3. Structural VARs - Theory
Structural Vector Autoregressions

Impulse response functions are interpreted under the assumption that *all the other shocks are held constant*. However in the Wold representation the shocks are not orthogonal. So the assumption is not very realistic!

This is why we need Structural VAR in order to perform policy analysis. Ideally we would like to have

1) orthogonal shock

2) shocks with economic meaning (technology, demand, labor supply, monetary policy etc.)
Statistical Orthogonalizations

There are two easy ways to orthogonalize shocks.

1) Cholesky decomposition

2) Spectral Decomposition
**Cholesky decomposition** Let us consider the matrix $\Omega$. The Cholesky factor, $S$, of $\Omega$ is defined as the unique lower triangular matrix such that $SS' = \Omega$. This implies that we can rewrite the VAR in terms of orthogonal shocks $\eta = S^{-1}\epsilon_t$ with identity covariance matrix

$$A(L)Y_t = S\eta_t$$

Impulse response to orthogonalized shocks are found from the MA representation

$$Y_t = C(L)S\eta_t$$

$$= \sum_{j=0}^{\infty} C_j S \eta_{t-j}$$

(39)

where $C_j S$ has the interpretation

$$\frac{\partial Y_{t+j}}{\partial \eta_t} = C_j S$$

(40)

That is, the row $i$, column $k$ element of $C_j S$ identifies the consequences of a unit increase in $\eta_k$ at date $t$ for the value of the $i$th variable at time $t + j$ holding all other $\eta_{-k}$ constant.
Spectral Decomposition Let $V$ and be a matrix containing the eigenvectors of $\Omega$ and $\Lambda$ a diagonal matrix with the eigenvalues of $\Omega$ on the main diagonal. Then we have that $V\Lambda V' = \Omega$. This implies that we can rewrite the VAR in terms of orthogonal shocks $\xi = (V\Lambda^{1/2})^{-1}\epsilon_t$ with identity covariance matrix

$$A(L)Y_t = V\Lambda^{1/2}\xi$$

Impulse response to orthogonalized shocks are found from the MA representation

$$Y_t = C(L)V\Lambda^{1/2}\xi_t$$

$$= \sum_{j=0}^{\infty} C_j S\eta_{t-j}$$

(41)

where $C_j V\Lambda^{1/2}$ has the interpretation

$$\frac{\partial Y_{t+j}}{\partial \xi_t} = C_j V\Lambda^{1/2}$$

(42)

That is, the row $i$, column $k$ element of $C_j V\Lambda^{1/2}$ identifies the consequences of a unit increase in $\xi_k$ at date $t$ for the value of the $i$th variable at time $t + j$ holding all other $\eta_{-k}$ constant.
**The Class of Orthonormal Representations** From the class of invertible MA representation of $Y_t$ we can derive the class of orthonormal representation, i.e. the class of representations of $Y_t$ in term of orthonormal shocks. Let $H$ any orthogonal matrix, i.e. $HH' = I$. Defining $w_t = (SH)^{-1} \epsilon_t$ we can recover the general class of the orthonormal representation of $Y_t$

$$
Y_t = C(L)SHw_t \\
= F(L)w_t
$$

where $F(L) = C(L)SH$ and $w_t \sim WN$ with

$$
E(w_tw_t') = E(HS^{-1} \epsilon_t \epsilon_t' S^{-1} H') \\
= HS^{-1} E(\epsilon_t \epsilon_t') S^{-1} H' \\
= HS^{-1} \Omega S^{-1} H' \\
= HS^{-1} SS'(S')^{-1} H' \\
= I
$$

*Problem H* can be any, so how should we choose one?
The Identification Problem

**Problem:** what is the economic interpretation of the orthogonal shocks? What is the economic information contained in the impulse response functions to orthogonal shocks?

Except for special cases not clear.

The idea is that structural economic shocks are linear combinations of the VAR innovations.

Identifying the VAR means fixing a particular matrix $H$, i.e. choosing one particular representation of $Y_t$ in order to recover the structural shocks from the VAR innovations.

In order to choose a matrix $H$ we have to fix $n(n - 1)/2$ parameters since there is a total of $n^2$ parameters and a total of $n(n + 1)/2$ restrictions implied by orthonormality.

