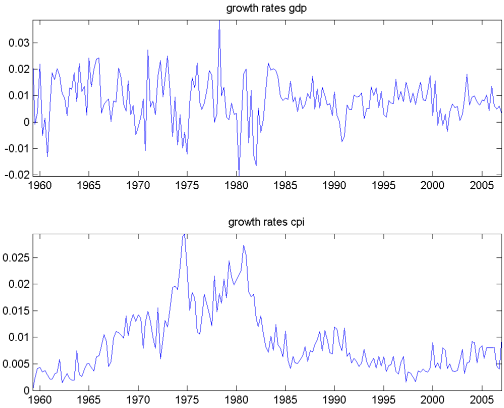
14. TVC-VARS

Question

Are the dynamic properties of the series constant over time?

Question


growth rates gdp
0.03
0.02
0.01
0
-0.01
-0.02
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
growth rates cpi
0.025
0.02
0.015
0.01
0.005
0
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005

Answer

For these series probably not.

- Changes in the variance of GDP growth.
- Changes in the mean of inflation.

Introduction

- More generally economic dynamics are evolving over-time.
- Many examples: Great Moderation, policy regime changes, financial innovations.
- Time-invariant VAR parameters, probably, not a too good idea.
- Better idea: allowing model dynamics to also vary over-time.
- Several ways to do it:
  - More or less smooth regime switches.
  - Continuously varying parameters
- In this lecture we focus on the second.

Introduction

- In this lecture we will study a class of models called Time-Varying Coefficients VAR with Stochastic volatility.
- Very general model: a VAR where both the VAR coefficients and the residuals covariance matrix are changing over time.
- Aim: to capture changes of various type in the economy.
- First we will see the model.
- Second we will see some applications.

The model

Time-varying coefficients VAR (TVC-VAR) represent a generalization of VAR models in which the coefficients are allowed to change over time.

Let Y_t be a n-vector of time series satisfying

$$Y_t = A_{0,t} + A_{1,t}Y_{t-1} + ... + A_{p,t}Y_{t-p} + \varepsilon_t \tag{1}$$

where

- ε_t is a Gaussian white noise with zero mean and time-varying covariance matrix Σ_t.
- A_{jt} $n \times n$ are matrices of coefficients.

The model

Law of motion of the VAR parameters.

Let $A_t = [A_{0,t}, A_{1,t} ..., A_{p,t}]$, and $\theta_t = vec(A'_t)$, ($vec(\cdot)$ is the stacking column operator).

We postulate

$$\theta_t = \theta_{t-1} + \omega_t \tag{2}$$

where

- ω_t is a Gaussian white noise with zero mean and covariance Ω.

The model

Covariance matrix.

Let

$$\Sigma_t = F_t D_t F_t' \tag{3}$$

where

- F_t is lower triangular, with ones on the main diagonal.
- D_t a diagonal matrix.

The model

Law of motion of the covariance matrix elements.

Let σ_t be the n-vector of the diagonal elements of $D_t^{1/2}$.

Let $\phi_{i,t}$, $i = 1, ..., n-1$ the column vector formed by the non-zero and non-one elements of the $(i+1)$-th row of F_t^{-1}.

We assume

$$\log \sigma_t = \log \sigma_{t-1} + \xi_t \tag{4}$$

$$\phi_{i,t} = \phi_{i,t-1} + \psi_{i,t} \tag{5}$$

where ξ_t and $\psi_{i,t}$ are Gaussian white noises with zero mean and covariance matrix Ξ and Ψ_i, respectively.

We assume that that ξ_t, ψ_{it}, ω_t, ε_t are mutually uncorrelated at all leads and lags.

A bivariate TVC-VAR(1)

Consider, as an example, the simplest possible case, a bivariate TVC-VAR(1).

$$\begin{pmatrix} Y_{1t} \\ Y_{2t} \end{pmatrix} = \begin{pmatrix} A_{11t} & A_{12t} \\ A_{21t} & A_{22t} \end{pmatrix} \begin{pmatrix} Y_{1t-1} \\ Y_{2t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \tag{6}$$

$$\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \sim N(0, \Sigma_t) \tag{7}$$

where

$$\Sigma_t = F_t D_t F_t' = \begin{pmatrix} 1 & 0 \\ \phi_{1t} & 1 \end{pmatrix}^{-1} \begin{pmatrix} \sigma_{1t}^2 & 0 \\ 0 & \sigma_{2t}^2 \end{pmatrix} \begin{pmatrix} 1 & \phi_{1t} \\ 0 & 1 \end{pmatrix}^{-1} \tag{8}$$

A bivariate TVC-VAR(1)

