The Real Effects of Monetary Policy: A New VAR Identification Procedure

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Abstract:

This paper proposes a new empirical method to analyze the real effects of monetary policy within a structural VAR approach. The monetary policy shock is the one having (i) zero impact effect on real GDP and prices; (ii) a large impact effect of opposite sign on non-borrowed reserves and federal funds rate. This definition provides a set of (a) *partial identifying* conditions and a set of (b) *quasi-identifying* conditions applied to US monthly data relative to the period 1965:1-1994:3. Results show that a contractionary monetary policy shock produces a large negative effect on the real GDP which reduces and vanishes in the long-run. We find strong empirical evidence in favor of money non-neutrality in the short-run and money neutrality in the long-run.

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1 Introduction

The purpose of this paper is to provide new empirical evidence on the real effects of monetary policy by means of VAR analysis. Money neutrality and the effectiveness of monetary policy have been one of the most discussed topics in economic theory for half a century. In recent years there seems to be a growing agreement between economists on the *facts* of monetary policy. Monetary policy is non-neutral in the short-term, because of imperfect information (Lucas, 1972) or nominal rigidities, (Fisher, 1977, Taylor, 1980, 1999), and is neutral in the long run, since the price level adjustment will offset real effects. In this paper we propose empirical evidence in favor of this view.
Starting from the original contribution by Sims (1980), VAR analysis has been widely used in empirical macroeconomics (see Canova, 1995, for a survey). The basic idea of VAR models is the propagation impulse mechanism of Slutsky (1937) and Frisch (1933) formalized by the Wold Representation Theorem. The economic cycle is seen as the sum of white noise shocks of different nature that, through complex propagation mechanisms, cause booms and recessions. By means of VAR analysis it is possible to separate the effects of any single shock and to study their relative weight over the cycle. In the last decade there have been numerous contributions aiming to recover the effects of monetary policy by means of structural VAR analysis; the research has been directed toward both more refined econometric techniques and new identification methods (see Christiano, Eichenbaum and Evans (1998) for a survey).
Resuming the recent contributions of Bernanke and Mihov (1998a, 1998b) and Uhlig (1999), this work proposes a new criterion for identifying the effects of monetary policy. Such a criterion involves a *Partial Identification* of the VAR−only the effects of the monetary policy shock are identified. The identification is based on hypotheses concerning the shock impact effect on real GDP, prices, non-borrowed reserves and the federal funds rate. No restrictions are imposed on the long run behavior. The monetary policy shock is the one having a zero impact effect on real GDP and prices and a large effect of opposite sign

on non-borrowed reserves and the federal funds rate, according to the theory of the liquidity effect. While the first hypothesis involves standard restrictions to zero of the impact coefficients for the impulse response functions of real GDP and prices, the second involves non-standard restrictions obtained by means of a joint constrained maximization of the impact effects on non-borrowed reserves and the federal funds rate. Once the shock is identified, we develop a *Quasi-Identification Criterion* (QIC); such a criterion does not exactly identify, but identifies a set of admissible impulse response functions for each variable. Since the shape and the sign of the impulse response functions are very similar, quasi-identification enables us to identify the sign and the shape; that finding, by confirming the identification results, strengthens our identification criterion. Our main findings are:

1. A contractionary monetary policy shock reduces real GDP temporaneously. Monetary policy is non neutral in the short-run and neutral in the long-run.

2. Inflation drops sluggishly. A contractionary shock succeed in reducing inflation, but with lag and not permanently. Commodity prices drop more quickly than inflation; the effect of the shock last for three years and vanishes in the long-run.

3. Reserves and the federal funds rate react immediately to the shock. From the first year the effect of the shock reduces and vanish in the long run.

This paper is organized as follows. Section 2 sets up the model. Section 3 discusses the theoretical hypotheses and describes the identification criterion. Section 4 illustrates the results of identification. Section 5 proposes a sensitivity analysis. Section 6 describes the quasi-identification criterion. Section 7 draws the conclusions.

We use US monthly data relative to the period 1965:1-1994:3. The data set includes real GDP (GDP), the inflation rate (ΔGDPD) a commodity price index (CP), total reserves (TR), non-borrowed reserves (NBR) and the federal funds rate (FFR).

We use routines constructed by the author in MATLAB.

2 The Model

Let X_t be a covariance stationary stochastic vector process $(n \times 1)$[1]; from the Wold Theorem we may represent X_t in terms of innovations

$$X_t = A(L)\varepsilon_t \tag{1}$$

where $A(L) = I + A_1L + A_2L^2 + ...$ is a matrix of polynomials in the lag operator L $(n \times n)$ and ε_t is a zero mean white noise vector process $(n \times 1)$ with variance $E\varepsilon_t\varepsilon_t' = \Sigma_\varepsilon$ and $E\varepsilon_t\varepsilon_{t-k}' = 0$ for $k \neq 0$. Let S be the Choleski factor of Σ_ε and H any matrix such that $HH' = I$. By postmultiplying $A(L)$ for SH and premulitiplying ε_t for $H'S^{-1}$ we obtain the orthonormal representation

$$X_t = C(L)e_t \tag{2}$$

where

$$C(L) = A(L)SH \tag{3}$$

$e_t = H'S^{-1}\varepsilon_t$, $Ee_te_t' = I$ and $C(0) = SH$. Equation (2) is the VAR structural form and the matrix $C(L)$ contains the effects of the structural shocks on the vector X_t. In order to obtain (2) from (1) we need to identify the model, that is, since S is given, we must choose the matrix H under the orthonormality constraint $HH' = I$.

