# The Effects of Monetary Policy on Stock Market Bubbles: Some Evidence *

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Abstract

We estimate the response of stock prices to monetary policy shocks using a time-varying coefficients VAR. Our evidence points to protracted episodes in which stock prices end up increasing persistently in response to an exogenous tightening of monetary policy. That response is at odds with the "conventional" view on the effects of monetary policy on bubbles, as well as with the predictions of bubbleless models. We also argue that it is unlikely that such evidence can be accounted for by an endogenous response of the equity premium to the monetary policy shock.

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The economic and financial crisis of 2008-2009 has been associated in many countries with a rapid decline in housing prices, following a protracted real estate boom. This has generated a renewed interest in the link between monetary policy and asset price bubbles, and revived the long standing debate on whether and how monetary policy should respond to perceived deviations of asset prices from fundamentals.[1]

The consensus view before the crisis was that central banks should focus on stabilizing inflation and the output gap, and ignore fluctuations in asset prices, even if the latter are perceived to be driven by bubbles.[2] The recent crisis has challenged that consensus and has strengthened the viewpoint that central banks should pay attention and eventually respond to developments in asset markets. Supporters of this view argue that monetary authorities should "lean against the wind," i.e. raise the interest rate to counteract any bubble-driven episode of asset price inflation, even at the cost of temporarily deviating from their inflation or output gap targets. Any losses associated with these deviations, it is argued, would be more than offset by the avoidance of the consequences of a future burst of the bubble.[3]

A central tenet of the case for "leaning against the wind" monetary policies is the presumption that an increase in interest rates will reduce the size of an asset price bubble. While that presumption may have become part of the received wisdom, no empirical or theoretical support seems to have been provided by its advocates.

In recent work (Galí (2014)), one of us has challenged, on theoretical grounds, the link between interest rates and asset price bubbles underlying the conventional view. The

[1] Throughout the paper we use the term "monetary policy" in the narrow sense of "interest rate policy." Thus we exclude from that definition policies involving macroprudential instruments which are sometimes controlled by central banks and which may also be used to stabilize asset prices.

[2] See, e.g. Bernanke and Gertler (1999, 2000) and Kohn (2006). Two arguments have been often pointed to in support of that view: (i) asset price bubbles are difficult to detect and measure, and (ii) interest rates are "too blunt" an instrument to prick a bubble, and their use with that purpose may have unintended collateral damages.

[3] See, e.g., Borio and Lowe (2002) and Cecchetti, Gensberg and Wadhwani (2000) for an early defense of "leaning against the wind" policies.

reason is that, at least in the case of *rational* asset price bubbles, the bubble component must grow, in equilibrium, at the rate of interest. If that is the case, an interest rate increase may end up enhancing the size of the bubble. Furthermore, and as discussed below, the theory of rational bubbles implies that the effects of monetary policy on asset prices should depend on the relative size of the bubble component. More specifically, an increase in the interest rate should have a negative impact on the price of an asset in periods where the bubble component is small compared to the fundamental. The reason is that an interest rate increase always reduces the "fundamental" price of the asset, an effect that should be dominant in "normal" times, when the bubble component is small or non existent. But if the relative size of the bubble is large, an interest rate hike may end up increasing the asset price over time, due to its positive effect on the bubble more than offsetting the negative impact on the fundamental component.

In the present paper we provide evidence on the dynamic response of stock prices to monetary policy shocks, and try to use that evidence to infer the nature of the impact of interest rate changes on the (possible) bubble component of stock prices. Our main goal is to assess the empirical merits of the "conventional" view, which predicts that the size of the bubble component of stock prices should decline in response to an exogenous increase in interest rates. Since the fundamental component is expected to go down in response to the same policy intervention, any evidence pointing to a positive response of *observed* stock prices (i.e. of the sum of the fundamental and bubble components) to an exogenous interest rate hike would call into question the conventional view regarding the effects of monetary policy on stock price bubbles.

Our starting point is an estimated vector-autoregression (VAR) on quarterly US data for GDP, the GDP deflator, a commodity price index, dividends, the federal funds rate, and a stock price index (S&P500). Our identification of monetary policy shocks is based on the approach of Christiano Eichenbaum and Evans (2005; henceforth, CEE), though our focus here is on the dynamic response of stock prices to an exogenous hike in the interest rate. Also, and in contrast with CEE, we allow for time-variation in the VAR coefficients, which results in estimates of time-varying impulse responses of stock prices to policy shocks.[4] In addition to the usual motivations for doing this (e.g. , structural change), we point to a new one which is specific to the issue at hand: to the extent that changes in interest rates have a different impact on the fundamental and bubble components, the overall effect on the observed stock price may change over time as the relative size of the bubble changes.

Under our baseline specification, which assumes no contemporaneous response of monetary policy to asset prices, the evidence points to protracted episodes in which stock prices increase persistently in response to an exogenous tightening of monetary policy. That response is clearly at odds with the "conventional" view on the effects of monetary policy on bubbles, as well as with the predictions of bubbleless models.

We assess a variety of alternative explanations for our findings. In particular, we argue that it is unlikely that such evidence be accounted for by an endogenous response of the equity premium to the monetary policy shocks.

When we allow for an endogenous contemporaneous response of interest rates to stock prices, and calibrate the relevant coefficient in the monetary policy rule according to the findings in Rigobon and Sack (2003), our findings change dramatically: stock prices decline substantially in response to a tightening of monetary policy, more so than our estimated fundamental components. That finding would seem to vindicate the conventional view on the effectiveness of leaning against the wind policies. Recent evidence by Furlanetto (2011), however, suggests that Rigobon and Sack's finding is largely driven by the Fed's response to the stock market crash of 1987, thus calling into question the relevance of this alternative specification for much of the sample period analyzed, while supporting instead our baseline specification.

Ultimately, our objective is to produce evidence that can improve our understanding of the impact of monetary policy on asset prices and asset price bubbles. That under-

[4] See, e.g. Primiceri (2005) and Gali and Gambetti (2009) for some macro applications of the TVC-VAR methodology.

standing is a necessary condition before one starts thinking about how monetary policy *should* respond to asset prices.

The remainder of the paper is organized as follows. In Section 1 we discuss alternative hypothesis on the link between interest rates and asset prices. Section 2 describes our empirical model. In Section 3 we report the main findings under our baseline specification. Section 4 provides alternative interpretations as well as evidence based on an alternative specification. Section 5 concludes.

# I Monetary policy and asset price bubbles: Theoretical issues

We use a simple partial equilibrium asset pricing model to introduce some key concepts and notation used extensively below.[5] We assume an economy with risk neutral investors and an exogenous, time-varying (gross) riskless real interest rate $R_t$.[6] Let $Q_t$ denote the price in period $t$ of an infinite-lived asset, yielding a dividend stream $\{D_t\}$.

We interpret that price as the sum of two components: a "fundamental" component, $Q_t^F$, and a "bubble" component, $Q_t^B$. Formally,

$$Q_t = Q_t^F + Q_t^B \tag{1}$$

where the *fundamental* component is defined as the present discounted value of future dividends:

$$Q_t^F \equiv E_t \left\{ \sum_{k=1}^{\infty} \left( \prod_{j=0}^{k-1} (1/R_{t+j}) \right) D_{t+k} \right\} \tag{2}$$

or, rewriting it in log-linear form (and using lower case letters to denote the logs of the

[5] See Galí (2014) for a related analysis in general equilibrium.

[6] Below we discuss the implications of relaxing the risk neutrality assumption and allowing for a risk premium.

original variables)[7]

$$q_t^F = const + \sum_{k=0}^{\infty} \Lambda^k \left[(1-\Lambda) E_t\{d_{t+k+1}\} - E_t\{r_{t+k}\}\right] \tag{3}$$

where $\Lambda \equiv \Gamma/R < 1$, with $\Gamma$ and $R$ are denoting, respectively, the (gross) rates of dividend growth and interest along a balanced growth path.

How does a change in interest rates affect the price of an asset that contains a bubble? We can seek an answer to that question by combining the dynamic responses of the two components of the asset price to an exogenous shock in the policy rate. Letting that shock be denoted by $\varepsilon_t^m$, we have:

$$\frac{\partial q_{t+k}}{\partial \varepsilon_t^m} = (1-\gamma_{t-1}) \frac{\partial q_{t+k}^F}{\partial \varepsilon_t^m} + \gamma_{t-1} \frac{\partial q_{t+k}^B}{\partial \varepsilon_t^m} \tag{4}$$

where $\gamma_t \equiv Q_t^B / Q_t$ denotes the share of the bubble in the observed price in period $t$.

