Lecture 1: Information and Structural VARs

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Introductio

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This type of analysis is important to understand the transmission mechanisms and to provide policymakers with information about the consequences of their policies.
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... do you remember these models?
Introduction: SVAR

Main idea
Introduction: SVAR

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Introduction: SVAR

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▶ The economy is driven by exogenous structural shocks.

▶ The shocks are dynamically propagated on the economy through the impulse response functions, coefficients which are the outcome of agents decisions.

▶ The economy is the result of these exogenous shocks plus the response of economic agents.
Introduction: SVAR

More formally

Vector of economic variables: $\mathbf{Y}_t$.

Structural economic shocks: $\mathbf{u}_t$, white noise ($E(\mathbf{u}_t - \mathbf{i} \mathbf{u}_t' - \mathbf{j}) = \Sigma$ for $i = j$ and $E(\mathbf{u}_t - \mathbf{i} \mathbf{u}_t' - \mathbf{j}) = 0$, for $j \neq j$).

Impulse response functions: $B(L) = B_0 + B_1 L + B_2 L^2 + \ldots$, where $B_j$ are matrix of coefficients and $L$ is the lag operator ($L \mathbf{X}_t = \mathbf{X}_t - 1$).

So the economy can be written as $\mathbf{Y}_t = B(L) \mathbf{u}_t$
Introduction: SVAR

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Introduction: SVAR

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The problem is that $u_t$ is not observed it must be estimated or identified.

... do you remember how it works?
Introduction: SVAR

- If $B(L)$ is invertible then $Y_t$ can be approximated by means of a VAR model.

\[ A(L) Y_t = e_t \]

or

\[ Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \ldots + A_p Y_{t-p} + e_t \] (1)
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- The model can be consistently estimated by OLS equation by equation. This provides an estimates of the residuals $\hat{e}_t$ and the parameters $\hat{A}_j$. 

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By taking the appropriate combination of the reduced form residuals we get the structural shocks $u_t = B_0^{-1} e_t$ and the structural IRF $A(L)^{-1} B_0$. 
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- Appropriate means that the elements of $B_0$ must be such that $u_t$ is the vector of shocks we are looking for.
Introduction: SVAR

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- The monetary policy shock is identified by the following assumptions:
  1. it does not affect contemporaneously GDP and the CPI;
  2. the interest rate (systematic policy) reacts to fluctuations in GDP and CPI contemporaneously.
Introduction: SVAR

To implement the identification

▶ Estimate a VAR with the variables in that order and estimate $\Sigma$.
▶ Take the Cholesky decomposition, $S$, of $\Sigma$ that is the unique lower triangular matrix such that $SS' = \Sigma$.
▶ The Cholesky impulse response functions are $A(L)^{-1}S$ and the Cholesky shocks are $S^{-1}e_t$.
▶ The monetary policy shock is the third one. The associated IRFs are the third column of $A(L)^{-1}S$. 
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Introduction: SVAR

Cholesky impulse response functions of a system with GDP inflation and the federal funds rate. Monetary shock is in the third column.
Introduction: SVAR

so, what can go wrong?
The problem

Economic agents take their decisions based on large information sets. For instance, central banks typically monitor a large amount of series. SVAR econometricians estimate models with a much reduced number of series (previous example). That means that the information set of the econometrician is likely to be narrower than that of the agents. What are the implications?
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- In this situation SVAR model cannot in general consistently estimate the impulse response functions of the structural shocks and the structural shocks.

Why? The present and past values of the series considered are not enough informative.

Intuition: the innovation of the econometrician does not coincide with that of the agents.

The problem is also known as nonfundamentalness (see Hansen and Sargent, 1991, Lippi and Reichlin, 1993).

Formally speaking in this case the MA representation in terms of structural shocks of the series is not invertible.

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- More recently it has been the focus of several research contributions because there are many relevant economic cases that can give rise to nonfundamentalness.

...let us see some examples.
Examples (I): Price puzzle

- We noticed before that prices increase after a contractionary monetary policy shock.
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- Sims (1992) shows that once a commodity price index is included in the VAR prices fall.

- The reason is that the an important variable that signals future inflation pressures is omitted from the econometrician information set.
Examples (II): Fiscal foresight

First of all it is instructive to review the definition of fundamentalness.
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Assume that the $n$-dimensional stochastic vector $\mu_t$ admits a moving average representation

$$\mu_t = K(L)v_t$$

where $K(L)$ is a $n \times q$ ($q \leq n$) polynomial matrix and $v_t$ is a $q \times 1$ white noise.
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If $n = q$ (the SVAR case) if and only if the roots of the determinant of $K(L)$ are larger than one in absolute value.
Examples (II): Fiscal foresight

Example: suppose

\[ K(L) = \begin{pmatrix} \theta - L & 0 \\ \theta & 1 \end{pmatrix} \]

the determinant \((\theta - L)\) is zero for \(L = \theta\). The representation is fundamental iff \(\theta \geq 1\)
Examples (II): Fiscal foresight

- Recent works argued that fiscal policy actions are anticipated (see e.g. Yang, 2008, Leeper, Walker and Yang, 2008, Mertens and Ravn, 2010).
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- The reason is the existence of legislative and implementation lags: it takes time for a policy action to be passed and implemented.

- The phenomenon is called "fiscal foresight".
Examples (II): Fiscal foresight

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... so let’s see the example in detail.
Examples (II): Fiscal foresight

The model is a standard growth model. Log-linearized equilibrium solution for the three state variables of the model is

\[ k_t = \alpha k_{t-1} + a_t - \kappa \sum_{i=0}^{\infty} \theta^i E_t \tau_{t+i+1} \]

\[ a_t = \varepsilon_{A,t} \]

\[ \tau_t = \varepsilon_{\tau,t-q} \]

where \( k_t \) is capital, \( a_t \) is the technology shock and \( \tau_t \) are taxes, \( \varepsilon_{\tau,t-q}, \varepsilon_{A,t} \) are i.i.d. shocks to taxes and technology, \( \theta = \alpha \beta (1 - \tau) < 1 \), \( \kappa = (1 - \theta)(\tau/(1 - \tau)) \), where \( 0 < \alpha < 1 \) is the \( 0 < \beta < 1 \), and \( 0 \leq \tau < 1 \) is the steady state tax rate, \( q \) is the period of foresight.

