economic recessions. We leave this issue for future research.

Table 2 shows the correlation coefficients between four selected uncertainty indexes, computed according to equation (7), namely the unemployment rate uncertainty index, 1-quarter ad 4-quarter ahead ($\hat{U}^{UN}_{1,t}$ and $\hat{U}^{UN}_{4,t}$) and the stock price uncertainty index, 1-quarter ahead and 4-quarter ahead ($\hat{U}^{S\&P}_{1,t}$ $\hat{U}^{S\&P}_{4,t}$), and (a) the VXO index, extended as in Bloom (2009),

(b) the LMN (2020) financial uncertainty index 3-months (LMN fin), (c) the JLN (2015) macroeconomic uncertainty index 12 months (JLN), (d) the LMN (2020) real uncertainty index 12-months (LMN real), (e) the Becker et al. (2016) US Economic Policy Uncertainty index (EPU) and (f) the Rossi and Sekhposyan (2015) 4-quarters ahead uncertainty index (RS).

The indexes, except $\hat{U}_{4,t}^{S\&P}$, are highly positively correlated with each other and with JLN and LMN indexes, which are consistent with ours as for the definition of uncertainty. In particular, $\hat{U}_{1,t}^{UN}$ and $\hat{U}_{4,t}^{UN}$ exhibit correlation coefficients with JLN uncertainty 12-months as high as 0.66 and 0.79, respectively. On the contrary, and in line with the above discussion $\hat{U}_{4,t}^{S\&P}$, displays substantially smaller correlations, even negative with US Economic Policy Uncertainty index.

3.3 Uncertainy shocks: Idenification I

To begin, we have to choose the relevant uncertainty. We choose unemployment uncertainty as a proxy for macroeconomic, or real economic activity uncertainty, and stock prices uncertainty as a proxy for financial uncertainty. We choose unemployment rather than GDP as a benchmark mainly because the R^2 reported in Table 1 are larger and more significant than those for GDP. In the robustness section we show results for GDP uncertainty. As for the horizon, we choose 1 quarter. In subsection we also show the results for $h = 4$.

The literature does not provide a widespread consensus about a set of identification restrictions for the exogenous uncertainty shock. In this section we identify the uncertainty shock (Identification I) as the VAR innovation of uncertainty u_t^*, see Section (2.3). Therefore, the only shock affecting uncertainty on impact is the uncertainty shock. As already observed, this scheme is questionable. On the other hand, it is quite common in the literature, hence results may be useful for comparison.

Figure 3 shows results obtained for macroeconomic uncertainty under for Identification I. The uncertainty shock is contractionary for real economic activity, significantly reducing output and increasing unemployment. The effects are very large, as in JLN (2015), but not that much persistent, since they vanish after about 4 years. This result is different from those in JLN and Carriero et al. (2018b). Inflation is not affected significantly. The federal funds rate reduces, reacting to the slowdown of real activity and prices. Stock prices reduce on impact. The confidence index goes down on impact, reflecting consumers' expectations. The BAA-GS10 spread increases, reflecting the increased risk premium of Baa Corporate bonds.

Figure 4 plots the results for the financial uncertainty shock. The shock is again contractionary although the effects are much smaller in magnitude than those obtained for macroeconomic uncertainty and barely significant. Table 4 shows variance decomposition. The macroeconomic uncertainty shock accounts for a very high fraction of GDP and especially the unemployment rate. The shock explains more than half of the fluctuations in unemployment at the one-year horizon. The effect on the risk premium is also large: according to this identification, the uncertainty shock explains about three quarters of the spread variance at the one-year horizon. On the contrary the financial uncertainty shock generates very small effects. It essentially plays no role for fluctuations in real economic activity variables and stock prices. The only variable which seems to be driven by the shock is the spread, the explained variance being around 30%.

3.4 Uncertainy shocks: Idenification II

In this section we repeat the nalysis using a different identification strategy, Identification II. We impose that the uncertainty shock is orthogonal to the long-run shock above and, in addition, to the VAR innovations of GDP, unemployment, CPI and the federal funds rate (hence, we add four rows to the matrix D). In this way we impose that (i) the uncertainty shock has transitory effects on output; (ii) the slow-moving variables (output, unemployment and prices) do not react to uncertainty on impact, as is assumed for the monetary policy shock *à la* Christiano et al. (1999); in addition, (iii) the federal funds rate does not react to uncertainty on impact. The last constraint is imposed because, given (ii), (iii) entails that the uncertainty shock is orthogonal to a monetary policy shock which moves on impact the federal funds rate and therefore cannot be confused with it. On the other hand, the monetary policy shock, as well as the long-run shock and, possibly, other unidentified transitory shocks, may affect uncertainty on impact. Again we focus on 1-quarter ahead uncertainty of unemployment and stock prices.

Figure 5 shows results for macroeconomic uncertainty. Results are very similar to those of Identification I: the uncertainty shock significantly depresses real economic activity but without any significant effect on the inflation rate. The variance explained by the uncertainty shock (see Table 4) is slightly reduced but still very high. As for the stock market, the effects are smaller, consistently with Carriero et al. (2018b): the uncertainty shock explains about 20% of volatility at the one year horizon. Finally, the shock explains more than 90% of uncertainty itself on impact, leaving a very limited role for the long-term shock.

Figure 6 shows results for financial uncertainty. The effects are negligible and not significant for all of the variables except the spread. By imposing the additional restrictions the conclusion for the financial uncertainty shock are reinforced: the shock plays a negligible role for economic fluctuations.

Overall the variance explained by the macroeconomic uncertainty shock (see Table 4) is now much smaller. Still, at the one-year and the 4-year horizons, uncertainty shocks explains about 10% of output volatility and about 30% of unemployment volatility. Exogenous uncertainty considerably reduces at all horizons; however, it is still close to 80% on impact and about 50% at medium- and long-term horizons. On the contrary, the financial uncertainty shock explains nothing of the variance of real economic activity variables,

3.5 Uncertainy shocks: 4-quarter ahead

We now repeat the analysis using Identification I but focusing on the 4-quarter ahead uncertainty. Figure 7 and Figure 8 plot the impulse response functions for the macroeconomic uncertainty shock and financial uncertainty shock respectively. Macroeconomic uncertainty generates effects which are very similar to those obtained for the 1-quarter ahead: a significant and protracted downturn of economic activity and stock prices which triggers and expansionary response of monetary policy authorities. As far as financial uncertainty is concerned, the results are different from before. The shock generates a significant but very delayed contraction. Indeed the trough of the downturn occurs four years after the shock. Going back to the discussion in section 3.2, the result seems to suggest that there is, at least to some extent, a casual link between the protracted increase of 4-quarter ahead stock prices uncertainty and economic recessions.

3.6 Robustness checks

We make several robustness checks. In the first exercise, reported in Figure 9, we change the number of lags and use 2 lags (blue dotted lines) and 4 lags (magenta dotted-dashed lines) instead of 1 lag (benchmark case, black solid lines). Results are somewhat different from those obtained in the baseline model, particularly because the effects on GDP and stock prices are more persistent. However, both the sign and the size of the responses are similar to those of the baseline specification.

In the second exercise we change the VAR specification, by removing stock prices and the spread BAA-GS10, and including two different forward-looking variables: the ISM New Order Index and another component of the Michigan Consumer Confidence Index, the expected business conditions for the next five years (E5Y).[9] We remove the spread mainly to avoid a possible contamination of uncertainty shocks with credit market shocks (Gilchrist and Zakrajsek, 2012, Caldara et al., 2016). Results are reported in Figure 10. The effects of uncertainty shocks on the variables which are included in both specifications are similar.

In the last two exercises we retain the baseline specification for the VAR, but change the

[9] The latter variable is studied in depth in Barsky and Sims, 2012.

way we estimate uncertainty. First, we use the squares of the prediction error in place of their logs, i.e. we do not use equation (7), but simply replace the conditional expectation appearing in equation (4) with the linear projection. The effects of the implied uncertainty shock are very similar to those of the baseline model (Figure 11). Second, we specify $q = 1$ instead of $q = 0$ in equation (7), so that we have $q = p$ and the results are identical to those obtained with the proxy SVAR approach. The results are reported in Figure **??**. The effects on GDP and stock prices are larger and more persistent than in the benchmark model, whereas those on unemployment are smaller. However, the main results are confirmed: a positive uncertainty shock has large negative effects on economic activity.

All in all the results appear to be robust to changes in several features of the model specification.

4 Conclusions

We have shown that it is possible to produce reliable uncertainty estimates with a standard VAR model, without modeling time-varying volatility and using only OLS. The basic idea is to compute the squares of the prediction errors implied by the VAR model and replace expected values with linear projections.