Partial Identification

Let us suppose that we are not interested in completely identifying $-$ in determining the effects of all the n shocks $-$ but are interested in a determinate subclass of e_t containing m shocks with $m = 1, ..., n-1$. In this case, we need only the first m columns of the matrix $C(L)$. In order to identify the first m columns of $C(L)$ it is sufficient to choose the first m columns of H. Partitioning H, $C_1(L)$ and e_t as follows, $H = (H_1|H_2)$, $C(L) = (C_1(L)|C_2(L))$ and $e_t' = (e_{1t}'|e_{2t}')$, our model will be

$$X_t = A(L)SH_1H_1'\eta_t + A(L)SH_2H_2'\eta_t = C_1(L)e_{1t} + C_2(L)e_{2t} \tag{4}$$

[1] See Appendix A for details of the model.

where $C_1(L) = A(L)SH_1$, $C_2(L) = A(L)SH_2$, $e_{1t} = H_1'\eta_t$[2] and $e_{2t} = H_2'\eta_t$; H_1 is the matrix we must choose for identifying, e_{1t} is the vector of m structural shocks to be identified and $C_1(L)$ is the matrix of the impulse response functions relative to e_{1t}. $C_2(L)e_{2t}$ is the model relative to the other $(n-m)$ shocks. The orthogonality condition provides us $(m-1)/2$ restrictions and the orthonormality condition m restrictions; since the number of free parameters is nm, in order to fix H_1 we need $m(2n-m-1)/2$ further restrictions. As with the exact identification, such restrictions can be short or long run restrictions imposed on the first m columns of $C_1(0)$ or $C_1(1)$.

3 Identification

Let us consider equation (4)[3]. Let X_t be the vector of variables[4] GDP[6], ΔGDPD, CP, NBR, FFR, TR and e_1 the monetary policy shock. Our purpose is to determine $C_1(L)$; since $m=1$, $C_1(L)$ is the first column of $C(L)$ and H_1 the first column of H. As already observed, in order to identify we have to fix H_1. Given the orthonormality condition we need $n-1$ further restrictions. We adopt two different sets of theoretical hypotheses. On the one hand we refer to macroeconomic assumptions, on the other hand we refer to the operating mechanism of the market for bank reserves. We assume that

[2]See Appendix A for the definition of η_t.

[3]We estimated the autoregressive form of representation (1), $A^1LX_t = \varepsilon_t$. In order to have the impulse response functions $A(L)$, we transformed the VAR model into an $AR(1)$ process, then inverted the $AR(1)$ in a $MA(\infty)$ process. $MA(\infty)$ being a finite variance process, we truncated it at $k = 100$.

[4]The data set is provided by Bernanke and Mihov and contains US monthly data for the following variables: real GDP (GDP), inflation rate (ΔGDPD)[5], commodity price index (CP), total reserves (TR), non-borrowed reserves (NBR) and federal funds rate (FFR). All variables are taken in logarithms−except FFR−and in levels since the existence of cointegrating relations may bias the estimates in first differences.

[6]The real GDP is a monthly interpolation of several monthly variables, see Bernanke Mihov (1998a) for details.

(i) prices are sticky: both the inflation rate and the commodity price index react to the monetary policy shock with a period of lag,

(ii) real GDP reacts to the monetary policy shock with a period of lag,

(iii) a contractionary monetary policy shock has a negative impact effect on non-borrowed reserves,

(iv) a contractionary monetary policy shock has a positive impact effect on the federal funds rate,

(v) the impact effects in (iii) and (iv) are both large.

Hypotheses (i) and (ii) are widely used in VAR literature (see e.g Bernanke and Blinder, 1992, Strongin, 1995 and Bernanke and Mihov, 1998a 1998b). Assumption on ΔGDPD is derived from sticky price models (see e.g. Fisher, 1977, Blanchard, 1984 and Taylor, 1998, for a survey). The assumption on commodity prices is derived from the time pattern of the international propagation mechanisms of monetary policy: exchange rate variations induced by monetary policy actions may produce effects on commodity prices *via* capital flows, but over time horizons longer than one month. Assumption (ii) refers to the lag in the transmission of monetary policy actions to real economy.

Assumption (iii) and (iv)[7] derive from the theory of the liquidity effect: for a given reserves demand, a change in non-borrowed reserves produces, in the short term, a change of opposite sign on nominal interest rate (see e.g. Cagan and Gandolfi, 1968, Leeper and Gordon, 1992, Christiano, Eichenbaum and Evans, 1992, Pagan and Robertson, 1995, Strongin, 1995, Bernanke and Mihov, 1998b). Assumption (v) states that the portion of variances of FFR and NBR

[7]As suggested by several authors (see e.g. Gordon and Leeper, 1994, Strongin, 1995, Bernanke and Mihov, 1998a), by considering both the effects we exclude endogenous effects induced by reserve demand. With an interest rate targeting policy, variations in the supply of reserves may be due to an accommodating behavior of the central bank in consequence of demand variations in order to keep the interest rate constant. On the other hand, with a non-borrowed reserves policy, changes in the interest rate may be due to variations in reserve demand in order to keep the supply constant.

explained by the monetary policy shock within the first month is large[8].

