Generalised External-Instrument SVAR Analysis

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Abstract

The External-Instrument SVAR-IV (Proxy-SVAR) framework is extended to accommodate noninvertible and nonrecoverable shocks, with formal tests provided for recoverability and invertibility. The approach allows the estimation of unit-variance shocks, absolute impulse responses, and variance decompositions for recoverable shocks, and it coincides with the standard method when shocks are invertible. Simulations demonstrate that it outperforms the Internal-Instrument approach due to greater flexibility. Application to a monetary policy VAR uncovers substantial effects of monetary policy on prices, differing from previous findings, and highlights the practical relevance of partial identification in structural VAR analysis.

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1 Introduction

Since the seminal contributions of Stock (2008), Stock and Watson (2012), Mertens and Ravn (2013) and Stock and Watson (2018), SVAR-IV (or Proxy-SVARs) models have become a popular approach to structural macroeconomic analysis. The methodology relies on VAR residuals and external instruments – variables that provide a possibly noisy measure of a particular structural shock but are excluded from the VAR, hence external – to achieve identification of the shock of interest. In this paper, we refer to the application of this method as the 'Standard External-Instrument SVAR', or 'Standard SVAR-EIV'.

A severe limitation of the method is that it requires either global or partial invertibility of the structural shocks. The commonly adopted conditions for external-IV identification in VARs, provided by Mertens and Ravn (2013) and Stock and Watson (2018), require global invertibility – meaning that each and every shock in the economy can be retrieved from the vector of contemporaneous VAR innovations. A number of influential papers discuss global invertibility, including Lippi and Reichlin (1994), Giannone and Reichlin (2006), Fernandez-Villaverde et al. (2007), Forni et al. (2009), Ramey (2011), Leeper et al. (2013), Forni and Gambetti (2014), Soccorsi (2016), and Canova and Hamidi Sahneh (2017).[1]

Miranda-Agrippino and Ricco (2023) show that the Standard SVAR-EIV procedure is also valid under partial invertibility, provided that the validity conditions for the instrument are suitably reinforced. Partial invertibility is a much less demanding property since it only requires that one or some of the structural shocks be a linear combination of the VAR residuals. In this work we adopt a partial-invertibility perspective: when using the term invertibility we mean partial invertibility, unless otherwise stated.

A long literature in empirical macro has pointed out that, while less demanding than global invertibility, partial invertibility is nonetheless a restrictive property that can fail in relatively common settings. For example, it may not be satisfied in the presence of news technology shocks (Forni et al., 2014), forward guidance (Ramey, 2016), or fiscal foresight (Mertens and Ravn, 2010, Ramey, 2011, Leeper et al., 2013), and necessarily fails for the so-called noise shocks (Blanchard et al., 2013, Forni et al., 2017).

Internalising the proxy by including it in the VAR, in an 'Internal-Instrument SVAR' (SVAR-IIV), has long been a common approach (see Ramey, 2016). It consists of a standard Cholesky identification in which the instrument enters the VAR as the first variable, and the IRFs to the first shock are those of interest. Plagborg-Møller and Wolf

[1] A helpful summary of these contributions is provided by Kilian and Lütkepohl (2017).

(2021) show that, with this procedure, the impulse response functions are consistently estimated even in the absence of invertibility. Other approaches that are valid under non-invertibility are the LP-IV (discussed in Stock and Watson, 2018), based on Jordà (2005)'s Local Projections, and the VARX, where the proxy plays the role of an exogenous variable (see Paul, 2020), provided that sufficient lags of the proxy are included in the regression, as suggested in Noh (2017).

However, such solutions come at a price. First, in empirical applications, the available instruments often have a short time span, as is the case for the existing instruments for monetary policy, tax, or oil shocks (Gertler and Karadi, 2015, Mertens and Ravn, 2013, Känzig, 2021). In the External Instrument approach (SVAR-EIV), the time span of the VAR can differ from that of the instrument. This is an important flexibility that is lost in the alternative methods mentioned above. Second, internalising the proxy entails that the proxy is projected onto the lags of the variables; a treatment that may not be necessary or may require a different number of lags and possibly different variables. Third, unlike in a SVAR-IIV, an external IV approach allows one to fix the model, estimate it only once, and compare results obtained from different instruments. Finally, LP-IV and SVAR-IIV produces estimates that are more volatile than those of SVAR-EIV, as documented in Li et al. (2022).

The main methodological contribution of this paper is to generalise the external-instrument SVAR approach to the case of non-invertibility and non-recoverabilty. We refer to our proposed method as the Generalised External-Instrument SVAR (Generalised SVAR-EIV). We show that all types of analysis that can be carried out with LP-IV and SVAR-IIV can also be undertaken with our approach, under similar validity conditions. Compared with the alternative approaches – SVAR-IIV, LP-IV, and VARX – our method retains the full flexibility of the external IV framework. Furthermore, according to our simulations, it performs well in terms of Mean Square Error in comparison to the SVAR-IIV method.

The basic idea is very simple. Instead of regressing the VAR residuals on the current proxy only, we regress the VAR residuals on the current proxy and its lags. The impulse response functions are then estimated by combining the coefficients of this regression with the reduced-form impulse response functions obtained from the VAR. Moreover, we show how to implement, within our setting, the upper and lower bounds for the 'absolute' IRFs and the variance contributions proposed by Plagborg-Møller and Wolf (2022).

While the relative impulse-response functions – i.e. dynamic responses normalised by the response of a variable of interest – can be estimated independently of whether the shock of interest is invertible or recoverable, the absolute response functions and the structural shock itself cannot be estimated unless the shock is recoverable. Recoverability (Chahrour and Jurado, 2021) is less demanding than invertibility. A shock is recoverable if it is a linear combination of the present, past, and future values of the VAR variables, or, equivalently, a linear combination of the present and future values of the VAR residuals.

This paper offers a procedure to test for recoverability and to estimate the structural shock of interest when it is recoverable. It consists in first regressing the instrument on the present and future values of the VAR residuals. If the shock is recoverable, the fitted value is a consistent estimate of the shock and therefore must be serially uncorrelated. It is hence possible to test for recoverability by testing for serial uncorrelation of this fitted value. Moreover, we show how to estimate the corresponding absolute impulse-response functions. Having an estimate of the shock, historical decomposition can be performed as usual. Standard variance decomposition is downward biased at short horizons when the model is not globally invertible. However, an unbiased variance decomposition can be obtained by integrating the spectral densities over specific frequency bands, as suggested in Forni et al. (2019).

The regression of the proxy on the present and future values of the VAR residuals also allows to test for invertibility. Under invertibility, the shock is a linear combination of current VAR residuals only. Hence, a simple F-test for the null of zero coefficients on the future residuals indicates whether the shock is invertible or not. Both the recoverability test and the invertibility test provide a valuable guidance for the choice of the VAR variables. If the null of invertibility is not rejected, our proposed procedure collapses to the Standard SVAR-EIV procedure.

A few Monte Carlo exercises validate our proposed estimation and testing method in small samples. Our main simulations are based on the DSGE model of Justiniano et al. (2010) and focus on the monetary policy shock. We show that (i) invertibility is a serious problem for the standard SVAR-EIV approach; (ii) our procedure can detect and address this problem; and (iii) the Generalised SVAR-EIV performs well compared with the SVAR-IIV method.

In the empirical application, we study the effects of US monetary policy using the proxy of Gertler and Karadi (2015). This instrument is based on surprises in federal funds futures with three-month maturity, so that it is likely to capture both conventional monetary policy shocks and shocks to forward guidance about the path of the short-term rate. This news component might induce noninvertibility, providing a strong motivation for our analysis.

Our main findings are as follows. First, in standard VAR specifications, the monetary policy shock turns out to be noninvertible according to our test. This is true even for specifications including the excess bond premium. Hence, the results obtained so far with the standard SVAR-EIV approach should be interpreted with caution.

Second, when using our External-Instrument method, a contractionary shock reduces inflation and output consistently across different VAR specifications, independently of the inclusion of financial variables. By contrast, when using the standard approach, results vary dramatically across VAR specifications, with large price and real activity puzzles emerging when financial variables are not included.

Finally, the monetary policy shock is recoverable, allowing us to perform variance decomposition. The variance decomposition shows that the contribution of the monetary policy shock to both output and prices is sizeable and larger than previously reported. This is a notable result, as it suggests that monetary policy can be effective in controlling prices.

The remainder of the paper is organised as follows. Section 2 presents our structural MA model and representation results. In Section 3, we present our identification results and the proposed estimation and testing procedure, which is summarised in Section 3.6. Section 4 reports our Monte Carlo exercises. Section 5 presents our empirical application. The final section concludes. The Online Appendix provides additional results, proofs of all propositions, illustrative examples, further simulations, and robustness checks for the empirical analysis.

2 Representation theory

In this section we introduce our theoretical framework and study the relation between the structural representation and the VAR representation when the structural shock of interest is not recoverable, recoverable but not invertible, and invertible.

2.1 The model

Let us start from our assumptions about the structural macroeconomic model and the VAR representation.

Assumption 1. (Structural MA representation) *The observable macroeconomic variables in the* n*-dimensional vector* y_t*, possibly after suitable transformations, have the representation*

$$y_t = B(L)u_t, \tag{1}$$

where (i) $B(L) = B_0 + B_1 L + B_2 L^2 + \cdots$ *is an* $n \times q$ *matrix of rational impulse-response functions in the lag operator* L*; (ii)* $n \leq q$ *and* $B(z)$*,* z *being a complex variable, has rank* n *on the unit circle; (iii)* u_t *is a* q*-dimensional white noise vector including the structural shocks, whose variance covariance matrix is* I_q*.*

The above model is sometimes referred to as the Slutsky-Frisch representation of the macro economy. It can be thought of as resulting from the linearisation of a DSGE model and can easily be derived from its state-space representation. Notice that we do not assume that the number of shocks is equal to the number of variables, so that the matrix $B(L)$ is not necessarily square. However, we assume that the number of variables cannot be larger than the number of shocks ($n \leq q$). This assumption can be justified by recognising that the variables are observed with error and such errors are allowed to enter the vector u_t together with the structural shocks.