The idea is to use economic theory in order to derive some restrictions on the effects of some shock on a particular variables to fix the remaining $n(n - 1)/2$. 
**Zero restrictions: contemporaneous restrictions**  An identification scheme based on zero contemporaneous restrictions is a scheme which imposes restrictions to zero on the matrix $F_0$, the matrix of the impact effects.

*Example.* Let us consider a bivariate VAR. We have a total of $n^2 = 4$ parameters to fix. $n(n + 1)/2 = 3$ are pinned down by the orthonormality restrictions so that there are $n(n - 1)/2 = 1$ free parameters. Suppose that the theory tells us that shock 2 has no effect on impact (contemporaneously) on $Y_1$ equal to 0, that is $F_{012} = 0$. This is the additional restriction that allows us to identify the shocks. In particular we will have the following restrictions:

$$HH' = I$$
$$S_{11}H_{12} + S_{12}H_{22} = 0$$

Since $S_{12} = 0$ the solution is $H_{11} = H_{22} = 1$ and $H_{12} = H_{21} = 0$.

A common identification scheme is the Cholesky scheme (like in this case). This implies setting $H = I$. Such an identification scheme creates a recursive contemporaneous ordering among variables since $S^{-1}$ is triangular.

This means that any variable in the vector $Y_t$ does not depend contemporaneously on the variables ordered after.

Results depend on the particular ordering of the variables.
Zero restrictions: long run restrictions An identification scheme based on zero long run restrictions is a scheme which imposes restrictions on the matrix $F(1) = F_0 + F_1 + F_2 + ...$, the matrix of the long run coefficients.

Example. Again let us consider a bivariate VAR. We have a total of $n^2 = 4$ parameters to fix. $n(n+1)/2 = 3$ are pinned down by the orthonormality restrictions so that there are $n(n-1)/2 = 1$ free parameters. Suppose that the theory tells us that shock 2 does not affect $Y_1$ in the long run, i.e. $F_{12}(1) = 0$. This is the additional restriction that allows us to identify the shocks. In particular we will have the following restrictions:

\[
 HH' = I
\]

\[
 D_{11}(1)H_{12} + D_{12}(1)H_{22} = 0
\]

where $D(1) = C(1)S$ represents the long run effects of the Cholesky shocks.
**Signs restrictions** The previous two examples yield just identification in the sense that the shock is uniquely identified, there exists a unique matrix $H$ yielding the structural shocks.

Sign identification is based on qualitative restriction involving the sign of some shocks on some variables. In this case we will have sets of consistent impulse response functions.

**Example.** Again let us consider a bivariate VAR. We have a total of $n^2 = 4$ parameters to fix. $n(n + 1)/2 = 3$ are pinned down by the orthonormality restrictions so that there are $n(n – 1)/2 = 1$ free parameters. Suppose that the theory tells us that shock 2, which is the interesting one, produce a positive effect on $Y_1$ for $k$ periods after the shock $F_{12}^j > 0$ for $j = 1, ..., k$. We will have the following restrictions:

\[
HH' = I \\
S_{11}H_{12} + S_{12}H_{22} > 0 \\
D_{j,12}H_{12} + D_{j,22}H_{22} > 0 \quad j = 1, ..., k
\]

where $D_j = C_jS$ represents the effects at horizon $j$.

In a classical statistics approach this delivers not exact identification since there can be many $H$ consistent with such a restriction. That is for each parameter of the impulse response functions we will have an admissible set of values.

Increasing the number of restrictions can be helpful in reducing the number of $H$ consistent with such restrictions.
**Partial Identification** In many cases we might be interested in identifying just a single shock and not all the \( n \) shocks.

Since the shock are orthogonal we can also partially identify the model, i.e. fix just one (or a subset of) column of \( H \). In this case what we have to do is to fix \( n - 1 \) elements of \( H \), all but one elements of a column of the identifying matrix. The additional restriction is provided by the norm of the vector equal one.