From equation (4) we define $C_1 = C_1(0) = SH_1$ the **impact vector** of the monetary policy shock. We have to fix the vector H_1 so that restrictions (i), (ii), (iii), (iv), (v) on C_1 can be respected. Denoting c_i the i^{th} element of C_1, and h_i the i^{th} element of H_1, from hypotheses (i) and (ii) we have $c_1 = c_2 = c_3 = 0$; since $c_1 = s_{11}h_1$, $c_2 = s_{21}h_1 + s_{22}h_2$ and $s_{21}h_1 + s_{22}h_2 + s_{23}h_3$ this implies $h_1 = h_2 = h_3 = 0$ (see Appendix B). Hypothesis (iii) involves $c_4 < 0$ and hypothesis (iv) $c_5 > 0$. Hypothesis (v), by assuming that c_4 and c_5 are large, provides us with a constrained maximization restriction. In order that c_4 and c_5 be jointly large, we need, first, an idea of their maximal size and, second, a technical procedure to keep them that way. Let us define $\theta_1 = c_{41}^2/\sigma_{NBR}^2$ and $\theta_2 = c_{51}^2/\sigma_{FFR}^2$[9]. In order to have an idea of the maximal size of the impact effect of the shock on FFR and NBR let us maximize separately θ_1 and θ_2. Let $\Theta_1 = max(\theta_1)$ and $\Theta_2 = max(\theta_2)$[10]. Θ_1, Θ_2 signify the largest contribution possible of the monetary policy shock in the first month to the variances of the two series FFR and NBR (see note 8).

In order to keep the impact effects jointly large, we proceed as follows: first,

[8]Since $Ee_te'_{t-k} = 0$ for $k \neq 0$ the variance of the i^th element x_i of the vector X_t will be

$$var(x_{it}) = \sigma_{x_i}^2 = \sum_{j=1}^{6}\sum_{k=0}^{\infty} \mathrm{var}(e_j)c_{ijk}^2 \qquad (5)$$

with $i = 1, ..., n$, $j = 1, ..., n$ and k the time horizon; since $var(e_j) = 1$

$$var(x_{it}) = \sum_{j=1}^{6}\sum_{k=0}^{\infty} c_{ijk}^2. \qquad (6)$$

By assuming that c_{ij0} is large, we assume that the contribution of the j^{th} shock to the variance of the i^{th} variable in $k = 0$ is large. That assumption is justified since FFR and NBR are under the Fed control and there are no lag in the transmission of monetary policy actions to these variable.

[9]We normalize the coefficients for the standard deviations of the first differences of the series.

[10]The only constraint is $\sum_{i=1}^{n} h_i^2 = 1$; $\Theta_1 = max(\theta_1)$ for $h_4^2 = 1$ and $\Theta_2 = max(\theta_2)$ for $h_4^2 + h_5^2 = 1$

we calculate the ratio $r = \Theta_1/\Theta_2$ and second we maximize θ_1 under the constraints $\theta_1/\theta_2 = r$. In such a way we obtain the constrained maximal values $\bar{\Theta}_1$ and $\bar{\Theta}_2$. From $\bar{\Theta}_1$, $\bar{\Theta}_2$ and from the sign constraints (iii) $c_4 < 0$ and (iv) $c_5 > 0$, it is possible to derive the impact coefficients $\hat{c}_4$ and $\hat{c}_5$. The constrained maximization implies two restrictions, on h_4 and h_5 (see Appendix B). From the hypothesis of orthonormality of the matrix H, the following condition must hold, $\sum_{i=1}^{n} h_i^2 = 1$; such a condition implies a restriction on h_6 which determines $\hat{c}_6$ and identifies $C_1(L)$ (see Appendix B).
The **monetary policy impact vector**, C_1, is .

$$C_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \hat{c}_4 \\ \hat{c}_5 \\ \hat{c}_6 \end{pmatrix} \tag{7}$$

4 Results

The results are shown in Figure 1; the impulse response functions are plotted with 90% confidence bands[11].
Inflation rate reacts very sluggishly and drops significantly only after 16 months. The decline in inflation persists for five years and than vanishes. The monetary contraction succeeds in reducing inflation but with 16-months of lag and not permanently. Commodity prices drop more quickly than inflation, since com-

[11]The confidence bands are constructed with the *bootstrapping* method; let $\{\hat{\varepsilon}_t\}_{t=1}^{T}$ be the vector of residuals of the VAR with T the number of observations. By extracting with introduction T times from $\{\hat{\varepsilon}_t\}_{t=1}^{T}$ we construct 1000 new residual matrices $\{\hat{\varepsilon}_t(j)\}_{i=1}^{T}$ with $j = 1, ..., 1000$. For each j matrix, given initial conditions, we construct a new set of series. For each set we estimate the VAR, and collect the impulse functions. For each variable at any lag we extract the 50^{th} lower value and the 950^{th} higher value. In so doing, for each original impulse function we obtain two 90% confidence bands.

modities are traded on markets with flexible prices, and for the first three years the decline is significant. The reduction of the commodity price operates presumably *via* dollar appreciation; a contractionary monetary shock, by increasing interest rate, causes external capital inflow, making the dollar appreciate. Such appreciation, the prices of commodities being expressed in dollars, sets off an international disinflation process which reduces the price of commodities.

Non-borrowed reserves and total reserves initially drop with non-borrowed reserves dropping more than total reserves, in accordance to an interest rate inelastic short-term reserves demand (Strongin, 1995). The drop of non-borrowed reserves is partially offset by an increase in borrowing by commercial banks. From one to two years both non-borrowed and total reserves turn positive and than converge to the initial values in the long-run. Our findings are consistent with the theory. After the initial contraction, monetary authorities ease the monetary policy tightness by progressively expanding reserves.

The federal funds rate reacts positively reaching its maximum during the second month and than reversing course and turning negative within the year. The behavior of the federal funds rate is also consistent with the theory. In the very short term the liquidity effect holds and the federal funds rate rises. With inflation declining, price expectations adjust and make the federal funds rate turn negative; over longer time horizons a Fisherian effect holds and the federal funds rate drops.

The response of real GDP seems conventional with the theory. After the initial 6-months, real GDP falls significantly and the decline persists for three years. In the short-term monetary policy succeeds in reducing output and reduction is significant at 90%. From the third year the effect of the shock on real GDP reduces, vanishing after four years in consequence of the gradual easening of the monetary policy tightness. We find evidence in favor of the traditional view: monetary policy is non-neutral in the short-run and neutral in the long-run.