Suppose that \( q = 2 \). The capital transition equation becomes

\[ k_t = \alpha k_{t-1} + a_t - \kappa (\varepsilon_{\tau,t-1} + \theta \varepsilon_{\tau,t}) \]
Examples (II): Fiscal foresight

Suppose the econometrician wants to use data for capital and technology to estimate a VAR in order to identify the fiscal shock. The solution of the model for the two variables is

\[
\begin{pmatrix}
a_t \\
k_t
\end{pmatrix} = \begin{pmatrix}
0 & 1 \\
-\kappa(L+\theta) & 1 - \alpha L
\end{pmatrix} \begin{pmatrix}
\varepsilon_{\tau,t} \\
\varepsilon_{A,t}
\end{pmatrix} = A_1(L)\varepsilon_t \tag{2}
\]

The determinant of $A_1(z)$ is $\frac{\kappa(z+\theta)}{1-\alpha z}$ which is zero for $z = -\theta < 1$. This implies that the shock cannot be recovered using a VAR with data for capital and technology.
Examples (II): Fiscal foresight

Now suppose that the econometrician decides to use data for capital and taxes. The solution of the model for the two variables is

\[
\begin{pmatrix}
k_t \\
\tau_t
\end{pmatrix} = \begin{pmatrix}
-\kappa(L+\theta) \\
1-\alpha L
\end{pmatrix} \begin{pmatrix}
1-\alpha L \\
0
\end{pmatrix} \begin{pmatrix}
\epsilon_{\tau,t} \\
\epsilon_{A,t}
\end{pmatrix} = A_2(L)\epsilon_t
\]  

(3)

The determinant of \(A_1(z)\) is \(\frac{z^2}{1-\alpha z}\) which is zero for \(z = 0\) meaning that the MA representation is non-invertible and the shock non-fundamental for \(\tau_t\) and \(k_t\).

Again the shock cannot be recovered using a VAR with data for capital and taxes.

If the shocks are nonfundamental then SVAR models are not useful for structural analysis.
Consider the simple Lucas tree model. The agent maximizes

$$E_t \sum_{t=0}^{\infty} \beta^t c_t,$$

where $c_t$ is consumption and $\beta$ is a discount factor, subject to the constraint

$$c_t + p_t s_{t+1} = (p_t + \theta_t)s_t,$$

where $p_t$ is the price of a share, $s_t$ is the number of shares and $(p_t + \theta_t)s_t$ is the total amount of resources available at time $t$.

TFP evolves as

$$\theta_t = \theta_{t-1} + \varepsilon_{t-2} + u_t$$

$\varepsilon_t$: news shock - $u_t$: TFP shock.
Examples (III): News shocks

The equilibrium value for asset prices is given by:

\[ p_t = E_t \sum_{j=1}^{\infty} \beta^j \theta_{t+j} \]

We have

\[ E_t \theta_{t+1} = \theta_t + \varepsilon_{t-1}, \]
\[ E_t \theta_{t+j} = \theta_t + \varepsilon_{t-1} + \varepsilon_t, \quad \text{for } j \geq 2, \]

so that the price equation reads

\[ p_t = \frac{\beta}{1 - \beta} \theta_t + \frac{\beta}{1 - \beta} (\beta\varepsilon_t + \varepsilon_{t-1}). \]
Examples (III): News shocks

Taking first differences we get the following structural MA representation

\[
\begin{pmatrix}
\Delta \theta_t \\
\Delta p_t
\end{pmatrix}
= \begin{pmatrix}
\frac{L^2}{1-\beta} + \beta L & \frac{1}{1-\beta} \\
\end{pmatrix}
\begin{pmatrix}
\varepsilon_t \\
u_t
\end{pmatrix}.
\]  

(4)
Examples (III): News shocks

The determinant is

$$- \frac{\beta^2}{1 - \beta} - \beta z + \frac{\beta}{1 - \beta} z^2$$

which vanishes for $z = 1$ (cointegrated variables) and $z = -\beta$.

As $\beta < 1$, the two shocks $u_t$ and $\varepsilon_t$ are non-fundamental for the variables $\Delta P_t$ and $\Delta \theta_t$.

Here the agents see the shocks. The econometrician only see the variables. In this case not even a very forward-looking variable like stock prices conveys enough information to recover the shock.
As an alternative explanation, the joint dynamics of \( \theta_t \) and \( p_t \) can be represented in state-space form as

\[
\begin{pmatrix}
\theta_t \\
\varepsilon_t \\
\varepsilon_{t-1}
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 1 \\
0 & 0 & 0 \\
0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
\theta_{t-1} \\
\varepsilon_{t-1} \\
\varepsilon_{t-2}
\end{pmatrix} +
\begin{pmatrix}
0 & 1 \\
1 & 0 \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
\varepsilon_t \\
u_t
\end{pmatrix}
\tag{5}
\]

\[
\begin{pmatrix}
\theta_t \\
p_t
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 1 \\
\delta & \delta & \delta
\end{pmatrix}
\begin{pmatrix}
\theta_{t-1} \\
\varepsilon_{t-1} \\
\varepsilon_{t-2}
\end{pmatrix} +
\begin{pmatrix}
0 & 1 \\
\delta \beta & \delta
\end{pmatrix}
\begin{pmatrix}
\varepsilon_t \\
u_t
\end{pmatrix}.
\tag{6}
\]

where \( \delta = \beta / (1 - \beta) \).
In fact the model for $\theta_t$ and $p_t$ is a VARMA with a nonfundamental MA component

$$
\begin{pmatrix}
\theta_t \\
p_t
\end{pmatrix}
= 
\begin{pmatrix}
1 & 0 \\
\delta & 0
\end{pmatrix}
\begin{pmatrix}
\theta_{t-1} \\
p_{t-1}
\end{pmatrix}
+ 
\begin{pmatrix}
L^2 & 1 \\
\delta \beta + \delta L + \delta L^2 & \delta
\end{pmatrix}
\begin{pmatrix}
\varepsilon_t \\
u_t
\end{pmatrix}.
$$

