

Lecture 3: The role of information

Introduction: SVAR

So, what can go wrong with VAR analysis?

The problem

- ▶ Economic agents take their decisions based on large information sets. For instance central banks typically monitor a large amount of series.
- ▶ SVAR econometricians estimate models with a much reduced number of series (previous example).
- ▶ That means that the information set of the econometrician is likely to be narrower than that of the agents.

... what are the implications?

The problem

- ▶ In this situation SVAR model cannot in general consistently estimate the impulse response functions of the structural shocks and the structural shocks.
- ▶ Why? The present and past values of the series considered are not enough informative.
- ▶ Intuition: the innovation of the econometrician does not coincide with that of the agents.
- ▶ The problem is also known as nonfundamentality (see Hansen and Sargent, 1991, Lippi and Reichlin, 1993).
- ▶ Formally speaking in this case the MA representation in terms of structural shocks of the series is not invertible.

... the (big) problem.

The problem

- ▶ The problem has been largely ignored in the macroeconometrics literature in the past.
- ▶ More recently it has been the focus of several research contributions because there are many relevant economic cases that can give rise nonfundamentalnes.

...let us see some examples.

Examples (I): Price puzzle

- ▶ We noticed before that prices increase after a contractionary monetary policy shock.
- ▶ Result is counterintuitive: Bernanke should reduce the fed funds rate to fight inflation.
- ▶ The result is known as *price puzzle*.
- ▶ Sims (1992) shows that once a commodity price index is included in the VAR prices fall.
- ▶ The reason is that the an important variable that signals future inflation pressures is omitted from the econometrician information set.

Examples (II): Fiscal foresight

- ▶ First of all it is instructive to review the definition of fundamentalness.
- ▶ Assume that the n -dimensional stochastic vector μ_t admits a moving average representation

$$\mu_t = K(L)v_t$$

where $K(L)$ is a $n \times q$ ($q \leq n$) polynomial matrix and v_t is a $q \times 1$ white noise.

- ▶ The above representation is fundamental if and only if the rank of $K(L)$ is q for all z such that $|z| < 1$.
- ▶ If $n = q$ (the SVAR case) if and only if the roots of the determinant of $K(L)$ are larger than one in absolute value.

Examples (II): Fiscal foresight

Example: suppose

$$K(L) = \begin{pmatrix} \theta - L & 0 \\ \theta & 1 \end{pmatrix}$$

the determinant $(\theta - L)$ is zero for $L = \theta$. The representation is fundamental iff $\theta \geq 1$

Examples (II): Fiscal foresight

- ▶ Recent works argued that fiscal policy actions are anticipated (see e.g. Yang, 2008, Leeper, Walker and Yang, 2008, Mertens and Ravn, 2010).
- ▶ Private agents receive signals about future changes in taxes and government spending before these changes actually take place.
- ▶ The reason is the existence of legislative and implementation lags: it takes time for a policy action to be passed and implemented.
- ▶ The phenomenon is called "fiscal foresight".

Examples (II): Fiscal foresight

- ▶ Leeper, Walker and Yang, (2008) shows theoretically that this raises big problems for VAR analysis.
- ▶ The intuition is that given that the shock does not affect fiscal variables immediately these variables lose informational content.

... so let's see the example in detail.

Examples (II): Fiscal foresight

The model is a standard growth model. Log-linearized equilibrium solution for the three state variables of the model is

$$\begin{aligned}k_t &= \alpha k_{t-1} + a_t - \kappa \sum_{i=0}^{\infty} \theta^i E_t \tau_{t+i+1} \\a_t &= \varepsilon_{A,t} \\ \tau_t &= \varepsilon_{\tau,t-q}\end{aligned}$$

where k_t is capital a_t is the technology shock and τ_t are taxes, $\varepsilon_{\tau,t-q}, \varepsilon_{A,t}$ are i.i.d. shocks to taxes and technology, $\theta = \alpha\beta(1 - \tau) < 1$, $\kappa = (1 - \theta)(\tau/(1 - \tau))$, where $0 < \alpha < 1$ is the $0 < \beta < 1$, and $0 \leq \tau < 1$ is the steady state tax rate, q is the period of foresight.

Suppose that $q = 2$. The capital transition equation becomes

$$k_t = \alpha k_{t-1} + a_t - \kappa(\varepsilon_{\tau,t-1} + \theta \varepsilon_{\tau,t})$$

Examples (II): Fiscal foresight

Suppose the econometrician wants to use data for capital and technology to estimate a VAR in order to identify the fiscal shock. The solution of the model for the two variables is

$$\begin{pmatrix} a_t \\ k_t \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{-\kappa(L+\theta)}{1-\alpha L} & \frac{1}{1-\alpha L} \end{pmatrix} \begin{pmatrix} \varepsilon_{\tau,t} \\ \varepsilon_{A,t} \end{pmatrix} = A_1(L)\varepsilon_t \quad (1)$$

The determinant of $A_1(z)$ is $\frac{\kappa(z+\theta)}{1-\alpha z}$ which is zero for $z = -\theta < 1$.

This implies that the shock cannot be recovered using a VAR with data for capital and technology.

Examples (II): Fiscal foresight

Now suppose that the econometrician decides to use data for capital and taxes. The solution of the model for the two variables is

$$\begin{pmatrix} k_t \\ \tau_t \end{pmatrix} = \begin{pmatrix} \frac{-\kappa(L+\theta)}{1-\alpha L} & \frac{1}{1-\alpha L} \\ L^2 & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_{\tau,t} \\ \varepsilon_{A,t} \end{pmatrix} = A_2(L)\varepsilon_t \quad (2)$$

The determinant of $A_1(z)$ is $\frac{z^2}{1-\alpha z}$ which is zero for $z = 0$ meaning that the MA representation is non-invertible and the shock non-fundamental for τ_t and k_t .

Again the shock cannot be recovered using a VAR with data for capital and taxes.

If the shocks are nonfundamental then SVAR models are not useful for structural analysis.

Examples (III): News shocks

Consider the simple Lucas tree model. The agent maximizes

$$E_t \sum_{t=0}^{\infty} \beta^t c_t,$$

where c_t is consumption and β is a discount factor, subject to the constraint

$$c_t + p_t s_{t+1} = (p_t + \theta_t) s_t,$$

where p_t is the price of a share, s_t is the number of shares and $(p_t + \theta_t) s_t$ is the total amount of resources available at time t .

TFP evolves as

$$\theta_t = \theta_{t-1} + \varepsilon_{t-2} + u_t$$

ε_t : news shock - u_t : TFP shock.

Examples (III): News shocks

The equilibrium value for asset prices is given by:

$$p_t = E_t \sum_{j=1}^{\infty} \beta^j \theta_{t+j}$$

We have

$$E_t \theta_{t+1} = \theta_t + \varepsilon_{t-1},$$

$$E_t \theta_{t+j} = \theta_t + \varepsilon_{t-1} + \varepsilon_t, \quad \text{for } j \geq 2,$$

so that the price equation reads

$$p_t = \frac{\beta}{1-\beta} \theta_t + \frac{\beta}{1-\beta} (\beta \varepsilon_t + \varepsilon_{t-1}).$$

Examples (III): News shocks

Taking first differences we get the following structural MA representation

$$\begin{pmatrix} \Delta\theta_t \\ \Delta p_t \end{pmatrix} = \begin{pmatrix} L^2 & 1 \\ \frac{\beta^2}{1-\beta} + \beta L & \frac{\beta}{1-\beta} \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ u_t \end{pmatrix}. \quad (3)$$

Examples (III): News shocks

The determinant is

$$-\frac{\beta^2}{1-\beta} - \beta z + \frac{\beta}{1-\beta} z^2$$

which vanishes for $z = 1$ (cointegrated variables) and $z = -\beta$.

As $\beta < 1$, the two shocks u_t and ε_t are non-fundamental for the variables ΔP_t and $\Delta \theta_t$.

Here the agents see the shocks. The econometrician only see the variables. In this case not even a very forward-looking variable like stock prices conveys enough information to recover the shock.

Noninvertibility and the state space: News shocks

As an alternative explanation, the joint dynamics of θ_t and p_t can be represented in state-space form as

$$\begin{pmatrix} \theta_t \\ \varepsilon_t \\ \varepsilon_{t-1} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \theta_{t-1} \\ \varepsilon_{t-1} \\ \varepsilon_{t-2} \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ u_t \end{pmatrix} \quad (4)$$

$$\begin{pmatrix} \theta_t \\ p_t \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ \delta & \delta & \delta \end{pmatrix} \begin{pmatrix} \theta_{t-1} \\ \varepsilon_{t-1} \\ \varepsilon_{t-2} \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ \delta\beta & \delta \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ u_t \end{pmatrix}. \quad (5)$$

where $\delta = \beta/(1 - \beta)$.

Noninvertibility and the state space: News shocks

In fact the model for θ_t and p_t is a VARMA with a nonfundamental MA component

$$\begin{pmatrix} \theta_t \\ p_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \delta & 0 \end{pmatrix} \begin{pmatrix} \theta_{t-1} \\ p_{t-1} \end{pmatrix} + \begin{pmatrix} L^2 & 1 \\ \delta\beta + \delta L + \delta L^2 & \delta \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ u_t \end{pmatrix}. \quad (6)$$

The root of the determinant of the MA component vanishes for $-\beta$

Noninvertibility and the state space: News shocks

- ▶ The structural shocks can be obtained as the residuals of a VAR on the state variables.
- ▶ Unfortunately, the state vector includes ε_t and ε_{t-1} , which are not observable.
- ▶ By observing p_t the econometrician can obtain some information about the missing states but cannot tell apart ε_t and ε_{t-1} .

Noninvertibility and the state space: Fernandez-Villaverde et al (AER 2011)

Suppose the economy is representable with the state-space representation

$$s_t = As_{t-1} + Bu_t \quad (7)$$

$$x_t = Cs_{t-1} + Du_t \quad (8)$$

where

- ▶ s_t is an r -dimensional vector of stationary state variables,
- ▶ $q \leq r \leq n$, A , B , C and D are conformable matrices of parameters,
- ▶ B has a left inverse B^{-1} such that $B^{-1}B = I_q$.

Noninvertibility and the state space: Fernandez-Villaverde et al (AER 2011)

Notice that

$$u_t = B^{-1}s_t - B^{-1}As_{t-1}.$$

Substituting in x_t we have

$$\begin{aligned}x_t &= Cs_{t-1} + Du_t \\&= Cs_{t-1} + DB^{-1}s_t - DB^{-1}As_{t-1} \\&= [DB^{-1} - (DB^{-1}A - C)L] s_t\end{aligned}$$

In the square case $q = n$ we have

$$x_t = DB^{-1} [I - (A - BD^{-1}C)L] s_t$$

Noninvertibility and the state space: Fernandez-Villaverde et al (AER 2011)

- ▶ s_t and therefore the shocks can be recovered as a square summable combination of the present and past of y_t iff the eigenvalues of $(A - BD^{-1}C)$ are strictly less than one in modulus.
- ▶ In this case a VAR representation in terms of the structural shocks exists

$$x_t = \sum_{j=0}^{\infty} (A - BD^{-1}C)^j BD^{-1} x_{t-j} + Du_t$$

- ▶ Useful only for theoretical model but no implications for empirical analysis.

In sum

In many cases VARs are likely to suffer of an informational problem with serious consequences.

So...

the key question is: is there a way to know whether a set of variable is informative enough to estimate the structural shocks of interest?

and the answer is...

yes.

Sufficient information in Structural VARs

- ▶ In the next slides we study the testing procedure proposed in Forni and Gambetti (2011) "Sufficient information in Structural VARs".
- ▶ Proofs of the propositions and other detail can be found in the paper.