In conclusion:

1. A contractionary monetary policy shock reduces real GDP temporaneously. Monetary policy is non neutral in the short run and neutral in the long run.
2. Inflation drops sluggishly. A contractionary shock succeeds in reducing inflation but with lag and not permanently. Commodity prices drops more quickly than inflation; the effect of the shock last for three years and vanishes in the long-run.
3. Reserves and the federal funds rate reacts immediately to the shock than from one to two years the effect begins to reduce and vanishes in the long run.

5 Sensitivity Analysis

Our identification criterion, as shown in Appendix B, depends on the ratio r. Since r does not derive from any theoretical assumption, this could affect the results. We relax the assumption that $r = \Theta_1/\Theta_2$ and we perform a sensitivity analysis of the results for different values of r. We set a range of possible values for r and for any value we repeat the identification (Section 3 and Appendix B). Since hypothesis (v) must hold, we choose the range $-1 + \Theta_1/\Theta_2 \leq r \leq \Theta_1/\Theta_2 + 1$; inside this range we choose eleven equally spanned values for r. From Figure 2 we may conclude that r does not affect significantly the results and our identification procedure is invariant for different values of r satisfying hypothesis (v).

6 Quasi-Identification Criterion

In Section 4 identification is achieved by jointly maximizing the square impact effects of non-borrowed reserves and the federal funds rate under the constraint r. It could be objected that our methodology has an arbitrary aspect: though the effects are large, they may not be the largest. If that were the case our criterion would not be correct.

In this Section we propose a method that, by relaxing the identification hypotheses[12], confirms the results described in Section 4 and strengthens the identification criterion. We call this method *Quasi-Identification Criterion* (QIC) since it does not identify but enables us to recover the sign and the shape of the effect. A set of possible impulse response functions is identified. Let us suppose that all the resulting admissible impulse response functions have the same shape or, more simply, that they all present the same property. In this case we can say nothing about the exact size of the effect, but we may recover both the sign and the shape.

This kind of analysis reduces the number of the *a priori* restrictions to be imposed; exact identification requires $n(n-1)/n$ restrictions, partial identification $m(2n-m-1)/2$, quasi identification one at least. The hypotheses of quasi-identification are the following: a contractionary monetary policy shock has (i) a zero impact effect on real GDP, (ii) a zero impact effect on prices, (iii) a positive impact effect on federal funds rate, (iv) a negative impact effect on non-borrowed reserves and (va) an impact effect on the federal funds rate and non-borrowed reserves to such an extent that the contribution of the shock to the variance of the two series in the first month is at least equal to one third of the largest contribution possible; in other words, that $\theta_1 \geq \Theta_1/3$ and $\theta_2 \geq \Theta_2/3$[13]. All the impact vectors which satisfy restrictions (i)-(va) conform to the definition of monetary policy impact vectors and their relative impulse response functions conform to the effects of the monetary policy shock.

From hypotheses (i) and (ii) $c_1 = c_1 = c_3 = 0$ and $h_1 = h_2 = h_3 = 0$ (see Appendix B and Section 3). From the orthonormality hypothesis the following condition holds $\sum_{i=1}^{n} h_i^2 = 1$. Incorporating the orthonormality condition in H_1 we parameterize the vector H_1 as follows[14]

[12] In Section 5 we relaxed the hypothesis $r = \Theta_1/\Theta_2$. Here we relax also the maximization restriction.

[13] Assumptions (i)-(iv) are the same as in section 3; assumption (va) replaces the maximization assumption (v).

[14] This parameterization is a variant of Uhlig (1999).

$$H_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ cos(\delta_1)cos(\delta_2) \\ cos(\delta_1)sin(\delta_2) \\ sin(\delta_1) \end{pmatrix} \delta \in \Re^2. \qquad (8)$$

We consider different values for the vector (δ_1, δ_2) in the interval $[0, 2\pi]$, i.e. $(\delta_1, \delta_2) = (\pi h/5, \pi k/5)$; $h = 1, ..., 20$; $k = 1, ..., 20$. 20^2 vectors δ^j will result, with $j = 1, ..., 400$ any of which generates an impact vector C_1^j. For some of those the restrictions will be respected, for others they will not. Denoting with Ω the set of δ^j for which the restrictions (iii), (iv) and (va) are respected, the aim of the quasi-identification criterion is to study the set of admissible impulses $C_1^j(L)$ generated by the δ^j-vectors of Ω in order to recover the sign and the shape of the effect of the monetary policy shock.
Results of quasi-identification are shown in Figure 2. We find 74 vectors δ^j for which restrictions (i)-(va) are respected. All the resulting impulse response functions present the same sign and the same shape both in the short and in the long run, except total reserves which have an undetermined impact effect. Moreover the results are in line with those described in Section 4. In so doing, the suspicion that maximization could produce distortions in the results is dispelled.

7 Conclusions

The main conclusion of this paper is that monetary policy is non-neutral in the short-run and neutral in the long-run. We find evidence in favor of the mainstream view and our results are in line with several empirical works (see e.g. Bernanke and Mihov, 1998a, 1998b, Uhlig, 1999). Our findings are obtained by means of a partial identification criterion based on restrictions on the sign and the size of the impact effect of real GDP, prices, federal funds rate and non-borrowed reserves. In addition, we propose a quasi-identification criterion that, by relaxing the identification hypothesis, enables the sign and the shape of the response functions to be recovered confirming the identification results.

References

Bernanke, B and Blinder, A (1992) "The Federal Funds Rate and the Channels of Monetary Transmission," *American Economic Review*, 82 (September): 901-921.