Using (2), we can derive the predicted response of the fundamental component

$$\frac{\partial q_{t+k}^F}{\partial \varepsilon_t^m} = \sum_{j=0}^{\infty} \Lambda^j \left( (1-\Lambda) \frac{\partial d_{t+k+j+1}}{\partial \varepsilon_t^m} - \frac{\partial r_{t+k+j}}{\partial \varepsilon_t^m} \right) \tag{5}$$

Both conventional wisdom and economic theory (as well as the empirical evidence discussed below) point to a rise in the real interest rate and a decline in dividends in response to an exogenous tightening of monetary policy, i.e. $\partial r_{t+k} / \partial \varepsilon_t^m > 0$ and $\partial d_{t+k} / \partial \varepsilon_t^m \leq 0$ for $k = 0, 1, 2, ...$Accordingly, the fundamental component of asset prices is expected to decline in response to such a shock, i.e. we expect $\partial q_{t+k}^F / \partial \varepsilon_t^m < 0$ for $k = 0, 1, 2, ...$

Under the "conventional view" on the effects of monetary policy on asset price bubbles we have, in addition:

$$\frac{\partial q_{t+k}^B}{\partial \varepsilon_t^m} \leq 0 \tag{6}$$

for $k = 0, 1, 2, ...$i.e. a tightening of monetary policy should cause a decline in the size of the bubble. Hence, the overall effect on the observed asset price should be unambiguously

[7] See, e.g., Cochrane (2001, p.395) for a derivation.

negative, independently of the relative size of the bubble:

$$\frac{\partial q_{t+k}}{\partial \varepsilon_t^m} < 0$$

for $k = 0, 1, 2, ...$

As argued in Galí (2014), however, the premise of a decline in the size of the bubble in response to a interest rate hike does not have a clear theoretical underpinning. In fact, the theory of *rational* asset price bubbles opens the door for a very different prediction. To see this, note that the following difference equation must hold in a rational expectations equilibrium:

$$Q_t R_t = E_t\{D_{t+1} + Q_{t+1}\} \tag{7}$$

It can be easily checked that (2) satisfies

$$Q_t^F R_t = E_t\{D_{t+1} + Q_{t+1}^F\} \tag{8}$$

Using (1), (7) and (8), it can be easily checked that the bubble component must satisfy:[8]

$$Q_t^B R_t = E_t\{Q_{t+1}^B\} \tag{9}$$

or, equivalently, in its log-linear version:

$$E_t\{\Delta q_{t+1}^B\} = r_t$$

Hence, an increase in the interest rate will raise the expected growth of the bubble component. Note that the latter corresponds to the bubble's expected return, which–under the risk neutrality assumption made here–must be equal to the interest rate. Accordingly, and as discussed in Galí (2014), any rule that implies a systematic positive response of the interest rate to the size of the bubble, will tend to amplify the movements

[8]Transversality conditions generally implied by optimizing behavior of infinite-lived agents are often used to rule out such a bubble component (see, e.g., Santos and Woodford (1997)). That constraint does not apply to economies with overlapping generations of finitely-lived agents (e.g., Samuelson (1958), Tirole (1985)).

in the latter –an outcome that calls into question the conventional wisdom about the relation between interest rates and bubbles.

Changes in interest rates, however, may also affect the bubble through a second channel: a possible systematic comovement between the (*indeterminate*) innovation in the bubble with the surprise component of the interest rate. To see this, evaluate the previous expression at $t-1$ and eliminate the expectational operator to obtain:

$$\Delta q_t^B = r_{t-1} + \xi_t \tag{10}$$

where $\xi_t \equiv q_t^B - E_{t-1}\{q_t^B\}$ is an arbitrary process satisfying $E_{t-1}\{\xi_t\} = 0$ for all $t$ (i.e. the martingale-difference property). Note that the unanticipated change ("innovation") in the size of the bubble, $\xi_t$, may or may not be related to fundamentals and, in particular, to the interest rate innovation, $r_t - E_{t-1}\{r_t\}$. Thus, and with little loss of generality, one can write:

$$\xi_t = \psi_t(r_t - E_{t-1}\{r_t\}) + \xi_t^* \tag{11}$$

where $\psi_t$ is a (possibly random) parameter and $\{\xi_t^*\}$ is a zero-mean martingale-difference process, respectively satisfying the orthogonality conditions . $E_{t-1}\{\psi_t(r_t - E_{t-1}\{r_t\})\} = 0$ and $E_{t-1}\{\xi_t^* \psi_t(r_t - E_{t-1}\{r_t\})\} = 0$. Note that neither the sign nor the size of $\psi_t$, nor its possible dependence on the policy regime, are pinned down by the theory. Accordingly, the *contemporaneous impact* of an interest rate innovation on the bubble is, in principle, indeterminate.

The dynamic response of the bubble component to a monetary policy tightening is given by

$$\frac{\partial q_{t+k}^B}{\partial \varepsilon_t^m} = \begin{cases} \psi_t \frac{\partial r_t}{\partial \varepsilon_t^m} & \text{for } k = 0 \\ \psi_t \frac{\partial r_t}{\partial \varepsilon_t^m} + \sum_{j=0}^{k-1} \frac{\partial r_{t+j}}{\partial \varepsilon_t^m} & \text{for } k = 1, 2, ... \end{cases} \tag{12}$$

for $k = 0, 1, 2, ....$Thus, and as discussed above, the initial impact on the bubble, captured by coefficient $\psi_t$, is indeterminate, both in sign and size. Yet, and conditional on $\partial r_{t+k}/\partial \varepsilon_t^m > 0$, for $k = 0, 1, 2, ..$ the subsequent growth of the bubble is predicted to be positive. The long run impact of the monetary policy shock on the size of the bubble,

$\lim_{k\to\infty} \partial q^B_{t+k}/\partial \varepsilon^m_t$ will be positive or negative depending on whether the persistence of the real interest rate response is more than sufficient to offset any eventual negative initial impact. Thus, when considered in combination with the predicted response of the fundamental component, the theory of rational bubbles implies that the sign of the response of observed asset prices to a tightening of monetary policy is ambiguous. Most importantly, however, the theory opens the door to the possibility that the observed asset price rises (possibly after some initial decline), as long as one or more of the following conditions are satisfied: (i) $\psi_t$ is not "too negative", (ii) the response of the real interest rate is persistent enough, and (iii) the relative size of the bubble $\gamma_t$ is large enough (so that the eventual positive response of the latter more than offsets the likely decline in the fundamental component).

To illustrate the previous discussion, consider an asset whose dividends are exogenous and independent of monetary policy. In response to an exogenous policy tightening the real interest rate is assumed to evolve according to $\partial r_{t+k}/\partial \varepsilon^m_t = \rho^k_r$, for $k = 0, 1, 2, ...$The response of the (log) asset price to a unit shock is then given by:

$$\frac{\partial q_{t+k}}{\partial \varepsilon^m_t} = -(1-\gamma_{t-1})\frac{\rho^k_r}{1-\Lambda\rho_r} + \gamma_{t-1}\left(\psi_t + \frac{1-\rho^k_r}{1-\rho_r}\right)$$

Figure 1 displays the dynamic responses of the asset price for alternative configurations of $\gamma$ and $\psi$. In all cases we assume $\Lambda = 0.99$ and $\rho_r = 0.8$. The black line (with circles) displays the asset price response in the absence of a bubble ($\gamma_{t-1} = 0$). The asset price declines on impact, and gradually returns to its original value. The blue line (with circles) shows the response for $\gamma_t = 0.5$ and $\psi_t = 0$. Note that the asset price also declines on impact, but now it recovers at a faster pace (due to the growing bubble) and ends up overshooting permanently its initial value and stabilizing at a higher level. The red line (with squares) corresponds to the case of $\gamma_t = 0.5$ and $\psi_t = -8$. Now the negative impact of the interest rate hike on the asset price is larger, due to its initial shrinking effect on the size of the bubble. Finally, the green line (with triangles) shows the response under $\gamma_t = 0.5$ and $\psi_t = 6$. Now the asset price already rises from the time of the shock, given that the positive response of the bubble on impact more than offsets the decline of the fundamental component.

The previous simulations make clear that the theory of rational bubbles is consistent with a broad range of responses of asset prices to a tightening of monetary policy. By contrast, the conventional view predicts an unambiguous decline in asset prices, for both the fundamental and bubble components are expected to go down in response to a policy tightening. Accordingly, any evidence of a decline in asset prices in response to that tightening would not be conclusive as to the validity of the two views on the effects of monetary policy on the bubble. On the other hand, any evidence of a positive impact on the asset price at some horizon subsequent to the same policy intervention would be clearly at odds with both the key premise and the implications of the "conventional view," while consistent (at least, qualitatively) with the theory of rational bubbles.

# II The empirical model

The present section describes our empirical model, which consists of a structural vector autoregression model with time-varying coefficients (TVC-SVAR). Beyond the usual concern for possible structural changes over the sample period considered, our main motivation for using a model with time-varying coefficients has to do with the dependence of the stock price response on the relative size of its (eventual) bubble component, which is likely to change over time.