Our estimate of uncertainty is a linear combination of the VAR variables. Therefore, the uncertainty shock is a linear combination of the VAR residuals and its effects can be computed by applying simple formulas to the reduced form impulse response functions. In this way, the same VAR model is used to estimate both uncertainty and its effects on the macro economy.

We have also provided simple formulas that can be used to impose suitable orthogonality constraints on the uncertainty shock.

The advantage of our procedure is twofold: on the one hand, we avoid the problematic choice of an external uncertainty measure; on the other hand, we avoid imposing restrictive assumption about the structure of conditional volatility.

Our procedure can be regarded as a variant of a proxy SVAR with the log of the squared prediction error taken as the relevant proxy. Under suitable conditions, the two methods yield the same results.

The procedure described here can easily be adapted to a factor model or a factor-augmented VAR. Moreover, it can be applied to survey-based forecast errors associated with local projection impulse-response functions estimation.

We have applied our procedure to a US macroeconomic quarterly data set. Our main conclusion is that macroeconomic uncertainty explains a large part of business cycle fluctuations while financial uncertainty plays a minor role.

Appendix A: A useful formula

If the unit-variance structural shock is $v'\varepsilon_t$, its impact effects are $d = \Sigma_\varepsilon v$. To see this, consider first the Cholesky representation with orthonormal shocks: $y_t = B(L)CC^{-1}\varepsilon_t$, where C is such that $CC' = \Sigma_\varepsilon$. Any other fundamental representation with orthogonal, unit-variance shocks will be given by

$$y_t = B(L)CUU'C^{-1}\varepsilon_t,$$

where U is a unitary matrix (i.e. $UU' = I$). Assuming, without loss of generality, that the structural shock of interest is the first one, the impact effects are $d = CU_1$, where U_1 is the first column of U, and the vector identifying the structural shock is $v' = U_1'C^{-1}$. Hence $U_1 = C'v$ and $d = CC'v = \Sigma_\varepsilon v$.

Appendix B: The relation with standard proxy SVAR

In the main text we have shown that in population our procedure is equivalent to the proxy-SVAR methodology.

Here we show that the OLS estimates are identical to those of Mertens and Ravn (2013) if the number of lags of y_t included in the regression of z_t is equal to the number of lags of the VAR for y_t (see equation (16)).

Let us begin with OLS estimation of the VAR in equation (1), which we report here for convenience:

$$y_t = \mu - A_1 y_{t-1} - \cdots - A_p y_{t-p} + \varepsilon_t. \tag{17}$$

We need some additional notation. Let

$$Y_k = \begin{pmatrix} y'_{p+1-k} \\ y'_{p+2-k} \\ \vdots \\ y'_{T-k} \end{pmatrix}, \quad \mathbb{1} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}, \quad X = \begin{pmatrix} \mathbb{1} & Y_1 & \dots & Y_p \end{pmatrix}, \quad \mathcal{E} = \begin{pmatrix} \varepsilon'_{p+1-k} \\ \varepsilon'_{p+2-k} \\ \vdots \\ \varepsilon'_{T-k} \end{pmatrix}.$$

Moreover, let $Y = Y_0$. Hence the VAR equation can be written as

$$Y = XA + \mathcal{E},$$

where $A = \begin{pmatrix} \mu & -A_1 & \dots & -A_p \end{pmatrix}'$. The OLS estimates of A and $\mathcal{E}$ are

$$\hat{A} = (X'X)^{-1}X'Y, \qquad \hat{\mathcal{E}} = Y - X(X'X)^{-1}X'Y.$$

Of course we have $X'\hat{\mathcal{E}} = 0$.

Mertens and Ravn (2013) focuses on the effects of the structural shock. Such effects are estimated by performing the OLS regression of $\hat{\varepsilon}_t$ onto the proxy z_t, which for ease of exposition and without loss of generality we assume to be zero-mean. Precisely, let $z = \begin{pmatrix} z'_{p+1} & z'_{p+2} & \dots & z'_T \end{pmatrix}'$, and consider the regression equation

$$\hat{\mathcal{E}} = z\phi' + V.$$

The vector of the impact effects is obtained as the OLS estimator of ϕ, suitably normalized (for instance to get unit variance for the corresponding structural shock). The OLS estimator of ϕ is

$$\hat{\phi} = \hat{\mathcal{E}}' z / z' z. \tag{18}$$

The vector of the impact effects is then obtained by normalizing the above vector in the desired way.

Our proposed procedure focuses on the estimation of the structural shock, rather than the estimation of the corresponding impulse-response functions. We compute the OLS regression of z onto the columns of Y and X:

$$z = Yc_0 + Xb + \nu,$$

where $b = (\theta' \; c'_1 \; \cdots \; c'_p)'$ (see equation 7). Letting $W = \begin{pmatrix} Y & X \end{pmatrix}$, the fitted value of z (which in our case is the estimate of uncertainty) is $W(W'W)^{-1}W'z$ and the residual is $\hat{\nu} = z - W(W'W)^{-1}W'z$. Clearly, $W'\hat{\nu} = 0$, so that $Y'\hat{\nu} = 0$ and $X'\hat{\nu} = 0$. Hence $\hat{\mathcal{E}}'\hat{\nu} = 0$. Pre-multiplying the above equation by $\hat{\mathcal{E}}'$ we get

$$\hat{c}_0 = (\hat{\mathcal{E}}'Y)^{-1}\hat{\mathcal{E}}'z = (\hat{\mathcal{E}}'\hat{\mathcal{E}})^{-1}\hat{\mathcal{E}}'z,$$

where the last equality is obtained by observing that $\hat{\mathcal{E}}'Y = \hat{\mathcal{E}}'\left(X(X'X)^{-1}X'Y + \hat{\mathcal{E}}\right) = \hat{\mathcal{E}}'\hat{\mathcal{E}}$. Hence $\hat{c}_0$ could be obtained equivalently by OLS regression of z_t onto ε_t. This makes sense: the estimated structural shock is nothing else than the OLS projection of the proxy z_t onto the VAR residuals. The reason why we do not follow this way is that it would not enable us to get an estimate of uncertainty.

We have shown above that the impact effects of $c_0'\varepsilon_t$ are proportional to $\Sigma_\varepsilon c_0$. Hence we estimate such impact effects as $\hat{\mathcal{E}}'\hat{\mathcal{E}}\hat{c}_0 = \hat{\mathcal{E}}'z$, up to a multiplicative constant which is fixed by the unit variance normalization. These effects are proportional to the ones in equation (18) and are equal once we impose the same normalization.

Appendix C: The bootstrap procedure

To construct confidence bands we draw randomly $T-p$ times (with replacement) from the uniform discrete distribution with possible values $p+1,\dots,T$, to get the sequence $t(\tau)$, $\tau = p+1,\dots,T$ and the corresponding sequences $\varepsilon_\tau = \hat{\varepsilon}_{t(\tau)}$, $r_\tau = \hat{r}_{t(\tau)}$, $\tau = p+1,\dots,T$. Then we set $y_\tau = y_t$ for $\tau = 1,\dots,p$. Moreover, according to (17), we set $y_\tau = \hat{\mu} - \hat{A}_1 y_{\tau-1} - \cdots, -A_p y_{\tau-p} + \varepsilon_\tau$, and, according to (7), $z_\tau = \hat{\theta} + \hat{c}_0' y_\tau + \cdots + \hat{c}_p' y_{\tau-p} + r_\tau$, for $\tau = p+1,\dots,T$. Having the artificial series y_τ, $\tau = 1,\dots,T$, and z_τ, $\tau = p+1,\dots,T$, we re-estimate the relevant impulse-response functions. We repeat the procedure N times to get a distribution of IRFs and take the desired point-wise percentiles to form the confidence bands.

The above procedure takes into account the parameter estimate uncertainty of both the VAR and the proxy equation (7). On the other hand, we treat z_t as an observed variable, whereas in our case it is estimated. This cannot be avoided since we do not have a fully specified stochastic volatility model enabling us to reproduce the correct covariances between the squared prediction errors and the lagged variables.