In this paper we are concerned with identification of a single shock of interest, u_{it}. To highlight the shock of interest and the corresponding response functions, it is convenient to re-write (1) in the form

$$y_t = b_i(L)u_{it} + \tilde{B}(L)\tilde{u}_t, \tag{2}$$

where $b_i(L) = b_{i0} + b_{i1}L + b_{i2}L^2 + \cdots$ is the i-th column of $B(L)$, $\tilde{B}(L)$ includes the other columns of $B(L)$ and $\tilde{u}_t = (u_{1t} \ \cdots \ u_{i-1,t} \ u_{i+1,t} \ \cdots \ u_{qt})'$.

Being stationary and purely nondeterministic by (1), y_t always admits the Wold representation

$$y_t = C(L)\varepsilon_t \tag{3}$$

where $C(L) = C_0 + C_1 L + C_2 L^2 + \cdots$ and ε_t is an n-dimensional vector white-noise process with covariance matrix Σ_ε. Notice that this representation is square, since $C(L)$ is $n \times n$.

By Assumption 1 (ii), $B(z)$ has rank n on the unit circle, so that the spectral density matrix of y_t, i.e. $S^y(\theta) = \frac{1}{2\pi}B(e^{-j\theta})B'(e^{j\theta}) = \frac{1}{2\pi}C(e^{-j\theta})\Sigma_\varepsilon C'(e^{j\theta})$, for j the imaginary unit, is nonsingular everywhere in $[-\pi, \pi)$. This implies that $C(z)$ vanishes only outside the unit disk in the complex plane, so that the Wold representation is invertible and y_t has the VAR representation

$$A(L)y_t = \varepsilon_t \tag{4}$$

where $A(L) = C(L)^{-1}$. The existence of a VAR representation is necessary for SVAR analysis.

It is worth to observe that while our discussion is centred on a model for a stationary vector y_t, our results are easily generalised to the case of a nonstationary $I(1)$ process Y_t. Assume that $y_t = (1-L)Y_t$. From equation (4) we obtain $A(L)(1-L)Y_t = \Theta(L)Y_t = \varepsilon_t$, so that there exists a VAR representation for the levels Y_t with the same innovation ε_t. Consequently, all our results – which concern the relationship between u_t and ε_t – remain valid in a nonstationary environment.

2.2 Relationship between structural and reduced form

The relation linking the structural representation (1), on the one hand, and the Wold representation (3), with the associated VAR representation (4), on the other hand, has been studied in detail for the case $n = q$ in Lippi and Reichlin (1994). In this case, all of the shocks in u_t are *recoverable*, i.e. they are linear combinations of the present, past and future values of the variables in y_t. The concept of recoverability is introduced and studied in detail in Chahrour and Jurado (2021). Remark 1 in the Appendix shows that u_t is (globally) recoverable if and only if $n = q$. The important case $n > q$ has not been exhaustively studied, but for the case of *partial invertibility*, i.e. the shock of interest u_{it} is a linear combination of the present and the past of the variables in y_t (see Forni et al. 2019 and Miranda-Agrippino and Ricco 2021). Here we provide general results for the case $n \geq q$; such results are used in the next section as the basis for our proposed identification method. The case $n > q$ is discussed in depth in Forni et al. (2020).

2.2.1 Two basic results

Let us anticipate here the representation results that we shall use in the next subsection to identify the impulse-response functions and, when possible, the shock of interest. From equations (1) and (4) we see that

$$\varepsilon_t = A(L)y_t = A(L)B(L)u_t = Q(L)u_t. \tag{5}$$

Hence the VAR residuals in ε_t are linear combinations of the current and lagged structural shocks u_t. Pre-multiplying both sides of (5) by $C(L) = A(L)^{-1}$ and equating coefficients we get $B(L) = C(L)Q(L)$. In particular, the IRFs of interest are given by

$$b_i(L) = C(L)q_i(L), \tag{6}$$

where $q_i(L)$ is the i-th column of the matrix $Q(L)$. Since $C(L)$ is unique, we see from the above equation that identification of $b_i(L)$ can be obtained by identifying $q_i(L)$, i.e. the coefficients linking ε_t to the present and past values of u_{it}. This is what we do in the next section by exploiting the instrument.

Equation (5) expresses the VAR residuals in ε_t as linear combinations of the structural shocks. But in order to identify and estimate the structural shocks we need the inverse relation, i.e. the one linking u_t to ε_t. Hence we consider the linear projection of u_t onto the present, past and future values of the Wold shocks ε_t. We have

$$u_t = D'(F)\varepsilon_t + s_t, \tag{7}$$

where s_t is the residual of the projection, $F = L^{-1}$ is the forward operator such that $F\varepsilon_t = \varepsilon_{t+1}$ and $D'(F)$ is a $q \times n$ matrix of linear filters. We show below that the past of ε_t cannot enter the above relation, so that $D(F)$ is one-sided in the positive powers of F. In particular, for the shock of interest we have

$$u_{it} = d_i(F)\varepsilon_t + s_{it}.$$

In the special case in which u_{it} is recoverable, the residual of the above projection vanishes, $s_{it} = 0$, so that u_{it} is an exact linear combination of the present and future values of the ε's. The above equation provides the basis for the method proposed in the next section to identify and estimate the shock of interest when it is recoverable; moreover, it provides the basis for our proposed invertibility and recoverability tests.

2.2.2 Formal definitions and statements

Let us now present our formal statements. All proofs are in the Appendix.

Proposition 1. (Impulse-response functions)
(i) $D(F)$ defined in (7) is one-sided in the non-negative powers of F.
(ii) $Q(L)$ defined in (5) is linked to the projection coefficients in (7) by the relations

$$Q(L) = \Sigma_\varepsilon D(L); \qquad D(L) = \Sigma_\varepsilon^{-1} Q(L). \tag{8}$$

(iii) The structural impulse-response functions are linked to the Wold impulse response functions by the relation

$$B(L) = C(L)Q(L) = C(L)\Sigma_\varepsilon D(L).$$

In particular, for the impulse-response functions of interest the relation is

$$b_i(L) = C(L)q_i(L) = C(L)\Sigma_\varepsilon d_i(L), \tag{9}$$

where $q_i(L)$ and $d_i(L)$ are the i-th columns of $Q(L)$ and $D(L)$, respectively.

Proposition 1 establishes a mapping between the Wold impulse-response functions $C(L)$ and the structural impulse-response functions $B(L)$ which holds true independently of invertibility or recoverability of the structural shocks.

Let us now introduce our definition of recoverability and invertibility.

Definition 1. (Recoverability) *Let $\mathcal{H}^y$ be the closed linear space spanned by present, past and future values of y_t: $\mathcal{H}^y = \overline{\text{span}}(y_{j,t-k}, j = 1, \ldots, n, k \in \mathbb{Z})$. We say that the structural shock u_{it} is recoverable with respect to y_t if and only if $u_{it} \in \mathcal{H}^y$. We say that u_t is (globally) recoverable with respect to y_t if and only if all of the structural shocks are recoverable.*

If u_{it} is recoverable, it may be the case that it fulfils a more demanding property, that is fundamentalness (invertibility). In the literature, fundamentalness is often regarded as a synonymous of invertibility. Indeed, fundamentalness is somewhat weaker than invertibility. For instance, if $y_t = (1-L)u_t$, then u_t is fundamental but not invertible. In our setting however we are assuming that y_t has a VAR representation, so that fundamentalness and invertibility coincide.

Definition 2. (Fundamentalness/invertibility) *Let $\mathcal{H}_t^-$ be the closed linear space spanned by present and past values of y_t: $\mathcal{H}_t^- = \overline{\text{span}}(y_{j,t-k}, j = 1, \ldots, n, k \geq 0)$. We say that the structural shock u_{it} is fundamental with respect to y_t if and only if $u_{it} \in \mathcal{H}_t^-$. We say that u_t is fundamental with respect to y_t if and only if all structural shocks are fundamental with respect to y_t.*

From the definition of fundamentalness we see that if u_{it} is fundamental for y_t, then u_{it} is recoverable, whereas the converse is not necessarily true.

Proposition 2. (Structural shocks and VAR residuals)
(i) If u_{it} is recoverable with respect to y_t,

$$u_{it} = d_i'(F)\varepsilon_t = q_i'(F)\Sigma_\varepsilon^{-1}\varepsilon_t, \tag{10}$$

where $d_i(F) = d_{i0} + d_{i1}F + d_{i2}F^2 + \cdots$ is the i-th column of $D(F)$ and $q_i(F) = q_{i0} + q_{i1}F + q_{i2}F^2 + \cdots$ is the i-th column of $Q(F)$. Moreover, $d_i'(F)\Sigma_\varepsilon d_i(L) = q_i'(F)\Sigma_\varepsilon^{-1}q_i(L) = 1$.
(ii) If u_{it} is fundamental for y_t, then $d_i(F) = d_{i0} = d_i$ and $q_i(F) = q_{i0} = q_i$, so that

$$u_{it} = d_i'\varepsilon_t = q_i'\Sigma_\varepsilon^{-1}\varepsilon_t \tag{11}$$

and

$$b_i(L) = C(L)q_i = C(L)\Sigma_\varepsilon d_i, \tag{12}$$

where $d_i'\Sigma_\varepsilon d_i = q_i'\Sigma_\varepsilon^{-1}q_i = 1$.
(iii) If both u_{it} and u_{jt} are recoverable, then $d_i(L)'\Sigma_\varepsilon d_j(L) = q_i(L)'\Sigma_\varepsilon^{-1}q_j(L) = 0$.

Proposition 2 implies that, if the structural shock is recoverable, then it is a linear combination of current and future values of the VAR residuals, with the polynomials being such that the spectral density is one at all frequencies; if the structural shock is fundamental, then it is a linear combination of current VAR residuals, and its impulse-response functions are linear combination of the Wold impulse-response functions.