Though focusing on different variables, the specification of our reduced form time-varying VAR follows closely that in Primiceri (2005). On the other hand our choice of variables and identification strategy follows that in CEE. Our constant coefficients VAR, for which we also report results below, can be seen as a limiting case of the model with time-varying coefficients, so we do not provide a separate description.

Let $y_t$, $p_t$, $p_t^c$, $i_t$, $q_t$, and $d_t$ denote, respectively, (log) output, the (log) price level, the (log) commodity price index, the short-term nominal interest rate controlled by the central bank, the (log) stock price index, and its corresponding (log) dividend series (both in real terms). We define $\mathbf{x}_t \equiv [\Delta y_t, \Delta d_t, \Delta p_t, \Delta p_t^c,\ i_t, \Delta q_t]'$. The relationship between those variables and the structural shocks is assumed to take the form of an autoregressive model with time-varying coefficients:

$$\mathbf{x}_t = \mathbf{A}_{0,t} + \mathbf{A}_{1,t}\mathbf{x}_{t-1} + \mathbf{A}_{2,t}\mathbf{x}_{t-2} + ... + \mathbf{A}_{p,t}\mathbf{x}_{t-p} + \mathbf{u}_t \tag{13}$$

where $\mathbf{A}_{0,t}$ is a vector of time-varying intercepts, and $\mathbf{A}_{i,t}$, for $i = 1, ...$, are matrices of time-varying coefficients, and where the vector of reduced form innovations $\mathbf{u}_t$ follows a white noise Gaussian process with mean zero and covariance matrix $\mathbf{\Sigma}_t$. We assume the reduced form innovations are a linear transformation of the underlying structural shocks $\boldsymbol{\varepsilon}_t$ given by

$$\mathbf{u}_t \equiv \mathbf{S}_t\boldsymbol{\varepsilon}_t$$

where $E\{\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\} = I$ and $E\{\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-k}'\} = 0$ for all $t$ and $k = 1, 2, 3, ....$ and $\mathbf{S}_t$ is such that $\mathbf{S}_t\mathbf{S}_t' = \mathbf{\Sigma}_t$.

Let $\boldsymbol{\theta}_t = vec(\mathbf{A}_t')$ where $\mathbf{A}_t = [\mathbf{A}_{0,t}, \mathbf{A}_{1,t}..., \mathbf{A}_{p,t}]$ and $vec(\cdot)$ is the column stacking operator. We assume $\boldsymbol{\theta}_t$ evolves over time according to the process

$$\boldsymbol{\theta}_t = \boldsymbol{\theta}_{t-1} + \boldsymbol{\omega}_t \tag{14}$$

where $\boldsymbol{\omega}_t$ is a Gaussian white noise process with zero mean and constant covariance $\mathbf{\Omega}$, and independent of $\mathbf{u}_t$ at all leads and lags.

We model the time variation of $\mathbf{\Sigma}_t$ as follows. Let $\mathbf{\Sigma}_t = \mathbf{F}_t\mathbf{D}_t\mathbf{F}_t'$, where $\mathbf{F}_t$ is lower triangular, with ones on the main diagonal, and $\mathbf{D}_t$ a diagonal matrix. Let $\boldsymbol{\sigma}_t$ be the vector containing the diagonal elements of $\mathbf{D}_t^{1/2}$ and $\boldsymbol{\phi}_{i,t}$ a column vector with the non-zero elements of the $(i+1)$-th row of $\mathbf{F}_t^{-1}$ with $i = 1, ..., 5$. We assume that

$$\log\boldsymbol{\sigma}_t = \log\boldsymbol{\sigma}_{t-1} + \boldsymbol{\zeta}_t \tag{15}$$

$$\boldsymbol{\phi}_{i,t} = \boldsymbol{\phi}_{i,t-1} + \boldsymbol{\nu}_{i,t} \tag{16}$$

where $\boldsymbol{\zeta}_t$ and $\boldsymbol{\nu}_{i,t}$ are white noise Gaussian processes with zero mean and (constant) covariance matrices $\mathbf{\Xi}$ and $\mathbf{\Psi}_i$, respectively. We assume that $\boldsymbol{\nu}_{i,t}$ is independent of $\boldsymbol{\nu}_{j,t}$, for $j \neq i$, and that $\boldsymbol{\omega}_t$, $\boldsymbol{\varepsilon}_t$, $\boldsymbol{\zeta}_t$ and $\boldsymbol{\nu}_{i,t}$ (for $i = 1, ..., 5$) are mutually uncorrelated at all leads and lags. Note that the constant coefficient VAR can be seen as a limiting case of the previous model with $\mathbf{\Omega} = 0, \mathbf{\Xi} = 0, \mathbf{\Psi}_i = 0$.

Our identification of the monetary policy shock is inspired by the strategy proposed by Christiano, Eichenbaum and Evans (2005). More specifically we assume that the monetary policy shock does not affect GDP, dividends or inflation contemporaneously. In addition, our baseline specification assumes that the central bank does not respond contemporaneously to innovations in real stock prices.[9] Letting the fifth element in $\boldsymbol{\varepsilon}_t$, denoted by $\varepsilon_t^m$, correspond to the monetary policy shock, the first assumption implies that the fifth column of $\mathbf{S}_t$ has zeros as its first four elements, while its two remaining elements are unrestricted. The second assumption implies that the last element in the fifth row of $\mathbf{S}_t$ is zero. Since our focus is on monetary policy shocks, we need not place any other restrictions on matrix $\mathbf{S}_t$. To facilitate implementation we just let $\mathbf{S}_t$ be the Cholesky factor of $\mathbf{\Sigma}_t$, i.e. $\mathbf{S}_t = \mathbf{F}_t \mathbf{D}_t^{1/2}$, but make no attempt to interpret the remaining "structural" shocks.

To define the impulse response functions let us rewrite (13) in companion form:

$$\tilde{\mathbf{x}}_t = \tilde{\mu}_t + \tilde{\mathbf{A}}_t \tilde{\mathbf{x}}_{t-1} + \tilde{\mathbf{u}}_t$$

where $\tilde{\mathbf{x}}_t \equiv [\mathbf{x}'_t, \mathbf{x}'_{t-1}, ..., \mathbf{x}'_{t-p+1}]'$, $\tilde{\mathbf{u}}_t \equiv [\mathbf{u}'_t, 0, ..., 0]'$, $\tilde{\mu}_t \equiv [\mathbf{A}'_{0,t}, 0, ..., 0]'$ and $\tilde{\mathbf{A}}_t$ is the corresponding companion matrix. We use a *local approximation* of the implied dynamic response to a $t$ period shock. Formally, the local response is given by

$$\frac{\partial \mathbf{x}_{t+k}}{\partial \mathbf{u}'_t} = [\tilde{\mathbf{A}}_t^k]_{6,6} \equiv \mathbf{B}_{t,k}$$

for $k = 1, 2, ..$where $[M]_{6,6}$ represents the first 6 rows and 6 columns of any matrix $M$, and where $\mathbf{B}_{t,0} \equiv \mathbf{I}$. Thus, the dynamic responses of the variables in $\mathbf{x}_t$ to a monetary

[9] That assumption is consistent with the evidence reported in Fuhrer and Tootell (2008), based on the estimates of empirical Taylor rules augmented with stock price changes. Below we discuss the implications of relaxing that assumption.

policy shock $\varepsilon_t^m$ hitting the economy at time $t$ are given by

$$\begin{aligned}\frac{\partial \mathbf{x}_{t+k}}{\partial \varepsilon_t^m} &= \frac{\partial \mathbf{x}_{t+k}}{\partial \mathbf{u}_t'}\frac{\partial \mathbf{u}_t}{\partial \varepsilon_t^m} \\ &= \mathbf{B}_{t,k}\mathbf{S}_t^{(5)} \equiv \mathbf{C}_{t,k}\end{aligned}$$

for $k = 0, 1, 2, ...$and where $\mathbf{S}_t^{(5)}$ denotes the fifth column of $\mathbf{S}_t$. In the case of the constant coefficients model the response is just given by $\partial \mathbf{x}_{t+k}/\partial \varepsilon_t^m = \mathbf{B}_k \mathbf{S}^{(5)} \equiv \mathbf{C}_k$, where $\mathbf{B}_k \equiv [\tilde{\mathbf{A}}^k]_{6,6}$.

We use Bayesian methods in order to estimate the model with time-varying coefficients. The goal of our estimation is to characterize the joint posterior distribution of the parameters of the model. To do that we use, following Primiceri (2005), the Gibbs sampling algorithm described in the online Appendix.

## A Relation with the Existing Literature

We are not the first to analyze empirically the impact of monetary policy changes on stock prices.