*Example* Suppose \( n = 3 \). We want to identify a single shock using the restriction that such shock has no effects on the first variable on impact a positive effect on the second variable and negative on the third variable.
Impulse Response Functions

Impulse response to identified shocks are found from the structural MA representation

\[ Y_t = C(L)SHw_t \]
\[ = \sum_{j=0}^{\infty} C_j SH w_{t-j} \]  \hspace{1cm} (43)

where \( C_j SH \) has the interpretation

\[ \frac{\partial Y_{t+j}}{\partial w_t} = C_j SH \]  \hspace{1cm} (44)

That is, the row \( i \), column \( k \) element of \( C_j SH \) identifies the consequences of a unit increase in \( w_k \) at date \( t \) for the value of the \( i \)th variable at time \( t+j \).

Confidence bands can be obtained using the bootstrapping procedure described in lecture 3. Now the additional step is that for any draw of the reduced form impulse response functions we have to implement the identification scheme adopted.
Variance Decomposition

The second type of analysis which is usually done in SVAR is the variance decomposition analysis.

The idea is to decompose the total variance of a time series into the percentages attributable to each structural shock.

Variance decomposition analysis is useful in order to address questions like "What are the sources of the business cycle?" or "Is the shock important for economic fluctuations?".
Let us consider the MA representation of an identified SVAR

\[ Y_t = F(L)w_t \]

The variance of \( Y_{it} \) is given by

\[
\text{var}(Y_{it}) = \sum_{k=1}^{n} \sum_{j=0}^{\infty} F_{ik}^{j} \text{var}(w_{kt}) \\
= \sum_{k=1}^{n} \sum_{j=0}^{\infty} F_{ik}^{j} \\
\]

where \( \sum_{j=0}^{\infty} F_{ik}^{j} \) is the variance of \( Y_{it} \) generated by the \( k \)th shock. This implies that

\[
\frac{\sum_{j=0}^{\infty} F_{ik}^{j}}{\sum_{k=1}^{n} \sum_{j=0}^{\infty} F_{ik}^{j}}
\]

is the percentage of variance of \( Y_{it} \) explained by the \( k \)th shock.
It is also possible to study the impact of the series explained by the shock at different horizons, i.e. short vs. long run. Consider the forecast error in terms of structural shocks. The horizon $h$ forecast error is given by

$$Y_{t+h} - Y_{t+h|t} = F_0 w_{t+1} + F_2 w_{t+2} + \ldots + F_k w_{t+h}$$

the variance of the forecast error is thus

$$\text{var}(Y_{t+h} - Y_{t+h|t}) = \sum_{k=1}^{n} \sum_{j=0}^{h} F_{ik}^2 \text{var}(w_{kt})$$

$$= \sum_{k=1}^{n} \sum_{j=0}^{h} F_{ik}^2$$

Thus the percentage of variance of $Y_{it}$ explained by the $k$th shock is

$$\frac{\sum_{j=0}^{h} F_{ik}^2}{\sum_{k=1}^{n} \sum_{j=0}^{h} F_{ik}^2}$$
4: Structural VARs - Applications
Monetary Policy Shocks (Christiano Eichenbaum and Evans, 1998)

Monetary policy shocks is the unexpected part of the equation for the monetary policy instrument ($S_t$).

\[ S_t = f(I_t) + w_{it}^{mp} \]

$f(I_t)$ represents the systematic response of the monetary policy to economic conditions, $I_t$ is the information set at time $t$ and $w_{it}^{mp}$ is the monetary policy shock.

The "standard" way to identify monetary policy shock is through zero contemporaneous restrictions. Using the standard trivariate monetary VAR (a simplified version of the CEE 98 VAR) including output growth, inflation and the federal funds rate we identify the monetary policy shock using the following restrictions:

1) Monetary policy shocks do not affect output within the same quarter

2) Monetary policy shocks do not affect inflation within the same quarter
These two restrictions are not sufficient to identify all the shocks but are sufficient to identify the monetary policy shock.

A simple way to implement the restrictions is to take simply the Cholesky decomposition of the variance covariance matrix in a system in which the federal funds rate is ordered last. The last column of the impulse response functions is the column of the monetary policy shock.
Cholesky impulse response functions of a system with GDP inflation and the federal funds rate. Monetary shock is in the third column.
Notice that after a monetary tightening inflation goes up which is completely counterintuitive according to the standard transmission mechanism. This phenomenon if known as the *price puzzle*. Why is this the case?