Bernanke, B. and Mihov, I (1998a) "Measuring Monetary Policy," *Quarterly Journal of Economics* 113 (August): 869-902.

Bernanke, B. and Mihov, I. (1998b) "The Liquidity Effect and the Long-Run Neutrality," *Carnegie-Rochester Conference Series on Public Policy*, 49 (December): 149-194.

Blanchard, O. (1994) "Price Asynchronisation and the Price Level Inertia," in Dornbush, R. and Simonsen, M. H. (ed.) *Inflation, Debt and Indexation.* Cambridge MA: The MIT Press.

Cagan, P. and Gandolfi, A. (1969) "The Lag in Monetary Policy as Implied by the Time Pattern of Monetary Effects on Interest Rate," *American Economic Review*, Papers and Proceedings, 59 (February): 191-205.

Canova, F. (1995) "The Economics of VAR Models," in Hoover, K. *Macroeconometrics*, Boston: Kluwer Academic Publisher.

Christiano, L. and Eichenbaum, M. (1992) "Identification and the Liquidity Effect of a Monetary Policy Shock," in Cukierman, A. Hercovitz, Z. e Leiderman, L. (ed.) *Political Economy, Growth, and Business Cycles.* Cambridge MA: The MIT Press.

Christiano, L. Eichenbaum, M. and Evans, C. (1998) "Monetary Policy Shocks: What We Have Learned and to What End?" *NBER Working Papers*, No. 6400.

Fisher, S. (1977) "Long Term Contracts, Rational Expectations, and the Optimal Money Supply Rule," *Journal of Political Economy*, 85 (February): 163-190.

Frisch, R. (1933) "Propagation Problems and Impulse Problems in Dynamic Economics," in *Economic Essays in Honor of Gustav Cassel*. London: Allen and Unwin.

Gordon, D. and Leeper, E. (1994) "The Dynamic Impacts of Monetary Policy: An Exercise in Tentative Identification," *Journal of Political Economy*, 102 (December): 1228-1247.

Leeper, E. and Gordon, D. (1992) "In Search of the Liquidity Effect." *Journal of Monetary Economics* 29 (June): 341-369.

Lippi, M. and Reichlin, L. (1993)"A Note on Measuring the Dynamic Effects of Aggregate Demand and Supply Disturbances." *American Economic Review* 83 (June): 644-652.

Lucas, R. (1972) "Expectations and the Neutrality of Money." *Journal of Economic Theory* 4 (April): 104-124.

Pagan, A. and Robertson, J. (1995) "Resolving the Liquidity Effect." *Federal Reserve Bank of St. Louis Review* 77 (May/June): 33-54.

Slutsky, E. (1937) "The Summation of Random Causes as the Source of Cyclical Process." *Econometrica* 5: 105-146.

Sims, C. (1980) "Macroeconomics and Reality." *Econometrica* 48 (January): 1-48.

Strongin, S (1995) "The Identification of Monetary Policy Disturbances: Explaining the Liquidity Puzzle." *Journal of Monetary Economics* 34 (June): 463-497.

Taylor, J. (1980) "Aggregate Dynamics and Staggered Contracts." *Journal of Political Economy* 88 (February): 1-23.

Taylor, J. (1998) "Staggered Contracts and Wage Setting in Macroeconomics." NBER *Working Papers Series* No. 6754.

Uhlig, H. (1999) "What Are the Effects of Monetary Policy? Results from an Agnostic Identification Procedure." CEPR *Discussion Papers* No. 2137.

Appendix A

The building block of VAR econometrics is the Wold Representation Theorem that states that any stationary stochastic process can be decomposed in two orthogonal components in the following manner:

$$X_t = \sum_{j=0}^{\infty} \alpha_j \varepsilon_{t-j} + \mu_t \tag{9}$$

where $\sum_{j=0}^{\infty} \alpha_j \varepsilon_{t-j}$ represents the stochastic component, with $\sum_{j=0}^{\infty} \alpha_j^2 < \infty$, $\{\varepsilon_{t-j}\}_{j=0}^{\infty}$ is a zero mean white noise process, that is (a) $E\varepsilon_t\varepsilon_t' = \sum_\varepsilon$, $E\varepsilon_t\varepsilon_{t-k}' = 0$ for $k \neq 0$, and μ_t represents the purely deterministic component, the one perfectly predictable by using past information. As usual in VAR literature, we will consider only *regular process*, that is those processes for which $\mu_t = 0$. Rewriting (9) in lag operator terms and assuming $\mu_t = 0$ we have the representation (1). Equation (1) is the Wold representation of the process X_t and the following conditions hold: (a), (b) all the roots of the determinant of $A(L)$ are outside the unit circle in the complex field, (c) $A(0) = I$. Conditions (a), (b) and (c) guarantee the unicity of the representation. From the Wold representation is possible to derive the class of *fundamental*[15] representations of the process X_t. Given any non singular matrix of constants R is possible to rewrite (1) as follows

$$X_t = A(L)RR^{-1}\varepsilon_t = B(L)u_t \tag{10}$$

where $B(L) = A(L)R$ and $u_t = R^{-1}\varepsilon_t$. Since R can be any non-singular matrix of constants, it follows that the class of fundamental representations defined by (10) has infinite representations that differ from each other for a particular R. From the class of fundamental representations we may define a subclass, that of orthonormal representations. Let S be the Choleski factor of $\sum_\varepsilon$ such that $SS' = \Sigma\varepsilon$. Postmultiplying $A(L)$ for S and premultiplying ε_t for S^{-1} in (1) we

[15]The representations for which condition (b) holds; for *non-fundamental* representations see Lippi and Reichlin (1993).

obtain

$$X_t = D(L)\eta_t \tag{11}$$

where $D(L) = A(L)S$ and $\eta_t = S^{-1}\varepsilon_t$. Equation (11) is the Choleski representation of X_t and has the following properties: $D(0) = S$, $D(L) = S + D_1L + D_2L + ...$, $\sum_\eta = E\eta_t\eta_t' = S^{-1}\sum_\varepsilon S'^{-1} = I$. As for the class of fundamental representations, even in this case it is possible to generalize to the whole class of orthonormal representations. For any matrix H such that $HH' = I$, by postmultiplying $A(L)S$ for H and premultiplying η_t for H' we obtain the representation (2). Representation (1) has the same properties of representation (11) and differs from (11) for H. The class of orthonormal representations, as subclass of fundamentals, contains infinite representations which differ from each others for a particular choice of H.