A few remarks are reported in the Appendix. Remarks 1 and 2 show that the above results reduce to a basic result in Lippi and Reichlin (1994) in the special case $q = n$. Remarks 3 and 5 introduce theoretical measures of recoverability and fundamentalness.

2.2.3 A general SVAR representation

Let u_t^f be the (possibly empty) sub-vector of the fundamental structural shocks, u_t^r of the recoverable (but nonfundamental) shocks, and u_t^n of the nonrecoverable ones. Moreover, let $Q^h(L)u_t^h$, for $h = f, r, n$, be the projection of ε_t onto u_{t-k}^h, with $k \geq 0$, and $D^h(F)\varepsilon_t$, for $h = f, r, n$, be the projection of u_t^h onto ε_{t+k}, with $k \geq 0$. The following result, which is an immediate consequence of Propositions 1 and 2, provides a general structural VAR representation of y_t.

Proposition 3. (General SVAR Representation) *Any vector process y_t satisfying Assumption 1 can be represented as*

$$\begin{aligned} A(L)y_t &= Q^f u_t^f + Q^r(L)u_t^r + Q^n(L)u_t^n \\ &= \Sigma_\varepsilon D^f u_t^f + \Sigma_\varepsilon D^r(L)u_t^r + \Sigma_\varepsilon D^n(L)u_t^n. \end{aligned} \tag{13}$$

Moreover, the following properties hold:

(i) D^f and Q^f are such that $D^{f\prime}\Sigma_\varepsilon D^f = Q^{f\prime}\Sigma_\varepsilon^{-1}Q^f = I$;

(ii) $D^r(L)$ and $Q^r(L)$ are such that $D^{r\prime}(F)\Sigma_\varepsilon D^r(L) = Q^{r\prime}(F)\Sigma_\varepsilon^{-1}Q^r(L) = I$.

Since $A(L)$ and Σ_ε are unique, equation (13) shows that the impulse response functions $B^h(L) = A(L)^{-1}Q^h(L)$, $h = f, r, n$, are identified whenever $Q^h(L)$ (or $D^h(L)$) is identified; moreover, we see from (ii) that, for $h = f, r$, $Q^h(L)$ (or $D^h(L)$) also identifies the shock $u_t^h = Q^h(F)\Sigma_\varepsilon^{-1}A(L)y_t$.

3 The Generalised External Instrument Approach

In this section we present our proposed identification and estimation procedure. First, we introduce the instrument. Next, we present our estimators for the relative IRFs, and, in the recoverability case, the absolute IRFs and the shock. Then we discuss variance decomposition and present our proposed recoverability and invertibility tests. In the last subsection, we summarise our proposed estimation and testing procedure.

3.1 The instrument

Let us begin by introducing the instrument.

Assumption 2. (The Instrument) *The researcher can observe the (scalar) instrument $\tilde{z}_t$, which fulfills the linear projection equation*

$$\tilde{z}_t = \beta(L)\tilde{z}_{t-1} + \mu'(L)x_{t-1} + \alpha u_{it} + w_t, \tag{14}$$

where $\alpha u_{it} + w_t = z_t$ is the residual of the projection of $\tilde{z}_t$ on $\tilde{z}_{t-k}, x_{t-k}$, $k > 0$, and w_t is an error orthogonal to u_{it} and ε_{t+k}, $k \geq 0$ (for simplicity we omit the constant term).

The above condition can be re-written in terms of conditional covariances as follows.

(i) $\text{cov}(\tilde{z}_t, u_{it}|\tilde{z}_{t-k}, x_{t-k}, k > 0) = \alpha \neq 0$ (relevance)

(ii) $\text{cov}(\tilde{z}_t, \varepsilon_t|u_{it}, \tilde{z}_{t-k}, x_{t-k}, k > 0) = 0$ (contemporaneous exogeneity)

(iii) $\text{cov}(\tilde{z}_t, \varepsilon_{t+h}|u_{it}, \tilde{z}_{t-k}, x_{t-k}, k > 0) = 0$ for $h > 0$ (dynamic exogeneity)

This condition is somewhat milder than the LP-IV$^\perp$ condition of Stock and Watson (2018), i.e. lead-lag exogeneity, conditional on the past history of $\tilde{z}_t$ and x_t. For, the latter condition requires that w_t in equation (14) is orthogonal to the present, past and future values of u_t, whereas we only require orthogonality with the present and future values of the Wold residuals ε_t.

In the following subsections, to exploit properly the above conditions, in place of $\tilde{z}_t$, we shall consider the residual of the projection of $\tilde{z}_t$ onto the past history of $\tilde{z}_t$ and x_t, i.e.

$$z_t = \alpha u_{it} + w_t. \tag{15}$$

Correspondingly, as a first step of our proposed procedure, we 'clean' $\tilde{z}_t$ by estimating (14) and then use the residual as our instrument. This of course is not needed if the instrument is already serially uncorrelated and cannot be predicted by past x's. Notice that in equation (14) the conditioning vector variable x_t is not necessarily equal to the VAR vector variable y_t. As a suggestion for practitioners, x_t might include the principal components of a large dataset or a few forward-looking financial variables, provided that they Granger cause $\tilde{z}_t$.

3.2 Impulse response functions and shocks

In this subsection, we present our main identification results. First we consider the case of nonrecoverable shocks. In this case the shock and the impulse response functions corresponding to a unit-variance shock (the absolute IRFs) are not identified, but the "relative" response functions are identified, meaning that the IRFs are identified up to a multiplicative constant. Moreover, we can estimate upper and lower bounds for the absolute IRFs. Then we turn to the case of a recoverable shock. In this case, both the shock and the absolute IRFs are identified. Finally, we consider the case of a fundamental shock. In this case the IRFs and the shock are identified by the standard External-Instrument SVAR formulas. The proofs for all propositions are reported in the Appendix.

3.2.1 Nonrecoverable shocks

Let us consider the projection of ε_t onto the current and past values of the proxy:

$$\varepsilon_t = \psi(L)z_t + e_t. \tag{16}$$

The following result holds.

Proposition 4. (Relative IRFs) *The coefficients of the projection (16) are related to $q_i(L)$ appearing in (6) by the equation*

$$\psi(L)\sigma_z^2 = q_i(L)\alpha. \tag{17}$$

Hence the impulse-response functions fulfil the relation

$$b_i(L)\alpha/\sigma_z^2 = C(L)\psi(L) = \gamma(L). \tag{18}$$

A consequence of Proposition 4 is that a possible strategy to estimate the relative IRFs is to perform the OLS regression of ε_t onto the present and past values of z_t until a maximum lag r to get an estimate of $\psi(L) = q_i'(L)\alpha/\sigma_z^2$, say $\widehat{\psi}(L)$, and estimate $b_i(L)\alpha$ as

$$b_i\widehat{(L)\alpha/}\sigma_z^2 = \widehat{C}(L)\widehat{\psi}(L) = \widehat{\gamma}(L). \tag{19}$$

Unfortunately, in the general case α cannot be found so that we cannot estimate the impulse response functions corresponding to a unit-variance shock. However, we can normalise the IRFs in equation (19) by dividing by the effect on a pre-specified variable at a given lag, as suggested in Stock and Watson (2018). For instance we can normalise the impulse-response functions by dividing by the impact effect on the first variable:

$$\widehat{\frac{b_i(L)}{b_{i1}(0)}} = \frac{\widehat{\gamma}(L)}{\widehat{\gamma}_1(0)}, \tag{20}$$

where $\widehat{\gamma}_1(L)$ is the first entry of $\gamma(L)$. The resulting IRFs are then the ones corresponding to a shock having impact effect 1 on the first variable.

Plagborg-Møller and Wolf (2022) show that, while it is impossible to estimate the absolute response functions, it is nonetheless possible to compute upper and lower bounds for the parameter α; in the Appendix we derive, within our setting, the upper and lower bounds for the absolute response functions $b_i(L)$.

3.2.2 Recoverable but nonfundamental shocks

If the shock is recoverable, we can identify and estimate the absolute IRFs and the shock itself. Let us begin with the absolute IRFs. The following result holds.

Proposition 5. (Absolute IRFs) *If u_{it} is recoverable, its (absolute) impulse response functions are given by the equation*

$$b_i(L) = \gamma(L)\sigma_z^2/\alpha = \frac{C(L)\psi(L)}{\sqrt{\sum_{k=0}^{\infty} \psi_k' \Sigma_\varepsilon^{-1} \psi_k}}.$$

From the above proposition we see that $b_i(L)$ can be estimated as

$$\hat{b}_i(L) = \frac{\widehat{C}(L)\widehat{\psi}(L)}{\sqrt{\sum_{k=0}^{r} \widehat{\psi}_k' \widehat{\Sigma}_\varepsilon^{-1} \widehat{\psi}_k}}. \tag{21}$$

Coming to the shock, let us consider the projection of z_t onto the space spanned by the present and the future of the VAR residuals:

$$z_t = \delta'(F)\varepsilon_t + v_t. \tag{22}$$

The following proposition holds.

Proposition 6. (The structural shock) *If u_{it} is recoverable, then*

$$u_{it} = d_i(F)\varepsilon_t = \frac{\delta'(F)\varepsilon_t}{\sqrt{\sum_{k=0}^{\infty} \delta_k' \Sigma_\varepsilon \delta_k}}.$$

From the above proposition we see that, if the shock is recoverable, it can be estimated as

$$\hat{u}_{it} = \frac{\hat{\delta}'(F)\hat{\varepsilon}_t}{\sqrt{\sum_{k=0}^{r} \hat{\delta}_k' \widehat{\Sigma}_\varepsilon \hat{\delta}_k}}. \tag{23}$$

Having an estimate of the shock, we can perform historical and variance decomposition as explained in Section 3.3 below.

3.2.3 Fundamental shocks

If we have fundamentalness, the following result holds.