The macroeconomy

The macroeconomy has the following MA representation.

$$x_t = F(L)u_t, \quad (9)$$

- ▶ u_t is a q -dimensional vector of structural macroeconomic shocks
- ▶ $F(L)$ is an $n \times q$ matrix of impulse response functions.

Equation (9) can be thought of as the representation of a macroeconomic equilibrium (Villaverde, Rubio-Ramirez, Sargent and Watson (2007)).

(9) is typically a 'tall' system, i.e. the number of variables on the LHS is larger than the number of shocks on the RHS.

Econometrician's and VAR information sets

The SVAR **econometrician** observes x_t , possibly with error, i.e. it observes

$$x_t^* = x_t + \xi_t = F(L)u_t + \xi_t, \quad (10)$$

where ξ_t is a (possibly zero) vector of errors, orthogonal to u_{jt-k} , $j = 1, \dots, q$, any k .

In practice however the econometrician needs to summarize the information. The **VAR information set** is then spanned by an s -dimensional subvector of x_t^* , or more, generally, an s -dimensional linear combination of x_t^* , say

$$z_t^* = Wx_t^* \quad (11)$$

Informational sufficiency

Now, consider the theoretical projection equation of z_t^* on its past history, i.e.

$$z_t^* = P(z_t^* | \mathcal{Z}_{t-1}^*) + \epsilon_t. \quad (12)$$

The SVAR methodology consists in (a) estimating a VAR to get ϵ_t ; (b) attempting to get the structural shocks as linear combinations of the estimated entries of ϵ_t .

Hence a very desirable property of z_t^* is the following:

Definition Let v_t be any subvector of u_t . We say that z_t^* and the related set \mathcal{Z}_t^* is “informationally sufficient for v_t ” if and only if there exist a matrix M such that $v_t = M\epsilon_t$. z_t^* is “informationally sufficient” if it is informationally sufficient for u_t .

A testable implication

Proposition 2. *If x_t^* Granger causes z_t^* , then z_t^* is not informationally sufficient.*

The intuition is that nothing can Granger cause variables which contain all of the relevant macroeconomic information.

Proposition 2 is derived (within somewhat different settings) in Forni and Reichlin (1996) and Giannone and Reichlin (2006).

If the econometrician believes that a given variable or vector in x_t^* , say y_t^* , may convey relevant information, he/she can check whether y_t^* Granger causes z_t^* .

Limitations of Proposition 2

Proposition 2, being only a necessary condition, can be used to reject informational sufficiency, but not to validate it.

Clearly, testing all of the variables in x_t^* would be close to a validation, but unfortunately this is not feasible, since in practice x_t^* is of high dimension.

On the one hand, we cannot use all of the variables simultaneously; on the other hand, testing each one of them separately would yield with very high probability to reject informational sufficiency even if z_t^* it is, owing to Type I error.

Assuming the factor model

We can provide a sufficient condition by assuming the state-space representation in Villaverde, Rubio-Ramirez, Sargent and Watson (2007), i.e.

$$s_t = As_{t-1} + Bu_t \quad (13)$$

$$x_t = Cs_{t-1} + Du_t \quad (14)$$

where

- ▶ s_t is an r -dimensional vector of stationary state variables,
- ▶ $q \leq r \leq n$, A , B , C and D are conformable matrices of parameters,
- ▶ B has a left inverse B^{-1} such that $B^{-1}B = I_q$.

Assuming the factor model

It can be seen that the model has a factor model representation. Indeed

$$u_t = B^{-1}s_t - B^{-1}As_{t-1}. \quad (15)$$

Substituting into the states

$$x_t = DB^{-1}s_t + (C - DB^{-1}A)s_{t-1}. \quad (16)$$

Therefore x_t^* has the **factor representation**

$$x_t^* = Gf_t + \xi_t, \quad (17)$$

where the $G = (DB^{-1} \quad C - DB^{-1}A)$ and $f_t = (s'_t \quad s'_{t-1})'$. **structural shocks are fundamental for the factors f_t .**

A necessary and sufficient condition

Proposition 3. z_t^* is informationally sufficient if and only if f_t does not Granger cause z_t^* .

The intuition for sufficiency is that if f_t does not help predicting z_t^* , then nothing can, since f_t is informationally sufficient.

Proposition 3 implies that we can summarize the information in the large dimensional vector x_t^* into a relatively small number of factors (the entries of f_t).

Such factors are unobservable, but can be consistently estimated by the **principal components** of x_t^* (Stock and Watson, 2002).

Testing for sufficient information

Proposition 3 provides the theoretical basis for the following testing procedure.

1. Take a large data set x_t^* , capturing all of the relevant macroeconomic information.
2. Set a maximum number of factors P and compute the first P principal components of x_t^* .
3. Perform Granger causation tests to see whether the first h principal components, $h = 1, \dots, P$, Granger cause z_t^* . If the null of no Granger causality is never rejected, z_t^* is informationally sufficient. Otherwise, sufficiency is rejected.

Structuralness of a single shock

Global sufficiency is needed to recover all of the structural shocks.

But the econometrician is often interested in identifying just a **single shock** within a VAR model. To this end, we propose a less demanding test.

The following example shows that, even if global sufficiency does not hold, z_t^* can be sufficient for a single shock:

$$z_{1t}^* = u_{1t} + u_{2t-1} \quad (18)$$

$$z_{2t}^* = u_{1t} - u_{2t-1}. \quad (19)$$

In this case z_t^* is not sufficient for u_t but is sufficient for u_{1t} , since $z_{1t}^* + z_{2t}^* = 2u_{1t}$.

The orthogonality test

Clearly the structural shock u_{jt} is orthogonal to f_{t-k} , $k > 0$.

Conversely, if a linear combination of the VAR residuals is orthogonal to the past of the factors, then it is a linear combination of the structural shocks.

Proposition 4. *Let $v_t = \alpha' \epsilon_t$, $\alpha \in \mathbb{R}^s$. if z_t^* is free of measurement error, i.e. $z_t^* = z_t$, and v_t is orthogonal to f_{t-k} , $k > 0$, then v_t is a linear combination of the structural shocks.*

Orthogonality does not guarantee that such linear combination be the desired shock; it will, only if identification is correct.

After having identified our shock of interest, we can therefore test whether it can be a structural shock by testing for **orthogonality with respect to the lags of the principal components**.

Solutions

What should we do if informational sufficiency or structuralness is rejected?

1. To add the principal components \hat{f}_t to the VAR information set and estimate an enlarged VAR with $w_t = \begin{pmatrix} z_t^{*'} \\ \hat{f}_t' \end{pmatrix}'$.
2. To use a FAVAR (Bernanke et al., 2005) .
3. To use a factor model (forni et al 2009).

Remember: these models provide an estimate of the state which is are the key variables.

Application: Technology shocks and the business cycle

- ▶ Now we will see an empirical application of the methods studied above.
- ▶ Question: **Do technology shock explain aggregate fluctuations?**
- ▶ The empirical evidence is mixed.
 - ▶ In his seminal paper, Gali (1999) finds a very modest role for technology shocks as a source of economic fluctuations.
 - ▶ On the contrary other authors, see for instance Christiano, Eichenbaum and Vigfusson (2003) and Beaudry and Portier (2006), provide evidence that technology shocks are capable of generating sizable fluctuations in macroeconomic aggregates.
- ▶ Most of the existing evidence about the effects of technology shocks is obtained using small-scale VAR models.

Technology shocks and the business cycle

- ▶ Following Barnichon (2010), we focus on the vector z_t^* including the growth rate of **total factor productivity** (TFP) and the **unemployment rate**.
- ▶ The state variables of the economy are estimated by using the principal components of a large dataset of 110 quarterly US macroeconomic series covering the period 1960-I to 2010-IV.
- ▶ To begin, we test for **informational sufficiency** of z_t^* .
- ▶ We find that sufficiency is rejected. To get sufficiency, we have to augment the VAR with 9 principal components.

Results: the Granger-causality test

j	FAVAR specifications									
	z_t^*	w_t^1	w_t^2	w_t^3	w_t^4	w_t^5	w_t^6	w_t^7	w_t^8	w_t^9
1	0.03									
2	—	0.14								
3	—	0.37	0.95							
4	—	0.25	0.07	0.06						
5	—	0.00	0.00	0.00	0.67					
6	—	—	—	—	0.90	0.03				
7	—	—	—	—	0.00	—	0.00			
8	—	—	—	—	—	—	—	0.35		
9	—	—	—	—	—	—	—	0.03	0.01	
10	—	—	—	—	—	—	—	—	—	0.64
11	—	—	—	—	—	—	—	—	—	0.13
12	—	—	—	—	—	—	—	—	—	0.10
13	—	—	—	—	—	—	—	—	—	0.51
14	—	—	—	—	—	—	—	—	—	0.59
15	—	—	—	—	—	—	—	—	—	0.17

Table 1: p-values of the out-of-sample Granger causality test for global sufficient information. w_t^h is the original VAR vector z_t^* , augmented with h principal components. j refers to the number of principal components used in the test.

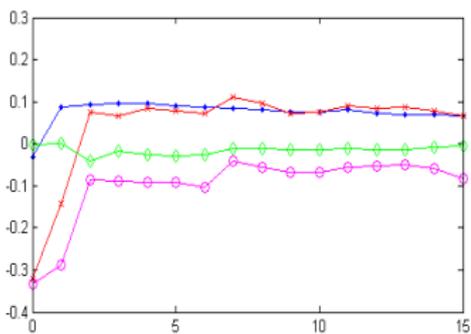
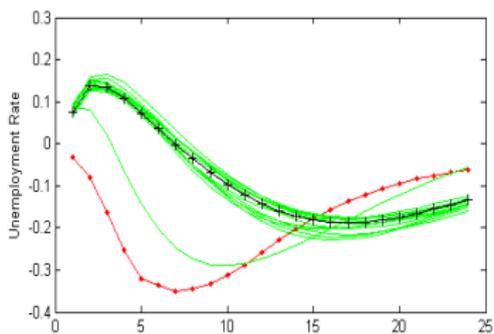
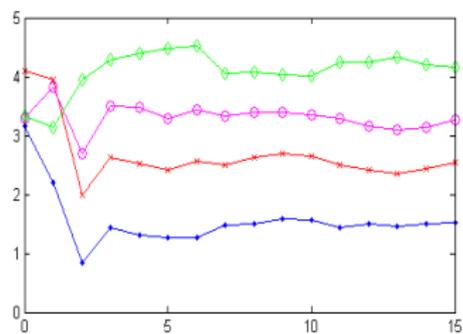
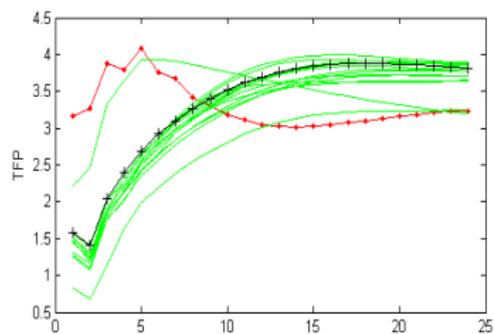
Testing for orthogonality of the estimated shock

- ▶ As a validation, we check for structuralness of the estimated technology shock.
- ▶ We identify the technology shock in both the original VAR with z_t^* and the FAVAR w_t^9 , by imposing the standard long-run restriction that technology is the only shock driving TFP.
- ▶ Then we test whether the estimated shock is orthogonal to the past of the principal components.
- ▶ Specifically we run a regression of the estimated shock on the past of the principal components and we perform an F-test of the null hypothesis that the coefficients are jointly zero.
- ▶ We find that the hypothesis is strongly rejected for z_t^* , but cannot be rejected for w_t^9 .