Given a matrix H, equation (1) and (2) set up our model: the first is the reduced form and the second the structural form of the VAR. The following relations hold: $C(L) = A(L)SH$ and $e_t = H'S^-1\varepsilon_t$.

Appendix B

Here we show technical aspects of the identification criterion and we show how to choose the vector H_1. The first step consists in transforming the model expressed by equation (1) into the orthonormal model expressed by equation (11) by postmultiplying $A(L)$ by the Choleski factor S of the variance-covariance matrix $\sum_\varepsilon$. The second step consists in choosing the vector H_1 in order to determine $C_1(L)$ by postmultiplying $A(L)S$ for H_1. H_1 is obtained as follows. Let us consider the impact vector $C_1 = SH_1$. Hypotheses (i) and (ii) entail $s_{11}h_1 = 0$, $s_{21}h_1 + s_{22}h_2 = 0$ and $s_{31}h_1 + s_{32}h_2 + s_{33}h_3 = 0$; since S is lower triangular, $s_{ij} \neq 0$ for $i \geq j$, $h_1 = h_2 = h_3 = 0$. Hypotheses (iii) and (iv) involve $s_{44}h_4 < 0$ and $s_{54}h_4 + s_{55}h_5 > 0$. From hypothesis (v) $s_{44}h_4$ and $s_{54}h_4 + s_{55}h_5$ must be jointly large. Recall that[16] $\theta_1 = (s_{44}h_4)^2$ and $\theta_2 = (s_{54}h_4 + s_{55}h_5)^2$, $\Theta_1 = max(\theta_1) = (s_{44}h_4^*)^2$ and $\Theta_2 = max(\theta_2) = (s_{54}h_4^* + s_{55}h_5^*)^2$. First we calculate the ratio, $r = \Theta_1/\Theta_2$, then we maximize θ_1 under the constraints $\theta_1/\theta_2 = r$ and the sign constraints $s_{44}h_4 < 0$, $s_{54}h_4 + s_{55}h_5 > 0$. Hence we have $\frac{s_{44}h_4}{s_{54}h_4+s_{55}h_5} = -\sqrt{r}$ Easy arithmetic passages lead to

$$h_5 = h_4 \left(-\frac{s_{44} + \sqrt{r}s_{54}}{\sqrt{r}s_{55}} \right). \tag{12}$$

Let

$$\left(-\frac{s_{44} + \sqrt{r}s_{54}}{\sqrt{r}s_{55}} \right) = \Gamma. \tag{13}$$

From the orthonormality condition, $\sum_{i=1}^{n} h_i^2 = 1$, the following restriction must hold

$$h_6 = \pm\sqrt{1 - h_4^2 - h_5^2} = \pm\sqrt{1 - h_4^2 - h_4^2\Gamma^2} \tag{14}$$

From equation (12) and (14) we have

$$(1 + \Gamma^2)h_4^2 \leq 1 \tag{15}$$

[16]Here we assume, for convenience of exposition, that the elements s_{ij} of S have been divided by the standard deviation, σ_i, of first differences of the i^{th} element x_i.

Since s_{44} is constant, the impact effect on non-borrowed reserves will be maximum when h_4 is maximum, hence when h_4^2 is maximum, and we will have

$$h_4 = \pm\frac{1}{\sqrt{1+\Gamma^2}} \tag{16}$$

In particular from (iii), since $s_{44} > 0$, see Table 2, h_4 must be negative. From equations (14) and (16) we have $h_6 = 0$; this last passage completes the identification. H_1 will result

$$H_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{1}{\sqrt{1+\Gamma^2}} \\ -\frac{\Gamma}{\sqrt{1+\Gamma^2}} \\ 0 \end{pmatrix}, \tag{17}$$

$\bar{\Theta}_1 = \left(\frac{s_{44}}{\sqrt{1+\Gamma^2}}\right)^2$, $\bar{\Theta}_2 = \left(\frac{s_{54}+s_{55}\Gamma}{\sqrt{1+\Gamma^2}}\right)^2$, $\hat{c}_4 = -\frac{s_{44}}{\sqrt{1+\Gamma^2}}$ and $\hat{c}_5 = -\frac{s_{54}+s_{55}\Gamma}{\sqrt{1+\Gamma^2}}$, since $s_{54} < 0$, $s_{55} > 0$ and $\Gamma < 0$, see Table 2. The vector C_1 will be

$$C_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{s_{44}}{\sqrt{1+\Gamma^2}} \\ -\frac{s_{54}+s_{55}\Gamma}{\sqrt{1+\Gamma^2}} \\ \frac{s_{65}\Gamma-s_{64}}{\sqrt{1+\Gamma^2}} \end{pmatrix}. \tag{18}$$

Tables

Table 1: Impact values for non-borrowed reserves and federal funds rate

Θ_1	Θ_2	$\bar{\Theta}_1$	$\bar{\Theta}_2$	$\Theta_1/3$	$\Theta_2/3$	r	$-\sqrt{r}$
0.69	0.43	0.61	0.39	0.23	0.14	1.5	-1.2