Patelis (1997) analyzes the role played by monetary and financial variables in predicting stock returns. He finds that increases in the federal funds rate have a significant negative impact on predicted stock returns in the short run, but a positive one at longer horizons. That predictability works largely through the effect of federal funds rate changes on anticipated excess returns down the road, rather than dividends or expected returns.

Bernanke and Kuttner (2005) use an event-study approach, based on daily changes observed on monetary policy decision dates, to uncover the effects on stock prices of unanticipated changes in the federal funds rate. They find that a surprise 25-basis-point cut in the Federal funds rate is associated with about a 1 percent increase in stock prices. Their analysis largely attributes that response to a persistent declines in the equity premium, and to a lesser extent of the relevant cash flows. They do not report, however, the dynamic response of stock prices to the monetary policy surprise. Rigobon and Sack (2004) obtain similar (but slightly larger) estimates of the response of stock prices to changes in interest rates using a heteroskedasticity-based estimator that exploits the increase in the volatility of interest rates on FOMC meeting and Humphrey-Hawkins testimony dates in order to control for possible reverse causality.

Gürkaynak, Sack and Swanson (2005) use intraday data to estimate the response of asset prices to two factors associated with FOMC decisions. The first factor corresponds, like in Bernanke and Kuttner (2005), to the unanticipated movements in the Federal funds rate target. The estimated effect on stock prices is very similar to that uncovered by Bernanke and Kuttner (2005).[10] The second factor is associated with revisions in expectations about future rates, given the funds rate target, and appears to be linked to the statement accompanying the FOMC decision. The impact of this second factor on stock prices is significant, but more muted than the first, possibly due to revisions in expectations on output and inflation which may partly offset the impact of anticipated changes in interest rates.

As far as we know, the literature contains no attempts to uncover the effects of monetary policy shocks on the bubble component of stock prices. Uncovering those effects requires that the response of the fundamental component stock prices be estimated using the estimated joint response of dividends and the real interest rate.

## III Evidence

In this section we report the impulse responses of a number of variables to a monetary policy shock, generated by our estimated VARs, both with constant and time-varying coefficients. We use quarterly U.S. time series for GDP and its deflator, the World Bank commodity price index, the federal funds rate, and the S&P500 stock price index and the corresponding dividend series (both deflated by the GDP deflator). Our baseline

[10] Similar results are obtained by D'Amico and Farka (2011) in their first-step, which involves the same intraday-data strategy as Gürkaynak, Sack and Swanson (2005).

sample period is 1960Q1-2011Q4. Due to the impact of the zero lower bound on the behavior of the federal funds rate since 2008 and its likely influence on our estimates we have also estimated the model ending the sample in 2007Q4 as a robustness check.

Figure 2 displays the estimated responses to a contractionary monetary policy shock, based on the estimated VAR with constant coefficients. The tightening of monetary policy leads to a persistent increase in both nominal and real rates, a decline in GDP and an (eventual) decline in the GDP deflator. The response pattern for dividends is similar to that of GDP. The stock price index is also seen to decline in the short run, but it recovers fast subsequently and ends up in slightly positive territory (though the confidence bands are too large to reject the absence of a long run effect). Figure 2.g displays the implied response of the "fundamental" component of the stock price, computed using (5). Not surprisingly, given the response of the real rate and dividends), the fundamental stock price is shown to decline sharply on impact, and to return only gradually to its initial value. Figure 2.f compares the latter with that of the observed price shown earlier.

Note that (4) implies

$$\frac{\partial(q_{t+k} - q^F_{t+k})}{\partial \varepsilon^m_t} = \gamma_{t-1}\left(\frac{\partial q^B_{t+k}}{\partial \varepsilon^m_t} - \frac{\partial q^F_{t+k}}{\partial \varepsilon^m_t}\right)$$

As Figure 2.h makes clear, the response of the gap $q_{t+k} - q^F_{t+k}$ is positive and, after one period, increasing, which points to (i) the existence of a non-negligible bubble component, and (ii) a substantial difference between the responses of the bubble and fundamental components of stock prices to a monetary policy shock.[11]

[11] In the simple example of a rational bubble considered above (with exogenous dividends and a geometric response of the real rate) we have:

$$\frac{\partial(q_{t+k} - q^F_{t+k})}{\partial \varepsilon^m_t} = \gamma_{t-1}\left(\frac{\rho_r^k}{1-\Lambda\rho_r} + \psi_t + \frac{1-\rho_r^k}{1-\rho_r}\right)$$

Thus, to the extent that a bubble is present to begin with ($\gamma_{t-1} > 0$) and its contemporaneous response to the interest rate innovation is not too negative ($\psi_t \gtrsim 0$), the gap between the response of the asset price and its fundamental should be positive and increasing over time in response to a

Figures 3a-f show the impulse responses of a number of variables to a monetary policy shock, based on our estimated VAR with time-varying coefficients. The estimated dynamic responses of nominal and real rates, shown in figures 3.a and 3.b respectively, appear to be relatively unchanged over time, though the former shows substantially greater persistence over the last few years of the sample period (possibly due to the "distortion" created by the zero lower bound). Figures 3.c and 3.d display the impulse responses of GDP and the GDP deflator. In both cases the impulse responses are relatively stable over time, with both GDP and its deflator displaying a persistent negative decline after the tightening of monetary policy. Broadly speaking, the same holds true for the response of dividends (shown in Figure 3.e), with the exception of a brief period in the early 1980s, when the tightening of policy appears to have a positive impact on dividends after about three years.

Our focus is, however, on the changing response of stock prices, displayed in Figure 3.f.. Note that the S&P500 generally declines on impact, often substantially, in response to an exogenous monetary policy tightening. Until the late 1970s that decline is persistent, in a way consistent with the response of stock prices in the absence of a bubble. By contrast, starting in the early 1980s, the initial decline is rapidly reversed with stock prices rising quickly (and seemingly permanently) above their initial value. That phenomenon is particularly acute in the 1980s and 1990s. The previous estimated response stands in contrast with that of the fundamental component, as implied by the impulse responses of the real rate and dividends, and shown in Figure 3.g Note that the pattern of the response of fundamental stock prices to a tightening of monetary policy has changed little over time, (roughly) corresponding to that obtained with the constant coefficient VAR. Figure 3.h displays the response of the gap between observed and fundamental stock prices. Note that with the exception of the early part of the sample that gap appears to be positive and growing, in a way consistent with the theory of rational bubbles, and in contrast with the "conventional" view. Figure 4 provides an alternative

tightening of monetary policy.

perspective to the same evidence, by displaying the evolution over time of the impact of the monetary policy shock on the log deviations between observed and fundamental stock prices at different horizons. Figure 5 shows the estimated (bootstrap-based) probability that the same gap is positive. Note that the probability is well above 50 percent (and often much closer to unity) since the mid-80s.

Figures 6.a-6.d illustrate the changing patterns of stock price responses by showing the *average* impulse responses of both observed and fundamental prices over four alternative three-year periods: 1967Q2-1970Q1, 1976Q1-1978Q4, 1984Q4-1987Q3, and 1997Q1-1999Q4. The changing pattern of the gap between the two variables emerges clearly. The response during the first episode, from the 1960s, points to a drop of the observed price larger than that of the fundamental. The evidence from the 1970s suggests a relatively similar pattern in both responses, though the observed price displays some overshooting relative to the fundamental. On the other hand, the estimated responses for the three-year periods before the crash of October 1987, as well as the period before the burst of the dotcom bubble, point to a very different pattern: the observed price declines less than the fundamental to begin with, and then recovers faster to end up in strongly positive territory, as the theory of rational asset price bubbles would predict when a large bubble is present.

We have re-estimated the model using an alternative sample period ending in 2007Q3, i.e. leaving out the period associated with the deeper financial crisis, a binding zero lower bound and the adoption of unconventional monetary policies. We have also examined the robustness of our results to the use of earnings instead of dividends. Even though the latter is, in principle, the appropriate variable, earnings are often used in applications due to their less erratic seasonal patterns. In both cases, our findings are largely unchanged. Figures 7 and 8 illustrates that robustness by showing the dynamic response of the gap between the stock price and the fundamental to an exogenous tightening of monetary policy using, respectively, the shorter sample period and the earnings-based VAR. Note that the observed pattern of responses is very similar to that found in Figure 3.h, at least qualitatively.

# IV Alternative interpretations

## A Time-varying equity premium

The theoretical analysis of section 1 has been conducted under the maintained assumption of risk neutrality or –equivalently, for our purposes– of a constant expected excess return (or equity premium). That assumption also underlies our definition of the fundamental component of stock prices and of the estimates of the latter's dynamic response to monetary policy shocks shown in the previous section. There is plenty of evidence in the literature, however, of time-varying expected excess return in stock prices, partly linked to monetary policy shocks.[12] Next we examine whether our estimated deviation between observed stock prices and the "measured" fundamental component can be plausibly interpreted as resulting from a time-varying equity premium, as an alternative to the bubble-based interpretation.