The root of the determinant of the MA component vanishes for $-\beta$. 

$$
(7)
$$
Noninvertibility and the state space: News shocks

- The structural shocks can be obtained as the residuals of a VAR on the state variables.
Noninvertibility and the state space: News shocks

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- Unfortunately, the state vector includes $\varepsilon_t$ and $\varepsilon_{t-1}$, which are not observable.
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Unfortunately, the state vector includes $\varepsilon_t$ and $\varepsilon_{t-1}$, which are not observable.

By observing $p_t$ the econometrician can obtain some information about the missing states but cannot tell apart $\varepsilon_t$ and $\varepsilon_{t-1}$. 
Suppose the economy is representable with the state-space representation

\[ s_t = A s_{t-1} + B u_t \] (8)
\[ x_t = C s_{t-1} + D u_t \] (9)

where
Noninvertibility and the state space: Fernandez-Villaverde et al (AER 2011)

Suppose the economy is representable with the state-space representation

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\[ x_t = C s_{t-1} + D u_t \]  \hspace{1cm} (9)

where

- \( s_t \) is an \( r \)-dimensional vector of stationary state variables,
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- \( s_t \) is an \( r \)-dimensional vector of stationary state variables,
- \( q \leq r \leq n \), \( A \), \( B \), \( C \) and \( D \) are conformable matrices of parameters,
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where

- \( s_t \) is an \( r \)-dimensional vector of stationary state variables,
- \( q \leq r \leq n \), \( A \), \( B \), \( C \) and \( D \) are conformable matrices of parameters,
- \( B \) has a left inverse \( B^{-1} \) such that \( B^{-1} B = I_q \).
Noninvertibility and the state space: Fernandez-Villaverde et al (AER 2011)

Notice that

\[ u_t = B^{-1}s_t - B^{-1}A_{t-1}. \]

Substituting in \( x_t \) we have

\[ x_t = Cs_{t-1} + Du_t \]
\[ = Cs_{t-1} + DB^{-1}s_t - DB^{-1}A_{t-1} \]
\[ = [DB^{-1} - (DB^{-1}A - C)L] s_t \]

In the square case \( q = n \) we have

\[ x_t = DB^{-1} [I - (A - BD^{-1}C)L] s_t \]
Noninvertibility and the state space: Fernandez-Villaverde et al (AER 2011)

- \( s_t \) and therefore the shocks can be recovered as a square summable combination of the present and past of \( y_t \) iff the eigenvalues of \((A - BD^{-1}C)\) are strictly less than one in modulus.
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$$x_t = \sum_{j=0}^{\infty} (A - BD^{-1}C)^j BD^{-1}x_{t-j} + Du_t$$
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- Useful only for theoretical model but no implications for empirical analysis.
In many cases VARs are likely to suffer of an informational problem with serious consequences.
So...

the key question is: is there a way to know whether a set of variable is informative enough to estimate the structural shocks of interest?
and the answer is...

yes.
In the remainder of the lecture we will

- study a procedure to test whether a set of variables is sufficiently informative to estimate the structural shocks;
- study a solution in the case where a VAR cannot be used;
- see two applications, one about technology shocks and the labor market; one about news shocks.
Remainder of the lecture

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Sufficient information in Structural VARs

In the next slides we study the testing procedure proposed in Forni and Gambetti (2011) "Sufficient information in Structural VARs".
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- Proofs of the propositions and other detail can be found in the paper.
The macroeconomy

The macroeconomy has the following MA representation.

\[ x_t = F(L)u_t, \quad (10) \]

- \( u_t \) is a \( q \)-dimensional vector of structural macroeconomic shocks
- \( F(L) \) is an \( n \times q \) matrix of impulse response functions.

Equation (10) can be thought of as the representation of a macroeconomic equilibrium (Villaverde, Rubio-Ramirez, Sargent and Watson (2007)). Equation (10) is typically a 'tall' system, i.e. the number of variables on the LHS is larger than the number of shocks on the RHS.
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(10) is typically a 'tall' system, i.e. the number of variables on the LHS is larger than the number of shocks on the RHS.
The SVAR econometrician observes $x_t$, possibly with error, i.e. it observes

$$x_t^* = x_t + \xi_t = F(L)u_t + \xi_t,$$

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where $\xi_t$ is a (possibly zero) vector of errors, orthogonal to $u_{jt-k}$, $j = 1, \ldots, q$, any $k$. 

Econometrician’s and VAR information sets
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In practice however the econometrician needs to summarize the information. The VAR information set is then spanned by an $s$-dimensional subvector of $x_t^*$, or more, generally, an $s$-dimensional linear combination of $x_t^*$, say

$$z_t^* = Wx_t^*$$

(12)
Informational sufficiency

Now, consider the theoretical projection equation of $z_t^*$ on its past history, i.e.

$$z_t^* = P(z_t^* | Z_{t-1}^*) + \epsilon_t.$$  \hspace{1cm} (13)
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The SVAR methodology consists in (a) estimating a VAR to get $\epsilon_t$; (b) attempting to get the structural shocks as linear combinations of the estimated entries of $\epsilon_t$. 