Impulse-response functions

- ▶ Next we study the consequences of insufficient information in terms of impulse response functions.
- ▶ We investigate how the predicted effects of technology shocks change by augmenting the original VAR with one, two,..., sixteen principal components.
- ▶ Figure 1 show the impulse response functions. The left column plots the impulse response functions for total factor productivity and unemployment, for all the sixteen specification.
- ▶ The right column displays for the three variables the effects on impact (blue-points) at 1 year (red-crosses) 2 year (magenta-circles) and in the long run (green-diamonds). In the horizontal axis we put the number of principal components included in the VAR. The first value refers to the VAR without principal components.

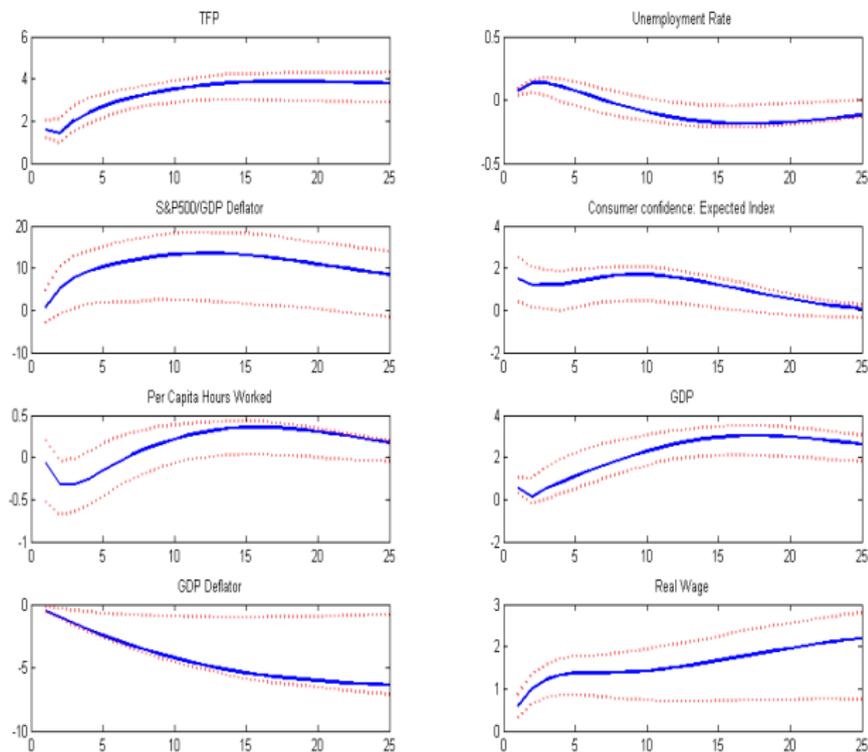
Figure 1: Impulse response functions



Comment for Figure 1

- ▶ The VAR without principal components predicts that the technology shock reduces unemployment, as predicted by standard RBC models. Total factor productivity reacts positively on impact and stays roughly constant afterward.
- ▶ The picture changes dramatically when adding the principal components. The effect of the technology shock on the unemployment rate becomes positive. Moreover, the impact effect on TFP reduces substantially, so that the diffusion process is much slower, in line with the S-shape view.
- ▶ As can be seen in the right panels of Figure 1, consistently with the results of the test, models including more than nine principal components all deliver similar impulse response functions.

Figure 2: Additional impulse response functions



Comment for Figure 2

- ▶ Figure 2 plots the FAVAR impulse response functions of some variables of interest.
- ▶ GDP increases significantly only after a few quarters.
- ▶ Per capita hours worked reduce in the short run.
- ▶ Such results are at odds with the standard RBC model.
- ▶ The GDP deflator significantly reduces, while real wages significantly increase.
- ▶ Somewhat surprisingly, the response of the S&P500 index is sluggish, whereas the consumer confidence (expected) index jumps immediately.

Amending the VAR with forward-looking variables

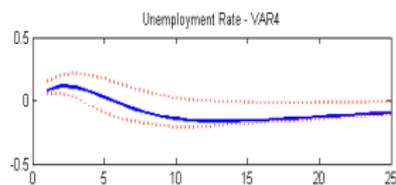
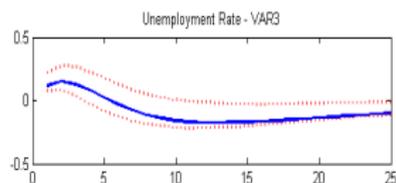
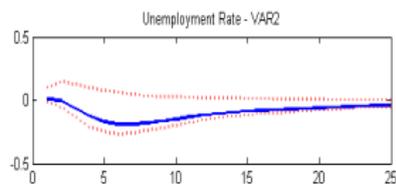
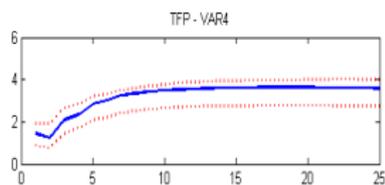
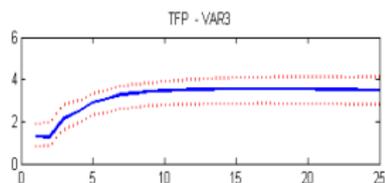
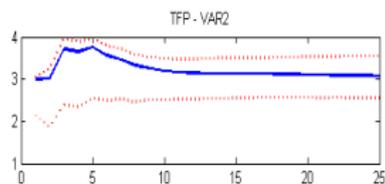
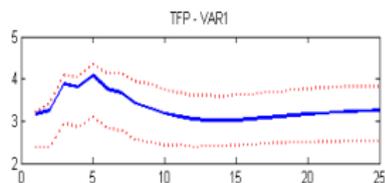
- ▶ As a further exercise, we try to amend the VAR by adding suitable variables.
- ▶ Natural candidates are forward-looking variables such as stock prices and consumer confidence indicators.
- ▶ We focus on the real Standard & Poor's 500 index, (in log differences), and a component of the Michigan University consumer confidence index, i.e. Business Conditions expected during the next 5 years.
- ▶ We test for orthogonality of the estimated shock with the lags of the principal components.
- ▶ Table 2 shows results for different specifications, including one or both of the above variables. The two specifications including the survey variable are not rejected.

Results: the orthogonality test

j	1 lag				2 lags			
	VAR1	VAR2	VAR3	VAR4	VAR1	VAR2	VAR3	VAR4
1	0.02	0.26	0.40	0.63	0.24	0.42	0.34	0.61
2	0.00	0.04	0.17	0.23	0.08	0.12	0.17	0.39
3	—	—	0.30	0.40	0.16	0.28	0.23	0.52
4	—	—	0.35	0.56	0.31	0.22	0.13	0.27
5	—	—	0.48	0.67	0.01	0.18	0.10	0.29
6	—	—	0.37	0.62	0.00	0.01	0.07	0.14
7	—	—	0.48	0.69	—	—	0.07	0.07
8	—	—	0.59	0.78	—	—	0.11	0.11
9	—	—	0.66	0.83	—	—	0.14	0.16
10	—	—	0.74	0.89	—	—	0.18	0.22
11	—	—	0.72	0.89	—	—	0.23	0.29
12	—	—	0.73	0.92	—	—	0.34	0.39
13	—	—	0.52	0.86	—	—	0.35	0.42
14	—	—	0.57	0.90	—	—	0.36	0.44
15	—	—	0.58	0.91	—	—	0.22	0.29

Table 2: p-values of the orthogonality test for different VAR specifications. VAR 1: TFP, Unemployment rate. VAR 2: TFP, Unemployment rate and the S&P500 stock price index. VAR 3: TFP, Unemployment rate and Business conditions expected during the next 5 years. VAR 4: TFP, Unemployment rate, S&P500, Business conditions expected during the next 5 years.

Figure 3: Impulse response functions



Comment for Figure 3

- ▶ Figure 3 shows the impulse response functions obtained with all four specifications.
- ▶ The impulse response functions of the rejected specifications VAR1 and VAR2 (lines 1 and 2, respectively) are similar to each other.
- ▶ The impulse response functions of VAR3 and VAR4 (lines 3 and 4, respectively) are very much similar to each other and to the one obtained with the 9-factor FAVAR model.
- ▶ For both specifications, the unemployment rate exhibits on impact a significant positive reaction.

Application: News and business cycles, Beaudry and Portier (2006 AER)

- ▶ Main idea back to Pigou: news about future productivity growth can generate business cycles since agents react to news by investing and consuming.
- ▶ Beaudry and Portier (2006 AER) finds news shocks are important for economic fluctuations. Output, investment, consumption and hours positively comove and the shocks explain a large fraction of the their variance.
- ▶ Use a VECM for TFP and stock prices.
- ▶ Standard model do not replicate the empirical finding since because of consumption comove negatively with investment and hours.
- ▶ Big effort in building models where news shocks generate business cycles (Jaimovich and Rebelo, 2009, Den Haan and Kaltenbrunner, 2009, Schmitt-Grohe and Uribe, 2008).

Application: News and business cycles, Beaudry and Portier (2006 AER)

Two identification procedures:

1. Technology shocks is the only shock driving TFP in the long run.
2. News shocks raise stock prices on impact but not TFP (lagged adjustment).

Application: News and business cycle, Beaudry and Portier (2006 AER)

Main finding:

- ▶ the two identified shocks are the same;
- ▶ such shocks generate positive comovement in consumption, investment, output and hours (consistently with business cycles comovements) and they explain a large portion of the variance of these series.

Conclusion: news shocks can generate business cycles.

Application: News and business cycle, Beaudry and Portier (2006 AER)

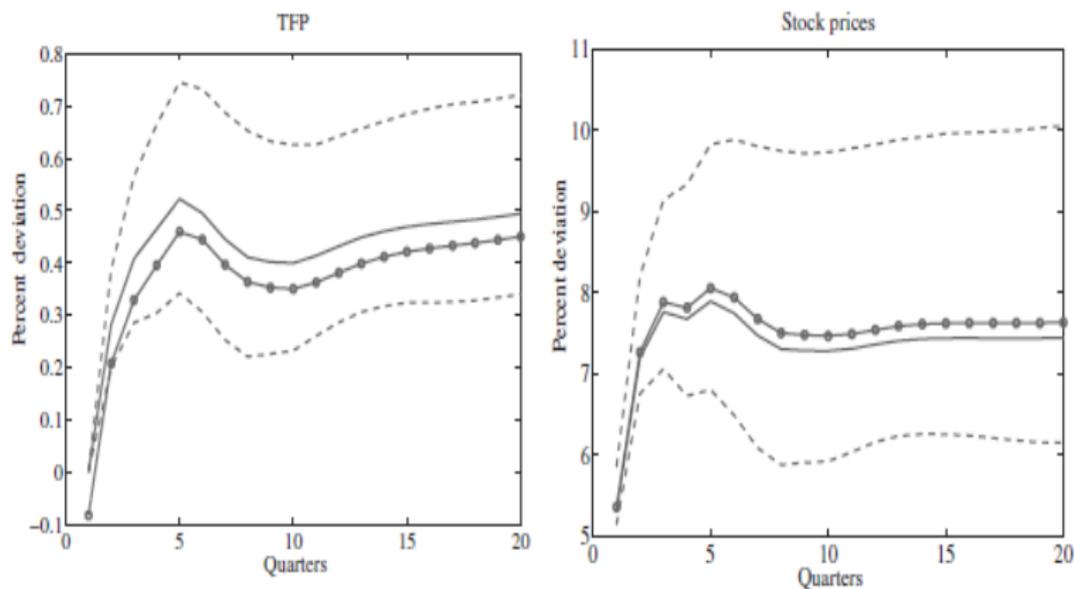


FIGURE 1. IMPULSE RESPONSES TO SHOCKS ε_2 AND $\tilde{\varepsilon}_1$ IN THE (TFP, SP) VECM

Source: Beaudry and Portier (AER 2006)

Application: News and business cycle, Beaudry and Portier (2006 AER)