The assumptions made before imply

$$\theta_t = \begin{pmatrix} A_{11t} \\ A_{12t} \\ A_{21t} \\ A_{22t} \end{pmatrix} = \begin{pmatrix} A_{11t-1} \\ A_{12t-1} \\ A_{21t-1} \\ A_{22t-1} \end{pmatrix} + \begin{pmatrix} \omega_{1t} \\ \omega_{2t} \\ \omega_{3t} \\ \omega_{4t} \end{pmatrix} \tag{9}$$

$$\log \sigma_t = \begin{pmatrix} \log \sigma_{1t} \\ \log \sigma_{2t} \end{pmatrix} = \begin{pmatrix} \log \sigma_{1t-1} \\ \log \sigma_{2t-1} \end{pmatrix} + \begin{pmatrix} \xi_{1t} \\ \xi_{2t} \end{pmatrix} \tag{10}$$

and

$$\phi_{1t} = \phi_{1t-1} + \psi_{1t} \tag{11}$$

Impulse response functions

- We will see next that the impulse response functions in this model are time varying.
- That means that the effects and the contributions to the variance of the series of a shock change over time.

Impulse response functions

Example: TVC-VAR(1). Consider the model

$$Y_t = A_t Y_{t-1} + \varepsilon_t \tag{12}$$

Ask: what are the effects of a shock occurring at time t on the future values of Y_t?

Impulse response functions

Substituting forward we obtain

$$\begin{aligned}
Y_{t+1} &= A_{t+1}A_tY_{t-1} + A_{t+1}\varepsilon_t + \varepsilon_{t+1} \\
Y_{t+2} &= A_{t+2}A_{t+1}A_tY_{t-1} + A_{t+2}A_{t+1}\varepsilon_t + A_{t+1}\varepsilon_{t+1} + \varepsilon_{t+2} \\
Y_{t+k} &= A_{t+k}...A_{t+1}A_tY_{t-1} + A_{t+k}...A_{t+2}A_{t+1}\varepsilon_t + ... + \varepsilon_{t+k}
\end{aligned}$$

the collection

$$I, A_{t+1}, (A_{t+2}A_{t+1}), ..., (A_{t+k}...A_{t+2}A_{t+1}),$$

represents the impulse response functions of ε_t. Clearly these will be different for ε_{t-k}.

Impulse response functions

In the general case of p lags we need to rely on the companion form of the VAR

$$\mathbf{Y}_t = \mathbf{A}_t \mathbf{Y}_{t-1} + \mathbf{e}_t$$

where

$$\mathbf{Y}_t = \begin{pmatrix} Y_t \\ Y_{t-1} \\ \vdots \\ Y_{t-p+1} \end{pmatrix} \quad \mathbf{e}_t = \begin{pmatrix} \varepsilon_t \\ 0_{n(p-1),1} \end{pmatrix}$$

and

$$\mathbf{A}_t = \begin{pmatrix} & A_t & \\ I_{n(p-1)} & & 0_{n(p-1),n} \end{pmatrix}$$

Impulse response functions

In this case the impulse response functions will be the upper left $n \times n$ sub-matrices of

$$\mathbf{I}, \mathbf{A}_{t+1}, (\mathbf{A}_{t+2}\mathbf{A}_{t+1}), ..., (\mathbf{A}_{t+k}...\mathbf{A}_{t+2}\mathbf{A}_{t+1}).$$

Second Moments

- The second moments of this process are hard to derive.
- People typically use local approximations.
- If coefficients are expected to remain constant and the VAR is stable for each t then we can approximate the dynamics of the process with a sequence of MAs.

Second Moments

Consider for simplicity again

$$Y_t = A_t Y_{t-1} + \varepsilon_t \tag{13}$$

and suppose that it is stable for each t, the eigenvalues of A_t are smaller than one in absolute value.

Then we can approximate the process at each point in time as

$$\begin{aligned} Y_t &= B_{0t}\varepsilon_t + B_{1t}\varepsilon_{t-1} + B_{2t}\varepsilon_{t-2} + \ldots \\ &= B_t(L)\varepsilon_t \end{aligned}$$

where

- $B_t(L) = B_{0t} + B_{1t}L + B_{2t}L^2 + \ldots$
- $B_{jt} = A_t^j$ are the impulse response functions under the assumption of no change in future coefficients.

Second Moments

Using (14) it is easy to derive the second moments.

The covariance matrix of Y_t is given by

$$Var(Y_t) = \sum_{j=0}^{\infty} B_{jt} \Sigma_t B'_{jt} \tag{14}$$

In the general case with p lags B_{jt} is the upper left $n \times n$ sub-matrix of $\mathbf{A}_t^j$ where $\mathbf{A}_t$ is again the VAR companion form matrix.

Identification of Structural Shocks

- So far the model is a reduced form model.
- As in time-invariant VAR we can identify the structural shocks.
- The only difference is that the shock has to be identified at each point in time to have the full history of impulse response functions.