**Informational sufficiency**

Let $v_t$ be any subvector of $u_t$. We say that $z_t^*$ and the related set $Z_t^*$ is "informationally sufficient for $v_t$" if and only if there exist a matrix $M$ such that $v_t = M\epsilon_t$. $z_t^*$ is "informationally sufficient" if it is informationally sufficient for $u_t$. 

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A testable implication

**Proposition 2.** If \( x_t^* \) Granger causes \( z_t^* \), then \( z_t^* \) is not informationally sufficient.

The intuition is that nothing can Granger cause variables which contain all of the relevant macroeconomic information. Proposition 2 is derived (within somewhat different settings) in Forni and Reichlin (1996) and Giannone and Reichlin (2006). If the econometrician believes that a given variable or vector in \( x_t^* \), say \( y_t^* \), may convey relevant information, he/she can check whether \( y_t^* \) Granger causes \( z_t^* \).
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Limitations of Proposition 2

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Clearly, testing all of the variables in \( x_t^* \) would be close to a validation, but unfortunately this is not feasible, since in practice \( x_t^* \) is of high dimension.

On the one hand, we cannot use all of the variables simultaneously; on the other hand, testing each one of them separately would yield with very high probability to reject informational sufficiency even if \( z_t^* \) it is, owing to Type I error.
Assuming the factor model

We can provide a sufficient condition by assuming the state-space representation in Villaverde, Rubio-Ramirez, Sargent and Watson (2007), i.e.

\[ s_t = A s_{t-1} + B u_t \]  
\[ x_t = C s_{t-1} + D u_t \]

(14)  
(15)

where

- \( s_t \) is an \( r \)-dimensional vector of stationary state variables,
- \( q \leq r \leq n \), \( A \), \( B \), \( C \) and \( D \) are conformable matrices of parameters,
- \( B \) has a left inverse \( B^{-1} \) such that \( B^{-1}B = I_q \).
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Assuming the factor model

It can be seen that the model has a factor model representation. Indeed

\[ u_t = B^{-1}s_t - B^{-1}A s_{t-1}. \]  \hspace{1cm} (16)

Substituting into the states

\[ x_t = DB^{-1}s_t + (C - DB^{-1}A) s_{t-1}. \]  \hspace{1cm} (17)

Therefore \( x_t^* \) has the factor representation

\[ x_t^* = G f_t + \xi_t, \]  \hspace{1cm} (18)

where the \( G = (DB^{-1} \hspace{1cm} C - DB^{-1}A) \) and \( f_t = (s_t' \hspace{1cm} s_{t-1}')' \). Structural shocks are fundamental for the factors \( f_t \).
A necessary and sufficient condition

**Proposition 3.** $z_t^*$ is informationally sufficient if and only if $f_t$ does not Granger cause $z_t^*$.
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Proposition 3 implies that we can summarize the information in the large dimensional vector \( x_t^* \) into a relatively small number of factors (the entries of \( f_t \)).

Such factors are unobservable, but can be consistently estimated by the principal components of \( x_t^* \) (Stock and Watson, 2002).
Testing for sufficient information

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1. Take a large data set $x_t^*$, capturing all of the relevant macroeconomic information.

2. Set a maximum number of factors $P$ and compute the first $P$ principal components of $x_t^*$.

3. Perform Granger causation tests to see whether the first $h$ principal components, $h = 1, \ldots, P$, Granger cause $z_t^*$. If the null of no Granger causality is never rejected, $z_t^*$ is informationally sufficient. Otherwise, sufficiency is rejected.
Structuralness of a single shock

Global sufficiency is needed to recover all of the structural shocks.
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But the econometrician is often interested in identifying just a single shock within a VAR model. To this end, we propose a less demanding test.

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\begin{align*}
  z^*_t &= u_{1t} + u_{2t-1} \\
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But the econometrician is often interested in identifying just a single shock within a VAR model. To this end, we propose a less demanding test.

The following example shows that, even if global sufficiency does not hold, $z^*_t$ can be sufficient for a single shock:

$$z^*_t = u_{1t} + u_{2t-1} \quad (19)$$

$$z^*_t = u_{1t} - u_{2t-1}. \quad (20)$$

In this case $z^*_t$ is not sufficient for $u_t$ but is sufficient for $u_{1t}$, since $z^*_t + z^*_t = 2u_{1t}$. 
The orthogonality test

Clearly the structural shock $u_{jt}$ is orthogonal to $f_{t-k}$, $k > 0$. 
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Conversely, if a linear combination of the VAR residuals is orthogonal to the past of the factors, then it is a linear combination of the structural shocks.
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**Proposition 4.** Let $\nu_t = \alpha' \epsilon_t$, $\alpha \in \mathbb{R}^s$. If $z_t^*$ is free of measurement error, i.e. $z_t^* = z_t$, and $\nu_t$ is orthogonal to $f_{t-k}$, $k > 0$, then $\nu_t$ is a linear combination of the structural shocks.
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Orthogonality does not guarantee that such linear combination be the desired shock; it will, only if identification is correct.

After having identified our shock of interest, we can therefore test whether it can be a structural shock by testing for orthogonality with respect to the lags of the principal components.
What should we do if informational sufficiency or structuralness is rejected?

1. To use directly a factor model.
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2. To add the principal components $\hat{f}_t$ to the VAR information set and estimate a FAVAR (Bernanke et al., 2005) with $w_t = \left(z_t^* \quad \hat{f}_t\right)'$. 
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2. To add the principal components $\hat{f}_t$ to the VAR information set and estimate a FAVAR (Bernanke et al., 2005) with $w_t = \left( z_t^* \quad \hat{f}_t \right)'$. Remember: these models provide an estimate of the state which is the key variables.
Now we will see an empirical application of the methods studied above.