Table 2: The matrix S

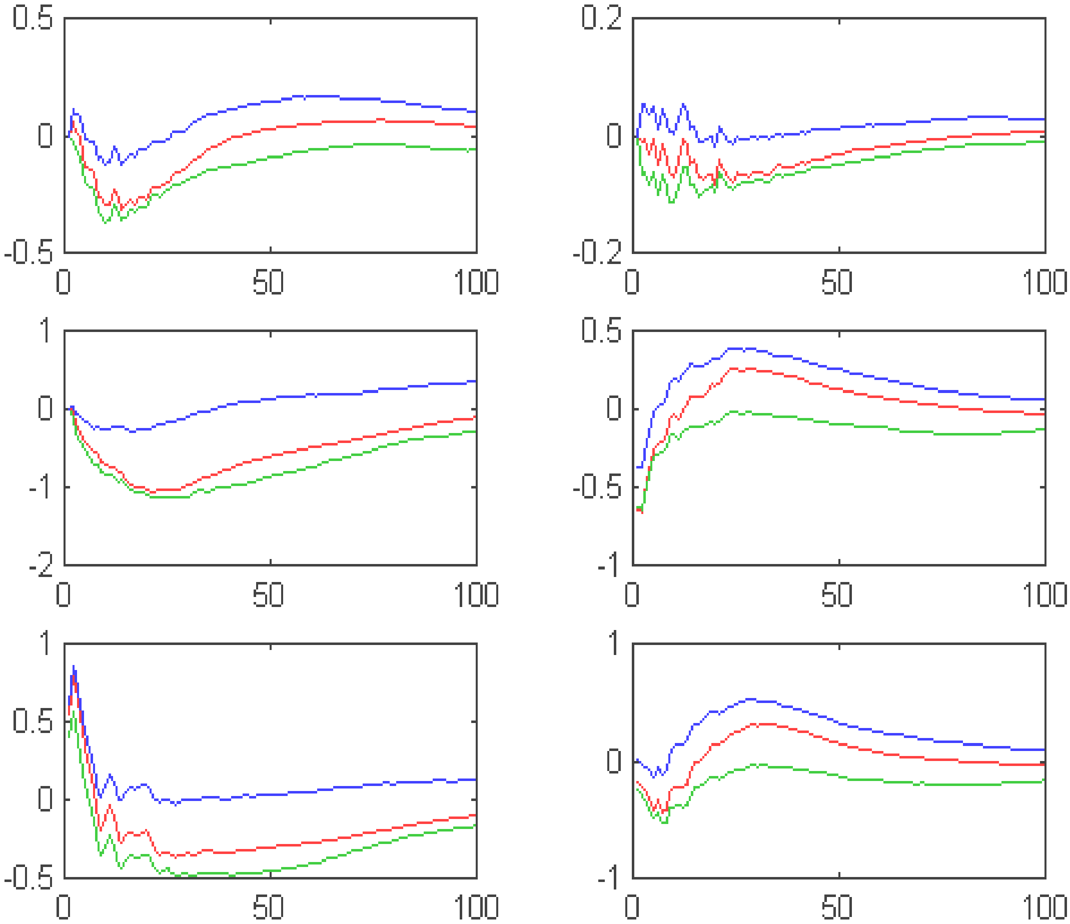
GDP	0.0071	0	0	0	0	0
GDPD	0.0001	0.0018	0	0	0	0
CP	0.0017	-0.0000	0.0172	0	0	0
NBR	-0.0010	-0.0006	-0.0027	0.0134	0	0
FFR	0.1042	0.0029	0.0662	-0.1321	0.4495	0
TR	0.0001	-0.0004	-0.0002	0.0051	0.0033	0.0069