Identification of Structural Shocks

Consider the MA representation

$$Y_t = B_t(L)\varepsilon_t$$

Let S_t the Cholesky factor of Σ_t, i.e. the unique lower triangular matrix such that $S_t S_t' = \Sigma_t$. Then

$$\begin{aligned} Y_t &= B_t(L)S_t S_t^{-1}\varepsilon_t \\ &= D_t(L)v_t \end{aligned}$$

where

- $D_t(L) = B_t(L)S_t$ are the Cholesky impulse response functions.
- $v_t = S_t^{-1}\varepsilon_t$ are the Cholesky shocks (with $E(v_t v_t') = I$).

Identification of Structural Shocks

Now let H_t be the orthogonal (i.e. $H_t H_t' = I$) identifying matrix, the matrix which imposes the identifying restrictions. Therefore

$$\begin{aligned} Y_t &= D_t(L) H_t H_t' v_t \\ &= F_t(L) u_t \end{aligned}$$

- $F_t(L) = D_t(L) H_t$ are the impulse response functions to the structural shocks.
- $u_t = H_t^{-1} v_t$ are the structural shocks.

The IRF will change over time.

Variance Decomposition

As in standard VAR the above MA representation allows us to run the variance decomposition analysis.

Let F_{kt}^{ij} be the i, j entry of F_{kt}. This denotes the effect of shock j on variable i.

The proportion of variance of variable i explained by the shock j is given by

$$\frac{\sum_{k=0}^{\infty}(F_{kt}^{ij})^2}{\sum_{i=1}^{n}\sum_{k=0}^{\infty}(F_{kt}^{ij})^2}$$

As the IRF also the variance decomposition will depend on t.

Estimation

- The easiest way to estimate the model is by using Bayesian MCMC methods, specifically the Gibbs sampler.
- Objective: we want to draws of the coefficients from the posterior distribution.
- Let ϕ t a vector containing all the ϕ_{it}, $i = 1, ..., n-1$. Let σ^T is a vector containing $\sigma_1, \sigma_2, ..., \sigma_T$ (same notation for the other coefficients).
- The posterior distribution is unknown. What is known are the conditional posteriors
  1. $p(\sigma^T | Y^T, \theta^T, \phi^T, \Omega, \Xi, \Psi)$
  2. $p(\phi^T | Y^T, \theta^T, \sigma^T, \Omega, \Xi, \Psi)$
  3. $p(\theta^T | Y^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi)$
  4. $p(\Omega | Y^T, \theta^T, \sigma^T, \phi^T, \Xi, \Psi)$
  5. $p(\Xi | Y^T, \theta^T, \sigma^T, \phi^T, \Omega, \Psi)$
  6. $p(\Psi | Y^T, \theta^T, \sigma^T, \phi^T, \Omega, \Xi)$

Estimation

- The Gibbs sampler works as follows. The coefficients are iteratively drawn from the above posteriors (1-6) conditioning on the previous draw of the remaining coefficients.
- After a burn-in period the draws converge to the draw from the joint posterior density.
- The objects of interests (IRF, variance decomposition, etc.) can be computed for each of the draws obtained.

Applications

We will see four applications:

- Cogley and Sargent (2001, NBER-MA) on unemployment-inflation dynamics.
- Primiceri (2005, ReStud) on monetary policy.
- Gali and Gambetti (2009) on the Great Moderation.
- D'Agostino, Gambetti and Giannone (forthcoming JEA) on forecasting.

Application 1: Cogley and Sargent (2001, NBER-MA) unemployment-inflation dynamics

- Very important paper. The first paper using a version of the model seen above.
- Aim: to provide evidence about the evolution of measures of the persistence of inflation, prospective long-horizon forecasts (means) of inflation and unemployment, statistics for a version of a Taylor rule.
- VAR for inflation, unemployment and the real interest rate.
- Main results: long-run mean, persistence and variance of inflation have changed. The Taylor principle was violated before Volcker (pre-1980). Monetary policy too loose.

Application 1: Cogley and Sargent (2001, NBER-MA) unemployment-inflation dynamics

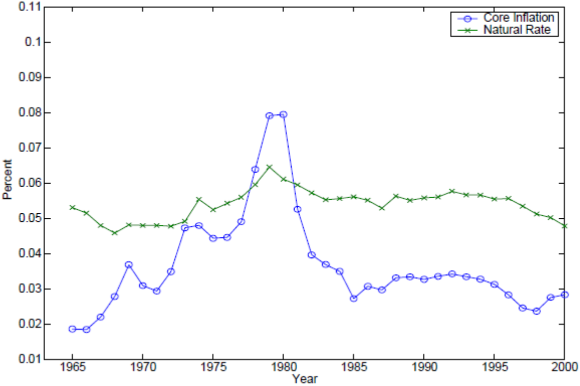


Figure 3.1: Core Inflation and the Natural Rate of Unemployment

T. Cogley and T.J. Sargent, (2002). "Evolving Post-World War II U.S. Inflation Dynamics," NBER Macroeconomics Annual 2001, Volume 16, pages 331-388.

