The Effects of the ECB's Expanded Asset Purchase Programme: Online supplementary appendix

Annex I: Model specification and estimation

A1: The TVC-VAR

Let $A_t = [A_{0,t}, A_{1,t}..., A_{p,t}]$, and $\theta_t = vec([A_{0,t}\ A_t]')$, $vec(\cdot)$ being the stacking column operator. The VAR coefficients evolve as a random-walk

$$\theta_t = \theta_{t-1} + \omega_t \tag{1}$$

where ω_t is a Gaussian white noise vector with covariance Ω.

We decompose the innovation variance as follows: $\Sigma_t = F_t D_t F_t'$, where F_t is a lower triangular matrix with ones on the main diagonal and D_t a diagonal matrix. Let σ_t be a column vector containing the diagonal elements of $D_t^{1/2}$ and let $\phi_{i,t}$, $i = 1, ..., 4$, be a column vector containing the first i elements of the $(i+1)$-th row of F_t^{-1}. We assume

$$\log \sigma_t = \log \sigma_{t-1} + \xi_t \tag{2}$$

$$\phi_{i,t} = \phi_{i,t-1} + \psi_{i,t} \tag{3}$$

where ξ_t and $\psi_{i,t}$ are Gaussian white noise vectors with zero mean and variance Ξ and Ψ_i respectively. Let us define $\phi_t = [\phi_{1,t}', \ldots, \phi_{n-1,t}']$, $\psi_t = [\psi_{1,t}', \ldots, \psi_{n-1,t}']$ and let Ψ be the covariance matrix of ψ_t. We make two additional assumptions. First, $\psi_{i,t}$ and $\psi_{j,t}$ are uncorrelated for $j \neq i$. Second ξ_t, ψ_t, ω_t, ε_t are mutually uncorrelated.

The time-varying impulse response functions are $C_t(L) = \sum_{k=0}^{\infty} C_{k,t} L^k$, with $C_{0,t} = I_n$ and $C_{k,t} = S_{n,n}(\mathbf{A}_t^k)$, where $\mathbf{A}_t = \begin{pmatrix} A_t \\ I_{n(p-1)} \quad 0_{n(p-1),n} \end{pmatrix}$ and $S_{n,n}(X)$ is a function which selects the first n rows and n columns of the matrix X. The structural shocks $e_t \sim WN(0, I)$ are related to reduced for shock as usual, $\varepsilon_t = Q_t e_t$. The structural impulse response functions are $B_t(L) = C_t(L) Q_t$.

Notice that in principle, a VAR with stochastic volatility and fixed parameters could already be enough to takle the problem of a time varying variance of the structural shock. Here we allow also for variation in the coeffifients for two main reasons. First, there might be variations in model parameters attributable to the adoption of the APP. Second, there might be also changes due to non-policy factors.

Estimation is standard and is performed along the lines of Gali and Gambetti (2015) which basically follows Del Negro and Primiceri (2015).

A2: Prior calibration

The VAR is estimated with four lags ($p = 4$). Following the literature we assume that Ω, Ξ, Ψ, θ_0, ϕ_0 and $\log \sigma_0$, are all independent from each other. Denoting $W(S, d)$ a

Wishart distribution with scale matrix S and degrees of freedom d, we make the following assumptions about the prior distributions:

$$\begin{aligned}
\theta_0 &\sim N(\hat{\theta}, \hat{V}_\theta) \\
\log \sigma_0 &\sim N(\log \hat{\sigma}_0, I_n) \\
\phi_{i0} &\sim N(\hat{\phi}_i, \hat{V}_{\phi_i}) \\
\Omega^{-1} &\sim W(\underline{\Omega}^{-1}, \underline{\rho}_1) \\
\Xi^{-1} &\sim W(\underline{\Xi}^{-1}, \underline{\rho}_2) \\
\Psi_i^{-1} &\sim W(\underline{\Psi}_i^{-1}, \underline{\rho}_{3i})
\end{aligned}$$

Scale matrices are parametrized as follows: $\underline{\Omega} = \underline{\rho}_1(\lambda_1 \hat{V}_\theta)$, $\underline{\Xi} = \underline{\rho}_2(\lambda_2 I_n)$ and $\underline{\Psi}_i = \underline{\rho}_{3i}(\lambda_3 \hat{V}_{\phi_i})$. The degrees of freedom $\underline{\rho}_1$ and $\underline{\rho}_2$ are equal to the number of rows $\underline{\Omega}^{-1}$ and I_n plus one, respectively, while $\underline{\rho}_{3i}$ is $i+1$ for $i = 1, ..., n-1$. The parameters $\hat{\phi}_i, \hat{V}_{\phi_i}, \log \hat{\sigma}_0, \hat{\theta}, \hat{V}_\theta$ are imposed equal to the OLS estimates obtained from a time invariant VAR estimated for the full sample. Finally, we assume $\lambda_1 = 0.00001$, $\lambda_2 = 0.05$ and $\lambda_3 = 0.05$. The choice of the λ's is relatively conservative especially for λ_1 and is motivated by the fact that we want time variations not to be inflated by our priors. The posterior distribution of the parameters is obtained with the Gibbs sampler. See the online appendix of Gali and Gambetti (2015) for the details of the seven steps involved in the algorithm.

References

Del Negro, M. and G. Primiceri (2015): "Time-varying structural vector autoregressions and monetary policy: a corrigendum," *The Review of Economic Studies*, 82: 1342-1345.

Galí, J. and L. Gambetti (2015): "The Effects of Monetary Policy on Stock Market Bubbles: Some Evidence," *American Economic Journal: Macroeconomics*, 7(1): 233-257.

Annex II: Data definitions and sources

Consumer prices

Harmonised index of consumer prices (HICP), Euro area (changing composition), seasonally adjusted, not working day adjusted, ECB calculation based on Eurostat data.

Alternative for robustness analysis: HICP - All-items excluding energy and food (HICPexEF), Euro area (changing composition), Eurostat, Neither seasonally nor working day adjusted.

Sources: European Central Bank and Eurostat.

Eurocoin

Eurocoin, or €-coin, is a real-time, monthly estimate of area-wide GDP growth. The index gives a monthly "smoothed" estimate of quarter-on-quarter GDP growth in the euro area and highlights the underlying trend by adjusting the growth rate for short-term fluctuations and measurement errors.

Sources: Banca d'Italia and CEPR.

Employment

Total number of persons employed (thousands of persons). Derived from unemployment rate (standardised unemployment rate, percentage of civilian workforce, Euro area 19, fixed composition, Seasonally adjusted, not working day adjusted) and total number of persons unemployed (standardised unemployment, thousands of persons, Euro area 19, fixed composition, Seasonally adjusted, not working day adjusted).

Source: Eurostat.

Government bond yields

Benchmark long-term interest rate: Composite Euro Area 10-year Government Benchmark bond yield, aggregated by the ECB, Euro area changing composition, monthly data derived as average of daily data.

Data for yield curve analysis: Yield curve spot rate, 10-year maturity and 1-year maturity - Government bond, nominal, all issuers whose rating is triple A - Euro area (changing composition). Svensson model - continuous compounding - yield error minimisation - Euro, provided by ECB: see https://www.ecb.europa.eu/stats/financial_markets_and_interest_rates/euro_area_yield_curves/html/index.en.html

Alternative data for yield curve robustness analysis: Euro area 10-year and 2-year government benchmark bond yield, percent per annum, Euro area (changing composition, GDP weighted aggregation), monthly data derived as average of daily values, data collected and compiled by the ECB.

Data for country yield curve analysis: 10-year and 1-year (2-year for Spain) government bond yield, percent per annum, monthly data derived as average of daily values, for Germany, France, Italy and Spain, data collected and compiled by the ECB.

Source: European Central Bank.

Eurosystem security purchases

Asset side of the (simplified) Eurosystem balance sheet, monthly data derived as end-of-month daily/weekly outstanding amounts. For the amounts of the Eurosystem public and private security purchases under the APP as well as the previous Eurosystem security purchase programmes for monetary policy purposes see http://www.ecb.europa.eu/mopo/implement/omt/html/index.en.html. For more details on the Eurosystem balance sheet see the ECB Economic Bulletin article entitled "The role of the central bank balance sheet in monetary policy", issue 4, 2015.

Source: European Central Bank.

Stock prices

Dow Jones Euro Stoxx Broad Stock Exchange Index, historical close, monthly data derived as monthly average of daily data.

Source: European Central Bank.

Exchange rate

Nominal effective exchange rate (NEER) against Euro, monthly data derived as monthly average of daily data, Euro area-19 countries vis-a-vis the EER-38 group of trading partners (AU, CA, DK, HK, JP, NO, SG, KR, SE, CH, GB, US, BG, CZ, HU, PL, RO, CN, DZ, AR, BR, CL, HR, IS, IN, ID, IL, MY, MX, MA, NZ, PH, RU, ZA, TW, TH, TR and VE) excluding the Euro.

Source: European Central Bank.

Credit to non-financial corporations

Sum of outstanding amounts of MFI loans to non-financial corporations (derived by rescaling indices of notional stocks with base equal to the outstanding amounts for December 2010) and outstanding amounts of securities (corporate bonds and equities) issued by non-financial corporations. Series adjusted to include loan sales and securitisation.

Source: European Central Bank.

Composite lending rate

Composite lending rate, derived as weighted average of interest rates charged on short-term and long-term loans to non-financial corporations, with weights based on the nominal outstanding amounts of corresponding loans to non-financial corporations.