"Sims (1992) conjectured that prices appeared to rise after certain measures of a contractionary policy shock because those measures were based on specifications of \( I_t \) that did not include information about future inflation that was available to the Fed. Put differently, the conjecture is that policy shocks which are associated with substantial price puzzles are actually confounded with non-policy disturbances that signal future increases in prices." (CEE 98)

Sims shows that including commodity prices (signaling future inflation increases) may solve the puzzle.
Uhlig (2005) JME’s monetary policy shocks

Uhlig (2005 JME) proposes a very different method to identify monetary policy shocks. Instead of using zero restrictions as in CEE he uses sign restrictions.

He identifies the effects of a monetary policy shocks using restrictions which are implied by several economic models.

In particular a contractionary monetary policy shock:

1. does not increase prices for k periods after the shock

2. does not increase money or monetary aggregates (i.e. reserves) for k periods after the shock

3. does not reduce short term interest rate for k periods after the shock.
Since just one shock is identified only a column of $H$ has to be identified, say column one.

If we order the variables in vector $Y_t$ as follows: GDP inflation, money growth and the interest rate the restrictions imply $F_{i1}^k < 0$ for $i = 2, 3$ and $F_{41}^k > 0$.

In order to draw impulse response functions he applies the following algorithm:

1. He assumes that the column of $H$, $H_1$, represents the coordinate of a point uniformly distributed over the unit hypersphere (in case of bivariate VAR it represents a point in a circle). To draw such point he draws from a $N(0, I)$ and divide by the norm of the vector.

2. Compute the impulse response functions $C_jSH_1$ for $j=1,...,k$.

3. If the draw satisfies the restrictions keep it and go to 1), otherwise discard it and go to 1). Repeat 1)-3) a big number of time $L$. 
Source: What are the effects of a monetary policy shock... JME H. Uhlig (2006)
Source: What are the effects of a monetary policy shock... JME H. Uhlig (2006)
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Source: What are the effects of a monetary policy shock... JME H. Uhlig (2006)
Monetary policy and housing

Central question: how does monetary policy affects house prices?

Jarocinski and Smets (2008) addresses this question.

Strategy:

1. Estimate a VAR nine variables (including: short term interest rate, interest rate spread, housing investment share of GDP, real GDP, real consumption, real hours prices, prices, commodity price index and a money indicator.

2. Identify the monetary policy shock using the restriction that the shock does not affect prices and output contemporaneously but affect the short term interest rate, the spread and the money stock and analyze the impulse response functions.

3. Shut down the identified shock and study the counterfactual path of housing prices over time.
Source: Jarocinski and Smets (2008)
Source: Jarocinski and Smets (2008)
Table 2A
Shares of Housing Demand, Monetary Policy, and Term Spread Shocks in Variance Decompositions, DVAR