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Tables

	R^2			p-value (F-test)		
	$h = 1$	$h = 4$	$h = 8$	$h = 1$	$h = 4$	$h = 8$
Per Capita GDP	0.15	0.08	0.06	0.00	0.01	0.04
Unemployment rate	0.19	0.15	0.13	0.00	0.00	0.00
CPI inflation	0.09	0.08	0.07	0.00	0.01	0.02
Federal Funds Rate	0.43	0.25	0.27	0.00	0.00	0.00
S&P500	0.10	0.09	0.05	0.00	0.00	0.08
E1Y	0.08	0.08	0.06	0.01	0.01	0.04
spread BAA-GS10	0.21	0.09	0.07	0.00	0.00	0.02

Table 1: R^2 of regression (7) and p-values of the F-test of the significance of the regression using the forecast error squared for the variables listed in the first column.

	$\hat{U}^{UN}_{1,t}$	$\hat{U}^{S\&P}_{1,t}$	$\hat{U}^{UN}_{4,t}$	$\hat{U}^{S\&P}_{4,t}$	VXO	LMN F3m	JLN 12m	LMN R12m	EPU	RS 4Q
$\hat{U}^{UN}_{1,t}$	1.00	-	-	-	-	-	-	-	-	-
$\hat{U}^{S\&P}_{1,t}$	0.63	1.00	-	-	-	-	-	-	-	-
$\hat{U}^{UN}_{4,t}$	0.80	0.69	1.00	-	-	-	-	-	-	-
$\hat{U}^{S\&P}_{4,t}$	0.25	0.27	0.37	1.00	-	-	-	-	-	-
VXO	0.34	0.43	0.56	0.12	1.00	-	-	-	-	-
LMN F12m	0.39	0.50	0.60	0.32	0.78	1.00	-	-	-	-
JLN 12m	0.66	0.48	0.79	0.58	0.47	0.52	1.00	-	-	-
LMN R12m	0.68	0.49	0.76	0.57	0.28	0.44	0.82	1.00	-	-
EPU	0.71	0.33	0.48	-0.41	0.35	0.38	0.29	0.25	1.00	-
RS 4q	-0.02	0.16	0.13	0.36	0.28	0.31	0.14	0.12	-0.14	1.00

Table 2: Correlation of unemployment uncertainty 1-quarter ($\hat{U}^{UN}_{1,t}$) and 4-quarter ahead ($\hat{U}^{UN}_{4,t}$) and S&P uncertainty 1-quarter ($\hat{U}^{S\&P}_{1,t}$) and 4-quarter ahead ($\hat{U}^{S\&P}_{4,t}$) with existing measures: VXO, LMN financial 12-month ahead (LMN F12m), JLN 12-month ahead (JLN 12m), LMN real 12-month ahead (LMN R12m), economic policy uncertainty (EPU), and Rossi and Sekhposyan 4-quarter ahead (RS 4q).

Identification I

	$\hat{U}^{UN}_{1,t}$				$\hat{U}^{S\&P}_{1,t}$			
	$h=0$	$h=4$	$h=16$	$h=40$	$h=0$	$h=4$	$h=16$	$h=40$
Per Capita GDP	20.1	44.5	30.7	16.9	0.9	3.3	2.7	1.5
Unemployment rate	30.9	66.4	61.6	48.8	0.5	2.9	4.1	3.1
CPI inflation	8.2	7.0	13.5	13.2	21.7	16.1	12.8	12.0
Federal Funds Rate	3.3	11.6	29.7	26.2	0.8	1.7	3.2	2.8
S&P500	6.0	9.5	4.7	5.1	3.5	5.9	3.6	2.2
E1Y	64.3	56.3	44.2	42.1	0.3	1.5	1.2	1.3
spread BAA-GS10	34.7	49.2	46.7	48.6	28.8	29.5	24.8	23.5
Uncertainty	100.0	94.2	71.7	69.0	100.0	79.1	65.1	61.0

Identification II

	$\hat{U}^{UN}_{4,t}$				$\hat{U}^{S\&P}_{4,t}$			
	$h=0$	$h=4$	$h=16$	$h=40$	$h=0$	$h=4$	$h=16$	$h=40$
Per Capita GDP	0.0	12.8	12.5	7.0	0.0	0.1	1.1	1.2
Unemployment rate	0.0	29.7	40.9	30.5	0.0	1.8	1.1	1.6
CPI inflation	0.0	0.6	2.4	2.7	0.0	0.5	0.8	1.9
Federal Funds Rate	0.0	3.1	10.4	9.2	0.0	0.7	0.9	2.3
S&P500	24.1	27.0	15.9	8.6	15.3	12.6	6.4	5.2
E1Y	61.3	50.6	38.8	36.1	1.1	1.7	2.1	3.1
spread BAA-GS10	41.3	54.3	50.0	48.8	52.1	44.9	35.2	32.8
Uncertainty	43.0	51.8	41.7	39.4	67.2	71.2	59.3	55.3

Table 3: Variance decomposition. Identification I: uncertainty innovation. Identification II: orthogonal to long run shock. Identification III: zero contemporaneous effects on GDO, unemployment rate, CPI and federal funds rate.

Identification I				
	$h = 0$	$h = 4$	$h = 16$	$h = 40$
Per Capita GDP	12.7	38.6	27.4	14.7
Unemployment rate	9.9	54.3	55.4	42.2
CPI inflation	1.1	1.3	6.4	6.4
Federal Funds Rate	1.2	8.9	22.6	19.2
S&P500	21.4	24.4	12.2	6.5
E1Y	62.2	50.6	38.8	36.4
spread BAA-GS10	61.3	75.9	68.2	67.6
Uncertainty	100.0	89.4	68.7	67.8

Identification II				
	$h = 0$	$h = 4$	$h = 16$	$h = 40$
Per Capita GDP	10.8	35.0	23.8	13.3
Unemployment rate	11.4	56.0	54.9	42.2
CPI inflation	0.7	0.9	6.8	6.7
Federal Funds Rate	1.4	9.8	24.4	20.7
S&P500	18.5	21.0	10.0	5.7
E1Y	60.7	48.5	37.4	35.1
spread BAA-GS10	63.3	77.7	69.5	68.8
Uncertainty	99.6	88.2	68.2	67.5

Identification III				
	$h = 0$	$h = 4$	$h = 16$	$h = 40$
Per Capita GDP	0.0	11.0	9.8	5.7
Unemployment rate	0.0	27.4	35.9	26.7
CPI inflation	0.0	0.4	1.9	2.4
Federal Funds Rate	0.0	3.2	9.4	8.5
S&P500	27.9	29.9	17.2	9.6
E1Y	47.7	38.6	29.4	27.6
spread BAA-GS10	54.0	65.2	58.1	55.9
Uncertainty	80.1	72.9	53.8	52.1

Table 4: Variance decomposition. Identification I: uncertainty innovation. Identification II: orthogonal to long run shock. Identification III: zero contemporaneous effects on GDO, unemployment rate, CPI and federal funds rate.