Source: European Central Bank.

Inflation expectations

Survey of Professional Forecasters (SPF) forecast of euro area HICP inflation 5-years ahead.

Source: European Central Bank.

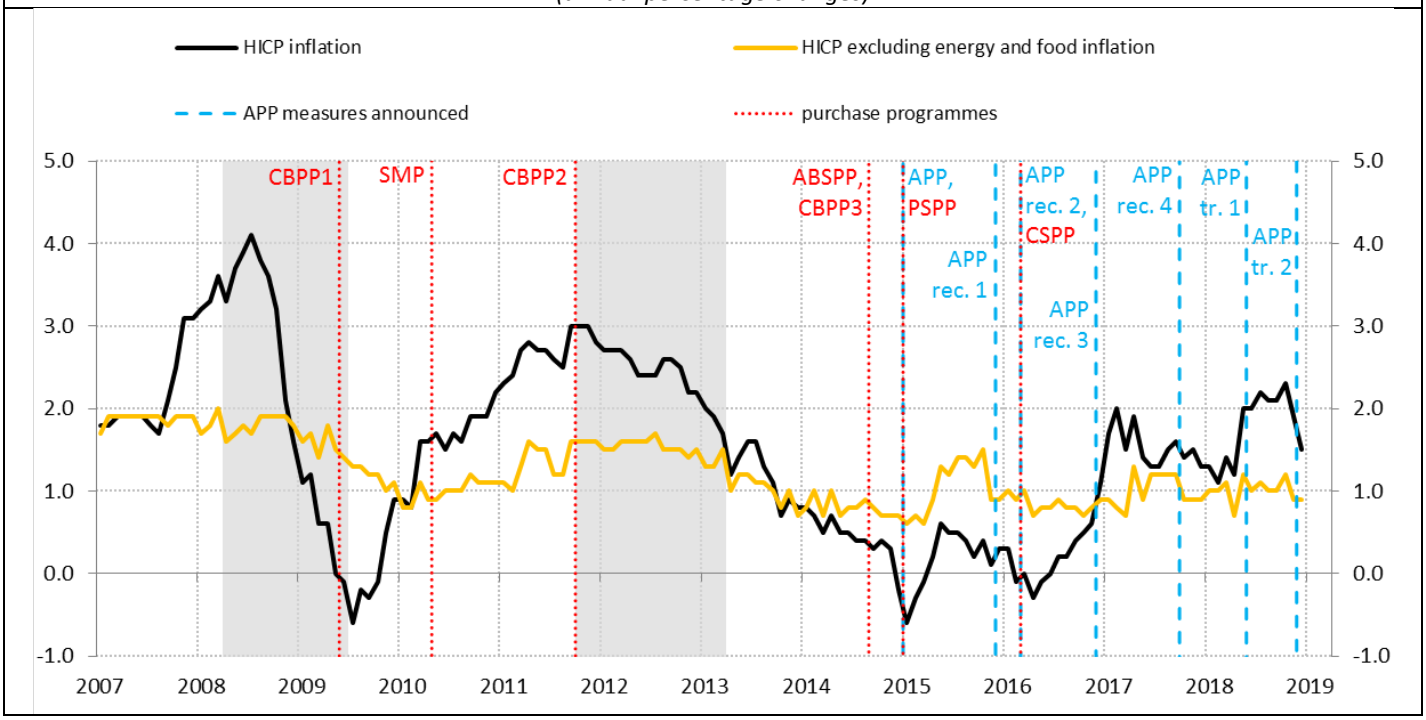
Short-term forward rates

Three months Eonia (OIS) forward rates two years ahead. Monthly data derived as monthly average of daily data.

Source: European Central Bank.

Chart A – HICP inflation, January 2007 to December 2018

(annual percentage changes)

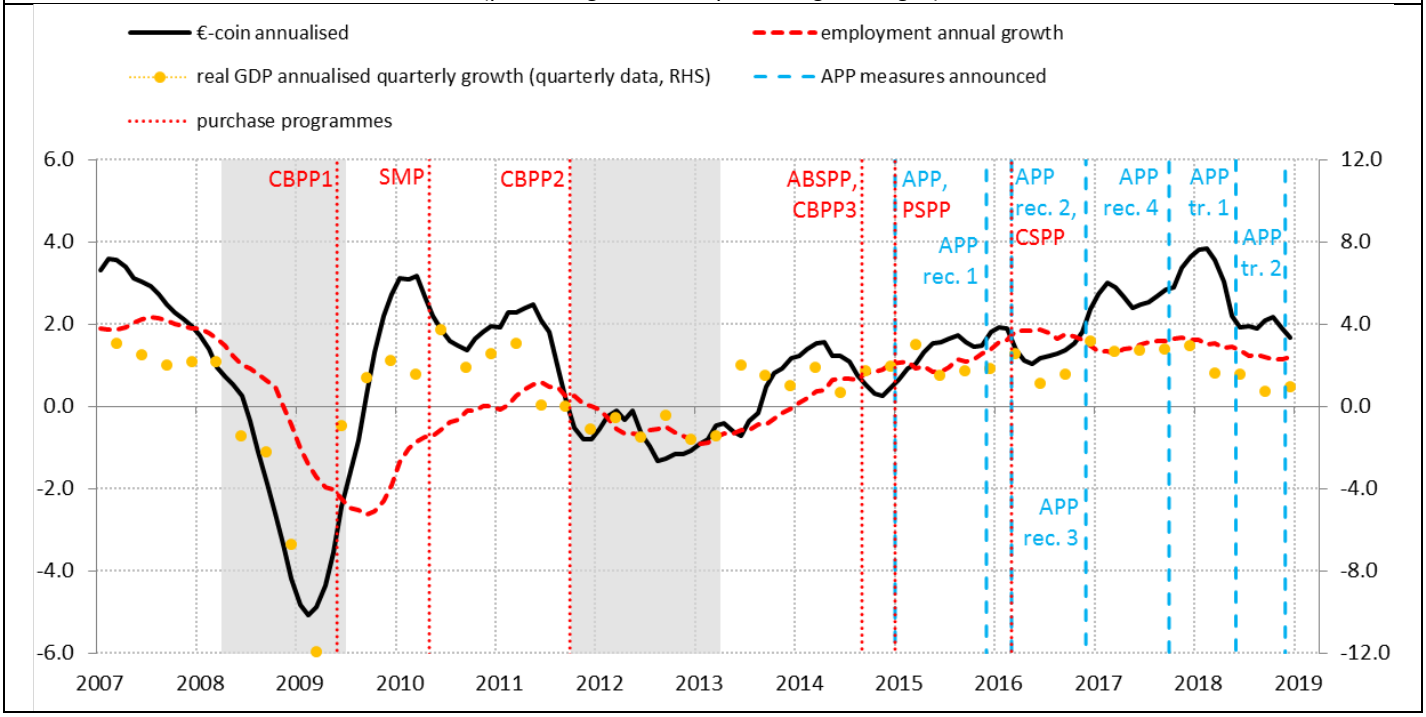


Source: CEPR, European Central Bank and Eurostat.

Note: Vertical red dotted lines and associated acronyms refer to security purchase programmes adopted by the ECB. Vertical dashed blue lines and associated text delimit the dates of the introduction and subsequent re-calibrations of the expanded asset purchase programme (APP). Shaded areas delimit Euro Area recessions as dated by the CEPR Euro Area Business Cycle Dating Committee.

Chart B – Eurocoin, total employment and real GDP in the euro area, January 2007 to December 2018

(percentages, annual percentage changes)