<table>
<thead>
<tr>
<th>Variable</th>
<th>Shock</th>
<th>0</th>
<th>3</th>
<th>11</th>
<th>23</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>Housing</td>
<td>0.016</td>
<td>0.034</td>
<td>0.052</td>
<td>0.062</td>
</tr>
<tr>
<td></td>
<td>Monetary policy</td>
<td>0.000</td>
<td>0.004</td>
<td>0.021</td>
<td>0.039</td>
</tr>
<tr>
<td></td>
<td>Term premium</td>
<td>0.000</td>
<td>0.003</td>
<td>0.015</td>
<td>0.020</td>
</tr>
<tr>
<td>Consumption</td>
<td>Housing</td>
<td>0.005</td>
<td>0.018</td>
<td>0.033</td>
<td>0.055</td>
</tr>
<tr>
<td></td>
<td>Monetary policy</td>
<td>0.000</td>
<td>0.003</td>
<td>0.015</td>
<td>0.029</td>
</tr>
<tr>
<td></td>
<td>Term premium</td>
<td>0.000</td>
<td>0.005</td>
<td>0.034</td>
<td>0.063</td>
</tr>
<tr>
<td>Prices</td>
<td>Housing</td>
<td>0.002</td>
<td>0.013</td>
<td>0.120</td>
<td>0.166</td>
</tr>
<tr>
<td></td>
<td>Monetary policy</td>
<td>0.000</td>
<td>0.003</td>
<td>0.014</td>
<td>0.037</td>
</tr>
<tr>
<td></td>
<td>Term premium</td>
<td>0.000</td>
<td>0.006</td>
<td>0.034</td>
<td>0.046</td>
</tr>
<tr>
<td>Housing investment</td>
<td>Housing</td>
<td>0.521</td>
<td>0.579</td>
<td>0.302</td>
<td>0.291</td>
</tr>
<tr>
<td></td>
<td>Monetary policy</td>
<td>0.000</td>
<td>0.015</td>
<td>0.175</td>
<td>0.136</td>
</tr>
<tr>
<td></td>
<td>Term premium</td>
<td>0.000</td>
<td>0.005</td>
<td>0.023</td>
<td>0.062</td>
</tr>
<tr>
<td>House prices</td>
<td>Housing</td>
<td>0.535</td>
<td>0.554</td>
<td>0.410</td>
<td>0.242</td>
</tr>
<tr>
<td></td>
<td>Monetary policy</td>
<td>0.000</td>
<td>0.010</td>
<td>0.068</td>
<td>0.083</td>
</tr>
<tr>
<td></td>
<td>Term premium</td>
<td>0.000</td>
<td>0.002</td>
<td>0.021</td>
<td>0.060</td>
</tr>
<tr>
<td>Commodity prices</td>
<td>Housing</td>
<td>0.027</td>
<td>0.028</td>
<td>0.041</td>
<td>0.085</td>
</tr>
<tr>
<td></td>
<td>Monetary Policy</td>
<td>0.000</td>
<td>0.012</td>
<td>0.167</td>
<td>0.222</td>
</tr>
<tr>
<td></td>
<td>Term premium</td>
<td>0.000</td>
<td>0.004</td>
<td>0.018</td>
<td>0.055</td>
</tr>
<tr>
<td>Interest rate</td>
<td>Housing</td>
<td>0.037</td>
<td>0.061</td>
<td>0.165</td>
<td>0.170</td>
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<tr>
<td></td>
<td>Monetary policy</td>
<td>0.752</td>
<td>0.496</td>
<td>0.192</td>
<td>0.166</td>
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<tr>
<td></td>
<td>Term premium</td>
<td>0.000</td>
<td>0.023</td>
<td>0.076</td>
<td>0.088</td>
</tr>
<tr>
<td>Spread</td>
<td>Housing</td>
<td>0.090</td>
<td>0.050</td>
<td>0.177</td>
<td>0.186</td>
</tr>
<tr>
<td></td>
<td>Monetary policy</td>
<td>0.223</td>
<td>0.303</td>
<td>0.214</td>
<td>0.206</td>
</tr>
<tr>
<td></td>
<td>Term premium</td>
<td>0.336</td>
<td>0.245</td>
<td>0.146</td>
<td>0.134</td>
</tr>
<tr>
<td>Money</td>
<td>Housing</td>
<td>0.060</td>
<td>0.044</td>
<td>0.062</td>
<td>0.099</td>
</tr>
<tr>
<td></td>
<td>Monetary policy</td>
<td>0.204</td>
<td>0.141</td>
<td>0.044</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>Term premium</td>
<td>0.013</td>
<td>0.042</td>
<td>0.129</td>
<td>0.135</td>
</tr>
</tbody>
</table>

NOTE: The reported shares are averages over the posterior distribution and relate to the (log) level variables.

Source: Jarocinski and Smets (2008)
Blanchard and Quah proposed an identification scheme based on long run restrictions.

In their model there are two shocks: an aggregate demand and an aggregate supply disturbance.

The restriction used to identify is that aggregate demand shocks have no effects on the long run levels of output, i.e. demand shocks are transitory on output. The idea behind such a restriction is the existence of a vertical aggregate supply curve.
Let us consider the following bivariate VAR

\[
\begin{pmatrix}
\Delta Y_t \\
U_t
\end{pmatrix} = \begin{pmatrix}
F_{11}(L) & F_{12}(L) \\
F_{21}(L) & F_{22}(L)
\end{pmatrix}
\begin{pmatrix}
w_t^s \\
w_t^d
\end{pmatrix}
\]

where \(Y_t\) is output, \(U_t\) is the unemployment rate and \(w_t^s, w_t^d\) are two aggregate supply and demand disturbances respectively.

The identification restriction is given by \(F_{12}(1) = 0\).