Application 1: Cogley and Sargent (2001, NBER-MA) unemployment-inflation dynamics

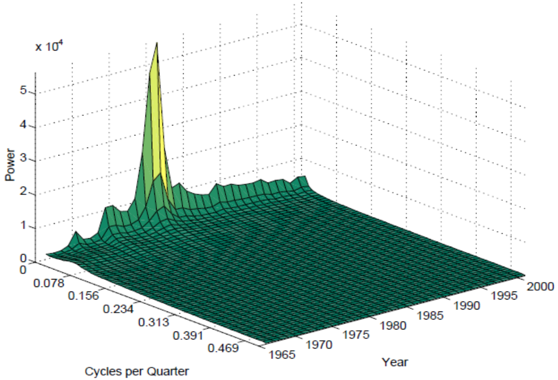


Figure 3.5: Median Posterior Spectrum for Inflation. Power is measured in basis points, the units of measurement for the variance of inflation.

T. Cogley and T.J. Sargent, (2002). "Evolving Post-World War II U.S. Inflation Dynamics," NBER Macroeconomics Annual 2001, Volume 16, pages 331-388.

Application 1: Cogley and Sargent (2001, NBER-MA) unemployment-inflation dynamics

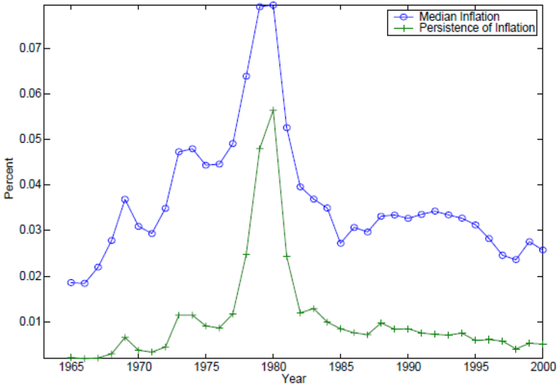


Figure 3.10: Core Inflation and Inflation Persistence

T. Cogley and T.J. Sargent, (2002). "Evolving Post-World War II U.S. Inflation Dynamics," NBER Macroeconomics Annual 2001, Volume 16, pages 331-388.

Application 1: Cogley and Sargent (2001, NBER-MA) unemployment-inflation dynamics

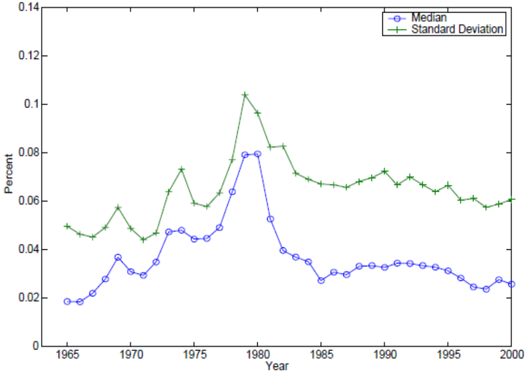


Figure 3.11: Core Inflation and the Standard Deviation of Inflation, 30 Years Ahead

T. Cogley and T.J. Sargent, (2002). "Evolving Post-World War II U.S. Inflation Dynamics," NBER Macroeconomics Annual 2001, Volume 16, pages 331-388.

Application 1: Cogley and Sargent (2001, NBER-MA) unemployment-inflation dynamics

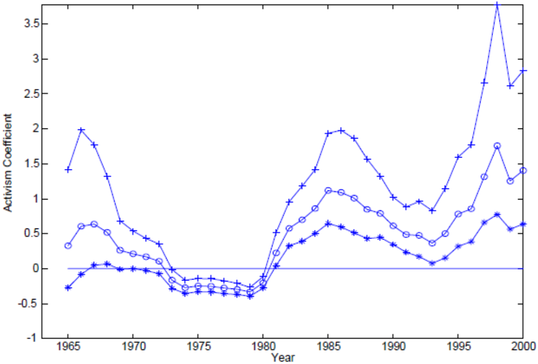


Figure 3.12: Posterior Median and Interquartile Range for the Activism Coefficient

T. Cogley and T.J. Sargent, (2002). "Evolving Post-World War II U.S. Inflation Dynamics," NBER Macroeconomics Annual 2001, Volume 16, pages 331-388.

Application 2: Primiceri (2005, ReStud) on monetary policy

- Very important paper: the first paper adding stochastic volatility.
- Aim: to study changes in the monetary policy in the US over the postwar period.
- VAR for inflation, unemployment and the interest rate.
- Main results:
  - systematic responses of the interest rate to inflation and unemployment exhibit a trend toward a more aggressive behavior,
  - this has had a negligible effect on the rest of the economy.