Figures

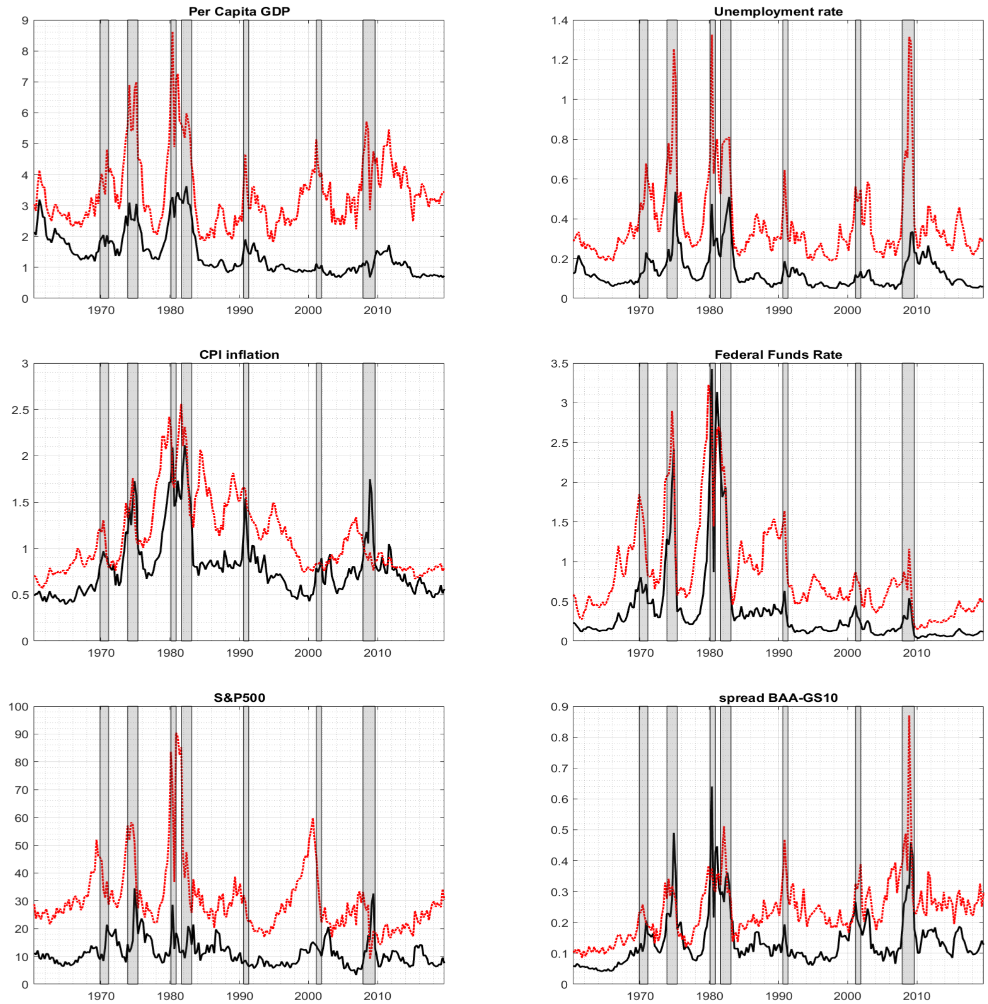

Figure 1: Estimated uncertainties. Black line 1-quarter ahead. Red line 4-quarter ahead. Gray vertical bands NBER recessions dares.

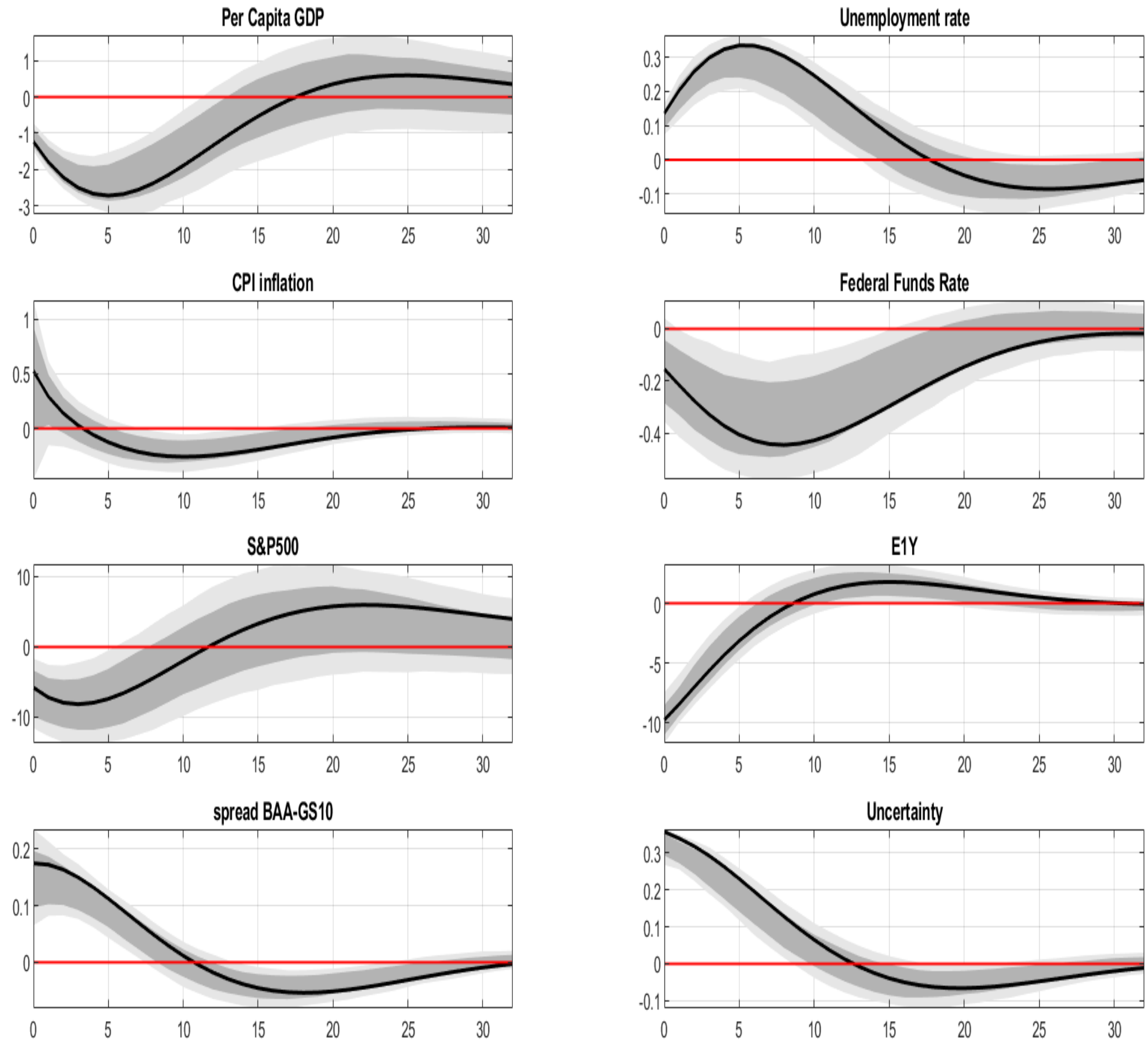

Figure 2: Impulse response functions of the unemployment rate uncertainty shock, 1-quarter ahead. The shock is identified as the innovation in uncertainty (Identification I). Solid line: point estimate. Light grey area: 90% confidence bands. Dark grey area: 68% confidence bands.

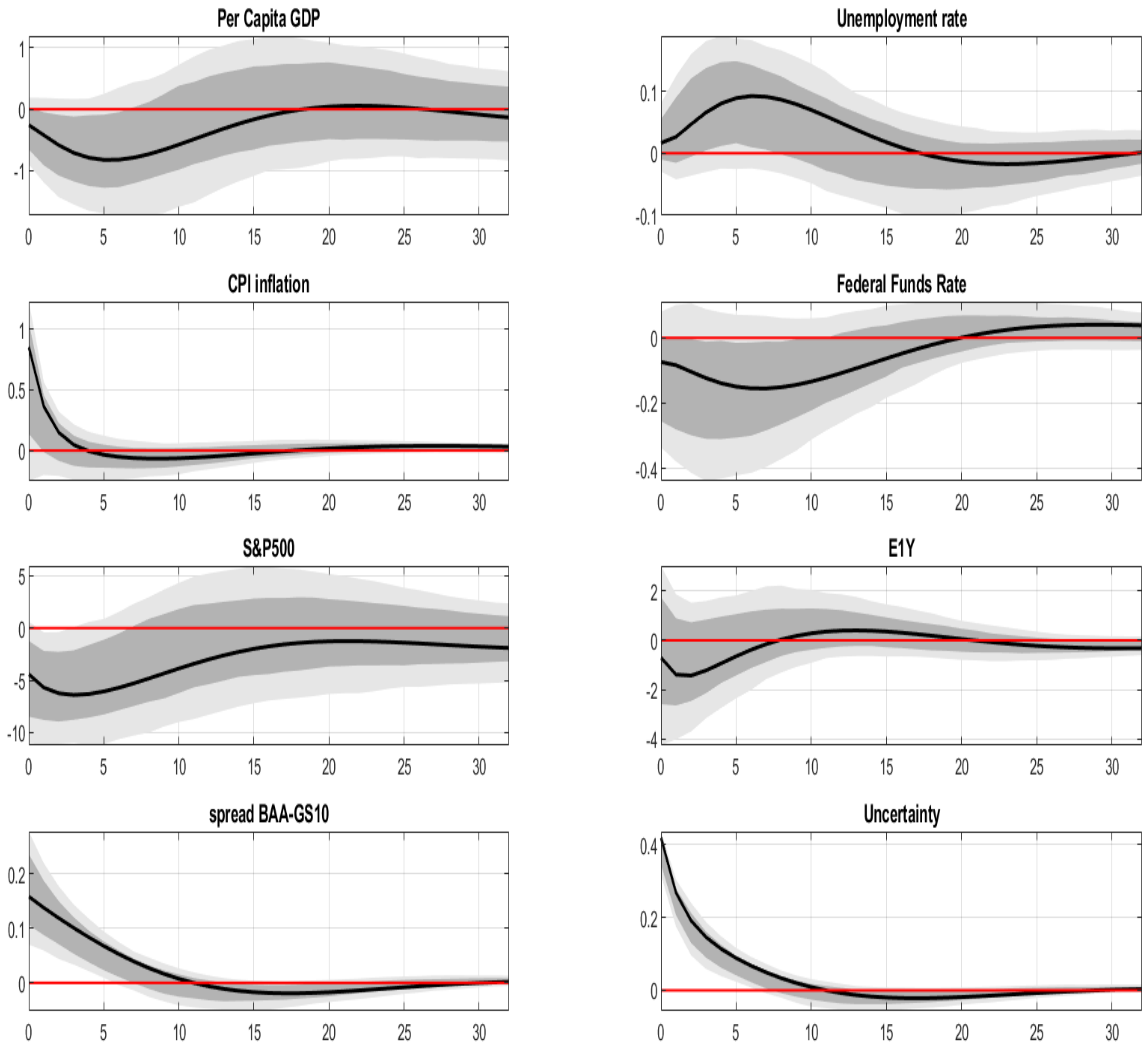

Figure 3: Impulse response functions of the S&P500 uncertainty shock, 1-quarter ahead. The shock is identified as the innovation in uncertainty (Identification I). Solid line: point estimate. Light grey area: 90% confidence bands. Dark grey area: 68% confidence bands.