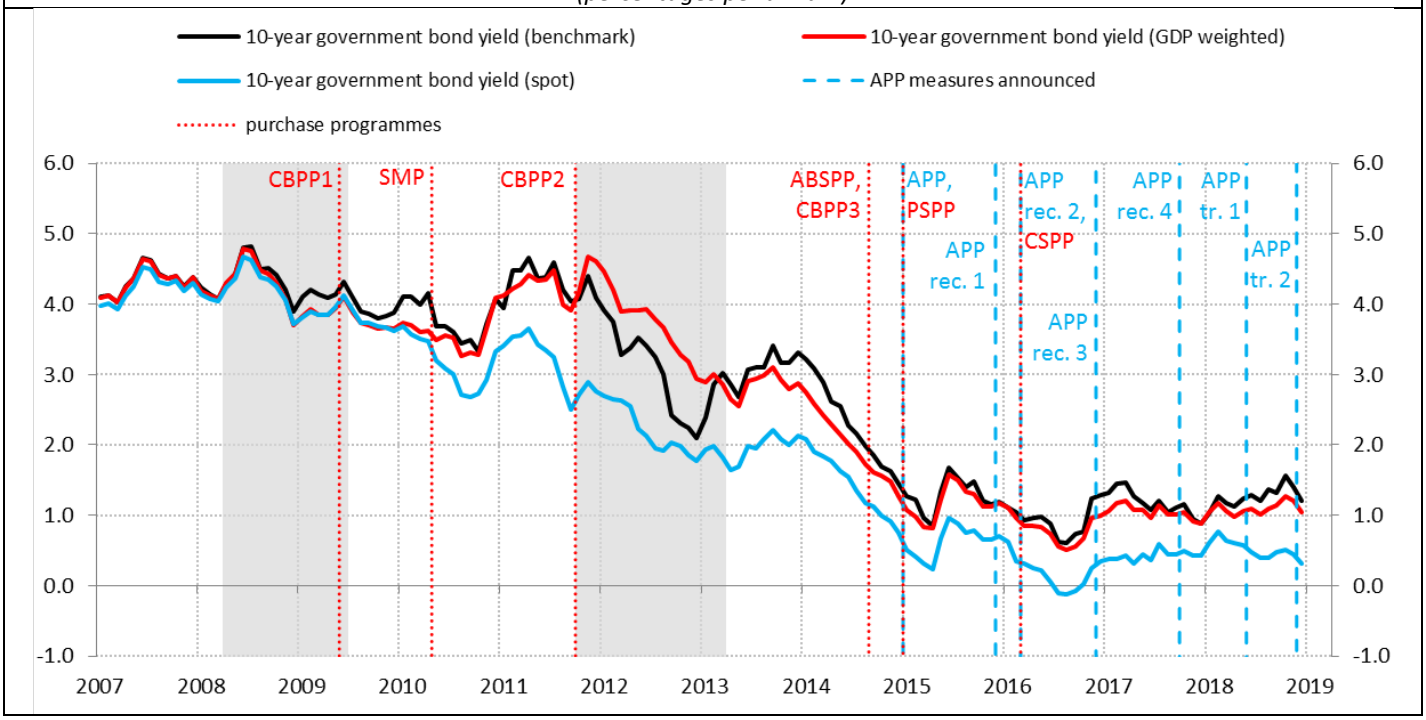


Source: Bank of Italy, CEPR and Eurostat.

Note: Vertical red dotted lines and associated acronyms refer to security purchase programmes adopted by the ECB. Vertical dashed blue lines and associated text delimit the dates of the introduction and subsequent re-calibrations of the expanded asset purchase programme (APP). Shaded areas delimit Euro Area recessions as dated by the CEPR Euro Area Business Cycle Dating Committee.

Chart C – Composite long-term euro area government bond yields, January 2007 to December 2018

(percentages per annum)

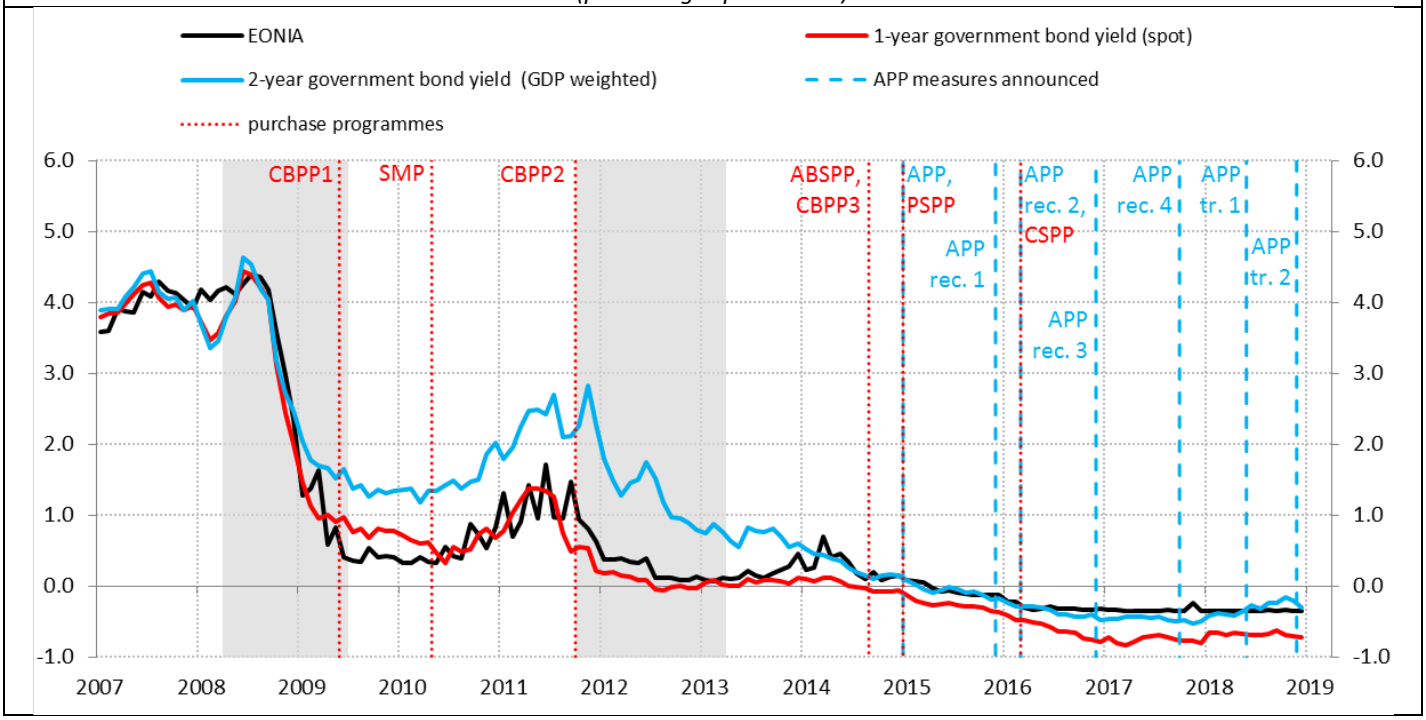


Source: CEPR and European Central Bank.

Note: Vertical red dotted lines and associated acronyms refer to security purchase programmes adopted by the ECB. Vertical dashed blue lines and associated text delimit the dates of the introduction and subsequent re-calibrations of the expanded asset purchase programme (APP). Shaded areas delimit Euro Area recessions as dated by the CEPR Euro Area Business Cycle Dating Committee.

Chart D – Composite short-term euro area government bond yields, January 2007 to December 2018

(percentages per annum)

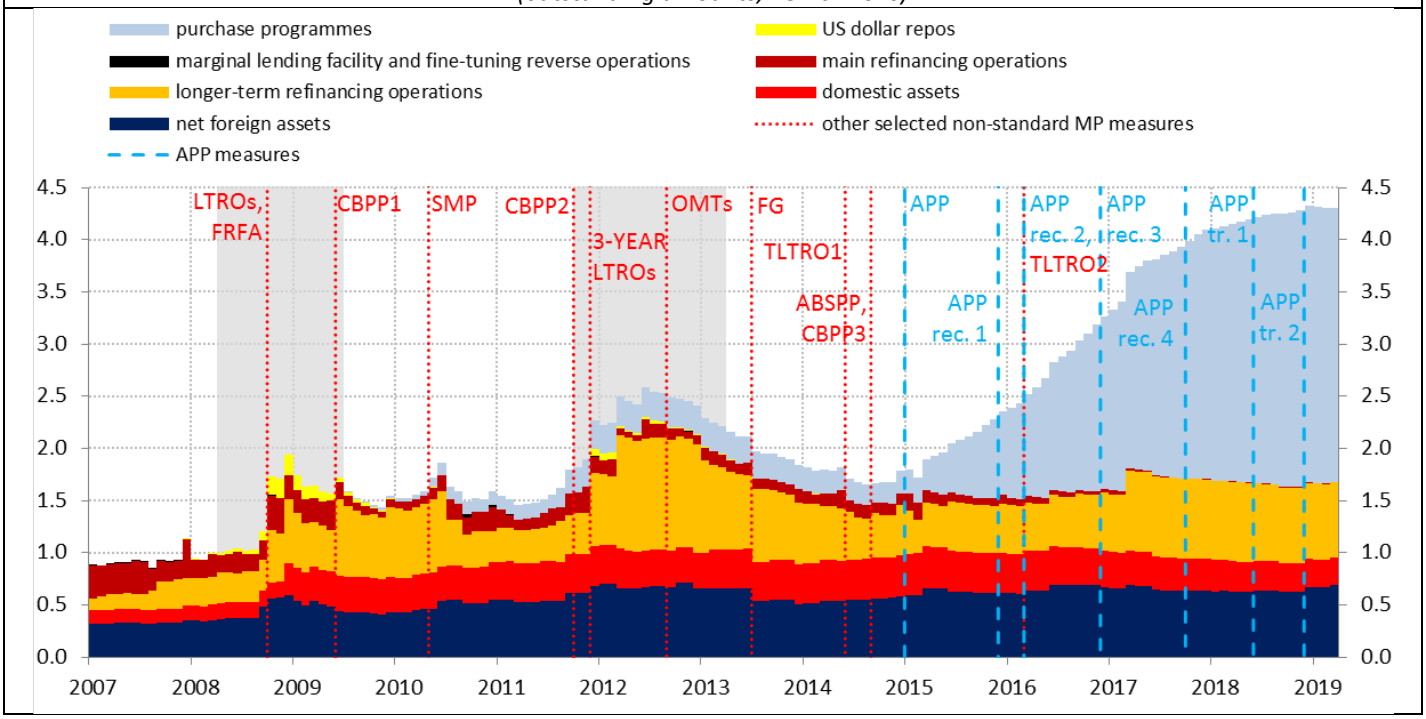


Source: CEPR and European Central Bank.

Note: Vertical red dotted lines and associated acronyms refer to security purchase programmes adopted by the ECB. Vertical dashed blue lines and associated text delimit the dates of the introduction and subsequent re-calibrations of the expanded asset purchase programme (APP). Shaded areas delimit Euro Area recessions as dated by the CEPR Euro Area Business Cycle Dating Committee.