The restriction can be implemented in the following way. Let us consider the reduced form VAR

\[
\begin{pmatrix}
\Delta Y_t \\
U_t
\end{pmatrix} = \begin{pmatrix}
A_{11}(L) & A_{12}(L) \\
A_{21}(L) & A_{22}(L)
\end{pmatrix}
\begin{pmatrix}
\epsilon_{1t} \\
\epsilon_{2t}
\end{pmatrix}
\]

where \(E(\epsilon_t\epsilon'_t) = \Omega\).
Let $S = \text{chol}(A(1)\Omega A(1)')$ and $K = A(1)^{-1}S$. The identified shocks are

$$w_t = K^{-1}\epsilon_t$$

and the resulting impulse response to structural shocks are

$$F(L) = A(L)K$$

notice that the restrictions are satisfied

$$F(1) = A(1)K$$

$$= A(1)A(1)^{-1}S$$

$$= S$$

which is lower triangular implying that $F_{12}(1) = 0$. 

Moreover we have that shocks are orthogonal since

\[ KK' = A(1)^{-1} S S' A(1)^{-1} \]
\[ = A(1)^{-1} A(1) \Omega A(1)' A(1)^{-1} \]
\[ = \Omega \]

(45)

And

\[ E(w_t w'_t) = E(K^{-1} \epsilon_t \epsilon'_t K^{-1}) \]
\[ = K^{-1} \Omega K^{-1} \]
\[ = K^{-1} K K' K^{-1} \]

(46)
Source: The Dynamic Effects of Aggregate Demand and Supply Disturbances, (AER) Blanchard and Quah (1989):
**Table 2—Variance Decomposition of Output and Unemployment**

*Change in Output Growth at 1973/1974; Unemployment Detrended*

<table>
<thead>
<tr>
<th>Percentage of Variance Due to Demand:</th>
<th>Output</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Horizon (Quarters)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>99.0</td>
<td>51.9</td>
</tr>
<tr>
<td></td>
<td>(76.9, 99.7)</td>
<td>(35.8, 77.6)</td>
</tr>
<tr>
<td>2</td>
<td>99.6</td>
<td>63.9</td>
</tr>
<tr>
<td></td>
<td>(78.4, 99.9)</td>
<td>(41.8, 80.3)</td>
</tr>
<tr>
<td>3</td>
<td>99.0</td>
<td>73.8</td>
</tr>
<tr>
<td></td>
<td>(76.0, 99.6)</td>
<td>(46.2, 85.6)</td>
</tr>
<tr>
<td>4</td>
<td>97.9</td>
<td>80.2</td>
</tr>
<tr>
<td></td>
<td>(71.0, 98.9)</td>
<td>(49.7, 89.5)</td>
</tr>
<tr>
<td>8</td>
<td>81.7</td>
<td>87.3</td>
</tr>
<tr>
<td></td>
<td>(46.3, 87.0)</td>
<td>(53.6, 92.9)</td>
</tr>
<tr>
<td>12</td>
<td>67.6</td>
<td>86.2</td>
</tr>
<tr>
<td></td>
<td>(30.9, 73.9)</td>
<td>(52.9, 92.1)</td>
</tr>
<tr>
<td>40</td>
<td>39.3</td>
<td>85.6</td>
</tr>
<tr>
<td></td>
<td>(7.5, 39.3)</td>
<td>(52.6, 91.6)</td>
</tr>
</tbody>
</table>

Source: The Dynamic Effects of Aggregate Demand and Supply Disturbances, (AER) Blanchard and Quah (1989):
### TABLE 2A—VARIANCE DECOMPOSITION OF OUTPUT AND UNEMPLOYMENT
(NO DUMMY BREAK, TIME TREND IN UNEMPLOYMENT)

<table>
<thead>
<tr>
<th>Percentage of Variance Due to Demand:</th>
<th>Output</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizon (Quarters)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>83.8</td>
<td>79.7</td>
</tr>
<tr>
<td></td>
<td>(59.4, 93.9)</td>
<td>(55.3, 92.0)</td>
</tr>
<tr>
<td>2</td>
<td>87.5</td>
<td>88.2</td>
</tr>
<tr>
<td></td>
<td>(62.8, 95.4)</td>
<td>(58.9, 95.2)</td>
</tr>
<tr>
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<td>(7.4, 23.5)</td>
<td>(56.9, 88.6)</td>
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Source: The Dynamic Effects of Aggregate Demand and Supply Disturbances, (AER) Blanchard and Quah (1989):
The technology shocks and hours debate

This is a nice example of how SVAR models can be used in order to distinguish among competing models of the business cycles.