Let $z_{t+1}$ denote the (log-linearized) excess return on stocks held between $t$ and $t+1$, given by

$$z_{t+1} = \Lambda q_{t+1} + (1-\Lambda) d_{t+1} - q_t - r_t$$

In the absence of a bubble, we can write the equilibrium stock price

$$q_t = const + \sum_{k=0}^{\infty} \Lambda^k \left[ (1-\Lambda) E_t\{d_{t+k+1}\} - E_t\{r_{t+k}\} - E_t\{z_{t+k+1}\} \right]$$

Thus, the dynamic response of the stock price to an exogenous monetary policy shock is given by

$$\frac{\partial q_{t+k}}{\partial \varepsilon_t^m} = \sum_{j=0}^{\infty} \Lambda^j \left( (1-\Lambda) \frac{\partial d_{t+k+j+1}}{\partial \varepsilon_t^m} - \frac{\partial r_{t+k+j}}{\partial \varepsilon_t^m} - \frac{\partial z_{t+k+j+1}}{\partial \varepsilon_t^m} \right)$$

[12]See, e.g. Thorbecke (1997), Patelis (1997).and Bekaert, Hoerova and Lo Duca (2013).

Then it follows that the gap between the response of the observed price and the response of the fundamental component computed under the assumption of risk neutrality are related to the equity premium response according to the equation:

$$\frac{\partial q_{t+k}}{\partial \varepsilon_t^m} - \frac{\partial q_{t+k}^F}{\partial \varepsilon_t^m} = -\sum_{j=0}^{\infty} \Lambda^j \frac{\partial z_{t+k+j+1}}{\partial \varepsilon_t^m}$$

for $k = 0, 1, 2, ...$and where, as above, $\frac{\partial q_{t+k}^F}{\partial \varepsilon_t^m} \equiv \sum_{j=0}^{\infty} \Lambda^j \left( (1-\Lambda) \frac{\partial d_{t+k+j+1}}{\partial \varepsilon_t^m} - \frac{\partial r_{t+k+j}}{\partial \varepsilon_t^m} \right)$ is the fundamental stock price under risk neutrality.

Thus, an interpretation of the evidence above that abstracts from the possibility of bubbles and relies instead on a time-varying equity premium requires that the latter declines substantially and persistently in response to a tightening of monetary conditions. That implication is at odds with the existing evidence on the response of excess stock returns (e.g. Patelis (1997), Bernanke and Kuttner (2005)) or variables that should be closely related to it, like the VIX (Bekaert, Hoerova and Lo Duca, 2013)).

# B Long term rate response

The evidence of a positive response of stock prices to a tightening of monetary policy could also be reconciled with a fundamentals-based explanation if the observed rise in the federal funds rate coexisted with a simultaneous decline in the long term interest rate, possibly due to a (mistaken) anticipation of sufficiently lower short term rates further down the road.[13] In order to assess that hypothesis we have re-estimated our VAR with the yield on the 10-year government bond replacing stock prices. Figure 9 displays the dynamic response of the long term rate to a tightening of monetary policy (i.e. to an orthogonalized innovation in the federal funds rate, as above). The Figure makes clear that the long-term rate rises persistently in response to the higher federal funds rate. The increase is particularly large in the period starting in the early 1980s, precisely when the gap between the observed stock price and its fundamental value shows a larger

[13]This possibility was suggested by our discussant Lucrezia Reichlin.

increase. Thus, the hypothesis that the observed rise in stock prices is due to a decline in long term rates is not supported by the evidence.

## C Simultaneity

The estimates reported above were obtained under the identifying assumption that the Federal Reserve did not respond contemporaneously (i.e. within the quarter) to stock price innovations. That assumption is consistent with the "pre-crisis consensus" according to which central banks should focus exclusively on stabilizing inflation and the output gap.[14]

Here we examine the robustness of our findings to relaxing that constraint, by allowing for some (contemporaneous) simultaneity in the determination of interest rates and stock prices. More specifically, we re-estimate our empirical model under the assumption that current log change in stock prices enters the interest rate rule with a coefficient 0.02. This implies that, ceteris paribus, a ten percentage point increase in stock prices within a quarter triggers a 20 basis points rise in the federal funds rate. The previous assumption is consistent with the estimated reaction of monetary policy to the stock market changes obtained by Rigobon and Sack (2003) using an approach that exploits heteroskedasticity in stock price shocks to identify the coefficient measuring that reaction.[15]

The estimated responses of interest rates and dividends to a monetary policy shock (not shown) are not much affected by the use of this alternative identification scheme. But the same cannot be said for stock prices: with the exception of a brief period in

[14]It is also consistent with formal evidence in Fuhrer and Tootell (2008) based on estimated interest rate rules using real time Greenbook forecasts, though that evidence does not rule out the possibility of an indirect response to stock prices, based on their potential ability to predict output or inflation developments.

[15]D'Amico and Farka (2011) use an alternative two-step procedure to identify the policy response to stock prices, obtaining a similar estimate of the response coefficient (about 0.02). Furlanetto (2011) revisits de Rigobon-Sack evidence and concludes that the positive estimated reaction is largely driven by the Fed response to the stock market crash of 1987.

the early 1980s, the latter now decline persistently throughout the sample period in response to a tightening of monetary policy, as shown in Figure 10.a. Furthermore, and most importantly for our purposes, the gap between the observed price and the estimated fundamental price also declines strongly in response to the same shock, as shown in Figure 10.b. The latter response is consistent, at least in a qualitative sense, with the conventional wisdom regarding the impact of monetary policy on stock price bubbles, and contrasts starkly with the evidence based on our baseline specification.

If one accepts this alternative identifying assumption as correct, the findings obtained in the previous section should be interpreted as spurious, and driven by biased estimates of matrices $\{S_t\}$ resulting from the imposition of an incorrect identifying assumption. Figure 11 displays the stock price response after four quarters to the tightening of monetary policy, for four alternative calibrations of the contemporaneous stock price coefficient in the interest rate rule: 0.0, 0.01, 0.02, and 0.03. We see that estimates of the effects of monetary policy on stock prices are rather sensitive to the calibration of that parameter. In a nutshell, the larger is the calibrated stock price coefficient in the interest rate rule, the smaller (i.e. more negative) is the estimated effect of an interest rate shock on stock prices. That negative conditional comovement is required in order to compensate for the strong positive comovement that arises as a result of non-monetary policy shocks, due to the endogenous policy response to stock price movements embedded in the rule.

The previous interpretation, however, is subject to an important caveat, which calls it into question. In a recent paper, Furlanetto (2011) has revisited the evidence of Rigobon and Sack (2003) using data that extends over a longer sample period (1988-2007) and focusing on the stability over time in the estimates of the monetary policy response to stock prices.[16] He shows that the main finding in Rigobon and Sack (2003)

[16] In addition, he also examines the evidence for six other economies (Australia, Canada, New Zealand, Norway, Sweden and the United Kingdom). He finds evidence of a significant endogenous response to stock prices only in Australia.

is largely driven by a single episode: the Fed's interest rate cuts in response to the stock market crash in 1987. When the same empirical model is re-estimated using post-1988 data, the estimated policy response is much smaller or insignificant. The Furlanetto evidence has an important implication for the present paper, for it suggests that our baseline specification is a good approximation, possibly with the exception of the period around 1987. Given that our empirical framework allows the model's coefficients to vary over time, that "transitory" misspecification should not distort the estimated responses for other "segments" of the sample. On the other hand, imposing a "fixed" stock price coefficient in the interest rate rule in the absence of an endogenous policy response would likely distort the estimated model for the entire sample period.

Thus, and conditional on Furlanetto's findings, our evidence pointing to an eventual positive (and growing) response of stock prices (in both levels and deviations from fundamentals) to a tightening of monetary policy should be viewed as valid, while the estimates using the alternative specification are likely to be distorted by the imposition of an identifying assumption that is invalid for much of the sample.

# V Concluding Remarks

Proposals for a "leaning against the wind" monetary policy in response to perceived deviations of asset prices from fundamentals rely on the assumption that increases in interest rates will succeed in shrinking the size of an emerging asset price bubble. Yet, and despite the growing popularity of such proposals, no evidence seems to be available providing support for that link.

In the present paper we have provided evidence on the response of stock prices to monetary policy shocks, and tried to use that evidence to evaluate the empirical merits of the "conventional" view according to which the size of the bubble component of stock prices should decline in response to an exogenous increase in interest rates.