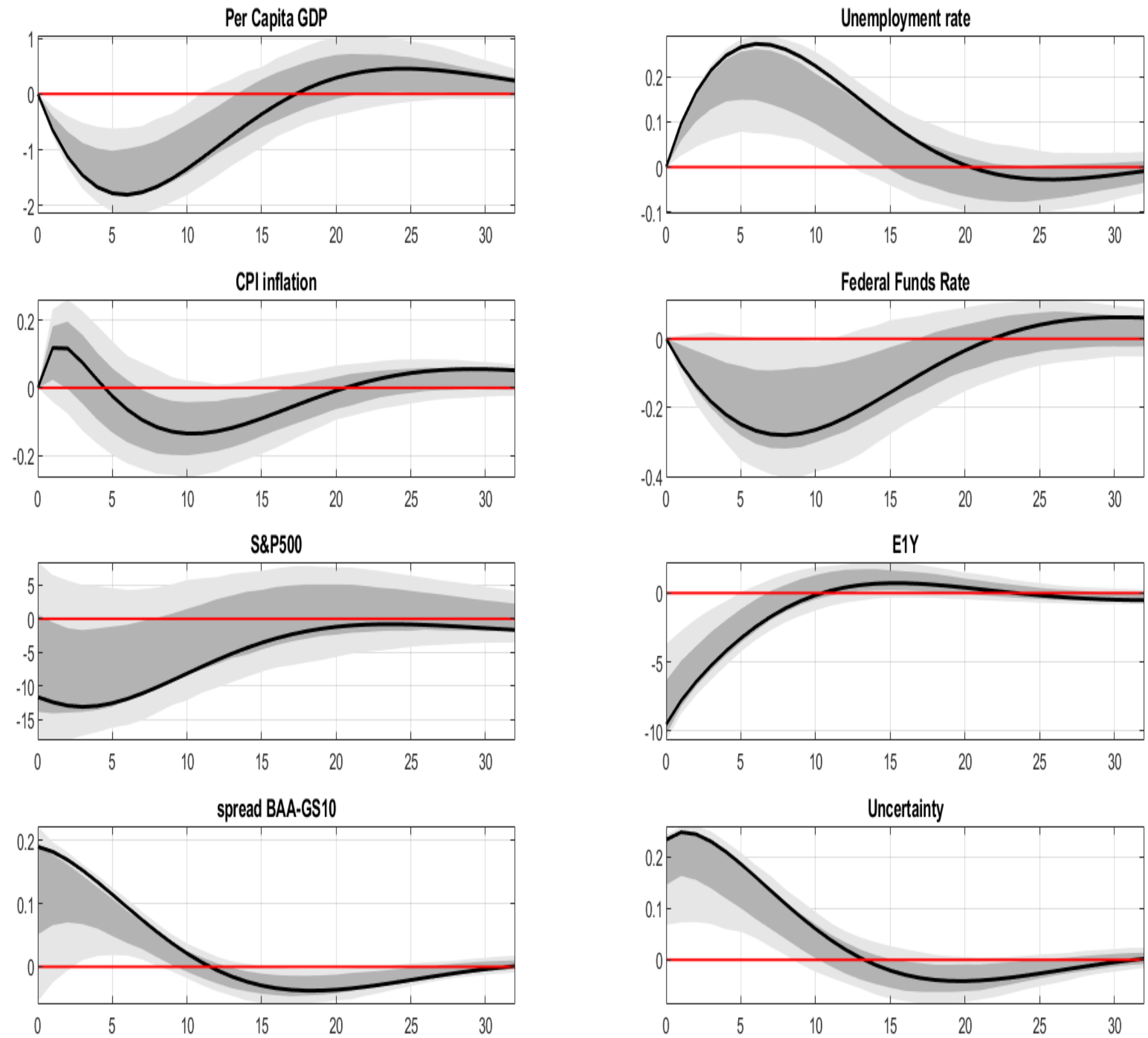

Figure 4: Impulse response functions of the unemployment rate uncertainty shock, 1-quarter ahead. The shock is identified as the residual of the projection of the uncertainty innovation onto the long-run shock, the GDP innovation, the unemployment rate innovation, the CPI innovation and the federal funds rate innovation (Identification II). Solid line: point estimate. Light grey area: 90% confidence bands. Dark grey area: 68% confidence bands.

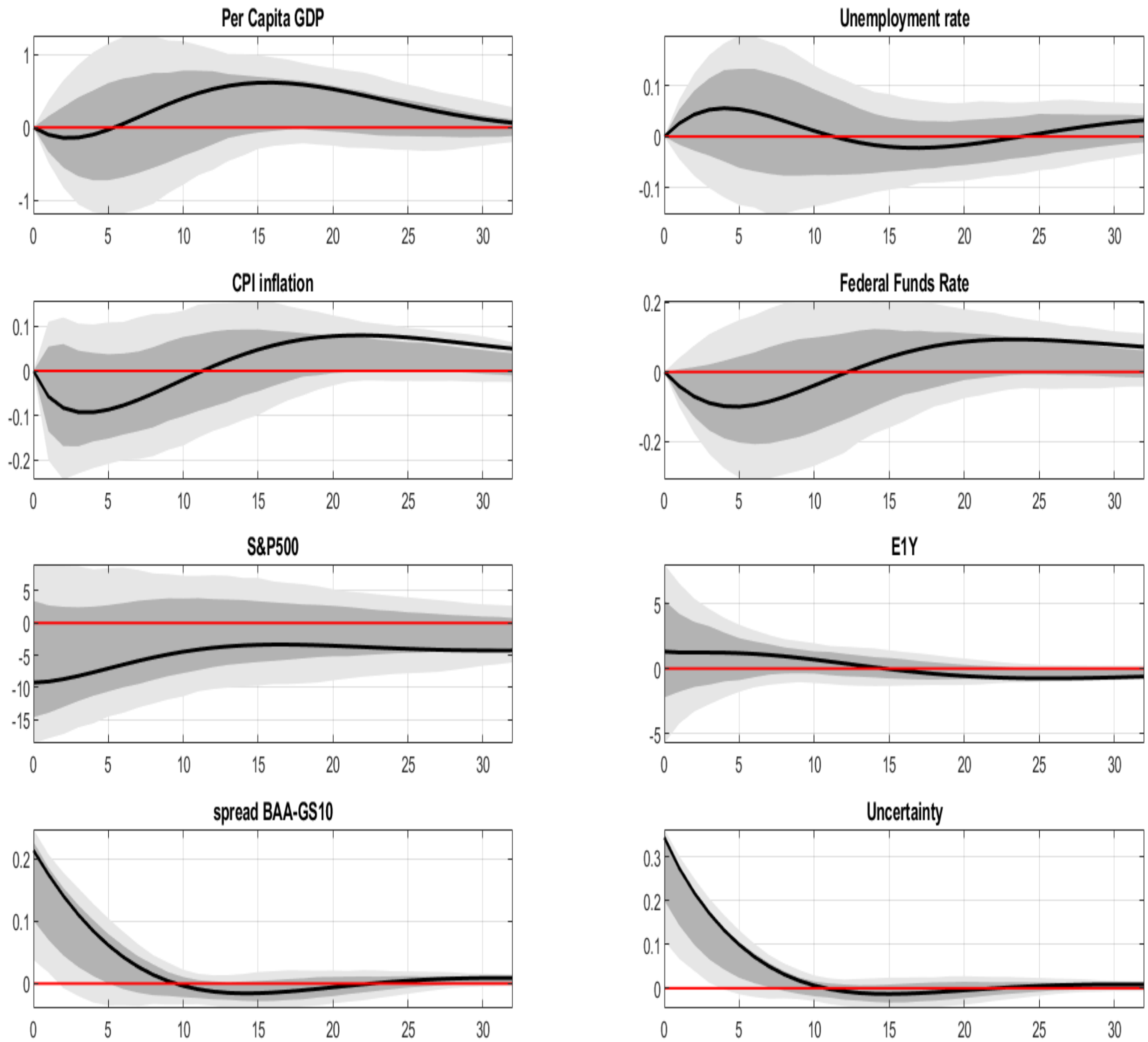

Figure 5: Impulse response functions of the S&P500 uncertainty shock, 1-quarter ahead. The shock is identified as the residual of the projection of the uncertainty innovation onto the long-run shock, the GDP innovation, the unemployment rate innovation, the CPI innovation and the federal funds rate innovation (Identification II). Solid line: point estimate. Light grey area: 90% confidence bands. Dark grey area: 68% confidence bands.