Application 2: Primiceri (2005, ReStud) on monetary policy

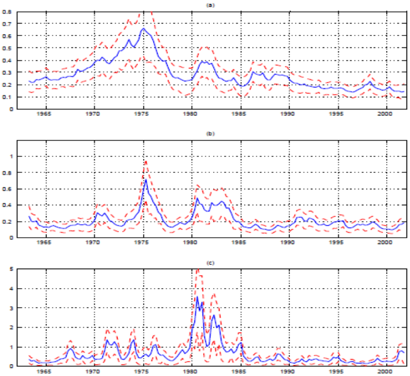


Figure 1: Posterior mean, 16th and 84th percentiles of the standard deviation of (a) residuals of the inflation equation, (b) residuals of the unemployment equation and (c) residuals of the interest rate equation or monetary policy shocks.

Source: G. Primiceri "Time Varying Structural Vector Autoregressions and Monetary Policy", The Review of Economic Studies, 72, July 2005, pp. 821-852

Application 2: Primiceri (2005, ReStud) on monetary policy

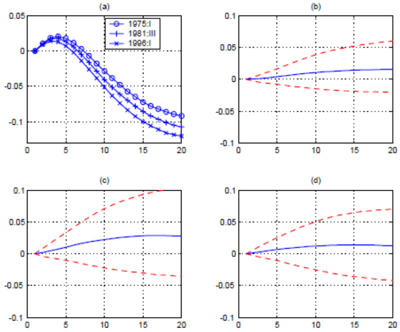


Figure 2: (a) impulse responses of inflation to monetary policy shocks in 1975:I, 1981:III and 1996:I, (b) difference between the responses in 1975:I and 1981:III with 16th and 84th percentiles, (c) difference between the responses in 1975:I and 1996:I with 16th and 84th percentiles, (d) difference between the responses in 1981:III and 1996:I with 16th and 84th percentiles.

Source: G. Primiceri "Time Varying Structural Vector Autoregressions and Monetary Policy", The Review of Economic Studies, 72, July 2005, pp. 821-852

Application 2: Primiceri (2005, ReStud) on monetary policy

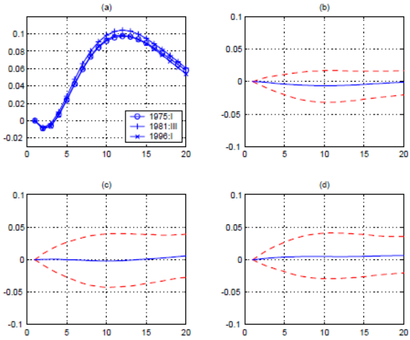


Figure 3: (a) impulse responses of unemployment to monetary policy shocks in 1975:I, 1981:III and 1996:I, (b) difference between the responses in 1975:I and 1981:III with 16th and 84th percentiles, (c) difference between the responses in 1975:I and 1996:I with 16th and 84th percentiles, (d) difference between the responses in 1981:III and 1996:I with 16th and 84th percentiles.

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Application 2: Primiceri (2005, ReStud) on monetary policy

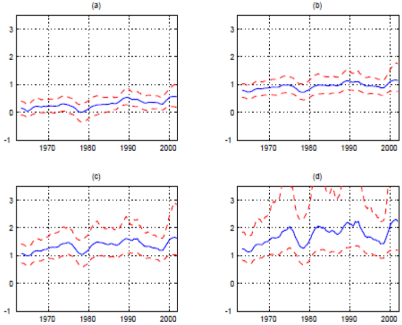


Figure 4: Interest rate response to a 1% permanent increase of inflation with 16th and 84th percentiles. (a) Simultaneous response, (b) response after 10 quarters, (c) response after 20 quarters, (d) response after 60 quarters.

Source: G. Primiceri "Time Varying Structural Vector Autoregressions and Monetary Policy", The Review of Economic Studies, 72, July 2005, pp. 821-852

Application 2: Primiceri (2005, ReStud) on monetary policy

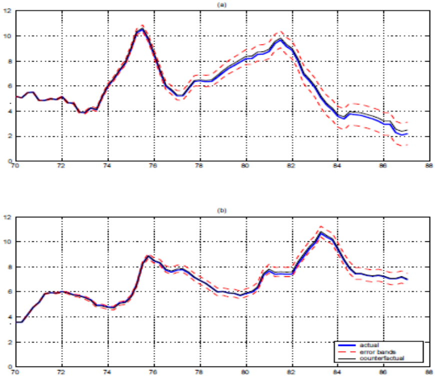


Figure 8: Counterfactual historical simulation drawing the parameters of the monetary policy rule from their 1991-1992 posterior. (a) Inflation, (b) unemployment.

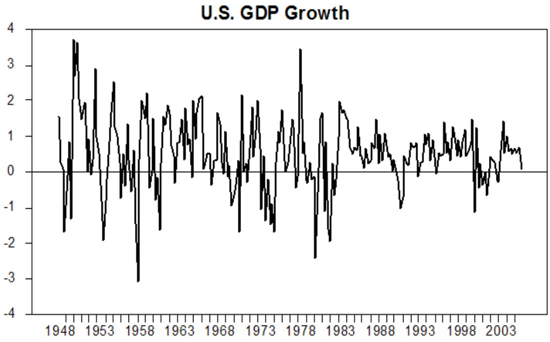
Source: G. Primiceri "Time Varying Structural Vector Autoregressions and Monetary Policy", The Review of Economic Studies, 72, July 2005, pp. 821-852

Application 3: Gali and Gambetti (2009, AEJ-Macro) on the Great Moderation

Sharp reduction in the volatility of US output growth starting from mid 80's.