Our evidence is based on an estimated vector-autoregression with time-varying coefficients, applied to quarterly US data. Under our baseline specification, which assumes no contemporaneous response of monetary policy to asset prices, the evidence points to protracted episodes in which stock prices increase persistently in response to an exogenous tightening of monetary policy. That response is clearly at odds with the "conventional" view on the effects of monetary policy on bubbles, as well as with the predictions of bubbleless models. We also argue that it is unlikely that such evidence be accounted for by an endogenous response of the equity premium to the monetary policy shocks or by "mistaken expectations" on the part of market participants that might drive long term interest rates down.

The previous findings are overturned when we impose a contemporaneous interest rate response to stock prices consistent with the evidence in Rigobon and Sack (2003): under this alternative specification our evidence points to a decline in stock prices in response to a tightening of monetary policy, beyond that warranted by the estimated response of the fundamental price. Recent independent evidence by Furlanetto (2011), however, calls into question the relevance of this alternative specification.

Further research seems to be needed to improve our understanding of the effect of interest rate changes on asset price bubbles. That understanding is a necessary condition before one starts thinking about how monetary policy *should* respond to asset prices. We hope to have contributed to that task by providing some evidence that calls into question the prevailing dogma among advocates of "leaning against the wind" policies, namely, that a rise in interest rates will help disinflate an emerging bubble.

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# Figures

Figure 1

Asset Price Response to an Exogenous Interest Rate Increase: Alternative Calibrations

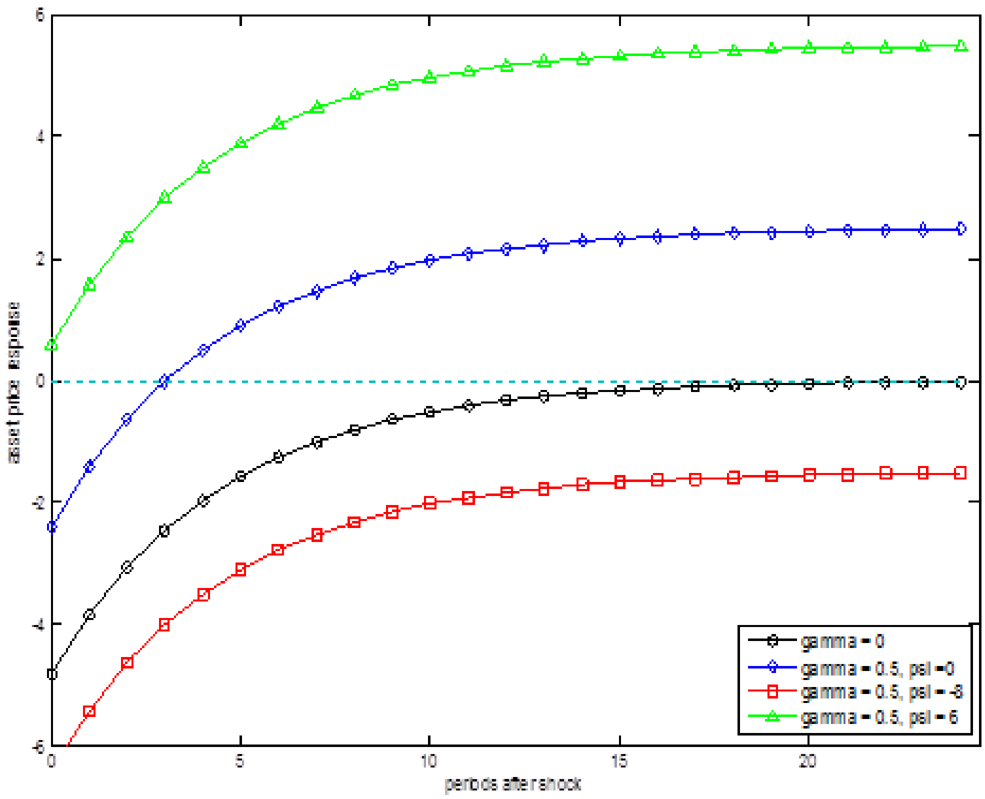
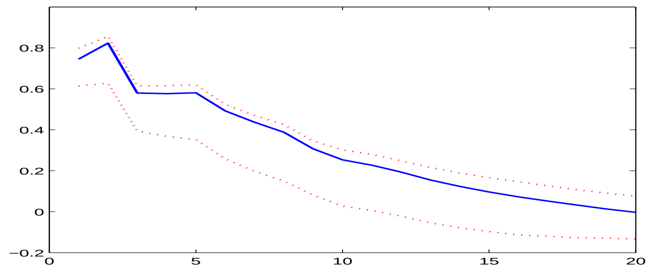
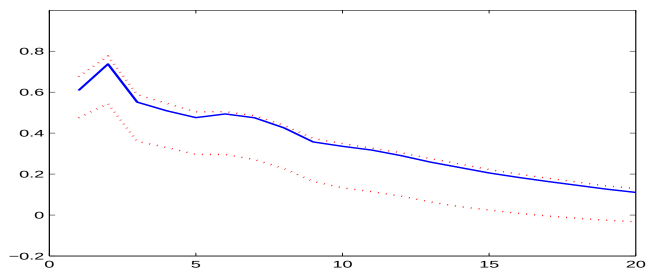


Figure 2

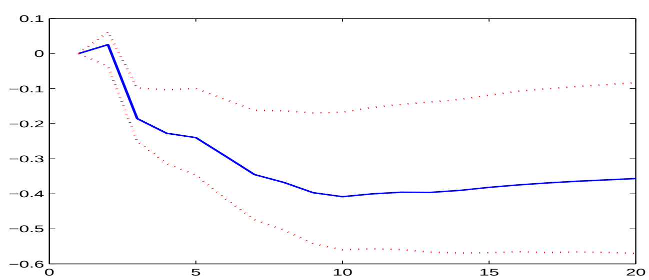
Estimated Responses to Monetary Policy Shocks: VAR with Constant Coefficients



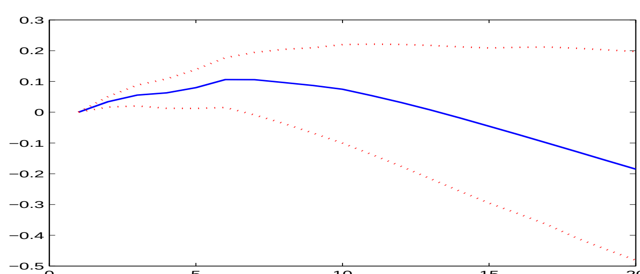

a. Federal funds rate




b. Real interest rate

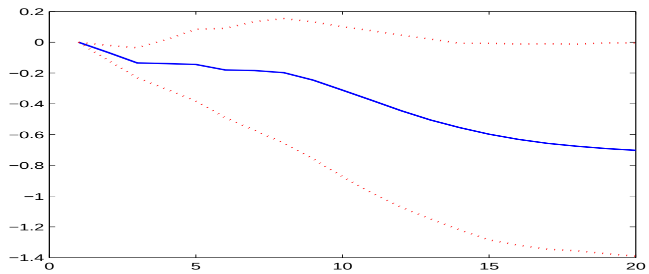



c. GDP

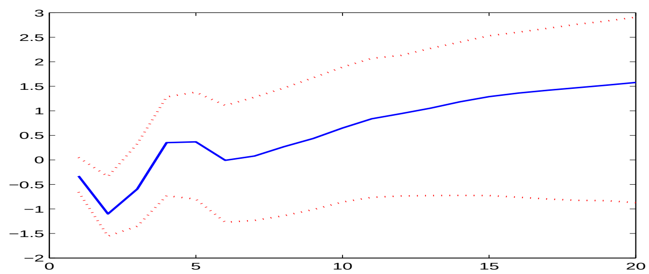



d. GDP deflator

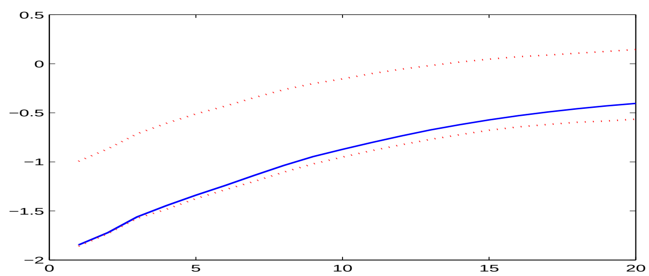
Figure 2 (cont.)