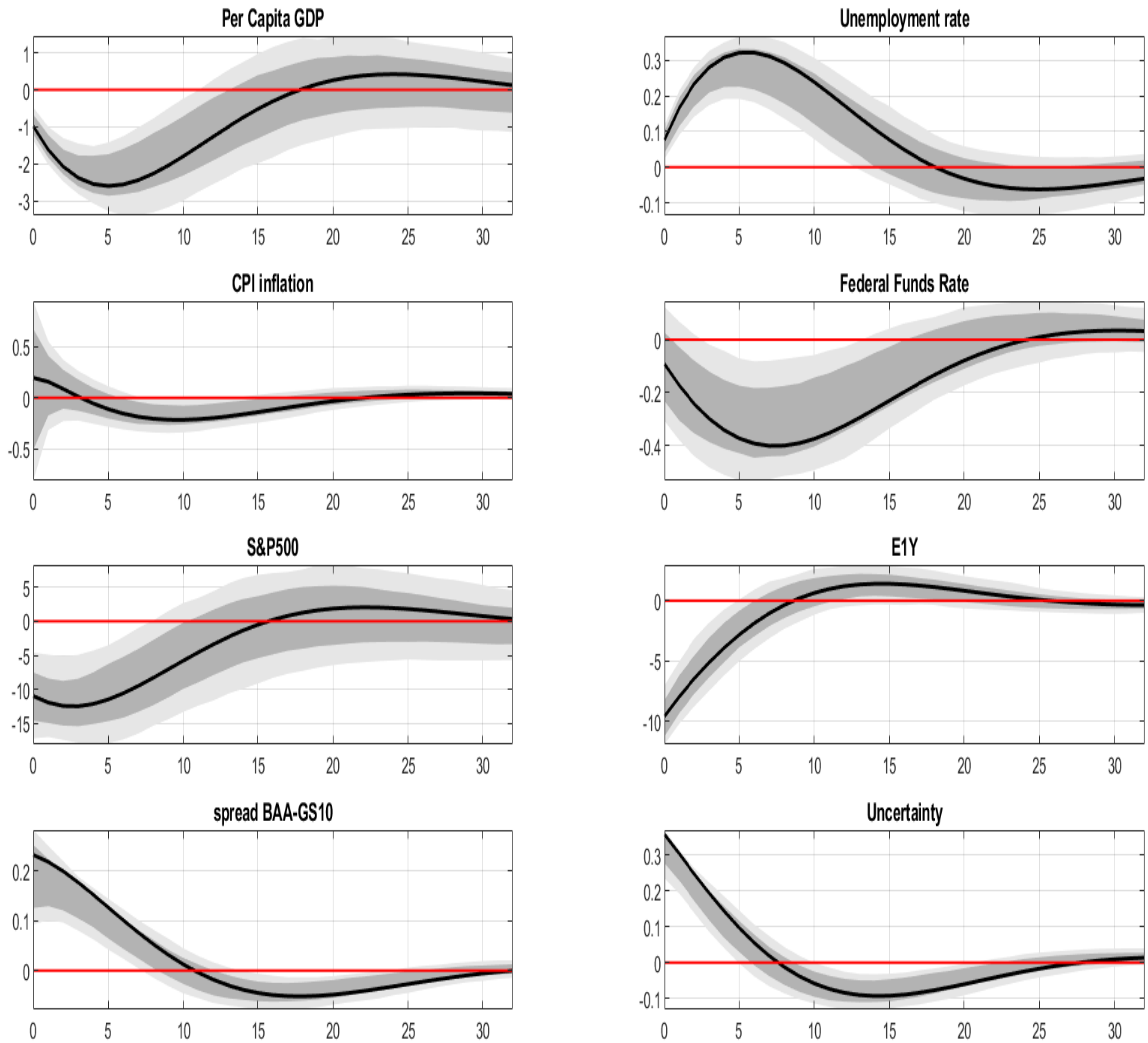

Figure 6: Impulse response functions of the unemployment rate uncertainty shock, 4-quarter ahead. The shock is identified as the innovation in uncertainty (Identification I). Solid line: point estimate. Light grey area: 90% confidence bands. Dark grey area: 68% confidence bands.

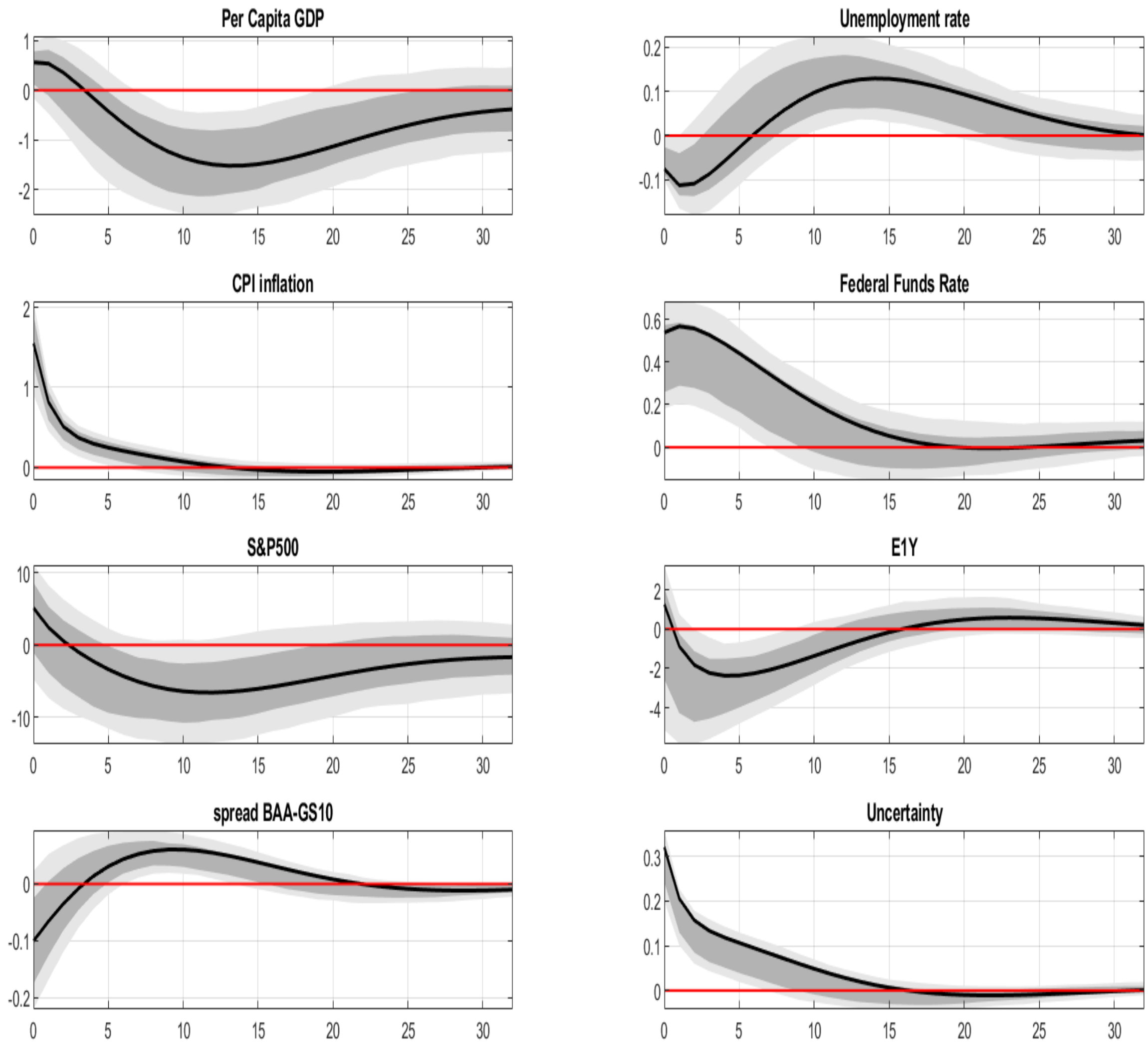

Figure 7: Impulse response functions of the S&P500 uncertainty shock, 4-quarter ahead. The shock is identified as the innovation in uncertainty (Identification I). Solid line: point estimate. Light grey area: 90% confidence bands. Dark grey area: 68% confidence bands.