Proposition 7. (IRFs and shocks under fundamentalness)
(i) Let us consider the projection equation $\varepsilon_t = \psi z_t + e_t$*. If* u_{it} *is fundamental, then*

$$b_i(L) = \frac{C(L)\psi}{\sqrt{\psi' \widehat{\Sigma}_\varepsilon^{-1} \psi}}.$$

(ii) Let us consider the projection equation $z_t = \delta' \varepsilon_t + e_t$*. If* u_{it} *is fundamental, then*

$$u_{it} = \frac{\delta' \varepsilon_t}{\sqrt{\delta' \Sigma_\varepsilon \delta}}.$$

From Proposition 7 (i) we see that, if the shock is fundamental, we can estimate (16) without including the lags of z_t, i.e. we can estimate by OLS the projection $\varepsilon_t = \psi z_t + e_t$ to get an estimate of ψ. The impulse-response functions can then be estimated as

$$\hat{b}_i(L) = \frac{\widehat{C}(L)\widehat{\psi}}{\sqrt{\widehat{\psi}' \widehat{\Sigma}_\varepsilon^{-1} \widehat{\psi}}}. \tag{24}$$

Notice that the above procedure is nothing else that the standard estimation procedure, which is usually applied without testing (see below for our proposed fundamentalness test).

Turning to estimation of the shock, by Proposition 7 (ii) we can estimate (22) including only the current ε_t among the regressors, in order to estimate δ; having $\hat{\delta}$, the unit variance shock can be estimated as

$$\hat{u}_{it} = \frac{\hat{\delta}' \hat{\varepsilon}_t}{\sqrt{\hat{\delta}' \widehat{\Sigma}_\varepsilon \hat{\delta}}}. \tag{25}$$

A few considerations about consistency of the estimators above are provided in Remark 6 in the Appendix. Remarks 7 and 8 discuss briefly alternative estimators for the IRFs and the shock, respectively, and explain why we prefer the estimators above.

3.3 Historical and variance decomposition

In this subsection we discuss historical decomposition and variance decomposition. We have shown that, if the shock of interest is recoverable, it can be estimated. Having an estimate of the shock and the corresponding impulse-response functions, historical decomposition can be performed in the standard way.

Variance decomposition is more problematic. The standard forecast error variance decomposition (FVD) can be computed only for globally invertible models. This is because the forecast error depends on all structural shocks and the corresponding impulse response functions. Having an estimate of the IRFs of u_{it} we can of course compute the numerator of the ratio, but we cannot estimate the denominator without estimating the whole structural model.

Plagborg-Møller and Wolf (2022) replace this denominator —i.e. the forecast error variance of the structural model— with the forecast error variance based on present and past values of y_t, which can be estimated, and name this ratio FVR. However, FVR might underestimate the variance contribution of the shock of interest at short horizons (See the online Appendix C.3). We therefore suggest to use the variance decomposition given by the integral of the spectral density over suitable frequency bands Forni et al. (2019).

Let $b_{ih}(L)$ be the h-th element of $b_i(L)$. The total variance of the component of y_{ht} which is attributable to u_{it} can be computed as $\int_0^\pi b_{ih}(e^{-j\theta})b_{ih}(e^{j\theta})d\theta/\pi$, where j denotes the imaginary unit. We can also compute the variance on a specific frequency band $[\theta_1\ \theta_2)$. If we are interested for instance in the variance of waves of business cycle periodicity, say between 8 and 32 quarters, the corresponding angular frequencies (with quarterly data) are $\theta_1 = \pi/16$ and $\theta_2 = \pi/4$ and the corresponding variance is $\int_{\pi/16}^{\pi/4} b_{ih}(e^{-j\theta})b_{ih}(e^{j\theta})d\theta/\pi$.

Our suggested measure is

$$c_h(\theta_1, \theta_2) = \frac{\int_{\theta_1}^{\theta_2} b_{ih}(e^{-j\theta})b_{ih}(e^{j\theta})d\theta}{\int_{\theta_1}^{\theta_2} S_h(\theta)d\theta}. \tag{26}$$

where $S_h(\theta) = C_h(e^{-j\theta})\Sigma_\varepsilon C_h(e^{j\theta})'$, $C_h(L)$ is the h-th row of the matrix $C(L)$, so that the integral at the denominator is the total variance of the series within the relevant band.

In the case of nonrecoverability only $\gamma(L)\sigma_z^2 = \alpha b_i(L)$ can be estimated according to Equation (19). Thus only upper and lower bounds for the variance decomposition can be obtained. Details about these bounds are discussed in the Appendix.

3.4 Testing for recoverability and fundamentalness

In this subsection we propose a test for recoverability and a test for fundamentalness.

3.4.1 Recoverability test

In the proof of Proposition 6 we have seen that, if u_{it} is recoverable, then the projection in (22), $\delta'(F)\varepsilon_t$, is equal to αu_{it} and therefore is a white noise process. By contrast, if recoverability does not hold, $s_{it} \neq 0$ and the projection is not equal to αu_{it}. Being a Moving Average of present and future VAR residuals, the projection will in general be autocorrelated.

Hence, to check whether the shock is recoverable or not, following a suggestion of Plagborg-Møller and Wolf (2022), we propose to test for zero serial correlation of the projection $\delta(F)\varepsilon_t$. Precisely, we propose to perform the OLS regression of z_t onto the present and future values of $\hat{\varepsilon}_t$, until a maximum lead r, to get an estimate of $\delta(F)$. Then apply the Ljung-Box Q-test to the estimated projection $\hat{\delta}(F)\hat{\varepsilon}_t$. The null hypothesis is recoverability (serial uncorrelation) and the alternative is nonrecoverability (serial correlation). In the following section we present a Monte Carlo exercise in which the autocorrelation test has a reasonably good power in rejecting recoverability when it is false.

When recoverability is rejected there are two options: (1) estimate the relative impulse response functions and the upper and lower bounds for variance decomposition; (2) amend the VAR specification by adding variables (or use a FAVAR model in place of the VAR) and perform the test with the novel VAR specification.

It is worth noticing that the above test is valid under the maintained hypothesis that Equation (14) is fulfilled. If it is not, the lags of u_{it} may appear in Equation (15) and the test may reject serial uncorrelation even if the shock is recoverable. Hence the serial uncorrelation test is indeed a joint test about recoverability and instrument validity.

If recoverability is not rejected, we can estimate the 'absolute' response function, the unit variance shock and the variance decomposition as explained above.

It should be stressed that global recoverability is never satisfied in the presence of measurement error (see the Online Appendix A.3, Remarks 1, 2, and 3). Since measurement error cannot be ruled out in any empirical application, the relevant question is not whether perfect global recoverability holds, but whether, empirically, it is possible to obtain a good approximation of the shock of interest.

3.4.2 Fundamentalness test

If u_{it} is fundamental with respect to y_t we see from Proposition 7 that in Equation (22) $\delta_k = 0$ for all positive k and $\delta(F)$ reduces to $\delta_0 = \delta$. Hence, we can test for the null of fundamentalness against the alternative of nonfundamentalness by estimating (22) as explained above and perform a standard F-test for the joint significance of the coefficients of the leads. Notice that the test is valid even if recoverability does not hold; hence, in principle we can test directly for fundamentalness without testing for recoverability. If fundamentalness is not rejected, we can estimate the 'absolute' response function with the standard method, the unit variance shock and the variance decomposition as explained in the previous subsections. In the Online Appendix, we report Monte Carlo exercises showing that the proposed F-test has a good performance in small samples.

Let us relate our test to existing fundamentalness tests in the literature. Giannone and Reichlin (2006) suggested testing whether selected variables, external to the VAR, Granger-cause the VAR residuals. If Granger causation is found, the predictability of the residuals indicates a failure of global invertibility and hence that they cannot be employed to retrieve the structural shocks. Forni and Gambetti (2014) proposed using, as external information, the principal components of a large macroeconomic dataset rather than arbitrary external variables. Canova and Hamidi Sahneh (2017) introduced a different formulation of the Granger-causation test. Chen et al. (2017) departed from external information and Granger causation altogether by assuming that the variables are non-Gaussian and the structural shocks are i.i.d. All of these procedures are designed to test for global fundamentalness. Forni and Gambetti (2014) also proposed a test for partial fundamentalness: first estimate the shock of interest and then verify whether it is predicted by the principal components: if it is, it cannot be a structural shock.

While the above tests are not based on the information provided by the instrumental variable, more recent works by Stock and Watson (2018) and Plagborg-Møller and Wolf (2022) have proposed fundamentalness tests related to IV identification. Both should be regarded as tests for partial, rather than global, fundamentalness, since the instrumental variable, under standard assumptions, provides information exclusively about the shock of interest. The test proposed by Stock and Watson (2018) compares VAR estimates with Local Projections estimates, whereas the procedure of Plagborg-Møller and Wolf (2022) verifies whether the proxy Granger-causes the VAR variables. This is essentially equivalent to checking whether the proxy predicts future VAR innovations, which is what our method does. Our new method is motivated by the fact that it emerges naturally as a by-product of our estimation procedure.

3.5 Inference

For inference purposes, we suggest the following bootstrap procedure. For simplicity, we assume that the sample size T is the same for z_t and y_t. The generalisation to the case of different time spans is straightforward.

First, draw with reintroduction $T-(p+r)$ integers $i(t)$, $t=1,\dots,T-(p+r)$, uniformly distributed between 1 and $T-(p+r)$, and construct the artificial sequences of shocks $\varepsilon_t^1=\hat{\varepsilon}_{i(t)}$ and $v_t^1=\hat{v}_{i(t)}$, $t=p+1,\dots,T-r$, $\hat{v}_t$ being the estimated residual of regression (22). Set the final conditions $\varepsilon_t^1=\hat{\varepsilon}_t$ for $t=T-r+1,\dots,T$. Repeat the procedure H times to get the sequences ε_t^h, $t=p+1,\dots,T$ and v_t^h, $t=p+1,\dots,T-r$, for $h=1,\dots,H$.