Chart E – Simplified Eurosystem balance sheet asset composition, January 2007 to December 2018

(outstanding amounts, EUR trillions)

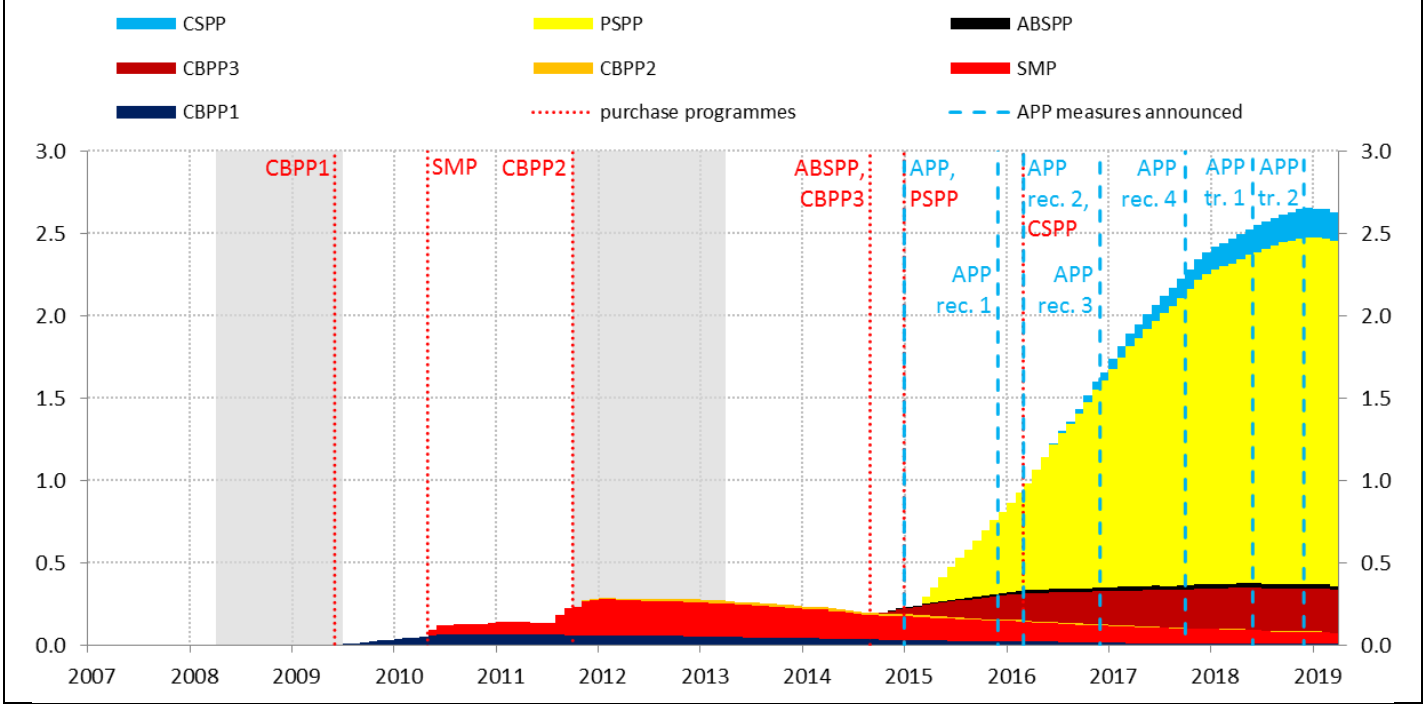


Source: CEPR and European Central Bank.

Note: Vertical red dotted lines and associated acronyms refer to security purchase programmes adopted by the ECB. Vertical dashed blue lines and associated text delimit the dates of the introduction and subsequent re-calibrations of the expanded asset purchase programme (APP). Shaded areas delimit Euro Area recessions as dated by the CEPR Euro Area Business Cycle Dating Committee.

Chart F – Eurosystem security purchases for monetary policy purposes, January 2007 to December 2018

(outstanding amounts, EUR trillions)

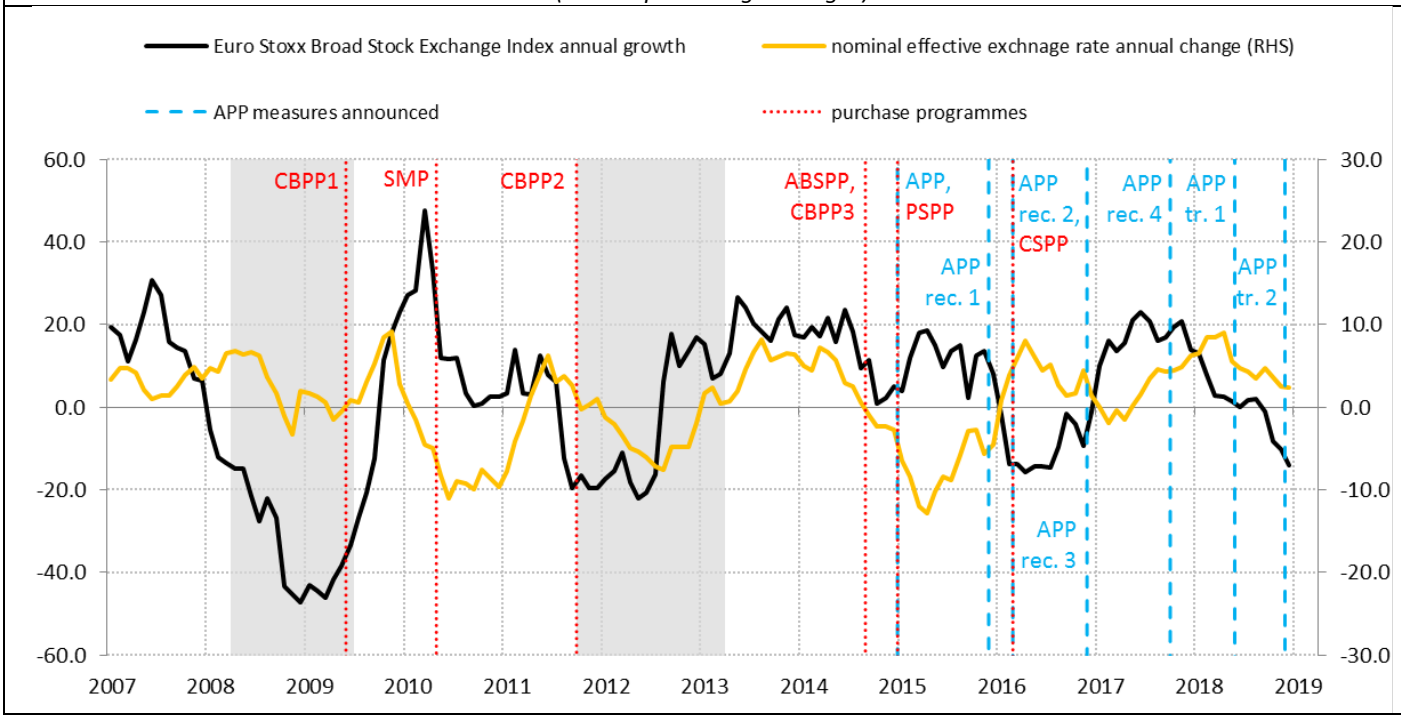


Source: CEPR and European Central Bank.

Note: Vertical red dotted lines and associated acronyms refer to security purchase programmes adopted by the ECB. Vertical dashed blue lines and associated text delimit the dates of the introduction and subsequent re-calibrations of the expanded asset purchase programme (APP). Shaded areas delimit Euro Area recessions as dated by the CEPR Euro Area Business Cycle Dating Committee.

Chart G – Stock prices and nominal effective exchange rate, January 2007 to December 2018

(annual percentage changes)