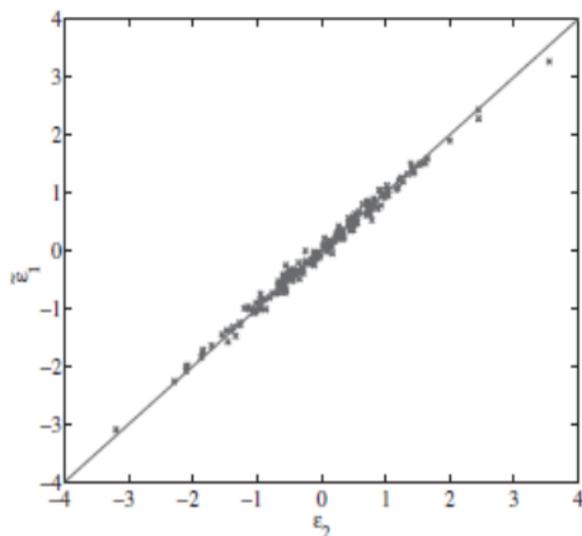


FIGURE 2. PLOT OF ε_2 AGAINST ε_1 IN THE
(TFP, SP) VECM

Source: Beaudry and Portier (AER 2006)

Application: News and business cycle, Beaudry and Portier (2006 AER)

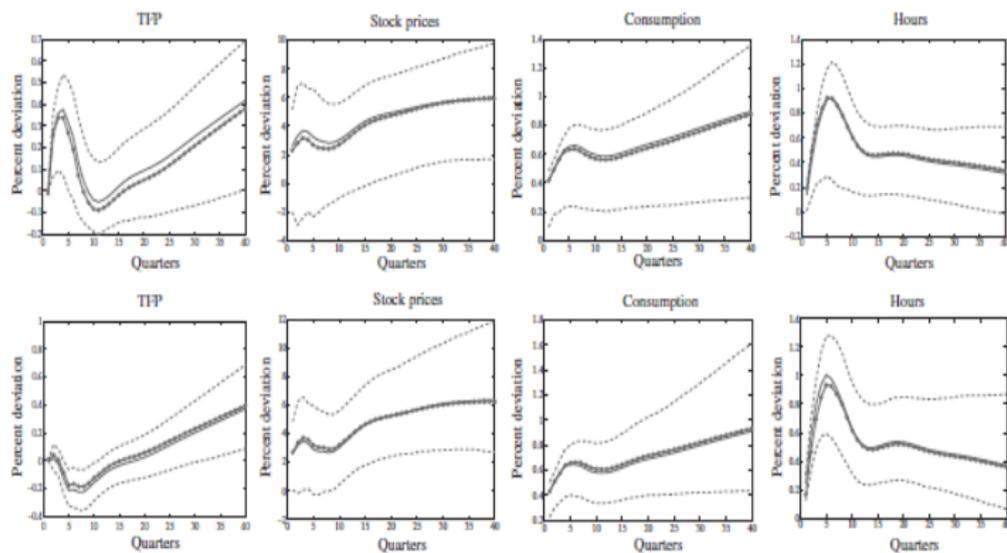


FIGURE 9. IMPULSE RESPONSES TO ε_2 AND $\bar{\varepsilon}_1$ IN THE (TFP, SP, C, H) VECM, WITHOUT (UPPER PANELS) OR WITH (LOWER PANELS) ADJUSTING TFP FOR CAPACITY UTILIZATION

Source: Beaudry and Portier (AER 2006)

Application: News and business cycle, Beaudry and Portier (2006 AER)

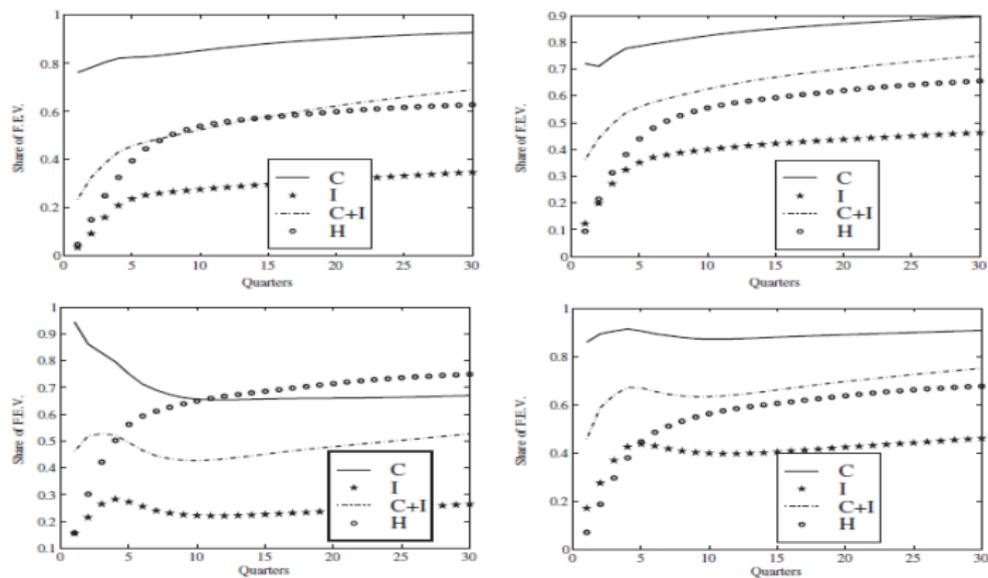


FIGURE 10. SHARE OF THE FORECAST ERROR VARIANCE (F.E.V.) OF CONSUMPTION (C), INVESTMENT I , OUTPUT ($C + I$), AND HOURS (H) ATTRIBUTABLE TO e_2 (LEFT PANELS) AND TO e_1 (RIGHT PANELS) IN VECMs, WITH NONADJUSTED TFP (TOP PANELS) OR ADJUSTED TFP (BOTTOM PANELS)

Source: Beaudry and Portier (AER 2006)

No news in business cycle, Forni, Gambetti and Sala (EJ 2014)

- ▶ Motivation: news shocks can give raise to nonfundamentalness (recall example at the beginning).
- ▶ VAR models like Beaudry and Portier AER can have an hard time in estimating news shocks.
- ▶ Here:
 - ▶ Test whether the news shock is fundamental for TFP and stock prices, i.e. are fundamental for the variable in BP.
 - ▶ Estimate the shocks using a FAVAR model.

Testing for fundamentalness

Use the Forni and Gambetti (2011) orthogonality test:

1. estimate a VAR with a given set of variables y_t and identify the relevant shock, w_t ;
2. test for orthogonality of w_t with respect to the lags of the factors (F-test);
3. the null of fundamentalness is rejected if and only if orthogonality is rejected.

The factors are not observed and we estimate them using the principal components of a dataset composed of 107 US quarterly macroeconomic series, covering the period 1960-I to 2010-IV.

Testing for fundamentalness

We consider the following VAR specifications.

2-variable VAR							
S1	TFP adj.	Stock P					
S2	TFP	Stock P					

4-variable VAR				
S3	TFP adj.	Stock P	Cons	Hours
S4	TFP	Stock P	Cons	Hours
S5	TFP adj.	Output	Cons	Hours

7-variable VAR							
S6	TFP adj.	Stock P	Output	Cons	Hours	Confidence	Inflation

Testing for fundamentalness

- ▶ We apply the Forni and Gambetti test.
- ▶ For each specification we use two identifications of the news shock:
 - ▶ The news shock is the shock that does not move TFP on impact and (for specifications from $S3$ to $S6$) has maximal effect on TFP at horizon 40.
 - ▶ The news shock is identified is the only shock with a non-zero effect on TFP in the long run.

Results of the test: Identification 1

		Principal components (from 1 to j)									
spec	lags	1	2	3	4	5	6	7	8	9	10
S1	1	0.12	0.30	0.07	0.02	0.04	0.04	0.02	0.04	0.06	0.06
	4	0.37	0.19	0.04	0.06	0.10	0.03	0.06	0.09	0.12	0.02
S2	1	0.31	0.60	0.03	0.01	0.01	0.01	0.00	0.00	0.01	0.01
	4	0.56	0.61	0.09	0.06	0.12	0.04	0.08	0.11	0.13	0.04
S3	1	0.02	0.01	0.02	0.03	0.06	0.02	0.02	0.03	0.04	0.03
	4	0.20	0.09	0.07	0.20	0.12	0.08	0.08	0.10	0.12	0.21
S4	1	0.21	0.02	0.04	0.04	0.06	0.02	0.03	0.02	0.02	0.03
	4	0.48	0.03	0.08	0.13	0.03	0.03	0.08	0.08	0.06	0.08
S5	1	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
S6	1	0.55	0.19	0.35	0.30	0.41	0.36	0.43	0.53	0.58	0.32
	4	0.43	0.24	0.53	0.52	0.49	0.72	0.58	0.66	0.72	0.72

Results of the fundamentalness test. Each entry of the table reports the p -value of the F -test in a regression of the news shock estimated using specifications S1 to S6 on 1 and 4 lags of the first differences of the first j principal components, $j = 1, \dots, 10$. The news shock is identified as the shock that does not move TFP on impact and (for specifications from S3 to S6) has maximal effect on TFP at horizon 60.

Results of the test: Identification 2

		Principal components (from 1 to j)									
spec	lags	1	2	3	4	5	6	7	8	9	10
S1	1	0.54	0.82	0.36	0.23	0.34	0.24	0.08	0.12	0.17	0.08
	4	0.18	0.02	0.00	0.01	0.01	0.00	0.01	0.02	0.03	0.01
S2	1	0.32	0.60	0.38	0.12	0.15	0.05	0.04	0.06	0.09	0.04
	4	0.34	0.02	0.01	0.02	0.02	0.01	0.03	0.05	0.08	0.03
S3	1	0.02	0.01	0.02	0.04	0.07	0.02	0.02	0.03	0.04	0.03
	4	0.20	0.08	0.06	0.19	0.10	0.06	0.08	0.09	0.11	0.20
S4	1	0.28	0.01	0.02	0.03	0.05	0.02	0.03	0.02	0.03	0.04
	4	0.52	0.02	0.07	0.12	0.03	0.05	0.13	0.14	0.10	0.09
S5	1	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
S6	1	0.54	0.22	0.37	0.26	0.37	0.36	0.42	0.51	0.57	0.37
	4	0.25	0.15	0.41	0.37	0.35	0.58	0.48	0.54	0.60	0.60

Results of the fundamentalness test. Each entry of the table reports the p -value of the F -test in a regression of the news shock estimated using specifications $S1$ to $S6$ on 1 and 4 lags of the first differences of the first j principal components, $j = 1, \dots, 10$. The news shock is identified as the only shock with a non-zero effect on TFP in the long run.

Identifying news shocks

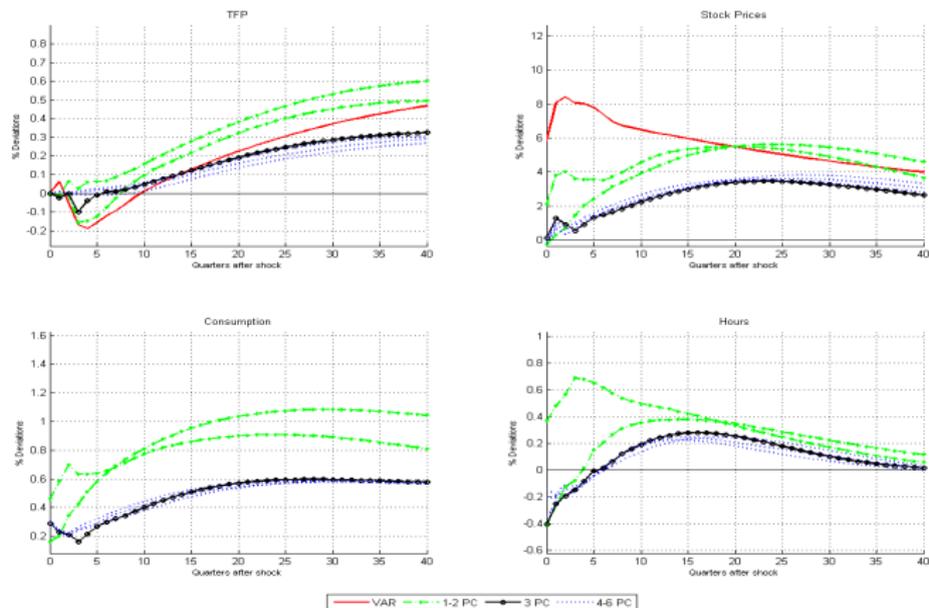
- ▶ We study the effects of news shocks with a FAVAR model.
- ▶ As in Beaudry and Portier (2006) stock prices and TFP are treated as observable factors.
- ▶ The news shock is identified by assuming that
 1. does not have a contemporaneous impact on TFP;
 2. has a maximal effect on the level of TFP at the 60-quarter horizon.
- ▶ We also identify a standard technology shock by assuming that
 1. is the only shock that affects TFP on impact.