Second, compute y_t^h, $h=1,\dots,H$, according to the VAR Equation (4). Precisely, set the initial conditions $y_t^h=y_t$, $t=1,\dots,p$, for all h. Then compute

$$y_t^h=-\sum_{k=1}^{p}\widehat{A}_k y_{t-k}^h+\varepsilon_t^h$$

for $t=p+1,\dots,T$. As for the proxy, set the initial and final conditions $z_t^h=z_t$, $t=1,\dots,p$ and $t=T-r+1,\dots,T$, for all h. Then compute

$$z_t^h=\sum_{k=0}^{r}\hat{\delta}_k\varepsilon_{t+k}^h+v_t^h$$

for $t=p+1,\dots,T-r$.

Finally, repeat the estimation procedure for any one of the artificial data sets $y_1^h,\dots,y_T^h$, $h=1,\dots,H$ to get the sequences of absolute IRFs $b_i^h(L)$, $h=1,\dots,H$, or the corresponding sequence of relative IRFs. Compute the confidence band as usual, by taking appropriate percentiles of the distribution of b_{ik}^h, for each lag k.

3.6 The proposed procedure

On the basis of the above considerations and results, we propose the following estimation and testing procedure.

1. As a first step, regress the available proxy $\tilde{z}_t$ onto the first m lags (m not necessarily equal to p) of $\tilde{z}_t$ itself and a set of regressors x_t —which can in principle be different from y_t— to get an estimate of the residual z_t, say $\hat{z}_t$. Step 1 is not needed if the

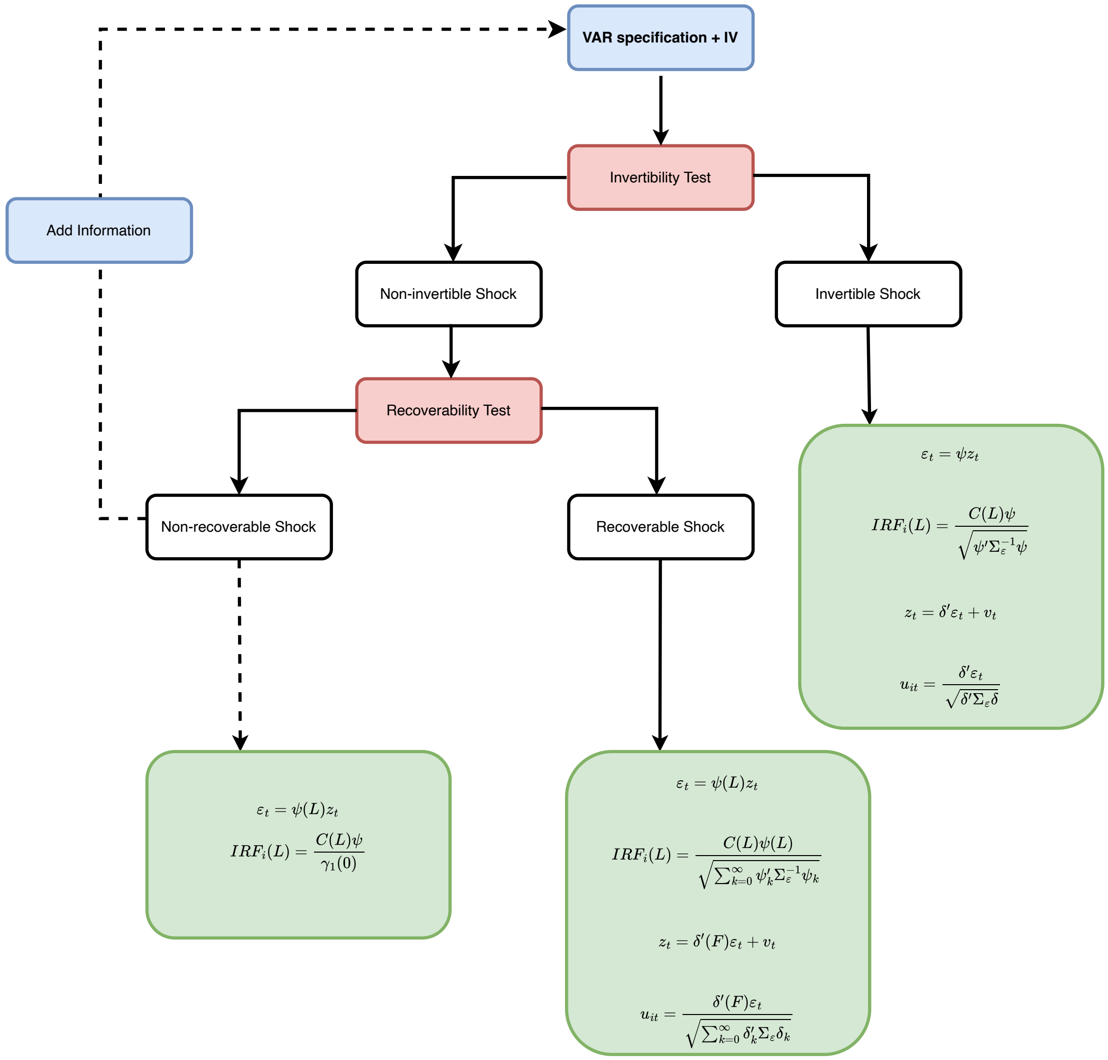

Figure 1: Flowchart summarising the proposed procedure.

instrument is already serially uncorrelated and cannot be predicted by past x's.

2. Estimate a VAR(p) with OLS to obtain $\widehat{A}(L)$, $\widehat{C}(L) = \widehat{A}(L)^{-1}$, $\hat{\varepsilon}_t$ and $\widehat{\Sigma}_\varepsilon$.

3. Regress with OLS the proxy $\hat{z}_t$ on the current value and the first r leads of the Wold residuals:

$$\hat{z}_t = \sum_{k=0}^{r} \hat{\delta}_k' \hat{\varepsilon}_{t+k} + \hat{v}_t = \hat{\delta}(F)\hat{\varepsilon}_t + \hat{v}_t.$$

Test for invertibility by performing the F-test for the null $H_0 : \delta_1 = \delta_2 = \cdots = \delta_r = 0$ against the alternative that at least one of the coefficients is non-zero.

4. Case 1: invertibility is not rejected. In this case estimate (22) without the leads of ε_t to get an estimate of δ and estimate the unit-variance shock according to (25). To estimate the corresponding IRFs, apply the standard procedure, i.e. estimate (16) without the lags of z_t to get $\hat{\psi}$ and estimate the IRFs according to (24). Estimate the variance decomposition according to equation (26) or (7).

4′. Case 2: invertibility is rejected. In this case perform the recoverability test by testing for the null of serial uncorrelation of the fitted value of the above regression $\hat{\eta}_t = \hat{\delta}(F)\hat{\varepsilon}_t$ by using the Ljung-Box Q-test.

5. Case 2a: recoverability is not rejected. In this case estimate the unit-variance shock according to (23) and the corresponding IRFs according to (21). Estimate the variance decomposition according to equation (26) or (7). Historical decomposition can be performed in the standard way.

5′. Case 2b: recoverability is rejected. In this case, either amend the VAR specification and repeat steps 2-4, or estimate (16) with a maximum lead r and the 'relative' IRFs according to (20). Estimate lower and upper bounds according to (2) and (3) and the corresponding variance contributions according to (5) and (6) or (8) and (9).

In this paper, we adopt a frequentist approach. An interesting extension would be to consider a Bayesian perspective. VARs are often overparameterised, and a Bayesian framework with informative priors could help address issues arising from the proliferation of parameters while making inference more stable and reliable. Moreover, it would allow the inclusion of additional lags, improving the approximation of the underlying Moving Average representation. The implementation of BVARs, however, requires a careful translation of the proposed procedures and is left for future research.

4 Simulations: Monetary policy in a DSGE model

In this Section, we evaluate the small-sample performance of our estimation and testing procedure through two Monte Carlo exercises. The results show that (i) invertibility is a serious issue for the standard SVAR-EIV approach; (ii) the Generalised SVAR-EIV effectively addresses this problem; and (iii) it performs well relative to the SVAR-IIV method. The Online Appendix presents two additional simulations: Simulation 3 examines the empirical size and power of the proposed invertibility test, while Simulation 4 provides guidance on lag selection.

In the simulated environment, the Data Generating Process of the economy is given by the medium-scale DSGE model of Justiniano et al. (2010) (JPT). The model features seven shocks and incorporates all the frictions deemed necessary to capture the persistence of macroeconomic data, including habit formation, investment adjustment costs, sticky prices, and sticky wages. We adopt the specification and parameterisation proposed in the original work.

The focus of our experiments is the effects of the monetary policy shock – u_{1t} – which we normalise to have unit variance. The proxy is generated as

$$\tilde{z}_t = u_{1t} + a y_{1t-1} + b w_t, \tag{27}$$

for different values of the parameters a and b. We assume y_{1t} to be output, while w_t is an i.i.d. normal white noise, independent of the other variables at all leads and lags. For all exercises, we generated 2000 datasets. The number of time observations is $T = 240$ for most exercises, but we also consider $T = 360$ and $T = 480$.

Additional Monte Carlo exercises, based on the fiscal foresight model of Leeper et al. (2013), are provided in the Online Appendix, in Section E.

4.1 Simulation 1: Does the method work?

In our first exercise, we specify three VAR models. VAR I includes four variables: the interest rate, GDP expressed in log deviations from technology, inflation, and the so-called 'GDP in the flexible-price allocation'. These variables are also the indicators entering the linear Taylor rule, so that the monetary policy shock can be obtained as a linear function of these variables and is therefore (partially) invertible in our VAR. VAR II includes the first three variables of VAR I. Since the fourth variable is omitted, the shock is no longer invertible. Finally, VAR III includes the same variables as VAR II, but the variables in this system are assumed to be measured with relatively small, independent measurement errors, with a variance equal to 5% of the total variance for each variable. It is important to note that measurement error worsens the quality of information, making the shock noninvertible in VAR III.