1) RBC technology important source of business cycles.

2) Other models (sticky prices) tech shocks not so important.

Response of hours worked very important in distinguish among theories

1) RBC hours increase.

2) Other hours fall
The model  Technology shock: \( z_t = z_{t-1} + \eta_t \) \( \eta_t = \) technology shock

Monetary Policy: \( m_t = m_{t-1} + \xi_t + \gamma \eta_t \) where \( \xi_t = \) monetary policy shock.

Equilibrium:

\[
\Delta x_t = \left( 1 - \frac{1}{\phi} \right) \Delta \xi_t + \left( \frac{1 - \gamma}{\phi} + \gamma \right) \eta_t + (1 - \gamma) \left( 1 - \frac{1}{\phi} \right) \eta_{t-1}
\]

\[
n_t = \frac{1}{\phi} \xi_t - \frac{(1 - \gamma)}{\phi} \eta_t
\]

or

\[
\begin{pmatrix}
\Delta x_t \\
n_t
\end{pmatrix} = \begin{pmatrix}
\left( \frac{1 - \gamma}{\phi} + \gamma \right) + (1 - \gamma) \left( 1 - \frac{1}{\phi} \right) L & \left( 1 - \frac{1}{\phi} \right) (1 - L) \\
\end{pmatrix} \begin{pmatrix}
\eta_t \\
\xi_t
\end{pmatrix}
\]

In the long run \( L = 1 \)

\[
\begin{pmatrix}
\Delta x_t \\
n_t
\end{pmatrix} = \begin{pmatrix}
\left( \frac{1 - \gamma}{\phi} + \gamma \right) + (1 - \gamma) \left( 1 - \frac{1}{\phi} \right) & 0 \\
-\frac{(1 - \gamma)}{\phi} & \frac{1}{\phi}
\end{pmatrix} \begin{pmatrix}
\eta_t \\
\xi_t
\end{pmatrix}
\]

that is only the technology shocks affects labor productivity.

Note the model prediction. If monetary policy is not completely accommodative \( \gamma < 1 \) then the response of hours to a technology shock \( -\frac{(1 - \gamma)}{\phi} \) is negative.
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Figure 1: Productivity and Hours
A. Labor productivity, business sector

Figure 2. Impulse Responses from Bivariate Specification
Response of Hours to a Technology Shock

A. No Trend Breaks

B. Pre-1973:3 and Post-1997:1 Break

The effects of government spending shocks

Understanding the effects of government spending shocks is important for policy authorities but also to assess competing theories of the business cycle.

Keynesian theory: government spending, GDP, consumption and real wage ↑, (because of the government spending multiplier).

RBC theory: government spending ↑, but consumption and the real wage ↓ because of a negative wealth effect.

Disagreement from the empirical point of view.
Government spending shocks: Blanchard and Perotti (2002) BP (originally) use a VAR for real per capita taxes, government spending, and GDP.

The shock is identified assuming that government spending does not react to taxes and GDP contemporaneously, Cholesky identification with government spending ordered first. The government spending shock is the first one (quadratic trend four lags).

When augmented with consumption consumption increases.

When augmented with investment, investment increases.

In a more recent version Perotti (2007) uses a larger VAR but results are confirmed. Consumption and real wage ↑ but investment ↓
Source: IDENTIFYING GOVERNMENT SPENDING SHOCKS: IT'S ALL IN THE TIMING

Focus on episodes where Business Week suddenly forecast large rises in defense spending induced by major political events that were unrelated to the state of the U.S. economy (exogenous episodes of government spending).


To identify government spending shocks, the military date variable is embedded in the standard VAR, but ordered before the other variables.
Both methodologies have problems.

VARs: shocks are often anticipated (fiscal foresight shocks may be not invertible)

War Dummy: few observations, subjective, relies on the construction of an exogenous time series.

Possible extensions.