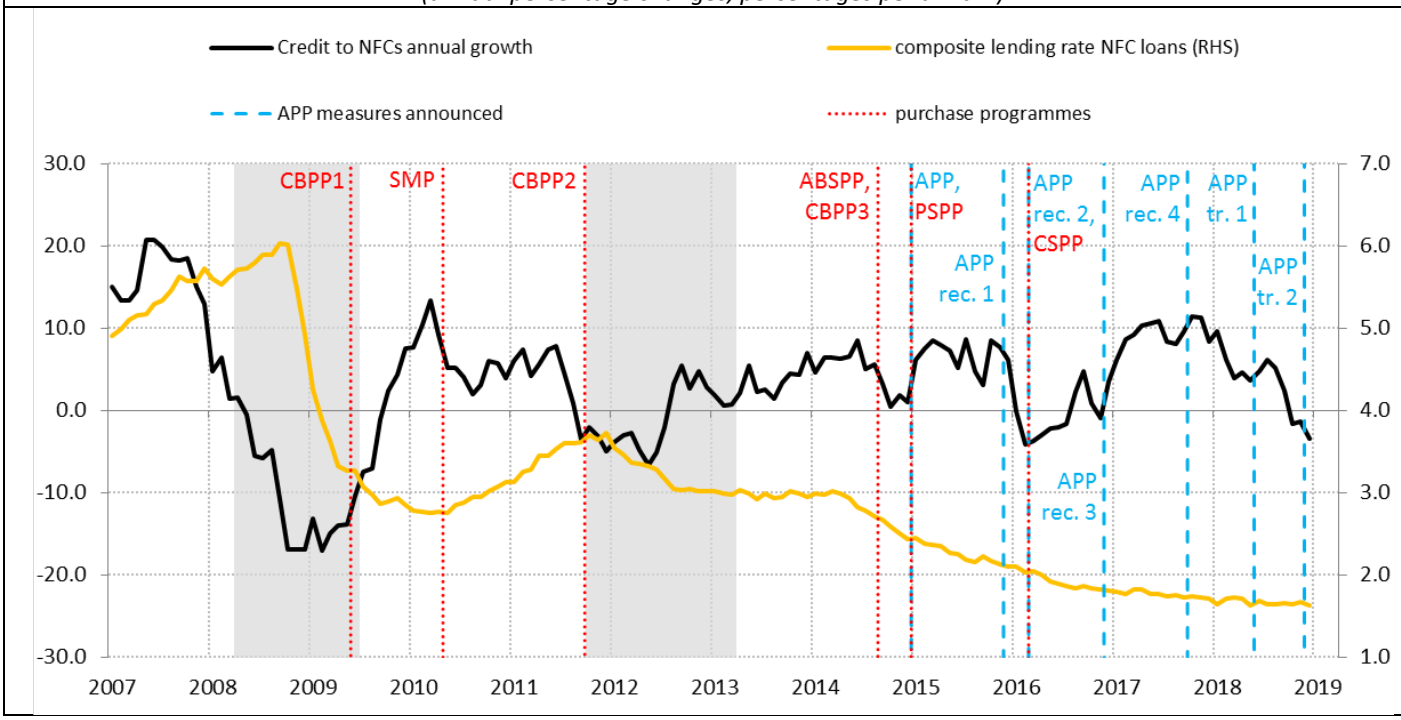


Source: CEPR and European Central Bank.

Note: Vertical red dotted lines and associated acronyms refer to security purchase programmes adopted by the ECB. Vertical dashed blue lines and associated text delimit the dates of the introduction and subsequent re-calibrations of the expanded asset purchase programme (APP). Shaded areas delimit Euro Area recessions as dated by the CEPR Euro Area Business Cycle Dating Committee.

Chart H – NFC credit and composite bank lending rates, January 2007 to December 2018

(annual percentage changes, percentages per annum)

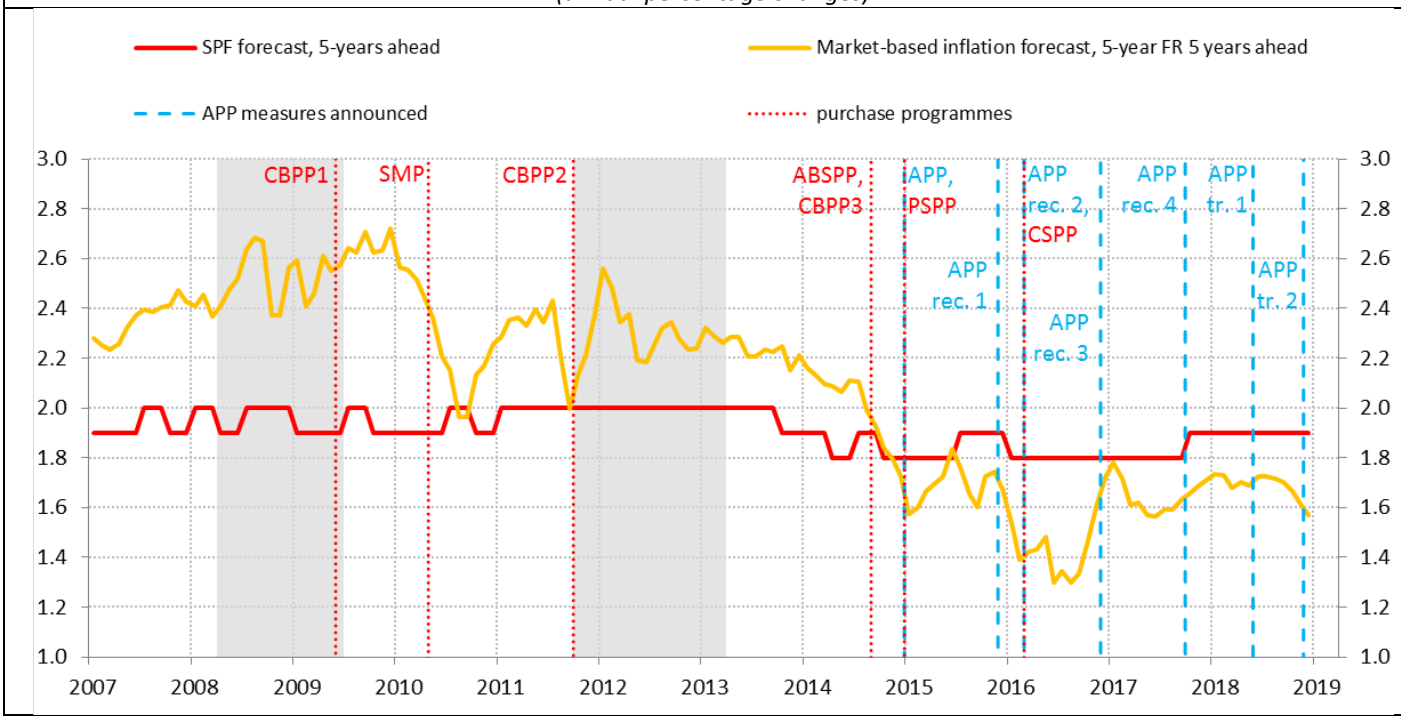


Source: CEPR and European Central Bank.

Note: Vertical red dotted lines and associated acronyms refer to security purchase programmes adopted by the ECB. Vertical dashed blue lines and associated text delimit the dates of the introduction and subsequent re-calibrations of the expanded asset purchase programme (APP). Shaded areas delimit Euro Area recessions as dated by the CEPR Euro Area Business Cycle Dating Committee.

Chart I – HICP inflation longer term expectations, January 2007 to December 2018

(annual percentage changes)

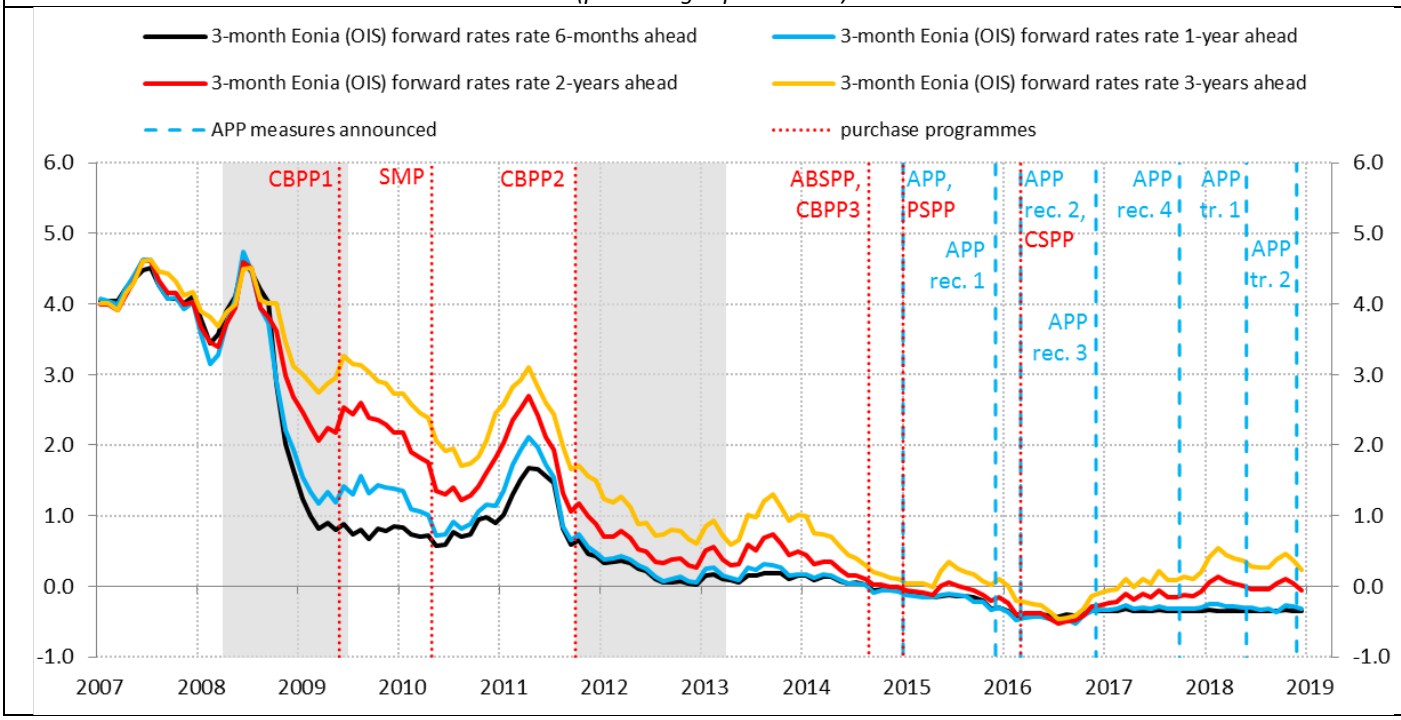


Source: CEPR, Consensus Economics and European Central Bank.

Note: Vertical red dotted lines and associated acronyms refer to security purchase programmes adopted by the ECB. Vertical dashed blue lines and associated text delimit the dates of the introduction and subsequent re-calibrations of the expanded asset purchase programme (APP). Shaded areas delimit Euro Area recessions as dated by the CEPR Euro Area Business Cycle Dating Committee.