We simulate 2000 datasets with $T = 240$ and estimate all VAR models. Results are shown in Figure 2. Red lines represent the true IRFs, black lines are the averages of the

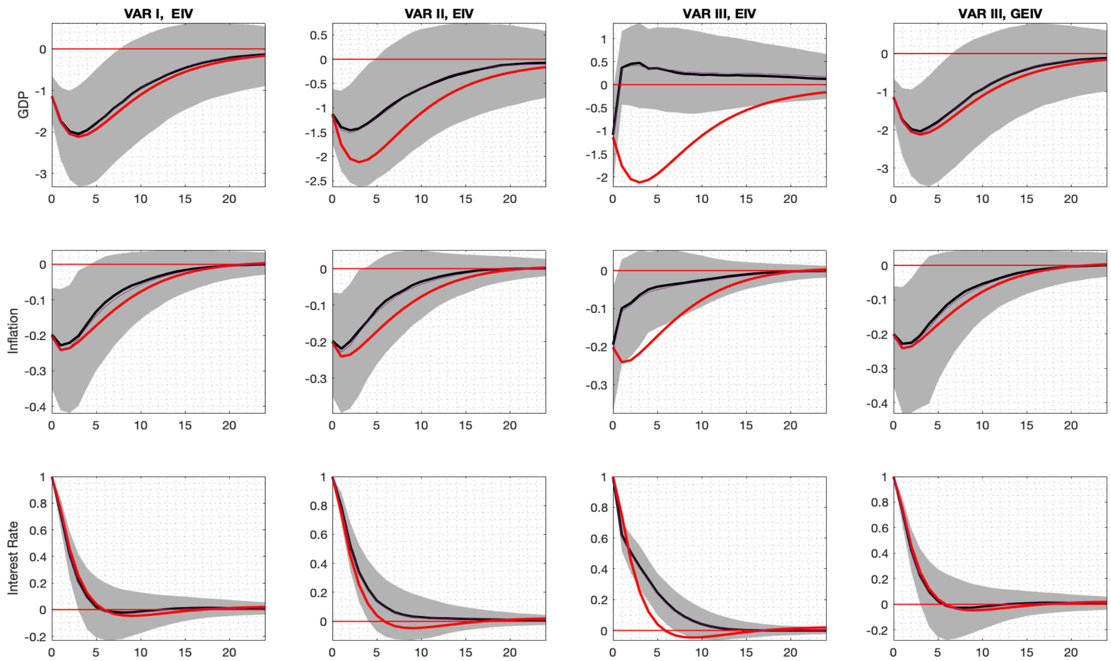

Figure 2: Simulation 1: JPT DSGE model, IRFs of the monetary policy shock. First column: VAR I EIV, IRF estimation with the standard EIV method, $p = 4$. Second column: VAR II EIV, IRF estimation with the standard EIV method, $p = 4$. Third column: VAR III EIV, IRF estimation with the standard EIV method, $p = 4$. Fourth column: VAR III GEIV, IRF estimation with the generalised EIV method, $p = r = 4$. Red lines are true response functions; black lines are the averages of the 2000 estimated response functions; grey areas, 16th -84th percentiles.

estimated IRFs, and the grey areas indicate the 16th to 84th percentiles. The first three columns report the IRFs of VAR I, VAR II, and VAR III, respectively, estimated with the standard SVAR-EIV method, $p = 4$.

In the first column, the standard method performs well, since the VAR includes sufficient information (there is a small truncation bias due to the true model being an ARMA). In the second column, the quality of the estimates deteriorates because of the omission of the fourth variable. In the third column, the estimated IRFs are dramatically incorrect, particularly for GDP. Interestingly, even relatively small measurement errors produce a large bias.

The fourth column reports the IRFs for VAR III, estimated with our Generalised SVAR-EIV method, $r = p = 4$. The estimated IRFs are very close to the true IRFs, demonstrating that the method performs well despite noninvertibility.

4.2 Simulation 2: A comparison with SVAR-IIV

Let us now compare the performance of the Generalised SVAR-EIV procedure with the SVAR-IIV method. To this end, we consider three specifications of the proxy: (i) $a = b = 0$ in equation (27), so that the proxy is equal to the normalised shock u_t; (ii) $a = 0$ and $b = 0.5$; and (iii) $a = 0.1$ and $b = 0.5$. We consider samples of size $T = 240$ and $T = 360$. For each sample size, we also consider the case of a "short proxy", where the first one-third of the sample is missing.

For the Generalised External-Instrument method, denoted by GEIV, both p and r are selected according to the BIC criterion, which provides the best results according to Simulation 4 reported in the Online Appendix, specifically designed to guide lag selection. For r, the criterion is applied to regression (22) ($\max p = \max r = 12$). The use of equation (22) in place of (16) is motivated by the results of Simulation 4. In the case of proxy (iii) above, we include a cleaning step by regressing it onto m lags of y_t and the proxy itself, with m determined by the BIC.

For the internal-instrument SVAR, denoted by IIV, we set the number of lags p according to the AIC ($\max p = \max r = 12$), since the AIC provides the best results according to Simulation 4. For the "short proxy" cases, internal IV is implemented using two methods: (i) truncating the sample to match the proxy (IIV) and (ii) filling the missing values with zeros, censored IIV (IIVc), as suggested in Noh (2017).

We normalise all IRFs by dividing by the impact effect of the shock on the interest rate, so that the contemporaneous effect on the interest rate is equal to 1. The estimation error of the competing methods is measured as the sum of squared errors of the GDP IRF divided by the sum of squared coefficients of the true IRF:

$$100 \times \frac{\sum_{k=0}^{K} (\widehat{\gamma}_{1k} - \gamma_{1k})^2}{\sum_{k=0}^{K} \gamma_{1k}^2}. \tag{28}$$

We set $K = 10$. This ratio equals 100 for the flat estimate $\widehat{\gamma}(L) = 0$.

Results are reported in Table 1. For the long proxy, the performance of IIV-AIC and GEIV-BIC is similar. In the short proxy case, GEIV outperforms both IIV and IIVc in all scenarios. Interestingly, IIVc consistently improves upon IIV.

	$T = 240$					$T = 360$				
	Long proxy		Short proxy			Long proxy		Short proxy		
	IIV	GEIV	IIV	IIVc	GEIV	IIV	GEIV	IIV	IIVc	GEIV
proxy: $\tilde{z}_t = u_{1t}$										
	61.2	**60.5**	107.7	81.7	**73.7**	**41.4**	42.5	61.2	53.6	**51.4**
proxy: $\tilde{z}_t = u_{1t} + 0.5w_t$										
	70.5	**67.6**	125.0	100.8	**84.0**	**46.5**	46.7	68.9	64.1	**57.1**
proxy: $\tilde{z}_t = u_{1t} + 0.1y_{1t-1} + 0.5w_t$										
	70.1	**66.4**	123.8	96.2	**83.8**	**46.0**	46.4	68.8	57.6	**56.8**

Table 1: Results of Simulation 2. Mean Square Errors for the relative IRFs of GDP, normalised according to (28), for the internal instrument SVAR-IIV, the censored internal instrument SVAR-IIVc and our proposed procedure (GEIV). For the SVAR-IIV p is determined by the AIC; for the SVAR-EIV, p determined by the BIC and r is set according to the BIC applied to (22). We consider three different proxy, $T = 240$ and $T = 360$. Short proxy means that we have missing values in the fist one-third of the sample. Boldface numbers are the best results obtained for each case.

5 The effects of monetary policy shocks

In this Section, we provide an empirical application of our method and study the propagation of monetary policy shocks. We first show that the monetary policy shocks identified using standard high-frequency surprises at the short end of the yield curve are likely to be nonfundamental but recoverable in a few routinely used VAR specifications. We then demonstrate that the standard External-Instrument SVAR procedure produces price and output puzzles. By contrast, when using our proposed procedure for noninvertible shocks, the results align with the textbook effects of monetary policy.

5.1 Data, VAR specification, instruments

Our baseline VAR specification includes three variables at monthly frequency: the 1-year government bond rate (1YB), industrial production (IP) in growth rates and CPI inflation (Specification I). We also present results for two additional specifications, one including Gilchrist and Zakrajšek (2012)'s excess bond premium (EBP) (Specification II); the other including EBP along with the mortgage spread (MS) and the commercial paper spread (CPS) (Specification III). We use 1YB as the policy indicator variable.[2]

[2]We use monthly data taken from the FRED-MD data set of McCracken and Ng (2015). Specifically, we use industrial production (FRED mnemonic INDPRO, IP from now on), taken in log differences, the CPI index (FRED mnemonic CPIAUCSL), taken in log differences, and the 1-year government bond rate (FRED mnemonic GS1, 1YB from now on). In addition, we use the excess bond premium (EBP), the mortgage spread (MS) and the commercial paper spread (CPS) taken from the replication files of Gertler and Karadi (2015).

Our benchmark sample spans the period 1983:1-2008:12. In two robustness exercises, we consider alternative initial dates, i.e. 1979:7, 1987:8, and 1990:1, as well as two alternative ending dates, i.e. 2012:6 and 2019:6.[3] The trending variables, CPI and IP, are taken in differences since these variables are unlikely to be cointegrated. In a robustness exercise, we consider a VAR specification with all the trending variables in levels.

The instrument for monetary policy shocks consists of the Gürkaynak et al. (2005)'s intra-daily monetary policy surprises triggered by Federal Open Market Committee (FOMC) decisions in the three month ahead monthly Fed Funds futures (FF4), as proposed in Gertler and Karadi (2015) (GK from now on). The use of this instrument provides scope for testing our approach to noninvertibility since, as discussed in Gertler and Karadi (2015) and Ramey (2016), surprises in futures with a three month maturity are likely to capture both conventional monetary policy shocks, and shocks to forward guidance about the path rate at short horizon. We 'clean' the instrument by regressing it onto its own lags and the lags of the three variables of Specification I, using 6 lags.[4] The instrument turns out to be relevant, the measure of relevance $\widehat{\mathrm{IR}}$ (the correlation between the estimated shock and the instrument, see Remark 9 in the Online Appendix) being between 0.4 and 0.6, depending on the specification adopted.