How many factors?

FAVAR with h factors	Principal components (from $h + 1$ to j)									
	1	2	3	4	5	6	7	8	9	10
0	0.12	0.30	0.07	0.02	0.04	0.04	0.02	0.04	0.06	0.06
1	-	0.17	0.04	0.08	0.10	0.15	0.19	0.20	0.02	0.01
2	-	-	0.79	0.96	0.96	0.97	0.79	0.47	0.16	0.04
3	-	-	-	0.49	0.46	0.56	0.65	0.52	0.60	0.69
4	-	-	-	-	0.95	0.99	0.98	0.82	0.91	0.95
5	-	-	-	-	-	0.71	0.84	0.75	0.87	0.92
6	-	-	-	-	-	-	0.76	0.40	0.59	0.74
7	-	-	-	-	-	-	-	0.10	0.26	0.43
8	-	-	-	-	-	-	-	-	0.59	0.72
9	-	-	-	-	-	-	-	-	-	0.63

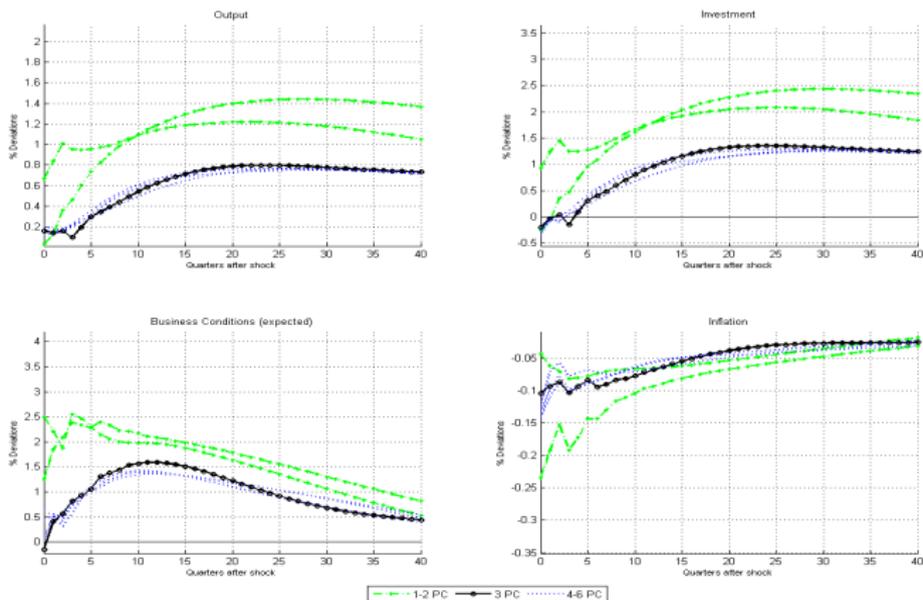
Results of the test for the number of principal components to be included in FAVAR models in specification S1. Each entry of the table reports the p -value of the F -test in a regression of the news shock on two lags of the principal components from the $h + 1$ -th to j -th, $j = h + 1, \dots, 10$. The news shock is estimated from a FAVAR with h principal components; it is identified as the shock that does not move TFP on impact and has maximal effect on TFP at horizon 60.

The effects of enlarging the information set



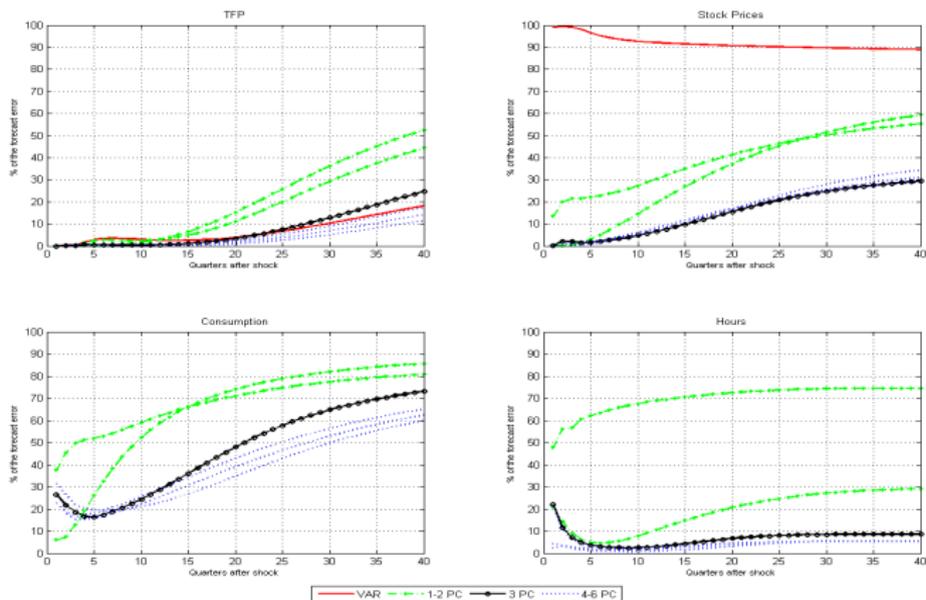
Impulse response functions to a news shock. Solid (only in the upper boxes): VAR, specification S1. Dash-dotted: FAVAR 1-2 principal components. Solid with circles: FAVAR, 3 principal components. Dashed: FAVAR, 4-6 principal components.

The effects of enlarging the information set



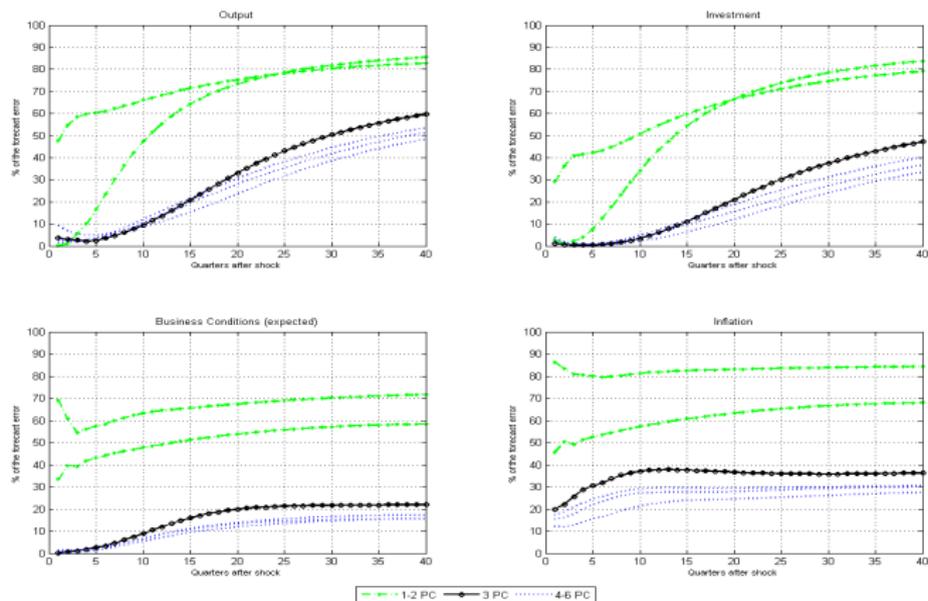
Impulse response functions to a news shock (continued). Dash-dotted: FAVAR 1-2 principal components. Solid with circles: FAVAR, 3 principal components. Dashed: FAVAR, 4-6 principal components.

The effects of enlarging the information set



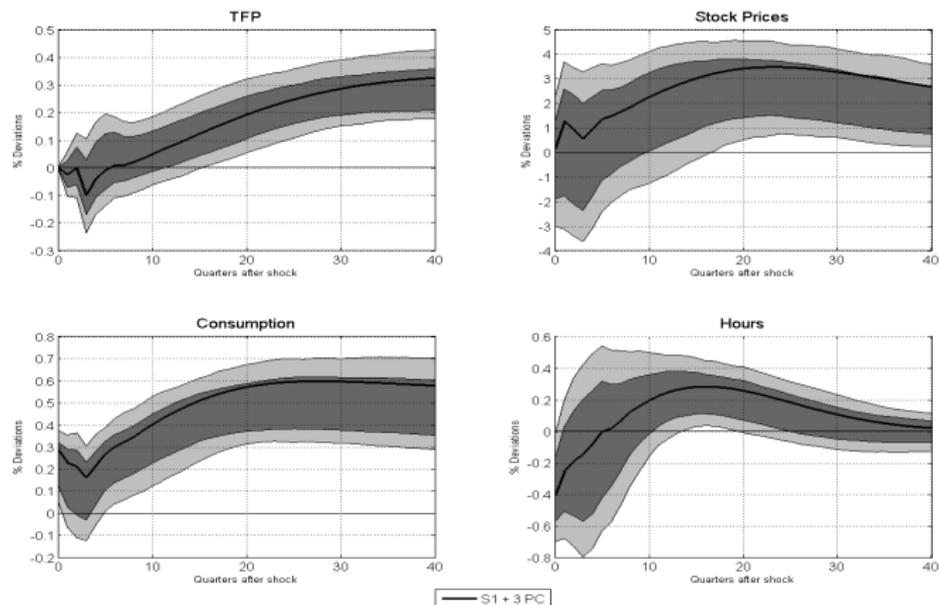
Variance decomposition for the news shock. Solid (only in the upper boxes): VAR, specification S1. Dash-dotted: FAVAR 1-2 principal components. Solid with circles: FAVAR, 3 principal components. Dashed: FAVAR, 4-6 principal components.

The effects of enlarging the information set



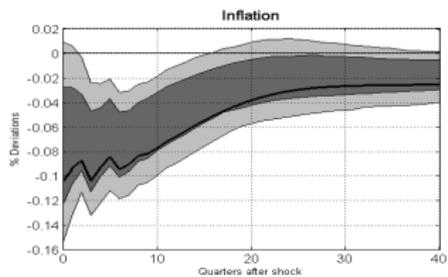
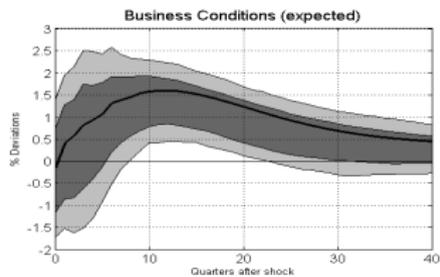
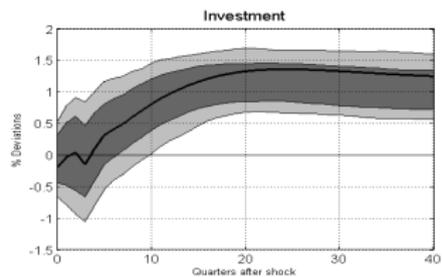
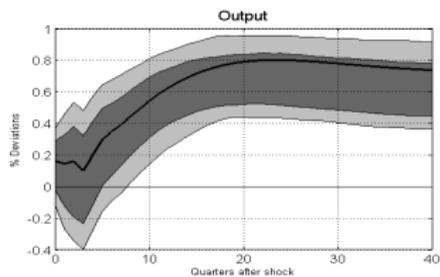
Variance decomposition for the news shock. Dash-dotted: FAVAR 1-2 principal components. Solid with circles: FAVAR, 3 principal components. Dashed: FAVAR, 4-6 principal components.