- Kim and Nelson, (REStat, 99).
- McConnel and Perez-Quiros, (AER, 00).
- Blanchard and Simon, (BPEA 01).
- Stock and Watson (NBER MA 02, JEEA 05).

Application 3: Gali and Gambetti (2009, AEJ-Macro) on the Great Moderation



Application 3: Gali and Gambetti (2009, AEJ-Macro) on the Great Moderation

Table 1. The Great Moderation

	Standard Deviation		
	Pre-84	Post-84	$\frac{\text{Post-84}}{\text{Pre-84}}$
First-Difference			
GDP	1.21	0.54	0.44
Nonfarm Business Output	1.57	0.68	0.43
BP-Filter			
GDP	2.01	0.93	0.46
Nonfarm Business Output	2.61	1.21	0.46

Application 3: Gali and Gambetti (2009, AEJ-Macro) on the Great Moderation

The literature has provided three different explanations:

- Strong good luck hypothesis $\Rightarrow$ same reduction in the variance of all shocks (Ahmed, Levin and Wilson, 2002).
- Weak good luck hypothesis $\Rightarrow$ reduction of the variance of some shocks (Arias, Hansen and Ohanian, 2006, Justiniano and Primiceri, 2005).
- Structural change hypothesis $\Rightarrow$ policy or non-policy changes (monetary policy, inventories management).

Application 3: Gali and Gambetti (2009, AEJ-Macro) on the Great Moderation

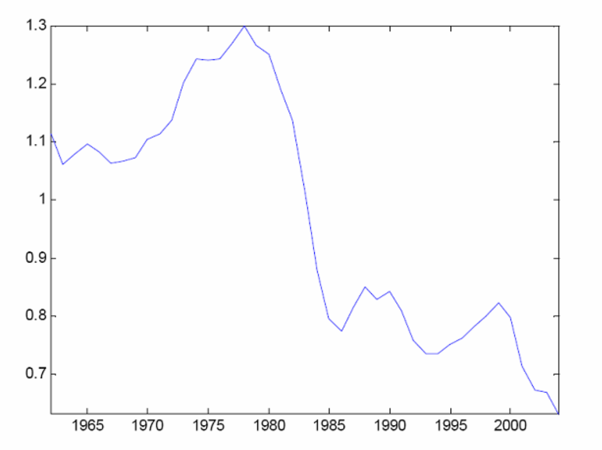
- Aim: to assess, using this class of model, the causes of this reduction in volatility.
- Idea of the paper very simple: to exploit different implications in terms of conditional and unconditional second moments of the different explanations.
  - Strong good luck hypothesis $\Rightarrow$ scaling down of all shocks variances, no change in conditional (to a specific shock) and unconditional correlations.
  - Weak good luck hypothesis $\Rightarrow$ change in the pattern of unconditional correlations, no change in conditional correlations.
  - Structural change hypothesis $\Rightarrow$ changes in both unconditional and conditional correlations.

Application 3: Gali and Gambetti (2009, AEJ-Macro) on the Great Moderation

- We estimate a TVC-VAR for labor productivity growth and hours worked for the US.
- We identify a technology shock using the assumption that is the only shock driving long run labor productivity.
- We study the second moments.

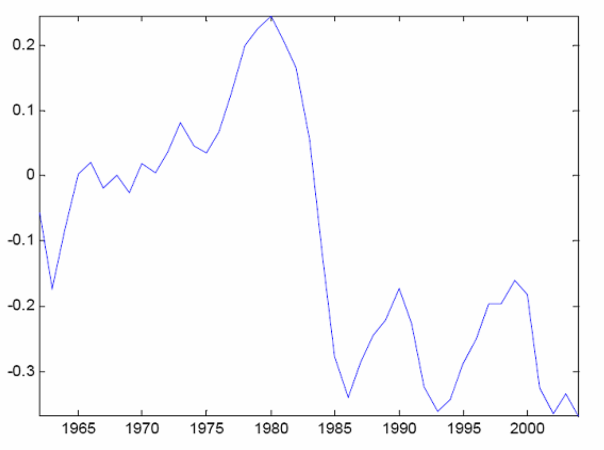
Application 3: Gali and Gambetti (2009, AEJ-Macro) on the Great Moderation