5.2 Fundamentalness and recoverability

We start our analysis by applying our fundamentalness test (Table 2a) to verify whether our specifications turns out to be fundamental or not when using our instrument GK. The main takeaway is that the results obtained with the standard proxy-SVAR approach in the monetary policy literature should be taken with caution, since the VAR specification might be affected by nonfundamentalness. In fact, results in Table 2a show that, for $r \geq 6$, fundamentalness is rejected at the 1% level with Specifications I and II and for $r \geq 7$ is rejected either at the 5% level or the 10% level with Specification III. The degree of fundamentalness $\widehat{R}_f^2$ (see Remark 5 in the Appendix) is below 0.5 for all specification for $r \geq 6$. We conclude that the inclusion of financial variables in the VAR may not be sufficient to solve fundamentalness problems. These results are in line with the findings

[3]The samples starting in 1983:1 and 1987:8 are chosen in line with Sims and Zha (2006). Moreover, 1979:7 is the beginning of Volcker's mandate; 1987:8 is the beginning of Greenspan's mandate; 2008:12 is the first month in which the 1-year bond rate falls below 1%, so that cutting our sample to 2008:12 excludes the zero lower bound period.

[4]The regression is significant at the 5% level and the residual is serially uncorrelated according to the Ljung-Box Q-test.

	Number of leads r					
	$r=4$	$r=5$	$r=6$	$r=7$	$r=8$	$r=9$
Specification I						
$p=6$	0.008	0.028	0.002	0.003	0.001	0.001
$p=9$	0.016	0.051	0.003	0.003	0.002	0.001
$p=12$	0.011	0.045	0.003	0.002	0.001	0.000
Specification II						
$p=6$	0.080	0.195	0.027	0.001	0.000	0.000
$p=9$	0.180	0.351	0.034	0.002	0.000	0.000
$p=12$	0.221	0.457	0.059	0.003	0.000	0.000
Specification III						
$p=6$	0.060	0.184	0.089	0.003	0.001	0.002
$p=9$	0.184	0.362	0.220	0.020	0.002	0.003
$p=12$	0.215	0.353	0.250	0.060	0.031	0.027

(a) Fundamentalness test

	Number of leads r					
	$r=4$	$r=5$	$r=6$	$r=7$	$r=8$	$r=9$
Specification I						
$p=6$	0.619	0.662	0.251	0.469	0.037	0.060
$p=9$	0.350	0.571	0.114	0.435	0.050	0.042
$p=12$	0.880	0.944	0.324	0.820	0.466	0.285
Specification II						
$p=6$	0.441	0.473	0.308	0.777	0.394	0.357
$p=9$	0.119	0.186	0.104	0.517	0.222	0.193
$p=12$	0.472	0.558	0.269	0.913	0.701	0.575
Specification III						
$p=6$	0.034	0.315	0.446	0.608	0.738	0.546
$p=9$	0.005	0.064	0.148	0.046	0.391	0.103
$p=12$	0.032	0.037	0.065	0.057	0.343	0.022

(b) Recoverability test

Table 2: *P*-values for the invertibility (a) and the recoverability tests (b), for different values of p and r. Specification I includes the 1-year bond rate (1YB, industrial production growth (IP) and CPI inflation (CPI). Specification II includes 1YB, IP, CPI and the excess bond premium (EBP). Specification III includes 1YB, IP, CPI, EBP, the mortgage spread and the commercial paper spread. The proxy is the one of Gertler and Karadi (2015).

of Plagborg-Møller and Wolf (2022) and the arguments in Ramey (2016), who cautions against the standard SVAR-IV approach.

On the other hand, the shock is recoverable. The p-values of the Ljung-Box Q-test for serial correlation of the estimated monetary policy shock, for our three specifications, with different values of p and r (maximum lag 24) are reported in Table 2b. The result is that recoverability cannot be rejected at the 5% level for most parameter configurations. We conclude that, at least for our time span, the monetary policy shock is recoverable, even with the three-variable Specification I, and that financial variables are not needed to find the policy shock.

5.3 The three-variable VAR

In this subsection we compare the impulse response functions obtained with the standard method with those obtained with our proposed method. We choose Specification I with the GK instrument as our benchmark.

For the number of lags in the VAR, the BIC selects $p = 2$, which appears too parsimonious for monthly data. In contrast, the AIC, with a maximum of 24 lags, selects 24, which is likely excessive. Given these conflicting indications, we set $p = 12$, in line with standard practice in the literature. As for the number of lags r included in the regression

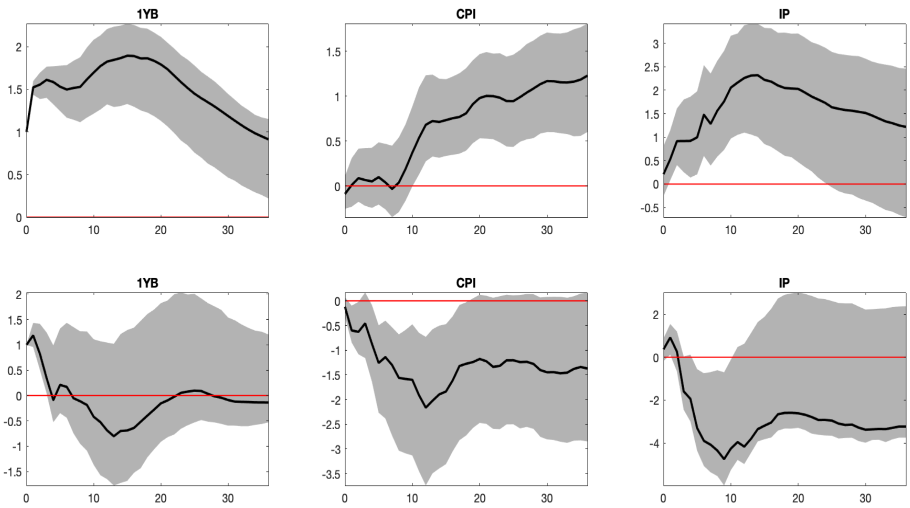

Figure 3: VAR results: Specification I, $p = 12$, GK instrument. Top panels: estimated response functions with $r = 0$ (standard method). Bottom panels: estimated response functions with our proposed method $r = 6$. Black line: point estimate. Grey area: 68% confidence bands.

of the proxy, the BIC applied to regression (22) suggests $r = 1$, which again seems too parsimonious, whereas the AIC applied to the same regression suggests $r = 11$, and the recursive F-tests indicate $r = 5$. We set $r = 6$, which appears to be a reasonable compromise between these conflicting indications. In a robustness exercise, we try different values of p and r (online Appendix, Section F).

The basic insight delivered by our exercise is that by incorrectly assuming invertibility without testing one can get dramatically misleading results. On the contrary, when the proposed procedure is applied, the estimation delivers results in line with textbook effects of monetary policy, even with a small VAR, not including the EBP or other financial variables.

The results from the baseline model are reported in Figure 3. All responses are normalised to have an impact effect of 100 basis points on the 1-year bond rate. The top panels show the estimated impulse response functions obtained with $r = 0$, i.e. the standard proxy SVAR procedure. The response of 1YB is hump-shaped and very persistent (the zero line is not reached after 3 years). Both prices and industrial production significantly increase after a tightening shock, so that we have both a large price puzzle and a large real activity puzzle.

The bottom panels show the result obtained with our proposed procedure with $r = 6$. Results are completely different and much more plausible. The reaction of the policy variable is much less persistent: after the significant impact effect it reaches the zero line in about 4 months and further on it is no longer significant. Both inflation and output puzzles disappear: prices and industrial production reduce significantly after an impact effect which is very close to zero. The effects on real activity are no longer significant after about one year, showing that the effects of monetary policy on real activity are transitory, in line with the consensus. In Section F of the Online Appendix, we report several robustness checks. The overall conclusion is that the above results are reasonably robust.

Figure 3 shows that our proposed procedure delivers meaningful results even with a small VAR that does not include the EBP. This should not be interpreted as exempting the researcher from considering a more appropriate specification, as we discuss in the following section.

5.4 Medium-size VAR specifications

Are results sensitive to the VAR information set? To answer this question, in this subsection we examine results for Specification II, that includes EBP, and Specification III, that incorporates the mortgage spread and the commercial paper spread, as well as EBP. We set $p = 12$, $r = 0$ and $r = 6$, as in the previous subsection. The main conclusion of this exercise is that results obtained with our proposed method are reasonably robust, whereas results obtained with the standard method are not.

In the top panels, Figure 4 reports the case $r = 0$ (standard method). The lines are respectively the point estimates for Specification I (black), Specification II (blue), and Specification III (red). We also report the confidence bands of Specification III (pink shaded areas). Results appear to be very sensitive to the set of variables included in the VAR. With Specification II, the effects on prices and real activity are essentially zero. With Specification III, the sign of the IRFs of prices and industrial production are negative, and the puzzles of Specification I disappear. Still, the effects are quantitatively small, especially for prices, and not significant.

The IRFs obtained with the Generalised External-Instrument approach ($r = 6$) of the first three variables are similar to those obtained with Specifications I and II (bottom panels in Figure 4). The reaction of prices is large and significant. The reaction of IP is barely significant, since the confidence bands are very large, however the point estimates consistently show a sizeable reduction of about 3-5 percentage points of production after one year, depending on the VAR specification.