Chart J – Selected market interest rates, January 2007 to December 2018

(percentages per annum)

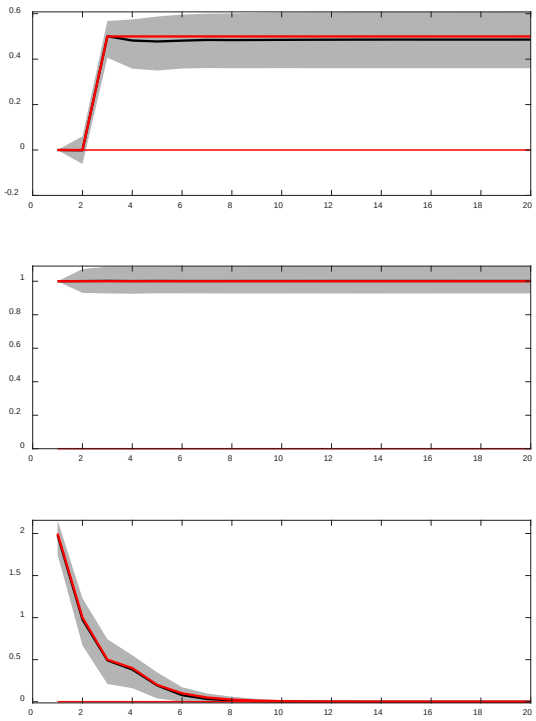
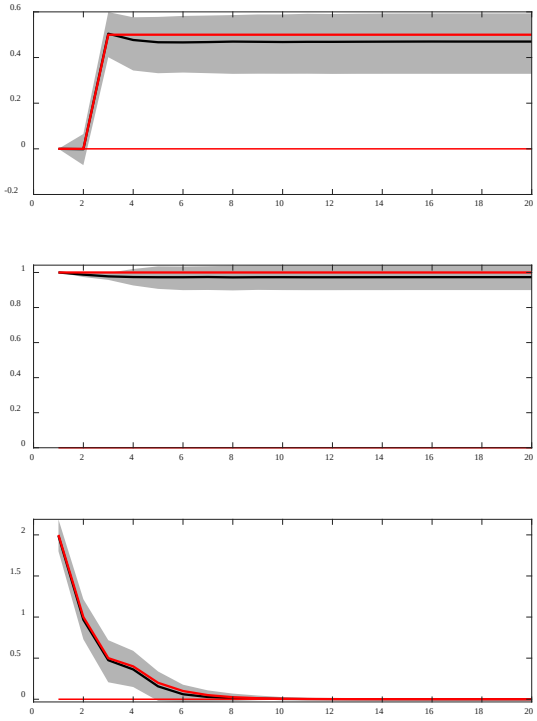


Source: CEPR and European Central Bank.

Note: Vertical red dotted lines and associated acronyms refer to security purchase programmes adopted by the ECB. Vertical dashed blue lines and associated text delimit the dates of the introduction and subsequent re-calibrations of the expanded asset purchase programme (APP). Shaded areas delimit Euro Area recessions as dated by the CEPR Euro Area Business Cycle Dating Committee.

Annex III: Simulation results

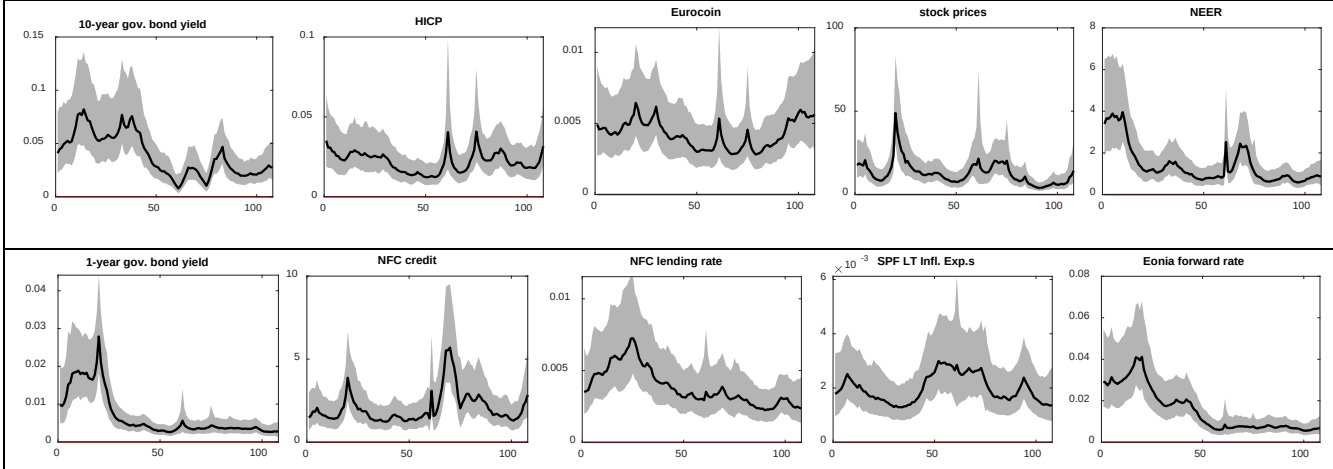
Chart K – Comparison of IRFs of VAR models with time-varying coefficients (TV-VAR) and VAR models with constant parameters (VAR) for simulation

TV-VAR model	VAR model
	

Note: Impulse response functions based on the time-varying coefficient VAR (TV-VAR) and fixed coefficient VAR (VAR) estimated on simulated data (see section 3.4 for more details on the simulation). The black solid lines are the mean of the point estimates obtained for the N samples. The grey areas delimit the 16% and 84% percentiles of the distribution of the point estimates (across samples). The red lines are the theoretical impulse response functions in the simulated model.

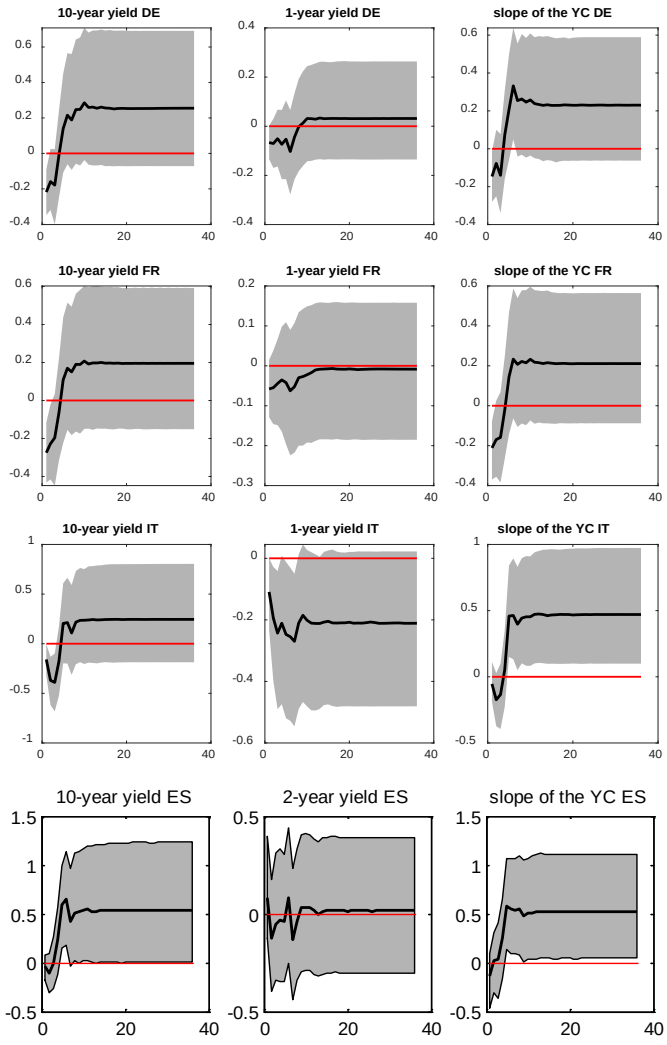
Annex IV: Further results

Chart L - Stochastic volatility



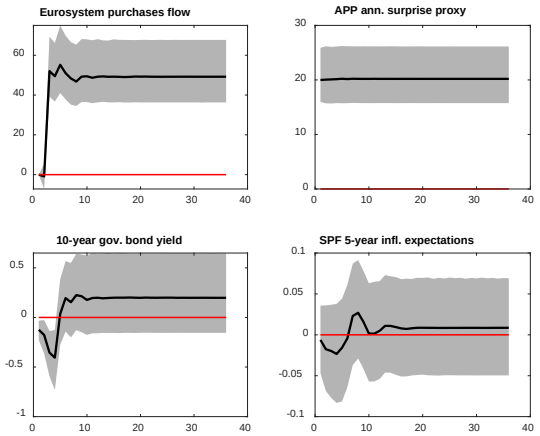
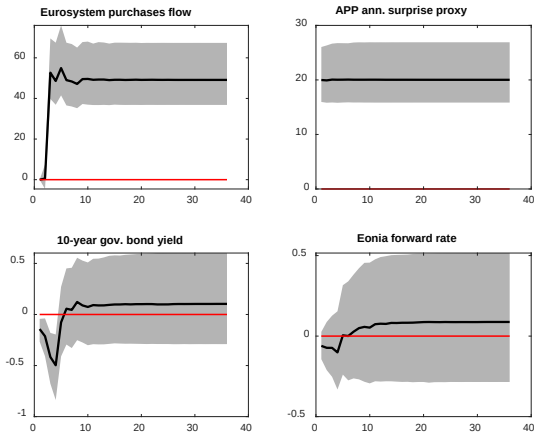
Note: Residual time-varying variances. Black full lines: posterior medians. Grey areas: areas delimited by the 16th and 84th percentiles.