Standard deviation of output growth



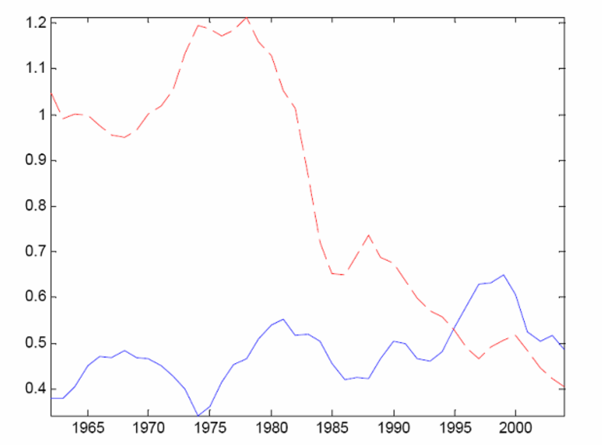
Application 3: Gali and Gambetti (2009, AEJ-Macro) on the Great Moderation

Unconditional moments: correlation of hours and labor productivity growth



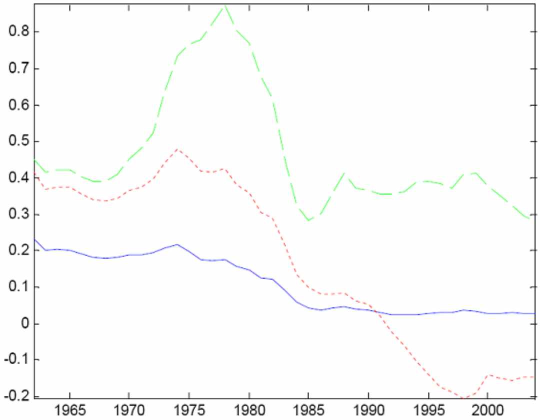
Application 3: Gali and Gambetti (2009, AEJ-Macro) on the Great Moderation

Technology and non-technology components of output growth volatility



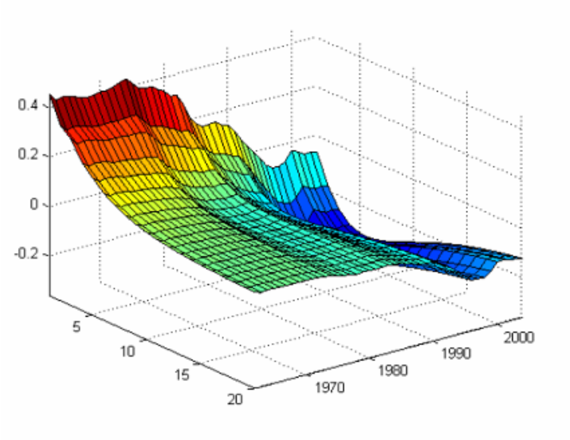
Application 3: Gali and Gambetti (2009, AEJ-Macro) on the Great Moderation

Non technology shock: variance decomposition of output growth



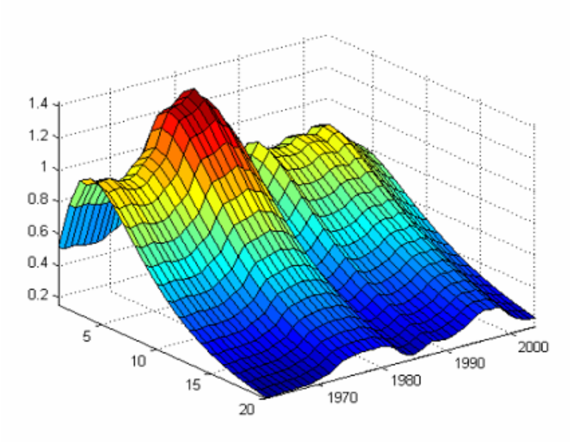
Application 3: Gali and Gambetti (2009, AEJ-Macro) on the Great Moderation

Non technology shock: labor productivity response



Application 3: Gali and Gambetti (2009, AEJ-Macro) on the Great Moderation

Non technology shock: hours response



Application 4: D'Agostino Gambetti and Giannone (2013 JAE) on forecasting

- Aim: to test the forecasting performance of the model see whether by modeling time-variations one can improve upon the forecast made with standard VAR models.
- The result is not trivial: time variations helpful but quite high number of parameter could worsen the forecasts.
- We estimate a sequence of TVC-VARs for unemployment, inflation and the federal funds rate using real time data from 1948:I-2007:IV.
- Real time out-of-sample forecast up to 12 quarters ahead.

Application 4: D'Agostino Gambetti and Giannone (2013 JAE) on forecasting

Table 1: Forecasting Accuracy over the sample 1970-2007: mean square forecast errors.