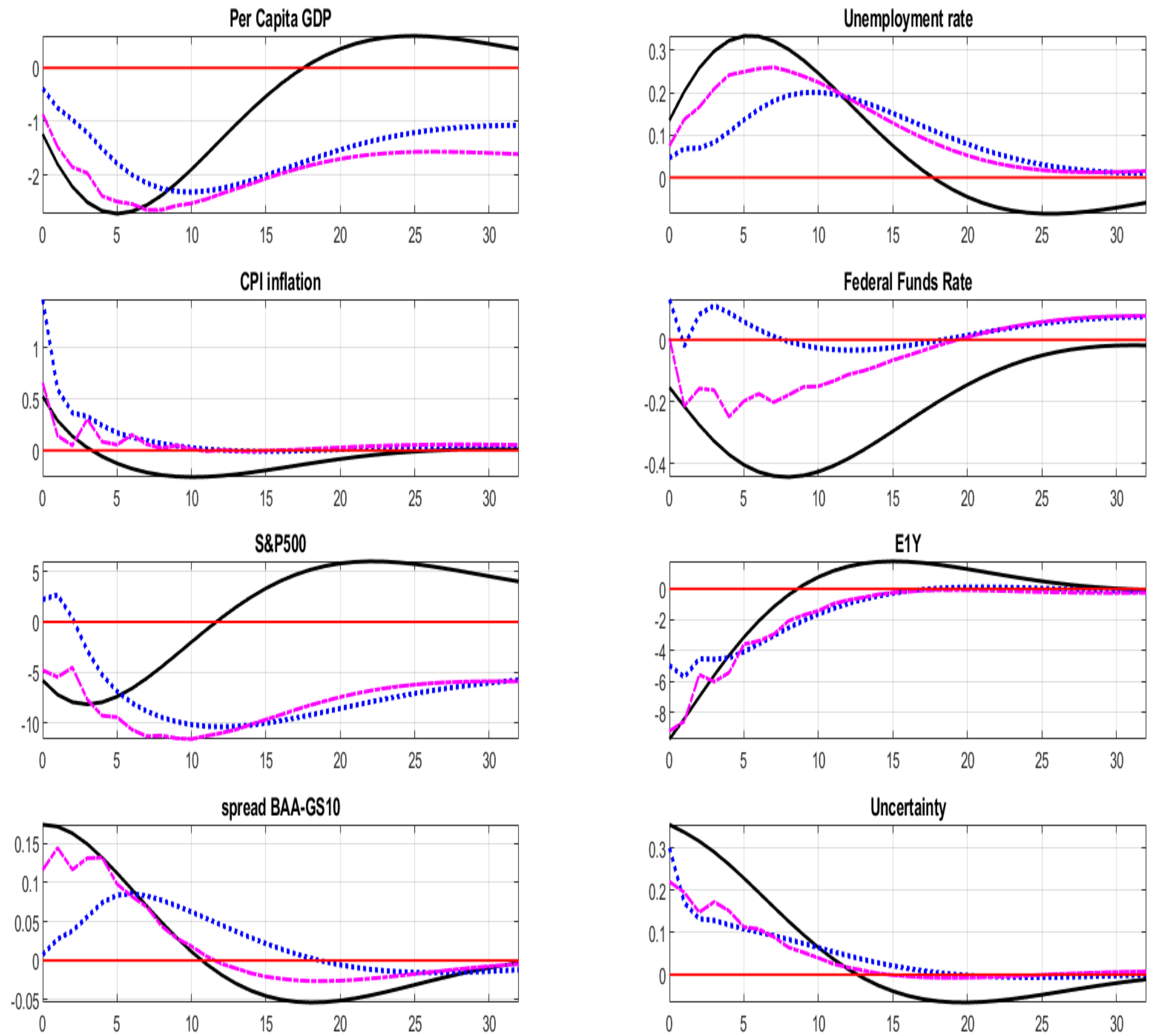

Figure 8: Comparison between the benchmark impulse response functions of Identification I (solid black lines), obtained with 1 lag in the VAR and the corresponding impulse response functions obtained with 2 lags (dotted blue lines) and 4 lags (dashed-dotted magenta lines).

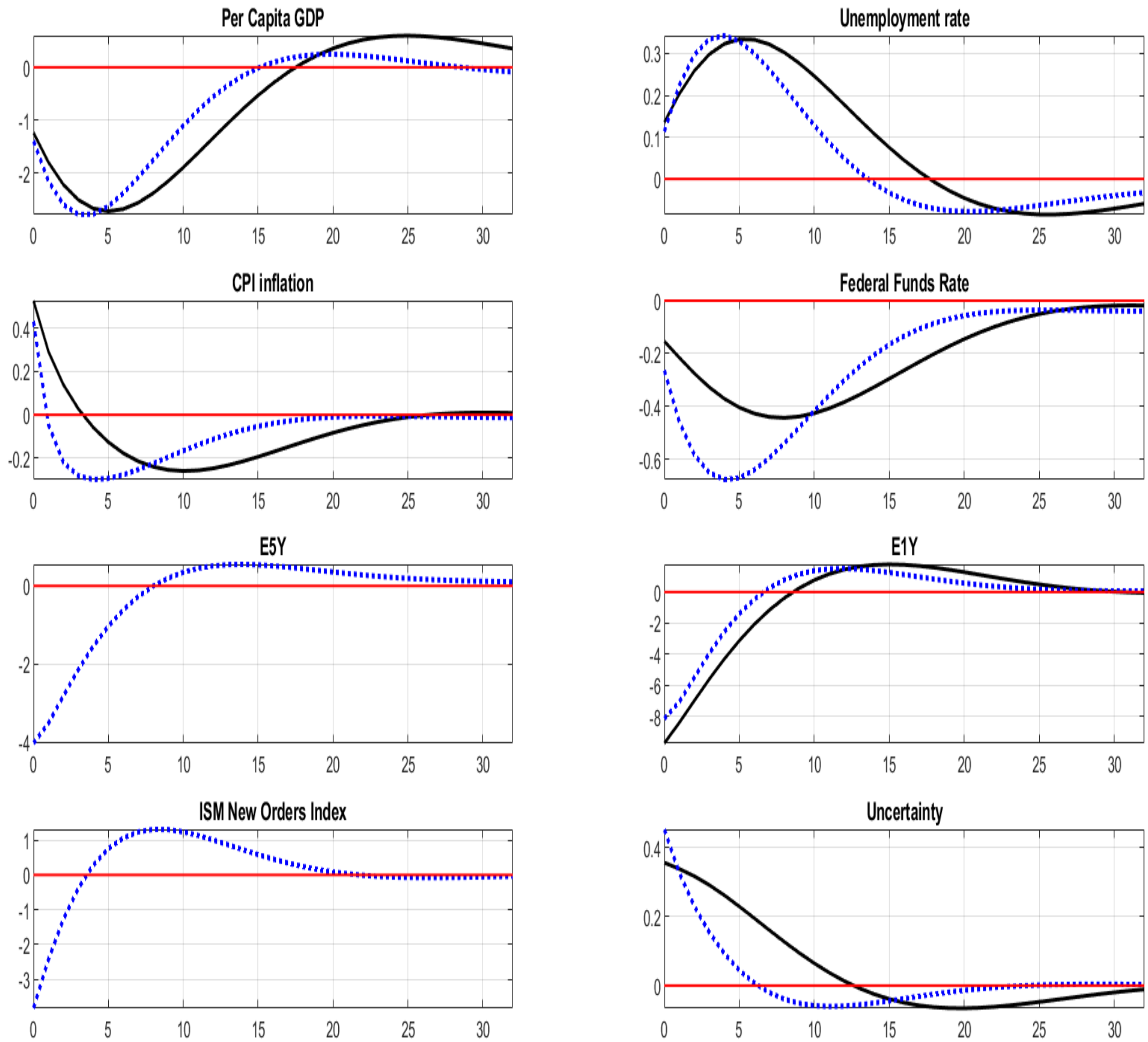

Figure 9: Comparison between the benchmark VAR impulse response functions, Identification I (solid black lines), and the impulse response function obtained with a different VAR specification, including E5Y (a component of the Michigan University Consumer Confidence Index) and the ISM New Order Index in place of S&P500 and the spread BAA-GS10 (dotted blue lines).