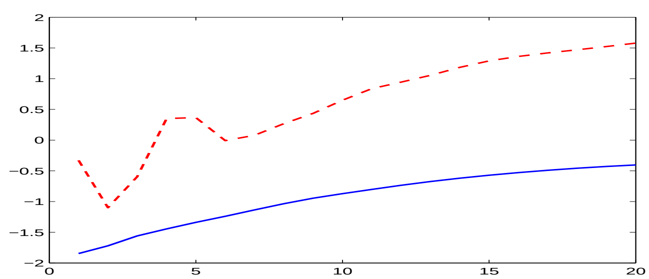

e. Dividends




f. Stock prices



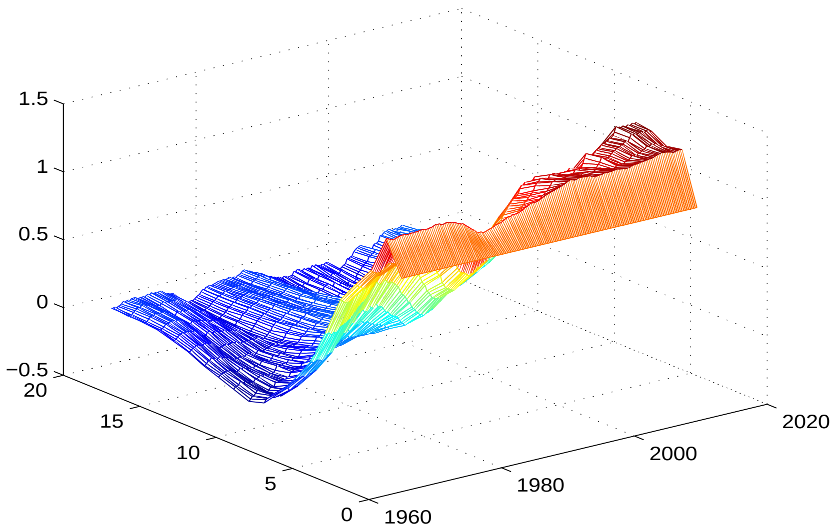

g. Fundamental component



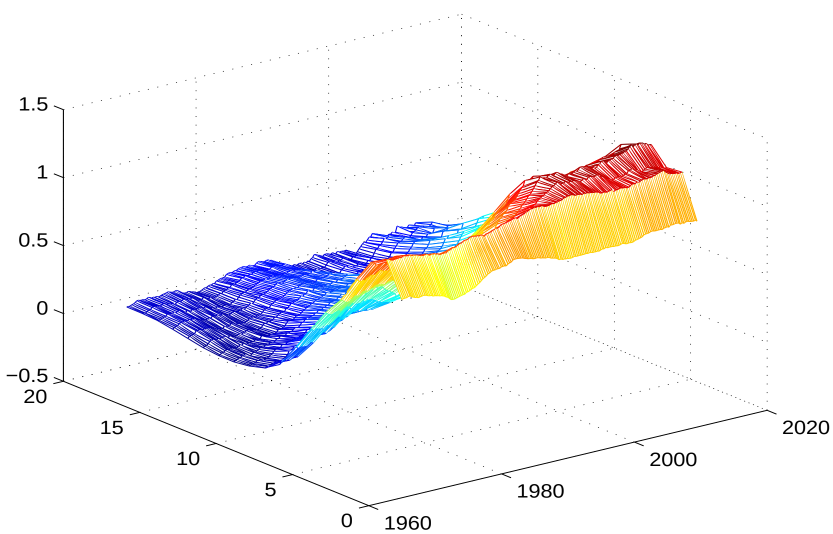

h. Price vs. fundamental

Figure 3

Estimated Responses to Monetary Policy Shocks: VAR with Time-Varying Coefficients

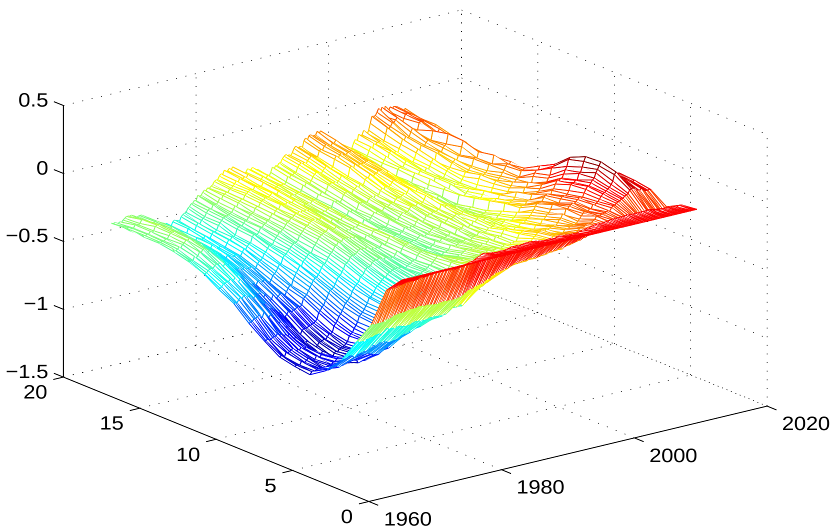



a. Federal Funds rate

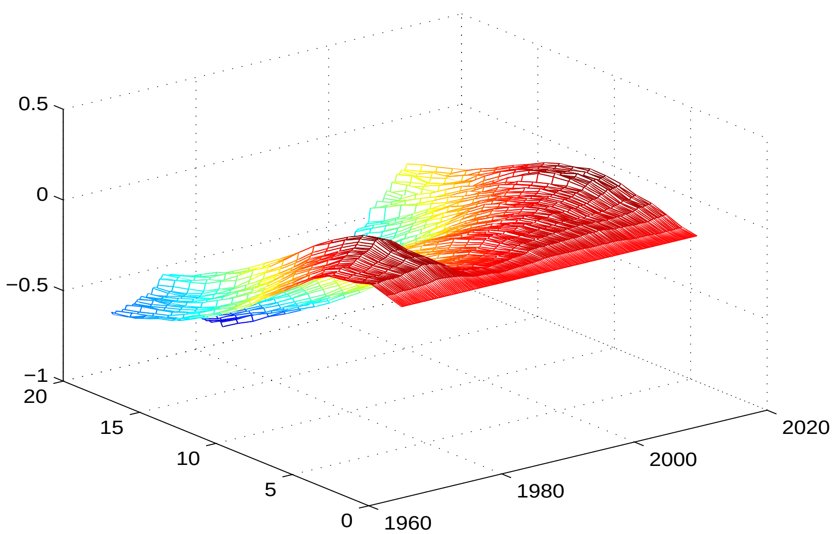



b. Real interest rate

Figure 3 (cont.)

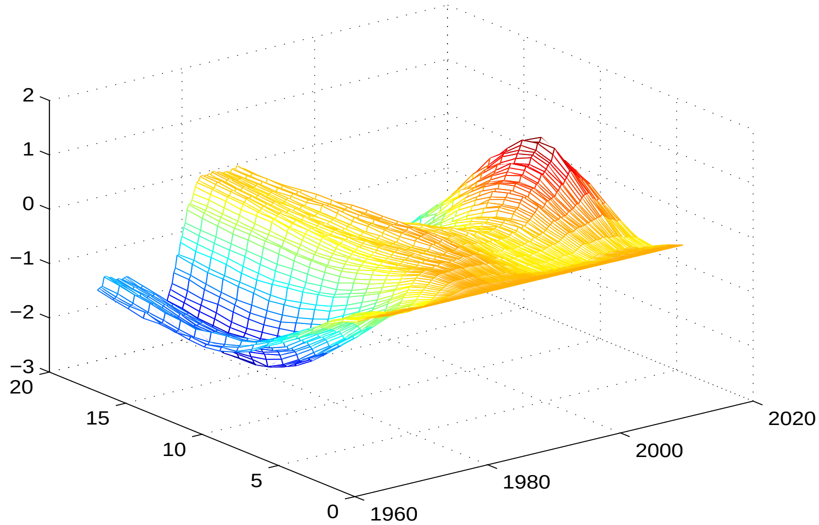



c. GDP

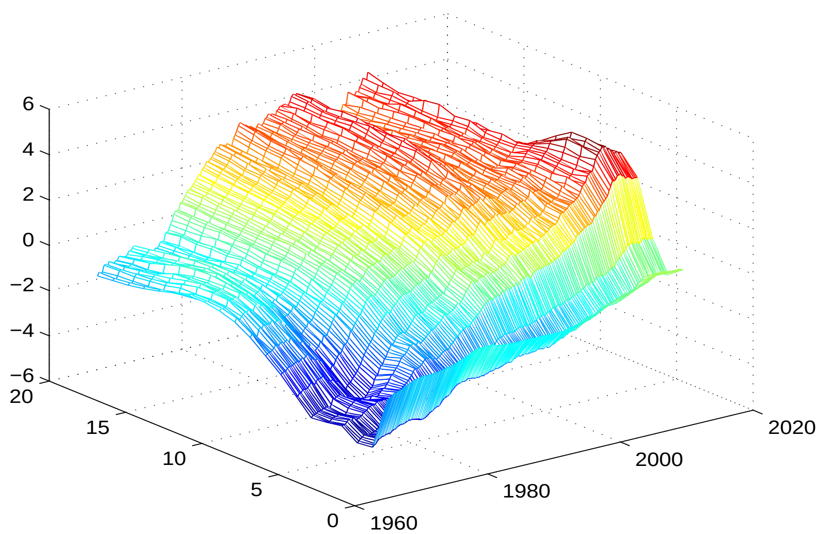



d. GDP deflator

Figure 3 (cont.)