Question: Do technology shocks explain aggregate fluctuations?

The empirical evidence is mixed. In his seminal paper, Gali (1999) finds a very modest role for technology shocks as a source of economic fluctuations. On the contrary, other authors, see for instance Christiano, Eichenbaum and Vigfusson (2003) and Beaudry and Portier (2006), provide evidence that technology shocks are capable of generating sizable fluctuations in macroeconomic aggregates.

Most of the existing evidence about the effects of technology shocks is obtained using small-scale VAR models.
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To begin, we test for informational sufficiency of $z_t^*$.

We find that sufficiency is rejected. To get sufficiency, we have to augment the VAR with 9 principal components.
Results: the Granger-causality test

<table>
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Table 1: p-values of the out-of-sample Granger causality test for global sufficient information. \( w_t^h \) is the original VAR vector \( z_t^* \), augmented with \( h \) principal components. \( j \) refers to the number of principal components used in the test.
Testing for orthogonality of the estimated shock

- As a validation, we check for structuralness of the estimated technology shock.
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- Specifically we run a regression of the estimated shock on the past of the principal components and we perform an F-test of the null hypothesis that the coefficients are jointly zero.

- We find that the hypothesis is strongly rejected for $z^*_t$, but cannot be rejected for $w^9_t$. 
Next we study the consequences of insufficient information in terms of impulse response functions.
Impulse-response functions

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- We investigate how the predicted effects of technology shocks change by augmenting the original VAR with one, two, ..., sixteen principal components.
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Figure 1 shows the impulse response functions. The left column plots the impulse response functions for total factor productivity and unemployment, for all the sixteen specification.
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The right column displays for the three variables the effects on impact (blue-points) at 1 year (red-crosses) 2 year (magenta-circles) and in the long run (green-diamonds). In the horizontal axis we put the number of principal components included in the VAR. The first value refers to the VAR without principal components.
Figure 1: Impulse response functions
Comment for Figure 1

- The VAR without principal components predicts that the technology shock reduces unemployment, as predicted by standard RBC models. Total factor productivity reacts positively on impact and stays roughly constant afterward.

- The picture changes dramatically when adding the principal components. The effect of the technology shock on the unemployment rate becomes positive. Moreover, the impact effect on TFP reduces substantially, so that the diffusion process is much slower, in line with the S-shape view.

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Figure 2: Additional impulse response functions
Comment for Figure 2

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Amending the VAR with forward-looking variables

▶ As a further exercise, we try to amend the VAR by adding suitable variables.

Natural candidates are forward-looking variables such as stock prices and consumer confidence indicators.

We focus on the real Standard & Poor's 500 index, (in log differences), and a component of the Michigan University consumer confidence index, i.e. Business Conditions expected during the next 5 years.

We test for orthogonality of the estimated shock with the lags of the principal components.

Table 2 shows results for different specifications, including one or both of the above variables. The two specifications including the survey variable are not rejected.
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## Results: the orthogonality test

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<th>VAR3</th>
<th>VAR4</th>
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<td>0.22</td>
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Table 2: p-values of the orthogonality test for different VAR specifications. VAR 1: TFP, Unemployment rate. VAR 2: TFP, Unemployment rate and the S&P500 stock price index. VAR 3: TFP, Unemployment rate and Business conditions expected during the next 5 years. VAR 4: TFP, Unemployment rate, S&P500, Business conditions expected during the next 5 years.
Figure 3: Impulse response functions
Figure 3 shows the impulse response functions obtained with all four specifications.
Comment for Figure 3

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- The impulse response functions of the rejected specifications VAR1 and VAR2 (lines 1 and 2, respectively) are similar to each other.
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- For both specifications, the unemployment rate exhibits on impact a significant positive reaction.
Application: News and business cycles, Beaudry and Portier (2006 AER)
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- Main idea back to Pigou: news about future productivity growth can generate business cycles since agents react to news by investing and consuming.
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Two identification procedures:

1. Technology shocks is the only shock driving TFP in the long run.
2. News shocks raise stock prices on impact but not TFP (lagged adjustment.
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Main finding:
Application: News and business cycle, Beaudry and Portier (2006 AER)

Main finding:
- the two identified shocks are the same;
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Conclusion: news shocks can generate business cycles.
Application: News and business cycle, Beaudry and Portier (2006 AER)

Source: Beaudry and Portier (AER 2006)
Application: News and business cycle, Beaudry and Portier (2006 AER)

Figure 2. Plot of $\varepsilon_2$ against $\tilde{\varepsilon}_1$ in the (TFP, SP) VECM

Source: Beaudry and Portier (AER 2006)
Application: News and business cycle, Beaudry and Portier (2006 AER)

Figure 9. Impulse responses to $\chi_2$ and $\delta_1$ in the (TFP, SP, C, H) VECM, without (upper panels) or with (lower panels) adjusting TFP for capacity utilization.

Source: Beaudry and Portier (AER 2006)
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Here:
- Test whether the news shock is fundamental for TFP and stock prices, i.e. are fundamental for the variable in BP.
- Estimate the shocks using a FAVAR model.
Testing for fundamentalness

Use the Forni and Gambetti (2011) orthogonality test:

1. estimate a VAR with a given set of variables \( y_t \) and identify the relevant shock, \( w_t \);
2. test for orthogonality of \( w_t \) with respect to the lags of the factors (F-test);
3. the null of fundamentalness is rejected if and only if orthogonality is rejected.

The factors are not observed and we estimate them using the principal components of a dataset composed of 107 US quarterly macroeconomic series, covering the period 1960-I to 2010-IV.
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We consider the following VAR specifications.