Table 3: Vector H_1

H_1
0
0
0
0.7825
-0.6227
0

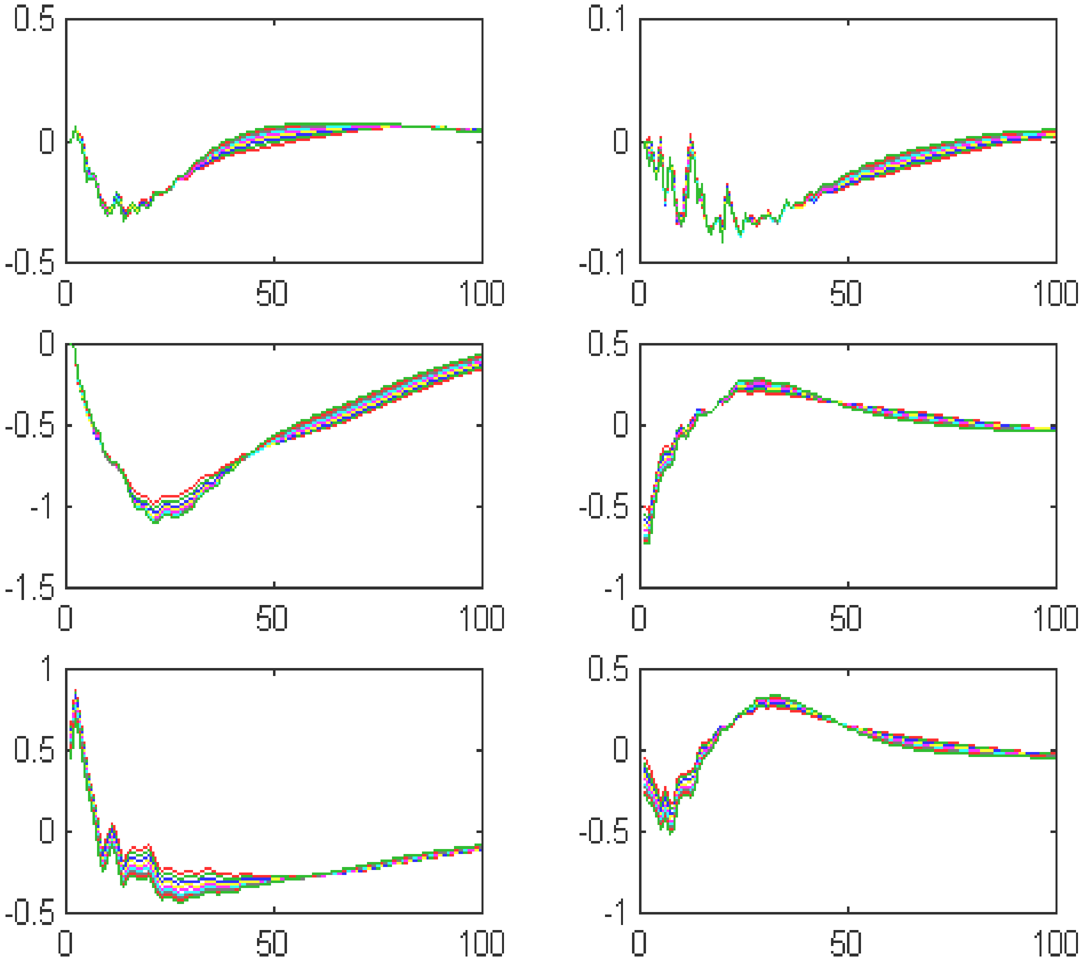
Figures

Figure 1



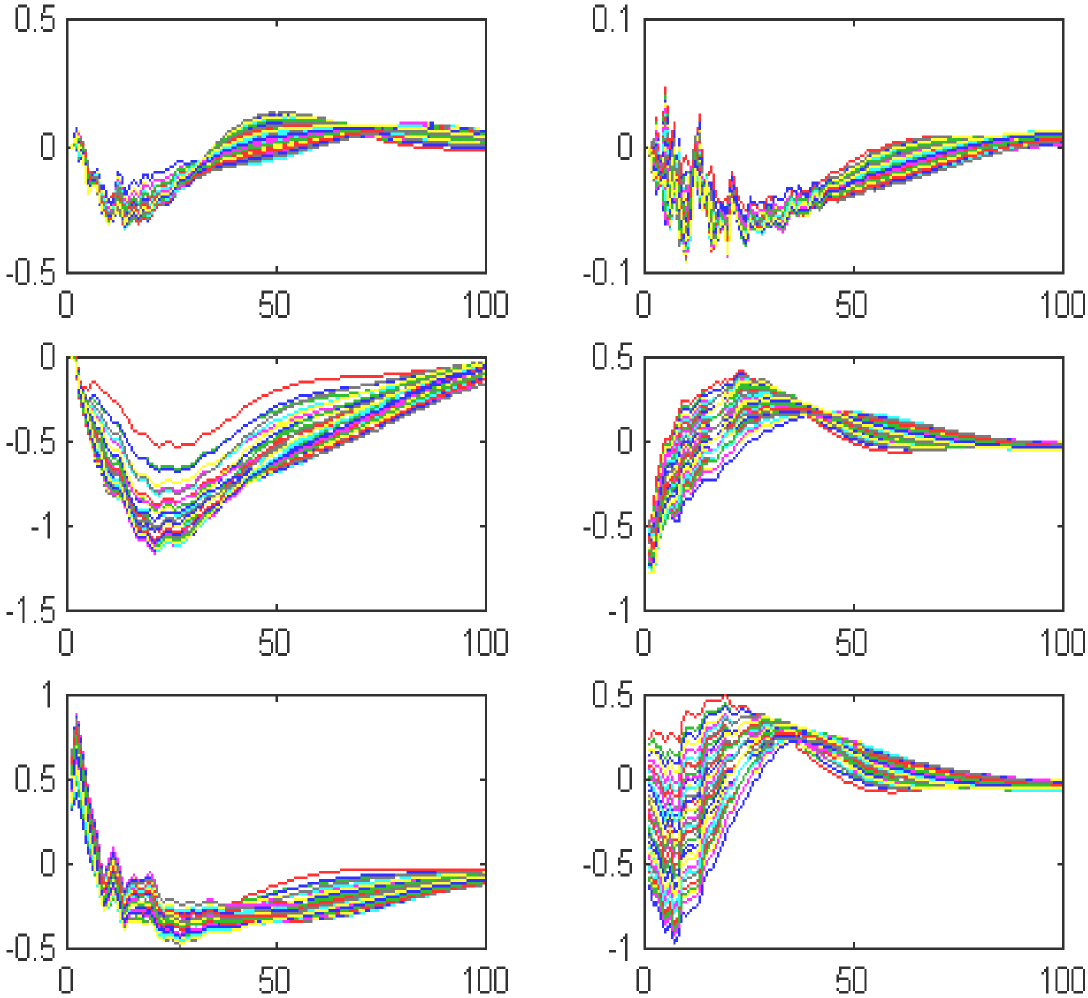
Results of identification. We plot, from the left to the right, the impulse response functions for GDP, DGDPD (first row), CP, NBR (second row), FFR and TR (third row) with 90

Figure 2



Impulse response functions for different values of r. We consider eleven equally spanned values of r of distance 0.2 in the interval [0.5, 2.5]. Results show that identification is invariant to r, since all the impulses response functions present the same shape and very similar values for any k, with k=1,,100. We plot 74 response functions for each variable. k=1,,100 are months.

Figure 3



Results of Quasi Identification. We plot the impulse response, functions, from left to right, for the variables: GDP, DGDPD (first row), CP, NBR (second row), FFR and TR (third row). $k = 1, , 100$ are months.