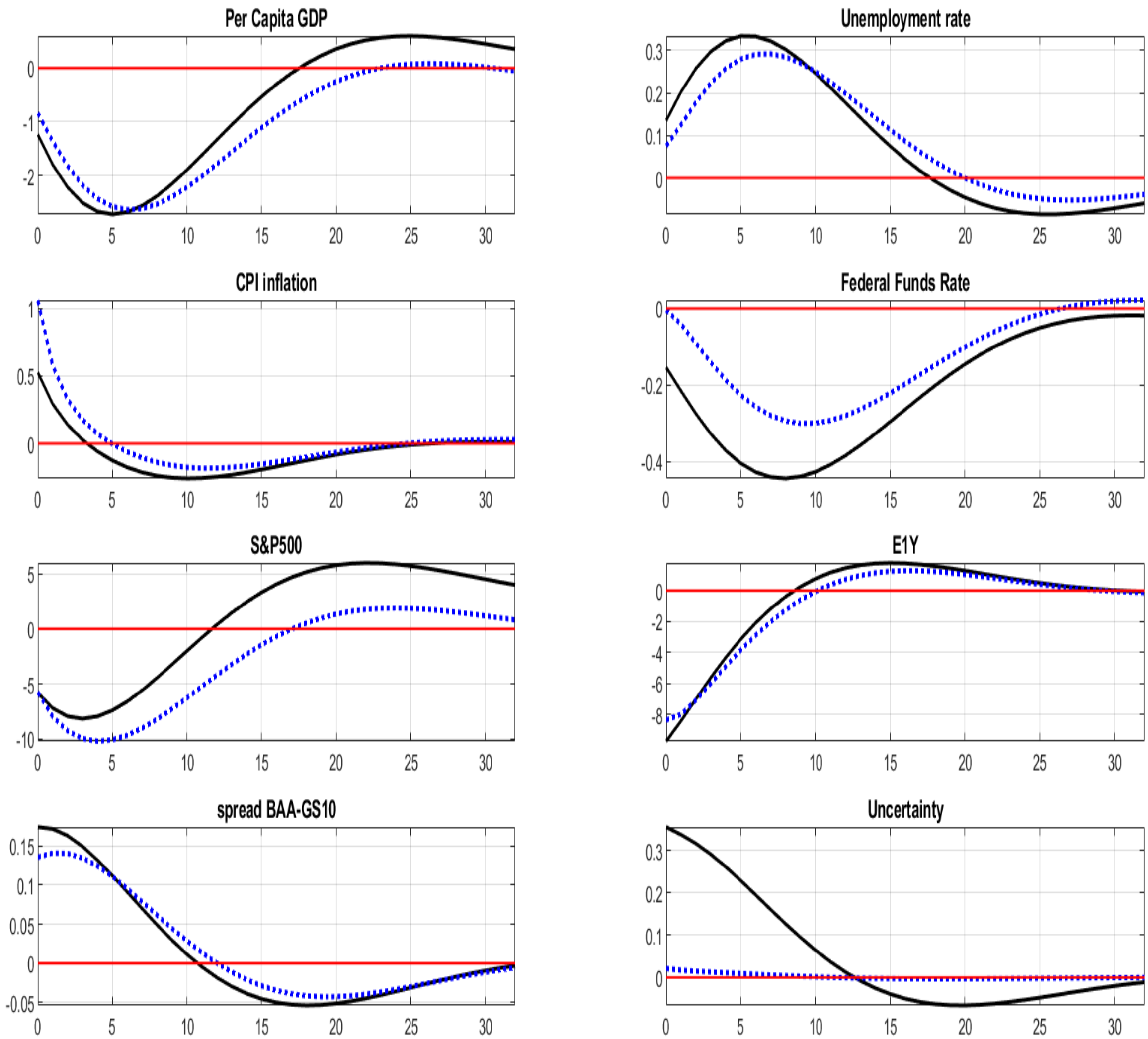

Figure 10: Comparison between the benchmark VAR impulse response functions, Identification I (solid black lines), and the impulse response function obtained when using the squared predictions error in place of the log of the squared prediction error to compute uncertainty (dotted blue lines).

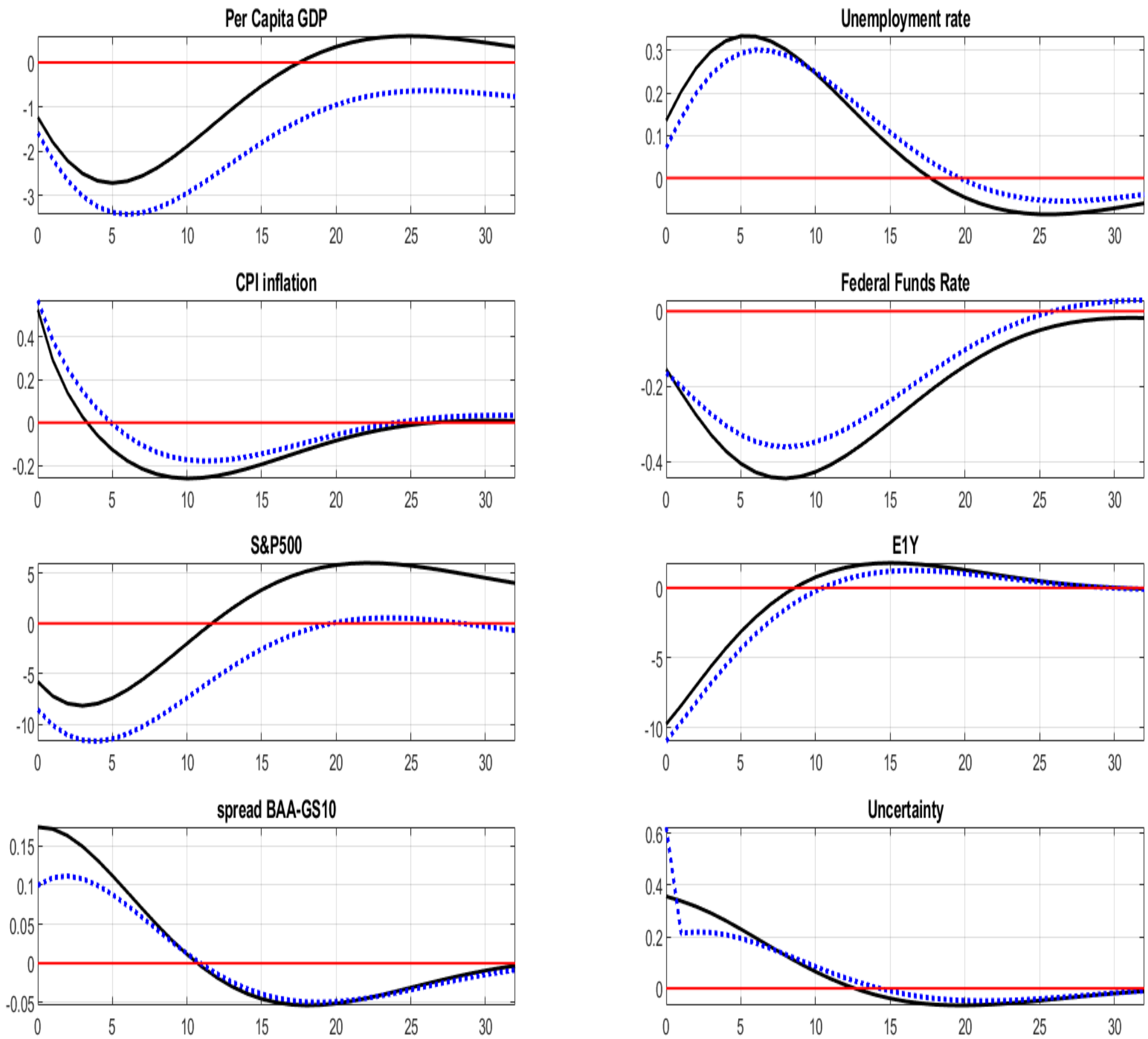

Figure 11: Comparison between the benchmark VAR impulse response functions, Identification I (solid black lines), and the impulse response function obtained when using 1 lag of the variables, in addition to the current values, to compute uncertainty (dotted blue lines).