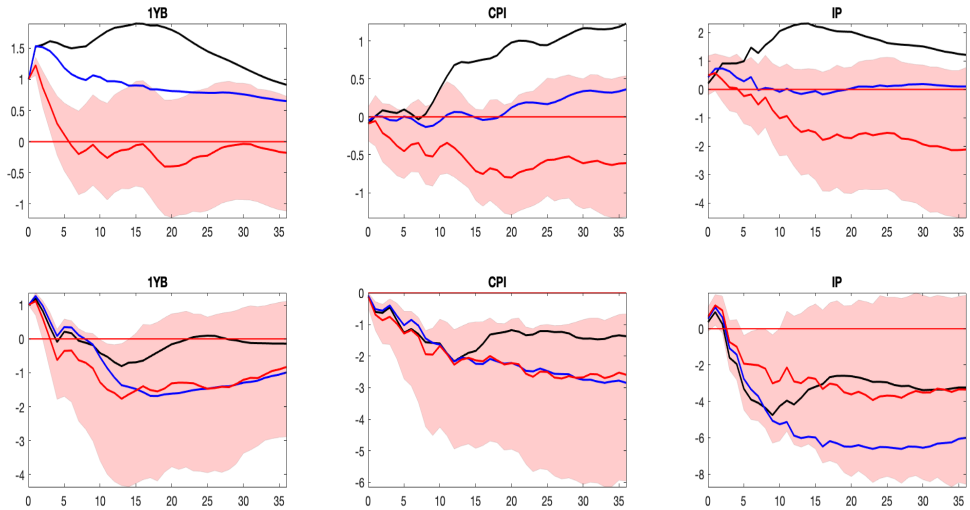

Figure 4: IRFs for Medium-size specifications. The instrument is GK. Red line: point estimates for Specification III; blue line: point estimates for Specification II; black line: point estimates for Specification I. Top panels: estimated response functions with $p = 12$, $r = 0$ (standard method). Bottom panels: estimated response functions with our proposed method, $p = 12$, $r = 6$. Pink shaded area: 68% confidence bands for Specification III.

5.5 Variance decomposition

Do monetary policy shocks account for a sizeable share of the variance of prices and output? To answer this question, it is useful to evaluate the variance decomposition VD of CPI inflation and industrial production growth obtained with our VAR specifications ($p = 12$, $r = 6$). We report results for waves of periodicity 2-18 months (short run), 18-96 month (business cycle) and 2+ months (overall variance). The main finding is that the effects of monetary policy on prices are much larger than previously reported, suggesting that it can be used successfully in controlling inflation.

Table 3 reports the point estimates and the 68% confidence bands (in brackets) for the percentage of variance explained by the monetary policy shock. The estimates for the cyclical variance (18-96 months) are not very reliable because of the large confidence bands, so that we focus mainly on the short-run and overall variances. The point estimates of the short-run volatility contributions range from 12.3% to 19.2% for inflation and from 16.1% to 27.7% for industrial production growth. As for the overall variance, the estimates range from 12.5% to 20.8% for inflation and from 13.0% to 28.3% for industrial production.

With Specification III, which provides the smallest estimates, the monetary policy shocks explains 12.5% of the overall variance of inflation and 13% of the overall variance

	Waves of periodicity		
	2 – 18 months	18 – 96 months	2+ months
Specification I			
CPI inflation	19.2	27.6	20.8
	(13.5—29.1)	(12.8—64.2)	(16.2—35.1)
IP growth	27.7	33.8	28.3
	(19.1—36.4)	(13.1—55.4)	(20.0—37.6)
Specification II			
CPI inflation	12.3	12.9	13.2
	(10.4—23.1)	(9.7—45.1)	(13.4—26.8)
IP growth	20.3	29.5	22.5
	(15.8—28.2)	(11.4—51.5)	(16.7—31.3)
Specification III			
CPI inflation	12.5	10.3	12.5
	(10.2—19.5)	(6.9—34.2)	(11.2—21.5)
IP growth	16.1	5.2	13.0
	(12.2—22.2)	(4.2—22.0)	(11.2—20.7)

Table 3: Percentage of variance accounted for by the monetary policy shock, for waves of periodicity 2-18 months (short run), 18-96 months (business cycle), 2+ months (overall variance). 68% confidence bands in brackets.

of production, with the 68% confidence bands ranging between a minimum of around 10% and a maximum of around 20% for both variables. We conclude that, contrary to previous findings, the effects of discretionary monetary policy on inflation are far from negligible. These results are at odds with the ones in Plagborg-Møller and Wolf (2022), where, according to FVR estimates, the contribution of policy shocks to inflation fluctuations is negligible at all horizons between 0 and 24 months.

To understand the sources of the difference, we compute the point estimates of the FVR of inflation, reported in Table 4. In the lower part of the table, we also report results for CPI in levels, as is common practice in the literature. At short horizons, the variance contributions are small. As argued in Subsection 3.3, these numbers should be taken with caution because of the downward bias. By contrast, the estimate of the FVR at the 24-month horizon is reliable, since we see in the upper part of the table that the numbers are almost identical to those of the overall VD, reported in the last column for convenience. This means that at horizon 24 all of the IRFs of inflations are already close to zero and the bias has disappeared. We conclude that the use of FVR in place of VD cannot explain the inconsistency of estimates. Coming to the bottom part of the table

	FVR Horizon					VD
	impact	3 months	6 months	12 months	24 months	2+ months
CPI inflation						
Specification I	0.5	7.2	15.3	18.4	20.7	20.8
Specification II	0.2	4.7	9.1	13.3	13.4	13.2
Specification III	0.3	5.6	7.4	12.5	12.4	12.5
CPI index in levels						
Specification I	0.5	4.2	9.9	20.0	21.5	
Specification II	0.2	2.6	5.3	13.7	22.5	
Specification III	0.3	4.4	7.1	13.8	18.5	

Table 4: Percentage of variance of CPI inflation and prices accounted for by the monetary policy shock, according to the FVR measure of Plagborg-Møller and Wolf (2022), on impact and at 3,6, 12, 24 months horizons.

and focusing on the 2-year horizon, we see that the variance contribution of monetary policy to the forecast error of prices is even larger when considering the price index taken in levels.

Another potential source of differences in the empirical estimates is the policy indicator. Following Plagborg-Møller and Wolf (2022), we consider a model (Specification IV) incorporating the same variables of Specification II but with the federal funds rate (FFR) in place of 1YB. In addition, we set $p = 6$, as in Plagborg-Møller and Wolf (2022) (instead of $p = 12$). We retain $m = 6$ and set $x_t = y_t$ for the preliminary treatment of the instrument. We report the point estimates of VD and FVR at horizon 24 in the first line of Table 5. The estimates are somewhat larger than the ones obtained above for Specification II. We conclude that the number of lags used in VAR estimation and the use of FFR in place of 1YB cannot explain the difference.

Finally, let us consider the results for different time spans, reported in Table 5. In the second row, we report the estimates for the time span 1990:1–2012:6, the same used in Plagborg-Møller and Wolf (2022). The explained variances for this sample are sizeably smaller than the ones of our time span: the overall VD is 8.0% as against 16.1%. This points to the fact that the different time spans explain part of the discrepancy. The remaining difference can only be due to the estimation methods.

The span of the GK instrument, 1990:1–2012:6, by excluding the 80's and including the first years of the zero-lower-bound period, exhibits little variation of both inflation and interest rates, which could be detrimental to the reliability of the estimates. To verify

Time span	VD: waves of periodicity 2 − 18 months	18 − 96 months	2+ months	FVR: horizon 24 months
1983:1–2008:12	10.4	22.0	16.1	15.5
1990:1–2012:6	6.3	15.5	8.0	8.1
1987:1–2008:12	7.3	15.4	11.3	10.6
1983:1–2012:6	10.0	24.6	12.7	12.8
1979:7–2012:6	17.2	19.3	17.4	17.5
1979:7–2019:6*	15.7	18.2	15.3	15.1

Table 5: Variance decomposition of inflation for different time spans, Specification IV: FFR, CPI inflation, IP growth, EBP. VD: percentage of inflation variance accounted for by the monetary policy shock, for waves of periodicity 2-18 months (short run), 18-96 months (business cycle), 2+ months (overall variance). FVR: percentage of forecast error variance of inflation accounted for by the monetary policy shock at the 2-year horizon. *For the sample 1979:7–2019:6 in place of the EBP series we use three financial variables: the 10-year treasury bond rate, the BAA corporate bond yield and the S&P500 stock price index.

how different spans affect the estimates, we report in the bottom part of the table the results for four additional time spans: 1987:1–2008:12, 1983:1–2012:6, 1979:7–2012:6 (the same of Gertler and Karadi, 2015) and 1979:7–2019:6. For the latter time span in place of the EBP series, which is not available, we use three financial variables: the 10-year treasury bond rate, the BAA corporate bond yield and the S&P500 stock price index. Notice that, for these time spans (as well as our benchmark 1983:1–2008:12) the Internal-Instrument method cannot be used, at least with the GK instrument. This is a nice illustration of the advantages of our proposed method. Despite results vary considerably across different samples, the overall picture emerging from Table 5 confirms our main finding: discretionary monetary policy has non-negligible effects on prices.

6 Concluding remarks

In this paper we propose a new estimation procedure for structural VARs with an external instrument. The procedure includes a test for invertibility and a test for recoverability, a method to estimate the relative impulse response functions when the shock is not recoverable and a method to estimate the absolute response functions and the shock itself when the shock is recoverable but not invertible. The procedure reduces to the standard method when the shock is invertible. Results reported in this paper indicate that all procedures work remarkably well under simulation, when the sample size is comparable with those typical of macroeconomic empirical analyses.

An application to monetary policy shocks, using the instrument of Gertler and Karadi, 2015, indicates that the policy shocks are not invertible in a few popular monetary policy VAR specifications. While the standard method produces puzzling results, our procedure delivers results in line with textbook effects. Finally, we find that the policy shock is recoverable, so that we can estimate its variance contributions. Variance decomposition shows that monetary policy has sizeable effects on both real activity and inflation, suggesting that monetary policy can be effective in controlling prices.

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