Chart M - Responses of the yield curve to the January 2015 APP announcement shock in the largest euro area countries



Note: Full black lines are the median impulse response functions, grey areas delimit the space between the 16th and 84th percentiles of impulse response functions. Median responses and percentiles multiplied by the estimated size of the shock in the respective month. Horizontal axes refer to number of months, while vertical axes refer to percentage points.

Chart N - Impulse response functions of long-term inflation expectations and short-term forward rates to the January 2015 APP announcement shock

Model with SPF long-term inflation expectations (5-years ahead)	Model with three month Eonia (OIS) forward rates (3-month forward rate 2-year ahead)
	

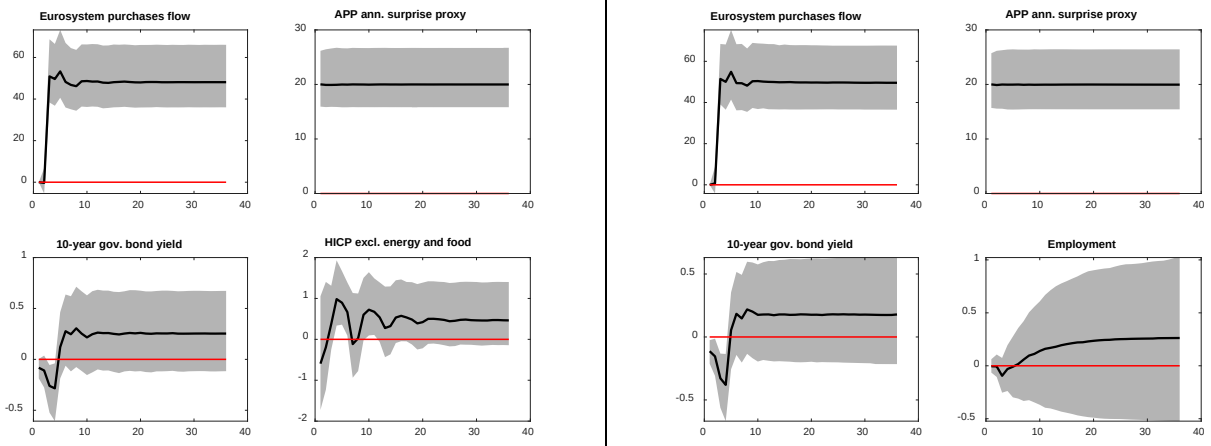
Note: Full black lines are the median impulse response functions, grey areas delimit the space between the 16th and 84th percentiles of impulse response functions. Median responses and percentiles multiplied by the estimated size of the shock in the respective month. Horizontal axes refer to number of months, while vertical axes refer to percentage points.

Annex V: Robustness analysis

A1: Alternative variables

Chart O - Impulse response functions of macroeconomic variables to the January 2015 APP announcement shock

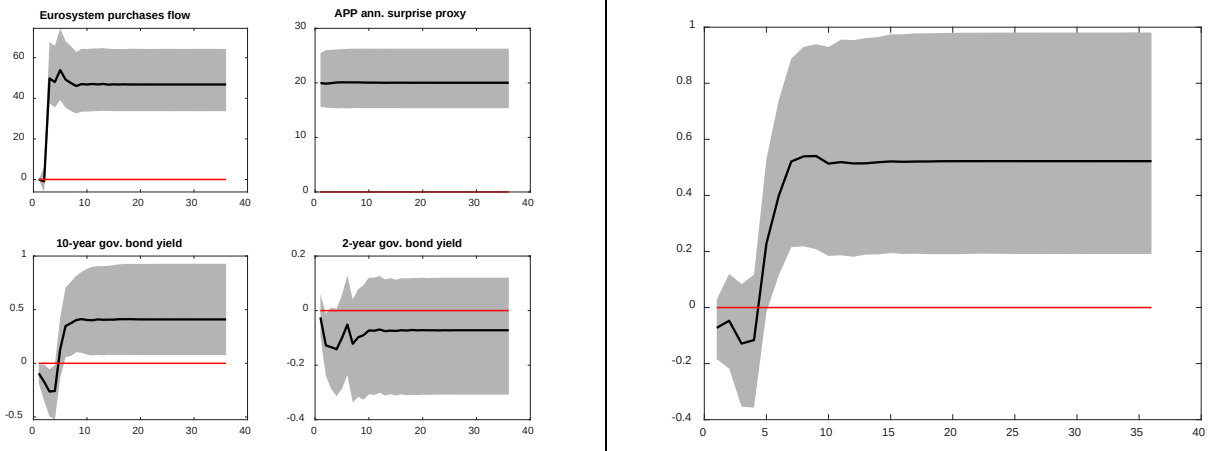
Model with HICP excluding energy and food	Model with employment



Note: Full black lines are the median impulse response functions, grey areas delimit the space between the 16th and 84th percentiles of impulse response functions. Median responses and percentiles multiplied by the estimated size of the shock in the respective month. Horizontal axes refer to number of months, while vertical axes refer to billions of euros (for Eurosystem purchase flow and APP announcement proxy), percentages (for the HICP and employment) or percentage points (for long-term interest rates and Eurocoin).

Chart P - Impulse response functions of the yield curve to the January 2015 APP announcement shock

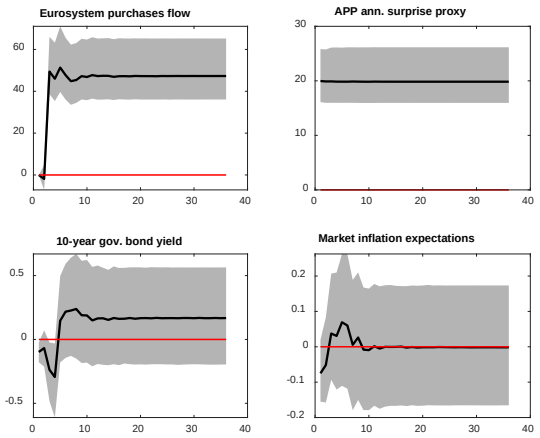
Model with the 10-year and 2-year yields	Slope of the yield curve (10Y yield – 2Y yield)

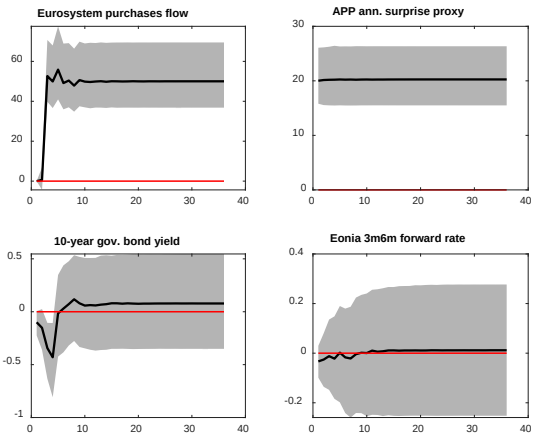


Note: Full black lines are the median impulse response functions, grey areas delimit the space between the 16th and 84th percentiles of impulse response functions. Median responses and percentiles multiplied by the estimated size of the shock in the respective month. Horizontal axes refer to number of months, while vertical axes refer to billions of euros (for Eurosystem purchase flow and APP announcement proxy) or to percentage points (for interest rates).

Chart Q - Impulse response functions of long-term inflation expectations and short-term forward rates to the January 2015 APP announcement shock

Model with capital market-based inflation forecast (5-year forward rate 5 years ahead)	Model with three month Eonia (OIS) forward rates (3-month forward rate 6- month ahead)





Note: Full black lines are the median impulse response functions, grey areas delimit the space between the 16th and 84th percentiles of impulse response functions. Median responses and percentiles multiplied by the estimated size of the shock in the respective month. Horizontal axes refer to number of months, while vertical axes refer to percentage points.