The effects of news shocks



Impulse response functions to a news shock. Solid: FAVAR model, specification S1 + 3 principal components. Dark gray areas denote 68% confidence intervals. Light gray areas denote 90% confidence intervals.

The effects of news shocks



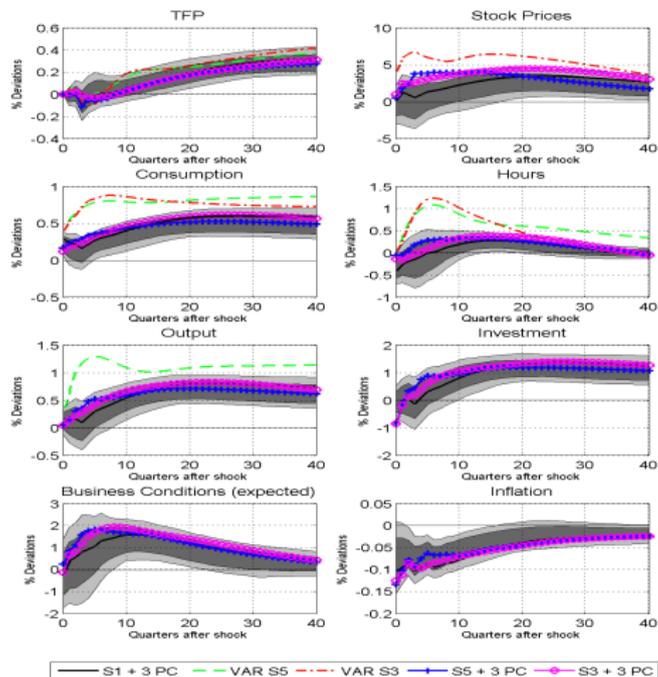
Impulse response functions to a news shock (continued). Solid: FAVAR model, specification S1 + 3 principal components. Dark gray areas denote 68% confidence intervals. Light gray areas denote 90% confidence intervals.

Variance decomposition

Variables	Horizons						BC freq.
	0	4	8	16	24	40	
TFP adj. [93]	0 (0)	0.6 (2.2)	0.4 (2.8)	1.8 (4.6)	7.5 (8.4)	25.8 (12.2)	15.1 (8.1)
Stock Prices [96]	0.0 (13.3)	1.7 (13.1)	4.0 (13.3)	12.0 (14.2)	20.8 (14.6)	29.7 (14.6)	8.6 (11.5)
Consumption [11]	26.5 (15.9)	16.4 (13.7)	22.4 (15.0)	40.9 (18.0)	57.8 (17.2)	73.7 (14.3)	43.2 (12.8)
Hours [26]	22.1 (16.5)	3.8 (9.9)	2.5 (8.9)	5.4 (8.3)	8.1 (8.4)	8.6 (8.3)	4.8 (9.6)
Output [5]	3.7 (7.9)	2.5 (9.4)	7.7 (11.6)	25.7 (14.7)	43.0 (14.4)	60.4 (13.1)	20.6 (9.7)
Investment [7]	1.1 (5.1)	0.3 (8.1)	2.5 (9.6)	14.9 (13.3)	30.2 (14.1)	47.9 (13.8)	16.5 (9.4)
Business Condition [104]	0.2 (7.8)	2.6 (10.5)	7.5 (11.7)	17.9 (11.7)	21.3 (11.0)	22.1 (11.0)	4.4 (8.8)
CPI Inflation [71]	19.9 (15.1)	30.6 (16.0)	36.2 (15.2)	37.2 (13.7)	36.0 (13.2)	36.4 (13.1)	20.3 (11.5)

Variance decomposition to a news shock. Columns 2-7: fraction of the variance of the forecast error at different horizon. Column 8: fraction of the variance at business cycle frequencies (between 2 and 8 years). It is obtained as the ratio of the integral of the spectrum computed using the impulse response functions of the news shock to the integral of the spectrum at frequencies corresponding to 6 to 32 quarters. Numbers in brackets are standard deviations across bootstrap simulations. Numbers in square brackets correspond to the series in the data appendix.

Robustness

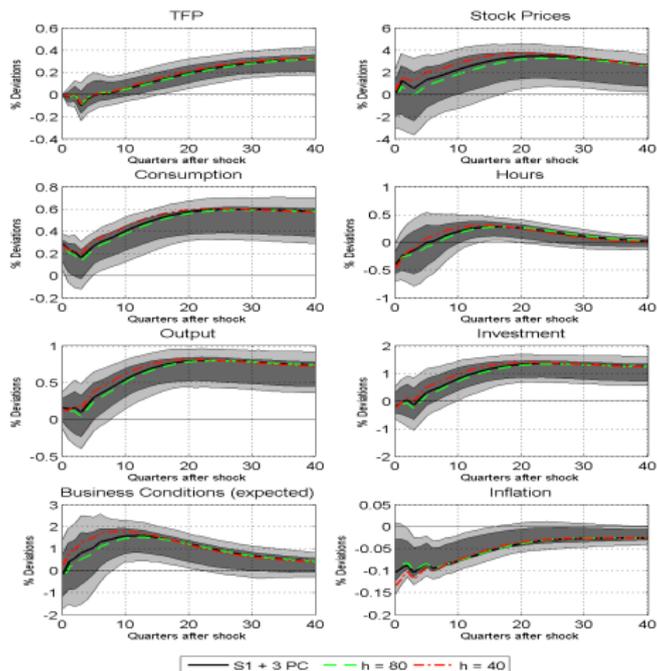


Impulse response functions to a news shock. Solid: Benchmark specification S1+3. Dark gray areas denote 68% confidence intervals.

Light gray areas denote 90% confidence intervals. Dotted: VAR S5. Dash-Dotted: VAR S3. Solid with point: FAVAR specification S5+3.

Solid with circles: FAVAR specification S3+3.

Robustness



Impulse response functions to a news shock. Solid: Benchmark specification $S1+3$. Dark gray areas denote 68% confidence intervals. Light gray areas denote 90% confidence intervals. Dotted: maximization horizon 80 quarters. Dash-Dotted: maximization horizon 40 quarters.

Application: Fiscal foresight

- ▶ Problem with Blanchard and Perotti identification: the shock is predictable (Ramey, 2011).
- ▶ Suppose the growth rate of government spending is

$$g_t = \phi_1 \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2}$$

with $\phi_2 > \phi_1$ (fiscal foresight: g_t increases slowly).

- ▶ Expectation is

$$E_t g_{t+1} = \phi_1 \varepsilon_t + \phi_2 \varepsilon_{t-1}$$

- ▶ But the process is not invertible.
- ▶ However the forecast revision gives the shock

$$E_t g_{t+1} - E_{t-1} g_{t+1} = \phi_1 \varepsilon_t$$

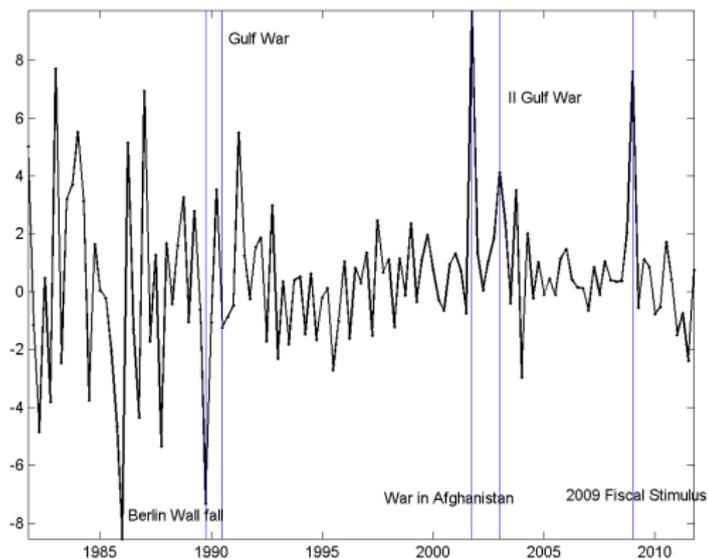
Application: Fiscal foresight

- ▶ Construct the agent's innovation set.
- ▶ Idea: changes in expectations of future government spending.
- ▶ Let $\hat{g}_{t+q|t}$ forecast of growth rate of government spending from period $t + q - 1$ to period $t + q$ using the information at time t .
- ▶ The measured forecast revision is

$$\hat{n}_t(1, 3) = \sum_{j=1}^3 (\hat{g}_{t+j|t} - \hat{g}_{t+j|t-1}). \quad (20)$$

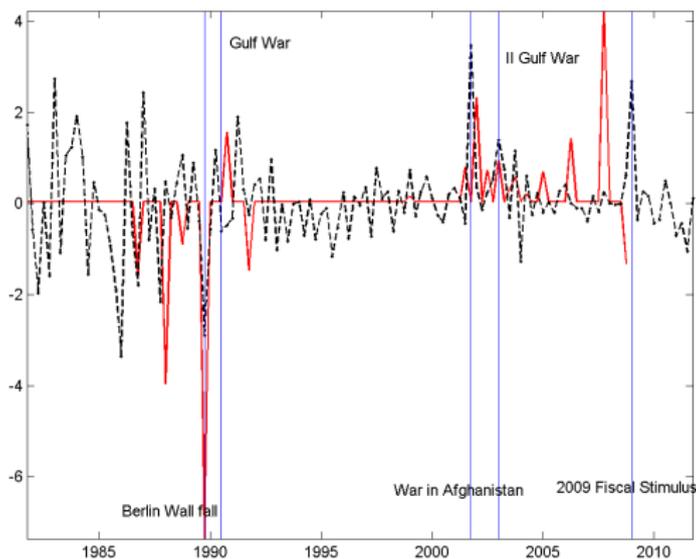
- ▶ The variable is the difference between two sums:
 - ▶ The first is the sum of the 1,2 and 3-period ahead forecasts made at time t , i.e. the growth rate of government spending from period $t + 1$ to $t + 3$ predicted in t .
 - ▶ The second is the sum of the 2,3 and 4-period ahead forecasts made at time $t - 1$, i.e. the growth rate of government spending from period $t + 1$ to $t + 3$ predicted in $t - 1$.

Application: Fiscal foresight



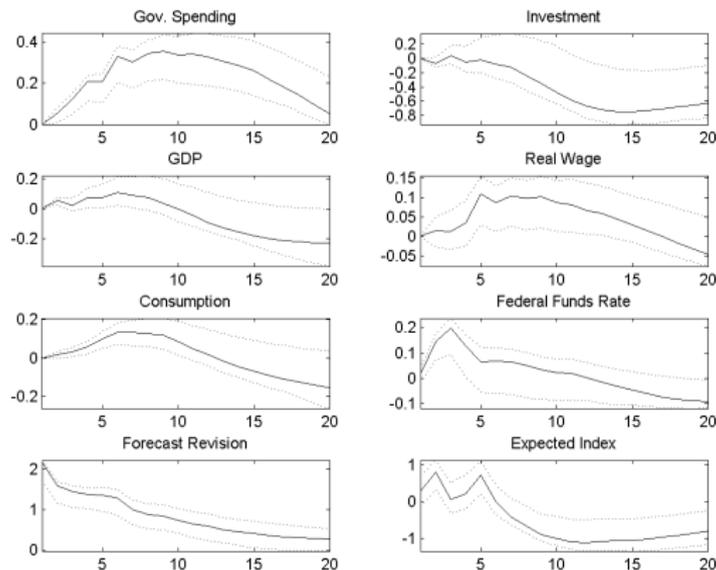
Plot of $\hat{n}_t(1, 3)$. The vertical lines are associated to the following episodes: fall of the Berlin Wall (1989:IV); the beginning of the Gulf War (1990:III); the beginning of the War in Afghanistan (2001:IV); the beginning of the Iraq War (2003:I); the approval of the Obama's fiscal stimulus package (2009:I).