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<th>2-variable VAR</th>
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<td>S2 TFP Stock P</td>
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<th>4-variable VAR</th>
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<td>S4 TFP Stock P Cons Hours</td>
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<td>S5 TFP adj. Output Cons Hours</td>
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<table>
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<th>7-variable VAR</th>
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<tr>
<td>S6 TFP adj. Stock P Output Cons Hours Confidence Inflation</td>
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Testing for fundamentalness

- We apply the Forni and Gambetti test.
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- For each specification we use two identifications of the news shock:
  - The news shock is the shock that does not move TFP on impact and (for specifications from \( S_3 \) to \( S_6 \)) has maximal effect on TFP at horizon 40.
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Results of the test: Identification 1

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<td>0.00</td>
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</tr>
<tr>
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<td>0.58</td>
<td>0.66</td>
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</table>

Results of the fundamentalness test. Each entry of the table reports the \( p \)-value of the \( F \)-test in a regression of the news shock estimated using specifications \( S_1 \) to \( S_6 \) on 1 and 4 lags of the first differences of the first \( j \) principal components, \( j = 1, \ldots, 10 \). The news shock is identified as the shock that does not move TFP on impact and (for specifications from \( S_3 \) to \( S_6 \)) has maximal effect on TFP at horizon 60.
## Results of the test: Identification 2

<table>
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<td>0.01</td>
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<td>0.48</td>
<td>0.54</td>
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</table>

Results of the fundamentalness test. Each entry of the table reports the $p$-value of the $F$-test in a regression of the news shock estimated using specifications $S_1$ to $S_6$ on 1 and 4 lags of the first differences of the first $j$ principal components, $j = 1, \ldots, 10$. The news shock is identified as the only shock with a non-zero effect on TFP in the long run.
Identifying news shocks

- We study the effects of news shocks with a FAVAR model.
Identifying news shocks

- We study the effects of news shocks with a FAVAR model.

- As in Beaudry and Portier (2006) stock prices and TFP are treated as observable factors.
Identifying news shocks

- We study the effects of news shocks with a FAVAR model.
- As in Beaudry and Portier (2006) stock prices and TFP are treated as observable factors.
- The news shock is identified by assuming that
  1. does not have a contemporaneous impact on TFP;
  2. has a maximal effect on the level of TFP at the 60-quarter horizon.
- We also identify a standard technology shock by assuming that
  1. is the only shock that affects TFP on impact.
Identifying news shocks

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- We also identify a standard technology shock by assuming that
  1. is the only shock that affects TFP on impact.
How many factors?

<table>
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<tr>
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<td>0.20</td>
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<td>-</td>
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<td>0.96</td>
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<td>5</td>
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</table>

Results of the test for the number of principal components to be included in FAVAR models in specification $S_1$. Each entry of the table reports the $p$-value of the $F$-test in a regression of the news shock on two lags of the principal components from the $h + 1$-th to $j$-th, $j = h + 1, \ldots, 10$. The news shock is estimated from a FAVAR with $h$ principal components; it is identified as the shock that does not move TFP on impact and has maximal effect on TFP at horizon 60.
The effects of enlarging the information set

The effects of enlarging the information set

The effects of enlarging the information set

The effects of enlarging the information set

The effects of news shocks

Impulse response functions to a news shock. Solid: FAVAR model, specification S1 + 3 principal components. Dark gray areas denote 68% confidence intervals. Light gray areas denote 90% confidence intervals.
The effects of news shocks

Impulse response functions to a news shock (continued). Solid: FAVAR model, specification $S_1 + 3$ principal components. Dark gray areas denote 68% confidence intervals. Light gray areas denote 90% confidence intervals.
Variance decomposition

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<td>(15.1)</td>
<td>(16.0)</td>
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Variance decomposition to a news shock. Columns 2-7: fraction of the variance of the forecast error at different horizon. Column 8: fraction of the variance at business cycle frequencies (between 2 and 8 years). It is obtained as the ratio of the integral of the spectrum computed using the impulse response functions of the news shock to the integral of the spectrum at frequencies corresponding to 6 to 32 quarters. Numbers in brackets are standard deviations across bootstrap simulations. Numbers in square brackets correspond to the series in the data appendix.
Robustness

The Factor Model

Forni, Giannone, Lippi and Reichlin (Econometric Theory 2009). Let us assume

\[ x_t = Af_t + \xi_t, \]  
\[ D(L)f_t = \epsilon_t \]  
\[ \epsilon_t = Ru_t \]

where

- \( x_t \) — a vector containing the \( n \) variables of the panel.
- \( Af_t \) — the common component.
- \( f_t \) — a vector containing \( r < n \) unobserved factors.
- \( u_t \) — a vector containing \( q < r \) structural macro shocks.
- \( R \) — a \( r \times q \) matrix of coefficients.
- \( D(L) \) — a \( r \times r \) matrix of polynomials in the lag operator.
- \( \xi_t \) — a vector of \( n \) idiosynchratic components (orthogonal to the common one, poorly correlated in the cross-sectional dimension.)
The Factor Model

From (1)-(2) We can derive the dynamic representation of the model (in terms of structural shocks)

\[ x_t = B(L)u_t + \xi_t \] (23)

where \( B(L) = AD(L)^{-1}R \) — a \( n \times q \) matrix of impulse response functions to structural shocks.

Notice that the fact that \( q < r \) makes \( D(L)^{-1} \) a rectangular where the conditions for fundamentalness are those described below.
Why is the factor model fundamental?

- Intuition: it uses a lot of information for a reduced number of shocks.
Why is the factor model fundamental?

- **Intuition**: it uses a lot of information for a reduced number of shocks.

- **Mathematical reason**: consider the common component
  \[
  \chi_t = B(L)u_t
  \]

  the system is fundamental \(\iff\) \(\text{rank}B(z) = q, \forall z, \text{s.t. } |z| < 1\).

  It is a rectangular, “tall” system: \(n\) variables, \(q\) shocks.