Horizon (quarters)	Variable	RW (MSFE)	AR-REC (RMSFE)	AR-ROL (RMSFE)	SV-AR (RMSFE)	TV-AR (RMSFE)	VAR-REC (RMSFE)	VAR-ROL (RMSFE)	SV-VAR (RMSFE)	TV-VAR (RMSFE)
	π	2.15	1.13	1.08	1.05	1.03	1.15	1.01	1.14	0.85
1	UR	0.15	1.00	1.08	0.98	1.00	0.99	1.18	0.96	1.04
	IR	0.87	1.12	1.23	1.01	1.04	0.99	1.09	0.94	0.99
	Avg.		1.08	1.13	1.01	1.02	1.04	1.09	1.01	0.96
	π	2.24	1.17	1.03	0.82	0.88	1.37	1.22	1.01	0.62
4	UR	1.07	1.03	1.24	0.95	1.01	0.67	0.91	0.67	0.77
	IR	3.46	1.05	1.20	0.93	0.95	0.96	1.39	0.93	0.93
	Avg.		1.08	1.16	0.90	0.95	1.00	1.17	0.87	0.77
	π	3.06	1.19	1.13	0.89	0.93	1.6	1.38	1.11	0.65
8	UR	2.39	0.95	1.14	0.88	0.95	0.45	0.63	0.42	0.61
	IR	7.54	1.05	1.18	1.11	0.92	0.99	1.44	0.85	0.89
	Avg.		1.06	1.15	0.88	0.93	1.01	1.15	0.80	0.72
	π	3.31	1.28	1.24	0.95	1.00	1.93	1.60	1.26	0.69
12	UR	3.22	0.85	1.12	0.79	0.86	0.47	0.85	0.40	0.51
	IR	10.28	1.08	1.15	0.82	0.91	1.03	1.32	0.79	0.86
	Avg.		1.07	1.17	0.85	0.92	1.14	1.26	0.82	0.72

The table reports the results relative to the forecasting accuracy using point forecasts. The variable we forecast are inflation (π_t), the unemployment rate (UR_t) and the interest rate (IR_t). The forecasting models are: RW - random walk; AR-REC - AR estimated recursively; AR-ROL - AR estimated with a rolling window; TV-VAR - time-varying VAR; VAR-REC - VAR estimated recursively; VAR-ROL - VAR estimated with a rolling window. For the random walk model we report the mean square forecast error (MSFE). For the other models we report the relative mean square forecast error (RMSFE), i.e. the ratio of the MSFE of a particular model to the MSFE of the naïve model. For each horizon it is also reported the average of the RMSFE across variables (Avg.).

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Table 2: Forecasting Accuracy over the sample 1985-2007: Mean square forecast errors.

Horizon (quarters)	Variable	RW (MSFE)	AR-REC (RMSFE)	AR-ROL (RMSFE)	SV-AR (RMSFE)	TV-AR (RMSFE)	VAR-REC (RMSFE)	VAR-ROL (RMSFE)	SV-VAR (RMSFE)	TV-VAR (RMSFE)
	π	0.93	2.61	1.19	1.23	1.21	1.29	1.35	1.28	0.98
1	UR	0.05	2.80	1.16	1.05	1.07	1.09	1.17	0.99	1.03
	IR	0.27	3.64	1.08	0.85	0.83	0.87	1.02	0.77	0.83
	Avg.		3.02	1.14	1.05	1.04	1.08	1.18	1.01	0.94
	π	0.45	5.76	1.54	1.19	1.16	2.22	2.64	1.42	0.94
4	UR	0.37	3.00	1.15	0.82	0.82	0.97	1.23	0.77	0.89
	IR	2.09	1.74	1.17	0.78	0.81	0.78	1.20	0.74	0.81
	Avg.		3.50	1.29	0.93	0.93	1.32	1.69	0.97	0.87
	π	0.57	6.39	2.09	1.10	1.08	3.03	3.11	1.55	0.72
8	UR	1.33	1.72	0.86	0.61	0.56	0.42	0.72	0.38	0.57
	IR	5.16	1.53	1.05	0.68	0.74	0.67	1.20	0.63	0.75
	Avg.		3.21	1.33	0.80	0.79	1.37	1.68	0.85	0.69
	π	0.92	4.61	2.10	0.91	0.86	3.47	2.51	1.34	0.46
12	UR	2.25	1.22	0.72	0.48	0.43	0.35	0.73	0.27	0.48
	IR	7.69	1.44	0.89	0.55	0.63	0.70	1.13	0.51	0.61
	Avg.		2.42	1.24	0.65	0.64	1.51	1.46	0.71	0.51

The table reports the results relative to the forecasting accuracy using point forecasts. The variable we forecast are inflation (π_t), the unemployment rate (UR_t) and the interest rate (IR_t). The forecasting models are: RW - random walk; AR-REC - AR estimated recursively; AR-ROL - AR estimated with a rolling window; TV-VAR - time-varying VAR; VAR-REC - VAR estimated recursively; VAR-ROL - VAR estimated with a rolling window. For the random walk model we report the mean square forecast error (MSFE). For the other models we report the relative mean square forecast error (RMSFE), i.e. the ratio of the MSFE of a particular model to the MSFE of the naïve model. For each horizon it is also reported the average of the RMSFE across variables (Avg.).