Application: Fiscal foresight



Ramey's variable (solid line) and $\hat{n}_t(1, 3)$ (dashed-pointed line). The vertical lines are associated to the following episodes: fall of the Berlin Wall (1989:IV); the beginning of the Gulf War (1990:III); the beginning of the War in Afghanistan (2001:IV); the beginning of the Iraq War (2003:I); the approval of the Obama's fiscal stimulus package (2009:I).

Application: Fiscal foresight



Impulse response functions to an anticipated government spending shock in BENCHVAR. BENCHVAR includes, in that order, the logs of real government spending, real GDP, real consumption and the cumulated sum of the forecast revision of the growth rate of government spending. The shock is the last of the Cholesky decomposition. Solid lines are point estimates, dotted line are 68% confidence bands.

Lecture 4: Factor models and FAVARs

The Factor Model

Forni, Giannone, Lippi and Reichlin (Econometric Theory 2009). Let us assume

$$x_t = Af_t + \xi_t, \quad (21)$$

$$D(L)f_t = \epsilon_t \quad (22)$$

$$\epsilon_t = Ru_t$$

where

- ▶ x_t – a vector containing the n variables of the panel.
- ▶ Af_t – the common component.
- ▶ f_t – a vector containing $r < n$ unobserved factors.
- ▶ u_t – a vector containing $q < r$ structural macro shocks.
- ▶ R – a $r \times q$ matrix of coefficients.
- ▶ $D(L)$ – a $r \times r$ matrix of polynomials in the lag operator.
- ▶ ξ_t – a vector of n idiosyncratic components (orthogonal to the common one, poorly correlated in the cross-sectional dimension).

The Factor Model

From (1)-(2) We can derive the dynamic representation of the model (in terms of structural shocks)

$$x_t = B(L)u_t + \xi_t \quad (23)$$

where $B(L) = AD(L)^{-1}R$ – a $n \times q$ matrix of impulse response functions to structural shocks.

Notice that the fact that $q < r$ makes $D(L)^{-1}$ a rectangular where the conditions for fundamentalness are those described below.

Identification

- ▶ $B(L)$ is identified up to an orthogonal ($q \times q$) matrix H (such that $HH' = I$) since $B(L)u_t = C(L)v_t$ where $B(L) = C(L)H$ and $v_t = H'u_t$.
- ▶ In this context identification consists in imposing economically-based restrictions on $B(L)$ to determine a particular H . This is the same as in VAR but restriction can be imposed on a $n \times q$ matrix of responses.
- ▶ In practice, given a matrix of nonstructural impulse response functions $C(L)$ obtained as described in the estimation one has to choosing H by imposing some restrictions on $B(L)$.
- ▶ Same types of restrictions used in VAR: Cholesky, long run, signs etc.

Consistent estimator of impulse response functions

- ▶ $\hat{\Gamma}^x$ the sample variance-covariance matrix of the data. Loadings $\hat{A} = (\hat{a}'_1 \hat{a}'_2 \cdots \hat{a}'_n)'$ is the $n \times r$ matrix having on the columns the normalized eigenvectors corresponding to the first largest \hat{r} eigenvalues of $\hat{\Gamma}^x$. Factors are $\hat{f}'_t = \hat{A}'(x_{1t} x_{2t} \cdots x_{nt})'$.
- ▶ VAR(p) for \hat{f}_t gives $\hat{D}(L)$.
- ▶ $\hat{\Gamma}^\epsilon$ the sample variance-covariance matrix of $\hat{\epsilon}_t$ $\hat{\mu}_j^\epsilon$ eigenvalue. \hat{M} the $q \times q$ diagonal matrix with $\sqrt{\hat{\mu}_j^\epsilon}$ as its (j, j) entry, \hat{K} the $r \times q$ matrix with the corresponding normalized eigenvectors on the columns.

$$\hat{C}(L) = \hat{A} \hat{D}(L)^{-1} \hat{K} \hat{M}. \quad (24)$$

- ▶ Finally, \hat{H} and $\hat{b}_i(L) = \hat{c}_i(L) \hat{H}$ $i = 1, \dots, n$ are obtained by imposing the identification restrictions on

$$\hat{B}(L) = \hat{C}(L) \hat{H}. \quad (25)$$

Why is the factor model fundamental?

- ▶ Recall the Rozanov condition for fundamentalness.
- ▶ Assume that the n -dimensional stochastic vector μ_t admits a moving average representation

$$\mu_t = K(L)v_t$$

where $K(L)$ is a $n \times q$ ($q \leq n$) polynomial matrix and v_t is a $q \times 1$ white noise.

- ▶ The above representation is fundamental if and only if the rank of $K(L)$ is q for all z such that $|z| < 1$.

Why is the factor model fundamental?

- ▶ In the case $n > q$ the condition is violated if and only if all the $q \times q$ submatrices of $K(L)$ share a common root smaller than one in modulus.
- ▶ Going back to the fiscal foresight example. The full system is fundamental

$$\begin{pmatrix} a_t \\ k_t \\ \tau_t \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{-\kappa(L+\theta)}{1-\alpha L} & \frac{1}{1-\alpha L} \\ L^2 & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_{\tau,t} \\ \varepsilon_{A,t} \end{pmatrix} \quad (26)$$

Why is the factor model fundamental?

Consider again the state space representation

$$\begin{aligned}x_t &= [DB^{-1} - (DB^{-1}A - C)L] s_t \\ &= [DB^{-1} - (DB^{-1}A - C)L] (I - AL)^{-1} Bu_t\end{aligned}\quad (27)$$

If $n > r$ then the representation is always fundamental and a (reduced rank) VAR representation always exists.

Why is the factor model fundamental?

Example: to get the intuition of how large information can mitigate the nonfundamentalness problem consider the two MA

$$X_t = (1 + 2L)\varepsilon_t,$$

$$Y_t = L\varepsilon_t$$

both are nonfundamental because the absolute root in the first is 0.5 and in the second is 0.

However the process

$$Z_t = X_t - 2Y_t = \varepsilon_t$$

is obviously fundamental.

Inference

Confidence bands are obtained by a standard non-overlapping block bootstrap technique.

- ▶ Let $X = [x_{it}]$ be the $T \times n$ matrix of data. Such matrix is partitioned into S sub-matrices X_s (blocks), $s = 1, \dots, S$, of dimension $\tau \times n$, τ being the integer part of T/S .
- ▶ An integer h_s between 1 and S is drawn randomly with reintroduction S times to obtain the sequence h_1, \dots, h_S .
- ▶ A new artificial sample of dimension $\tau S \times n$ is then generated as $X^* = [X'_{h_1} X'_{h_2} \dots X'_{h_S}]'$ and the corresponding impulse response functions are estimated.
- ▶ A distribution of impulse response functions is obtained by repeating drawing and estimation.

Determination of the number of factors

There are criteria available for the determination of the number of both static and dynamic factors.

- ▶ **# of static factors** r Bai and Ng (2002) proposes consistent criteria. The most common one is the $IC_{p2}(r)$. r should be chosen in order to minimize

$$IC_{p2}(r) = \ln V(r, \hat{f}_t) + r \left(\frac{n+T}{nT} \right) \ln(\text{Min}(n, T))$$

where $V(r, \hat{f}_t)$ is the sum of residuals (divided by (nT)) from the regression of x_i on the r factors for all i ,

$$V(r, \hat{f}_t) = \min_A \sum_{i=1}^N \sum_{t=1}^T (x_{it} - A_i^r f_t^r)^2$$

- ▶ **# of dynamic factors** q
 - ▶ Bai and Ng (2007) based on the rank of the residual covariance matrix.
 - ▶ Amengual and Watson (2008). Regress x on \hat{f}_t and apply Bai and Ng (2002) to the new obtained residuals to study the number of dynamic factors.

An application: Forni and Gambetti (2010, JME)

- ▶ Motivation: standard theory of monetary policy predicts that after a contractionary policy shock:
 - ▶ Prices fall
 - ▶ The real exchange rate immediately appreciates and then depreciates (overshooting theory)
- ▶ With VAR puzzling results:
 - ▶ Prices increase (price puzzle)
 - ▶ The real exchange rate appreciates with a long delay (delayed overshooting puzzle)
- ▶ Here: we study the effects of monetary policy shocks within a SFM.
- ▶ Why: information could be the key.
- ▶ Main result: both of them solved IRF behave like theory predicts.

An application: Forni and Gambetti (2010, JME)

- ▶ Data: 112 US monthly series from March 1973 to November 2007. Most series are those of the Stock-Watson, we added a few real exchange rates and short-term interest rate spreads between US and some foreign countries.
- ▶ The monetary policy shock is identified by the following assumptions:
 1. the monetary policy shock is orthogonal to all other structural shocks,
 2. the monetary policy shock has no contemporaneous effect on prices and output (Cholesky scheme).

An application: Forni and Gambetti (2010, JME)

Table 1: Variance decomposition SVAR (*)

	k=0	k=6	k=12	k=48
Ind. production	0 (0)	0.0361 (0.0634)	0.1129 (0.1388)	0.3062 (0.1737)
CPI	0 (0)	0.0483 (0.0300)	0.0461 (0.0364)	0.0170 (0.0358)
Federal funds rate	0.9209 (0.0205)	0.5435 (0.0182)	0.3996 (0.0208)	0.1854 (0.0322)
Swi/US real ER	0.0275 0.0313	0.0685 (0.0420)	0.0923 (0.0497)	0.1434 (0.0607)

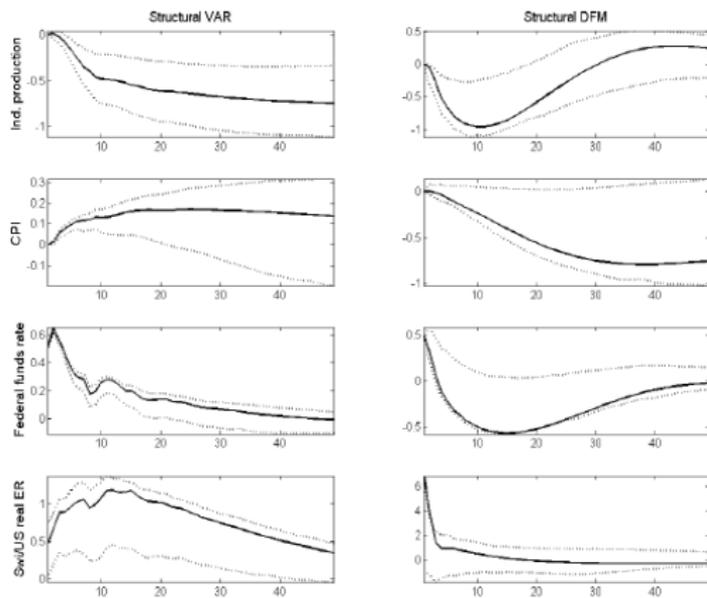
(*) Months after the shocks on the columns.