  In order to be non-fundamental, all the square \(q \times q\) submatrices must have reduced rank for a given \(z\), s.t. \(|z| < 1\).

  If there is enough heterogeneity in the dataset then factor models are fundamental.
Why is the factor model fundamental?

Example: to get the intuition of how large information can mitigate the nonfundamentalness problem consider the two MA

\[ X_t = (1 + 2L)\varepsilon_t, \]
\[ Y_t = L\varepsilon_t \]

both are nonfundamental because the absolute root in the first is 0.5 and in the second is 0.

However the process

\[ Z_t = X_t - 2Y_t = \varepsilon_t \]

is obviously fundamental.
Identification

\( B(L) \) is identified up to an orthogonal \((q \times q)\) matrix \( H \) (such that \( HH' = I \)) since \( B(L)u_t = C(L)v_t \) where \( B(L) = C(L)H \) and \( v_t = H' u_t \).
Identification

$B(L)$ is identified up to an orthogonal ($q \times q$) matrix $H$ (such that $HH' = I$) since $B(L)u_t = C(L)v_t$ where $B(L) = C(L)H$ and $v_t = H'u_t$.

In this context identification consists in imposing economically-based restrictions on $B(L)$ to determine a particular $H$. This is the same as in VAR but restriction can be imposed on a $n \times q$ matrix of responses.
Identification

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In practice, given a matrix of nonstructural impulse response functions \( \hat{C}(L) \) obtained as described in the estimation one has to choosing \( H \) by imposing some restrictions on \( B(L) \).
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In practice, given a matrix of nonstructural impulse response functions \( \hat{C}(L) \) obtained as described in the estimation one has to choosing \( H \) by imposing some restrictions on \( B(L) \).

Same types of restrictions used in VAR: Cholesky, long run, signs etc.
Consistent estimator of impulse response functions

- $\hat{\Gamma}$ the sample variance-covariance matrix of the data. Loadings $\hat{A} = (\hat{a}_1 \hat{a}_2 \cdots \hat{a}_n)'$ is the $n \times r$ matrix having on the columns the normalized eigenvectors corresponding to the first largest $\hat{r}$ eigenvalues of $\hat{\Gamma}$. Factors are $\hat{f}_t = \hat{A}'(x_{1t}x_{2t} \cdots x_{nt})'$.
Consistent estimator of impulse response functions

- $\hat{\Gamma}^x$ the sample variance-covariance matrix of the data. Loadings $\hat{A} = (\hat{a}_1 \hat{a}_2 \cdots \hat{a}_n)'$ is the $n \times r$ matrix having on the columns the normalized eigenvectors corresponding to the first largest $\hat{r}$ eigenvalues of $\hat{\Gamma}^x$. Factors are $\hat{f}_t = \hat{A}'(x_{1t}x_{2t} \cdots x_{nt})'$.

- $VAR(p)$ for $\hat{f}_t$ gives $\hat{D}(L)$.
Consistent estimator of impulse response functions

- \( \hat{\Gamma}^x \) the sample variance-covariance matrix of the data. Loadings \( \hat{A} = (\hat{a}_1 \hat{a}_2 \cdots \hat{a}_n)' \) is the \( n \times r \) matrix having on the columns the normalized eigenvectors corresponding to the first largest \( \hat{r} \) eigenvalues of \( \hat{\Gamma}^x \). Factors are \( \hat{f}_t = \hat{A}'(x_{1t}x_{2t} \cdots x_{nt})' \).

- \( \text{VAR}(p) \) for \( \hat{f}_t \) gives \( \hat{D}(L) \).

- \( \hat{\Gamma}^\epsilon \) the sample variance-covariance matrix of \( \hat{\epsilon}_t \) \( \hat{\mu}_{\hat{\epsilon}} \) eigenvalue. \( \hat{M} \) the \( q \times q \) diagonal matrix with \( \sqrt{\hat{\mu}_{\hat{\epsilon}}} \) as its \((j,j)\) entry, \( \hat{K} \) the \( r \times q \) matrix with the corresponding normalized eigenvectors on the columns.

\[
\hat{C}(L) = \hat{A}\hat{D}(L)^{-1}\hat{K}\hat{M}.
\] (24)
Consistent estimator of impulse response functions

- $\hat{\Gamma}^x$ the sample variance-covariance matrix of the data. Loadings $\hat{A} = (\hat{a}_1' \hat{a}_2' \cdots \hat{a}_n')'$ is the $n \times r$ matrix having on the columns the normalized eigenvectors corresponding to the first largest $\hat{r}$ eigenvalues of $\hat{\Gamma}^x$. Factors are $\hat{f}_t = \hat{A}'(x_{1t} x_{2t} \cdots x_{nt})'$.

- $\text{VAR}(p)$ for $\hat{f}_t$ gives $\hat{D}(L)$.

- $\hat{\Gamma}^\epsilon$ the sample variance-covariance matrix of $\hat{\epsilon}_t$ $\hat{\mu}_j^\epsilon$ eigenvalue. $\hat{M}$ the $q \times q$ diagonal matrix with $\sqrt{\hat{\mu}_j^\epsilon}$ as its $(j, j)$ entry, $\hat{K}$ the $r \times q$ matrix with the corresponding normalized eigenvectors on the columns.

$$\hat{C}(L) = \hat{A} \hat{D}(L)^{-1} \hat{K} \hat{M}. \quad (24)$$

- Finally, $\hat{H}$ and $\hat{b}_i(L) = \hat{c}_i(L) \hat{H}$ $i = 1, \ldots, n$ are obtained by imposing the identification restrictions on

$$\hat{B}(L) = \hat{C}(L) \hat{H}. \quad (25)$$
The benefit of being tall

- Recall the Rozanov condition for fundamentalness.
The benefit of being tall

- Recall the Rozanov condition for fundamentalness.