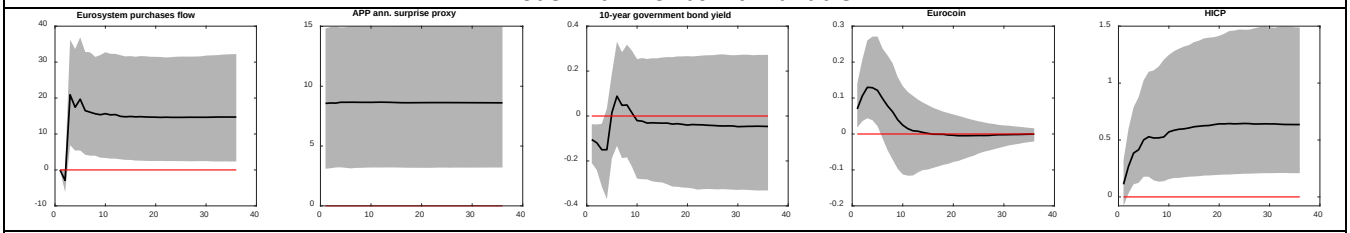
A2: Alternative identification

Table A – Alternative identification restrictions

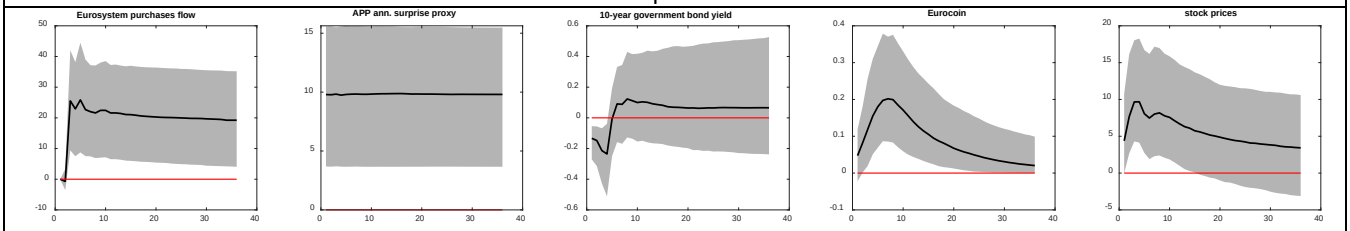
	on impact	lagged effects		
	0 Jan-15 (APP announced)	1 Feb-15	2 Mar-15 (purchases start)	3 Apr-15
Eurosystem security purchases	0		+	
APP ann. surprise proxy	+			
10-year government bond yield	-	-		
Eurocoin				+
A) HICP				+
B) stock prices				+
C) exchange rate				-
D) 1-year yield				-
E) NFC credit volumes				+
F) bank lending rate to NFCs				-

Chart R - Impulse response functions of variables in models with five variables to the January 2015 APP announcement shock

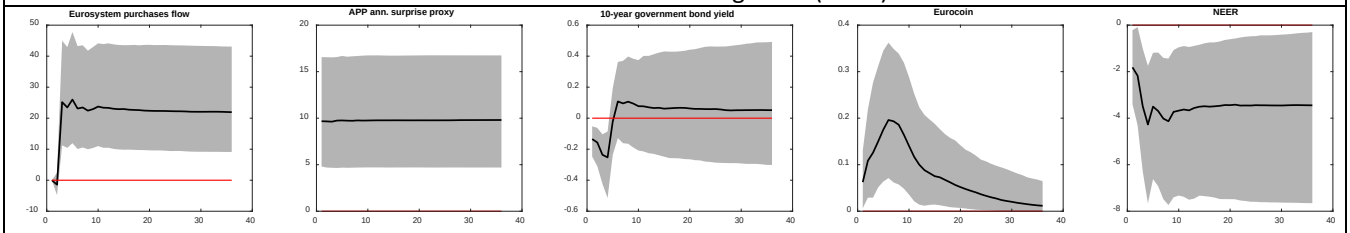
Model with HICP as fifth variable



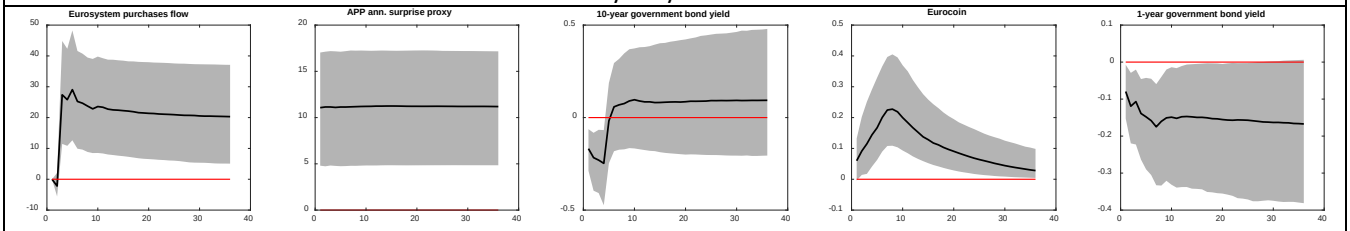
Model with stock prices as fifth variable



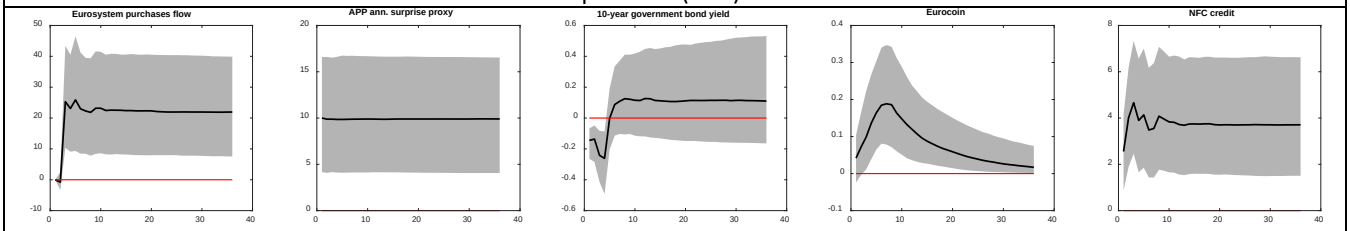
Model with the nominal effective exchange rate (NEER) as fifth variable



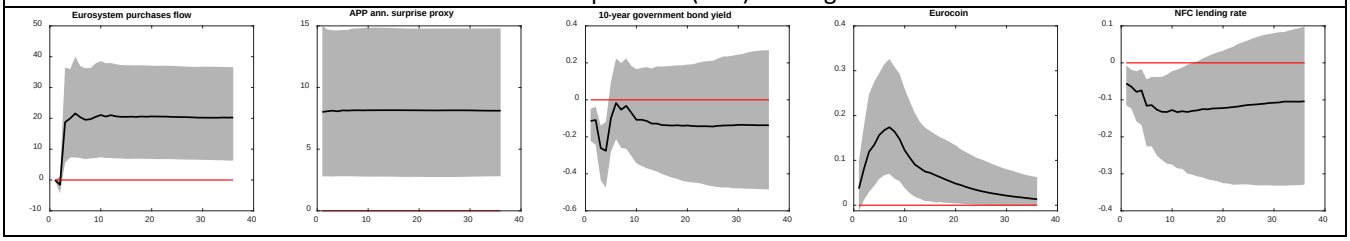
Model with 1-year yield as fifth variable



Model with non-financial corporation (NFC) credit volumes as fifth variable



Model with non-financial corporation (NFC) lending rates as fifth variable

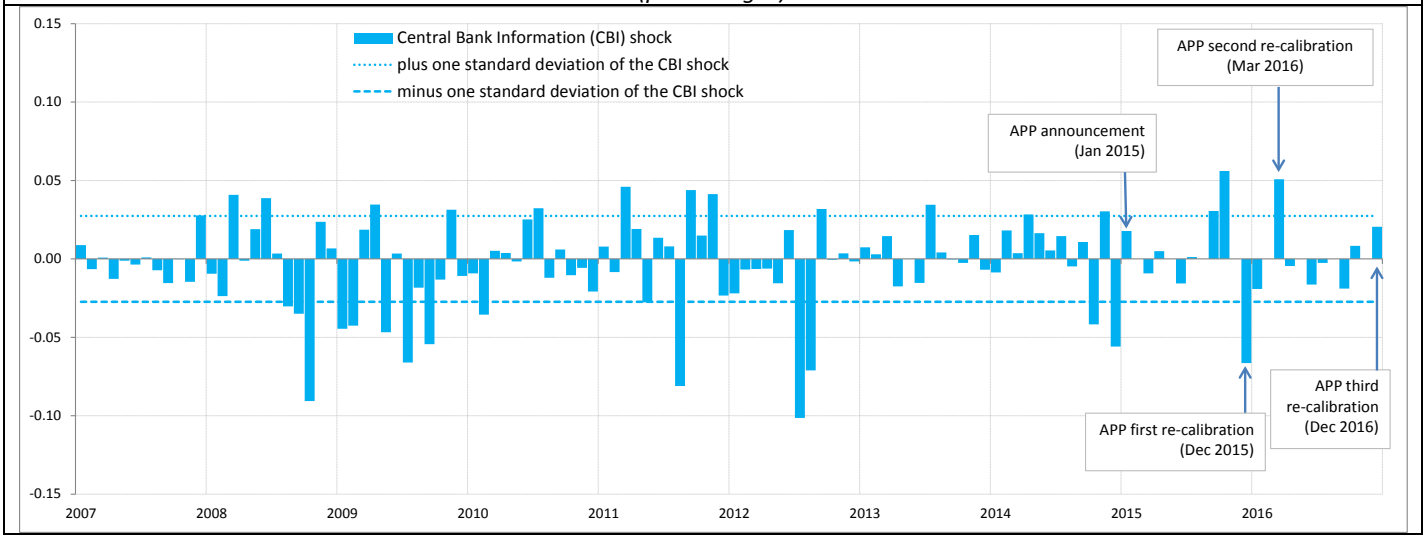


Note: Full black lines are the median impulse response functions, grey areas delimit the space between the 16th and 84th percentiles of impulse response functions. Median responses and percentiles multiplied by the estimated size of the shock in the respective month. Horizontal axes refer to number of months, while vertical axes refer to billions of euros (for Eurosystem purchase flow and APP announcement proxy), percentage points (for interest rates, lending rates and Eurocoin) or percentages (for the HICP, stock prices, the exchange rate and NFC credit).

A3: Central bank information shocks

Chart S – Estimated ECB information shock

(percentages)



Source: Jarocinski and Karadi (2020).

Note: Baseline estimate of the central bank information shock for the ECB ("Central bank information (sign restrictions)").