Table 2: Variance decomposition SDFM (*)

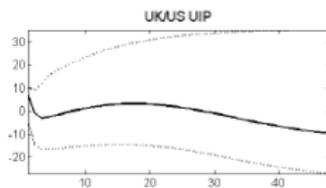
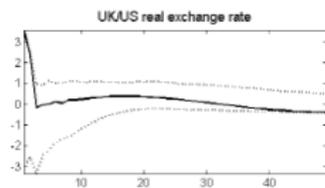
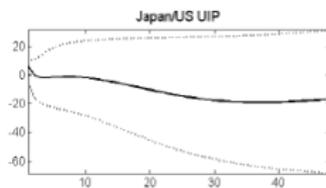
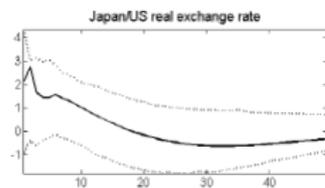
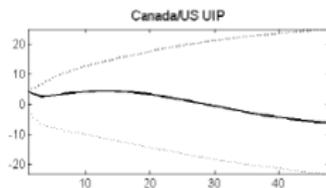
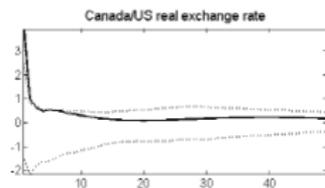
	k=0	k=6	k=12	k=48
Ind. production	0 (0)	0.0657 (0.0465)	0.1299 (0.0674)	0.1346 (0.0710)
CPI	0 (0)	0.0057 (0.0243)	0.0333 (0.0608)	0.1634 (0.1679)
Federal funds rate	0.5345 (0.2335)	0.1463 (0.2036)	0.1986 (0.1676)	0.2989 (0.1575)
Swi/US real ER	0.5227 (0.2704)	0.4330 (0.2123)	0.4041 (0.2028)	0.3836 (0.1666)
Can/US real ER	0.7541 (0.2605)	0.3474 (0.1825)	0.2523 (0.1794)	0.1643 (0.1580)
Jap/US real ER	0.1885 (0.2897)	0.2371 (0.2101)	0.2092 (0.2013)	0.1746 (0.1765)
UK/US real ER	0.2313 (0.2165)	0.1463 (0.1841)	0.1227 (0.1795)	0.1200 (0.1543)

(*) Months after the shocks on the columns.

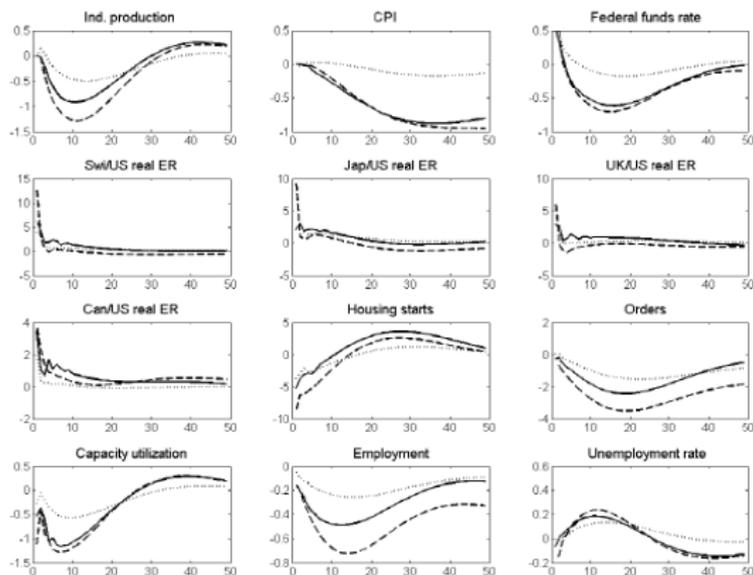
An application: Forni and Gambetti (2010, JME)



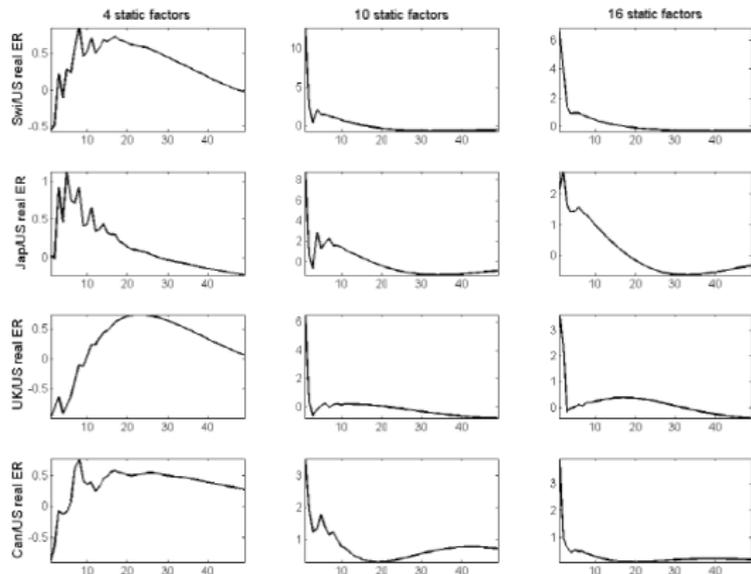
An application: Forni and Gambetti (2010, JME)



An application: Forni and Gambetti (2010, JME)



An application: Forni and Gambetti (2010, JME)



FAVAR

- ▶ Similar to factor models.
- ▶ Two main differences:
 1. Same number of dynamic and static factors $q = r$.
 2. Possibility of including observed factors in the VAR for the factors.

FAVAR and Monetary policy shocks - BBE

- ▶ Bernanke Boivin and Elias (2002) use a FAVAR model to study the effects of a monetary policy shock.
- ▶ x_t consists of a panel of 120 monthly macroeconomic time series. The data span from January 1959 through August 2001.
- ▶ The federal funds rate is the only observable factor.
- ▶ The model is estimated with 13 lags.
- ▶ 3 and 5 unobservable factors are used.
- ▶ Identification of the monetary policy shock similar to CEE

FAVAR and Monetary policy shocks - BBE

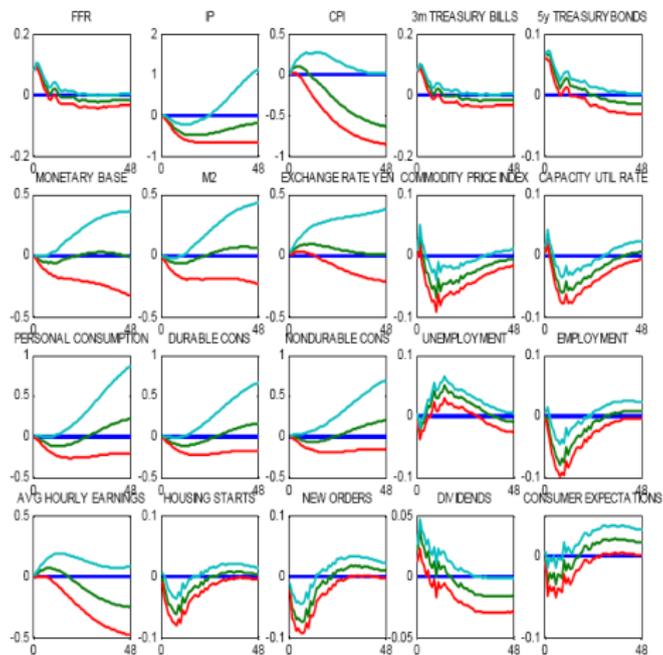


Figure 1. Impulse responses generated from FAVAR with 3 factors and FFR estimated by principal components with 2 step bootstrap.

FAVAR and Monetary policy shocks - BBE

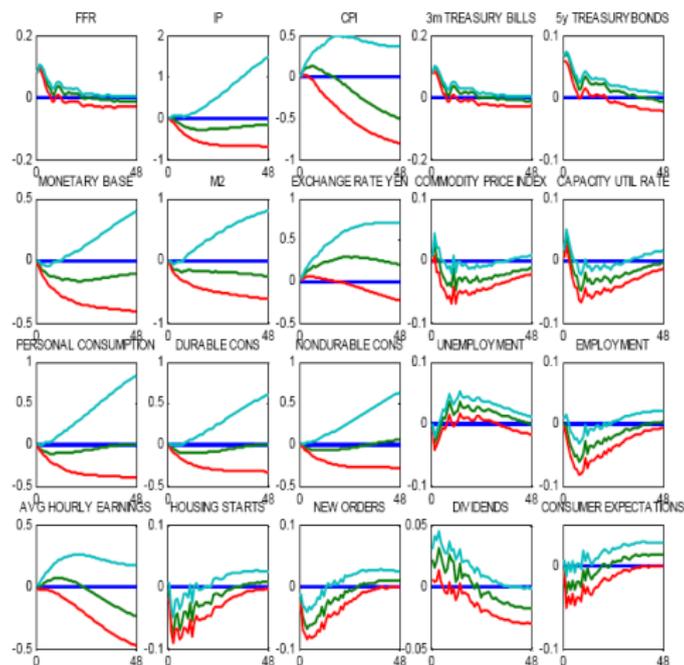


Figure 3. Impulse responses generated from FAVAR with 5 factors and FFR estimated by principal components with 2 step bootstrap.

FAVAR and Monetary policy shocks - BBE

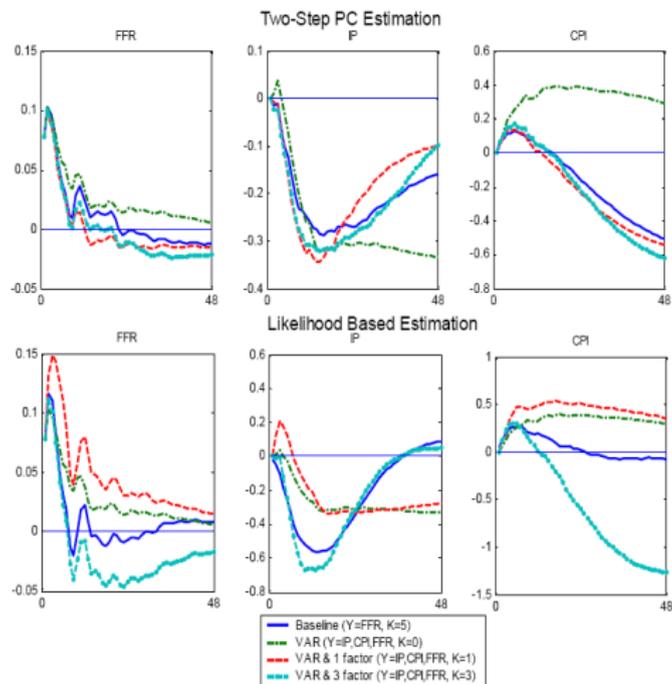


Figure 5. VAR – FAVAR comparison. The top panel displays estimated responses for the two-step principal component estimation and the bottom panel for the likelihood based estimation.

FAVAR and Monetary policy shocks - BBE

Table 1. Contribution of the policy shock to variance of the common component

Variables	Variance Decomposition	R ²
Federal funds rate	0.4538	*1.0000
Industrial production	0.0763	0.7074
Consumer price index	0.0441	0.8699
3-month treasury bill	0.4440	0.9751
5-year bond	0.4354	0.9250
Monetary Base	0.0500	0.1039
M2	0.1035	0.0518
Exchange rate (Yen/\$)	0.2816	0.0252
Commodity price Index	0.0750	0.6518
Capacity utilization	0.1328	0.7533
Personal consumption	0.0535	0.1076
Durable consumption	0.0850	0.0616
Non-durable cons.	0.0327	0.0621
Unemployment	0.1263	0.8168
Employment	0.0934	0.7073
Aver. Hourly Earnings	0.0965	0.0721
Housing Starts	0.0816	0.3872
New Orders	0.1291	0.6236
S&P dividend yield	0.1136	0.5486
Consumer Expectations	0.0514	0.7005

The column entitled "Variance Decomposition" reports the fraction of the variance of the forecast error of the common component, at the 60-month horizon, explained by the policy shock. "R²" refers to the fraction of the variance of the variable explained by the common factors, (\hat{F}_t, Y_t) . See text for details.

*This is by construction.