- Assume that the $n$-dimensional stochastic vector $\mu_t$ admits a moving average representation

  $$\mu_t = K(L)\nu_t$$

  where $K(L)$ is a $n \times q$ ($q \leq n$) polynomial matrix and $\nu_t$ is a $q \times 1$ white noise.
The benefit of being tall

- Recall the Rozanov condition for fundamentalness.

- Assume that the \( n \)-dimensional stochastic vector \( \mu_t \) admits a moving average representation

\[
\mu_t = K(L) v_t
\]

where \( K(L) \) is a \( n \times q \) \((q \leq n)\) polynomial matrix and \( v_t \) is a \( q \times 1 \) white noise.

- The above representation is fundamental if and only if the rank of \( K(L) \) is \( q \) for all \( z \) such that \( |z| < 1 \).
The benefit of being tall

- In the case $n > q$ the condition is violated if and only if all the $q \times q$ submatrices of $K(L)$ share a common root smaller than one in modulus.
The benefit of being tall

▶ In the case $n > q$ the condition is violated if and only if all the $q \times q$ submatrices of $K(L)$ share a common root smaller than one in modulus.

▶ Going back the the fiscal foresight example. The full system is fundamental

\[
\begin{pmatrix}
    a_t \\
    k_t \\
    \tau_t
\end{pmatrix} = \begin{pmatrix}
    0 & 1 \\
    \frac{-\kappa(L+\theta)}{1-\alpha L} & 1 \\
    \frac{1}{L^2} & 0
\end{pmatrix} \begin{pmatrix}
    \varepsilon_{\tau,t} \\
    \varepsilon_{A,t}
\end{pmatrix}
\]  

(26)
The benefit of being tall

Consider again the state space representation

\[
    x_t = [DB^{-1} - (DB^{-1}A - C)L] s_t \\
    = [DB^{-1} - (DB^{-1}A - C)L] (I - AL)^{-1}Bu_t
\]

(27)

If \( n > r \) then the representation is always fundamental and a (reduced rank) VAR representation always exists.
FAVAR

▶ Similar to factor models.
▶ Two main differences:
  1. Same number of dynamic and static factors \( q = r \).
  2. Possibility of including observed factors in the VAR for the factors.
Bernanke Boivin and Eliasz (2002) use a FAVAR model to study the effects of a monetary policy shock.

\( x_t \) consists of a panel of 120 monthly macroeconomic time series. The data span from January 1959 through August 2001.

The federal funds rate is the only observable factor.

The model is estimated with 13 lags.

3 and 5 unobservable factors are used.

Identification of the monetary policy shock similar to CEE.
Figure 1. Impulse responses generated from FAVAR with 3 factors and FFR estimated by principal components with 2 step bootstrap.
FAVAR and Monetary policy shocks - BBE

Figure 3. Impulse responses generated from FAVAR with 5 factors and FFR estimated by principal components with 2 step bootstrap.
FAVAR and Monetary policy shocks - BBE

Figure 5. VAR – FAVAR comparison. The top panel displays estimated responses for the two-step principal component estimation and the bottom panel for the likelihood based estimation.
Table 1. Contribution of the policy shock to variance of the common component

<table>
<thead>
<tr>
<th>Variables</th>
<th>Variance Decomposition</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Federal funds rate</td>
<td>0.4538</td>
<td>*1.0000</td>
</tr>
<tr>
<td>Industrial production</td>
<td>0.0763</td>
<td>0.7074</td>
</tr>
<tr>
<td>Consumer price index</td>
<td>0.0441</td>
<td>0.8699</td>
</tr>
<tr>
<td>3-month treasury bill</td>
<td>0.4440</td>
<td>0.9751</td>
</tr>
<tr>
<td>5-year bond</td>
<td>0.4354</td>
<td>0.9250</td>
</tr>
<tr>
<td>Monetary Base</td>
<td>0.0500</td>
<td>0.1039</td>
</tr>
<tr>
<td>M2</td>
<td>0.1035</td>
<td>0.0518</td>
</tr>
<tr>
<td>Exchange rate (Yen/$)</td>
<td>0.2816</td>
<td>0.0252</td>
</tr>
<tr>
<td>Commodity price Index</td>
<td>0.0750</td>
<td>0.6518</td>
</tr>
<tr>
<td>Capacity utilization</td>
<td>0.1328</td>
<td>0.7533</td>
</tr>
<tr>
<td>Personal consumption</td>
<td>0.0535</td>
<td>0.1076</td>
</tr>
<tr>
<td>Durable consumption</td>
<td>0.0850</td>
<td>0.0616</td>
</tr>
<tr>
<td>Non-durable cons.</td>
<td>0.0327</td>
<td>0.0621</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.1263</td>
<td>0.8168</td>
</tr>
<tr>
<td>Employment</td>
<td>0.0934</td>
<td>0.7073</td>
</tr>
<tr>
<td>Aver. Hourly Earnings</td>
<td>0.0965</td>
<td>0.0721</td>
</tr>
<tr>
<td>Housing Starts</td>
<td>0.0816</td>
<td>0.3872</td>
</tr>
<tr>
<td>New Orders</td>
<td>0.1291</td>
<td>0.6236</td>
</tr>
<tr>
<td>S&amp;P dividend yield</td>
<td>0.1136</td>
<td>0.5486</td>
</tr>
<tr>
<td>Consumer Expectations</td>
<td>0.0514</td>
<td>0.7005</td>
</tr>
</tbody>
</table>

The column entitled “Variance Decomposition” reports the fraction of the variance of the forecast error of the common component, at the 60-month horizon, explained by the policy shock. “$R^2$” refers to the fraction of the variance of the variable explained by the common factors, $(\hat{F}_t, Y_t)$. See text for details.

*This is by construction.