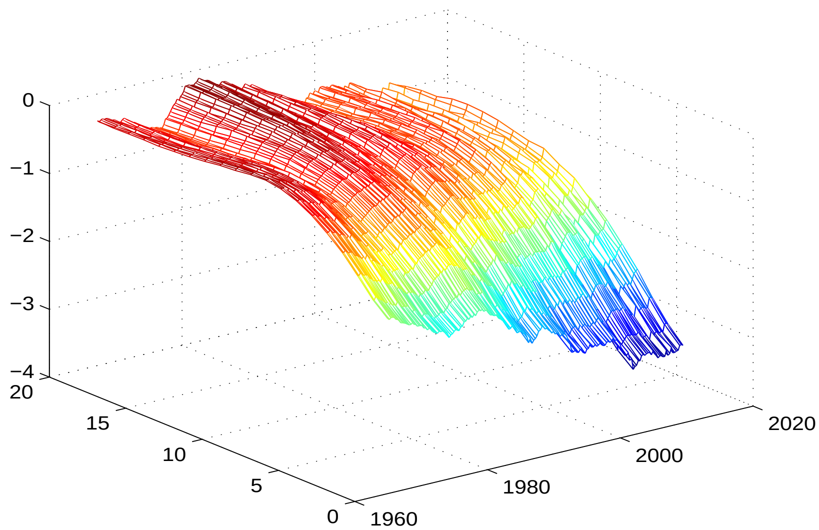



e. Dividends

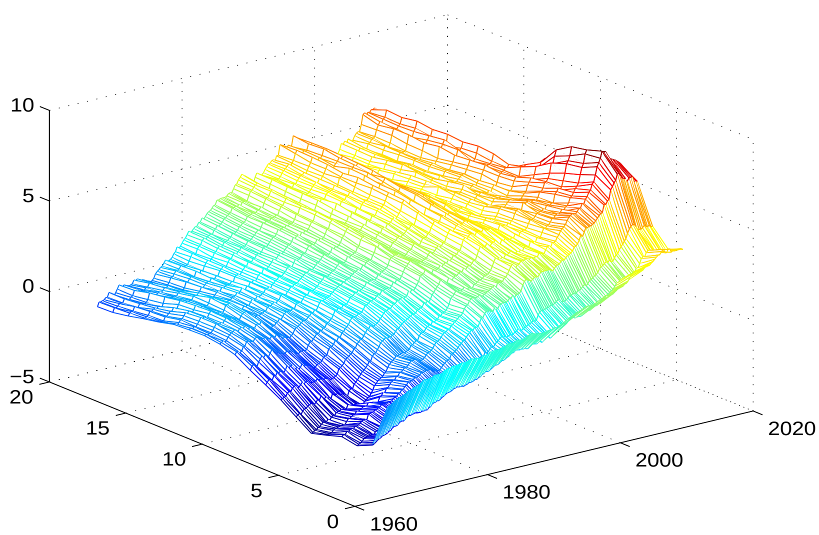



f. Stock prices

Figure 3 (cont.)




g. Fundamental component




h. Price minus fundamental

Figure 4

Estimated Responses of q minus qF at Selected Horizons

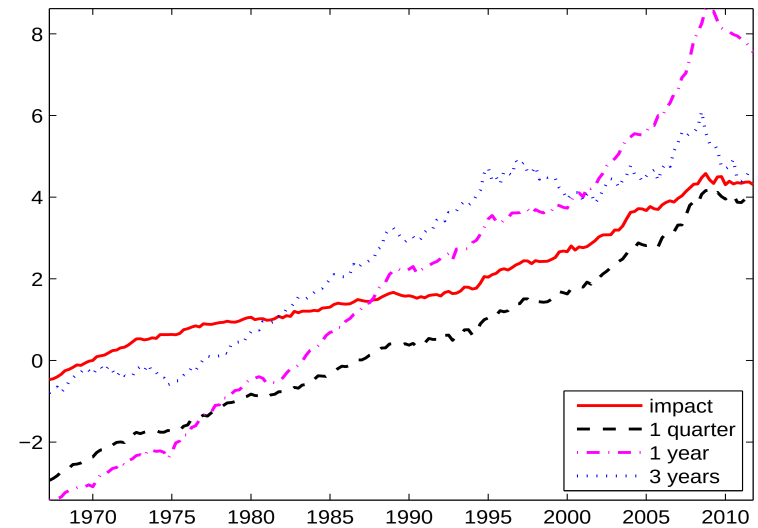



Figure 5

Probability of a Positive Response of q minus qF at Selected Horizons

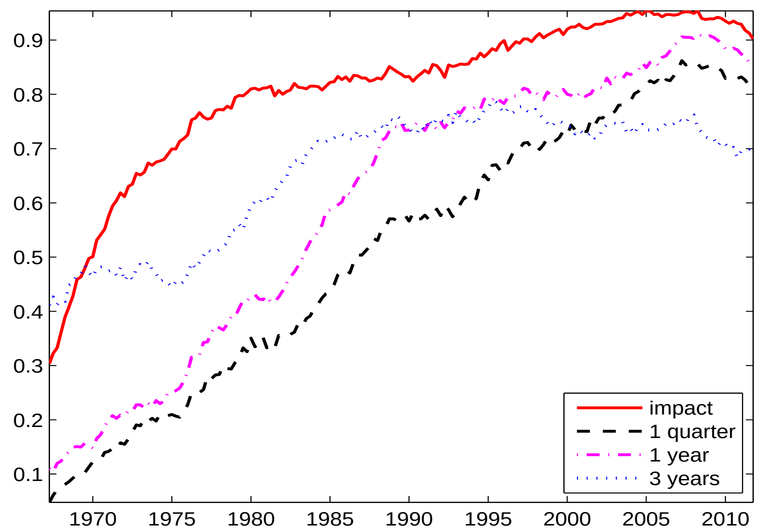
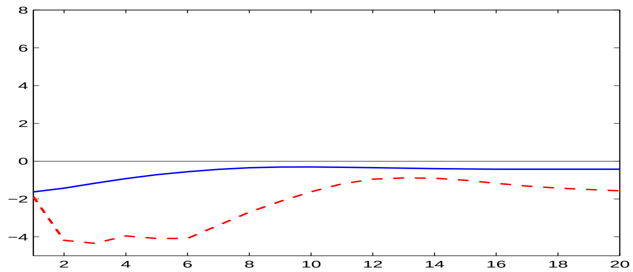
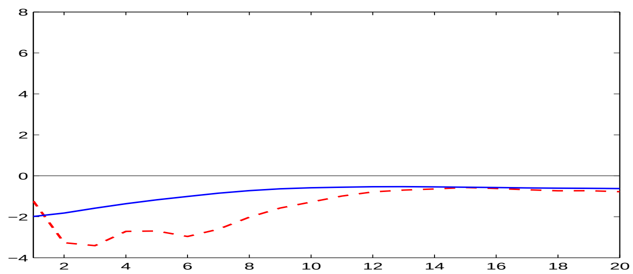


Figure 6

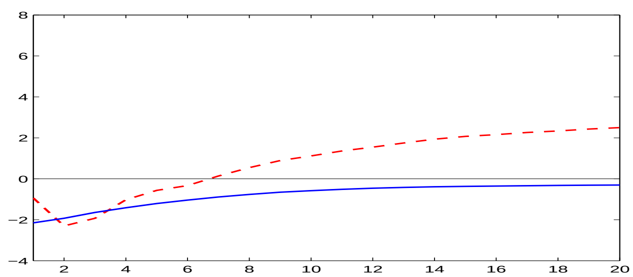
Estimated Responses to Monetary Policy Shocks: Analysis of Selected Episodes



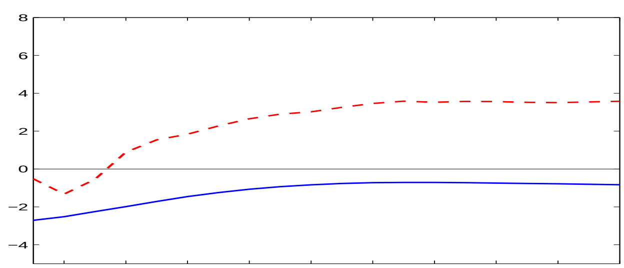

a. 1967Q2-1970Q1




b. 1976Q1-1978Q4




c. 1984Q4-1987Q3




d. 1997Q1-1999Q4

Figure 7

Estimated Responses of q minus qF to Monetary Policy Shocks: Shorter Sample Period

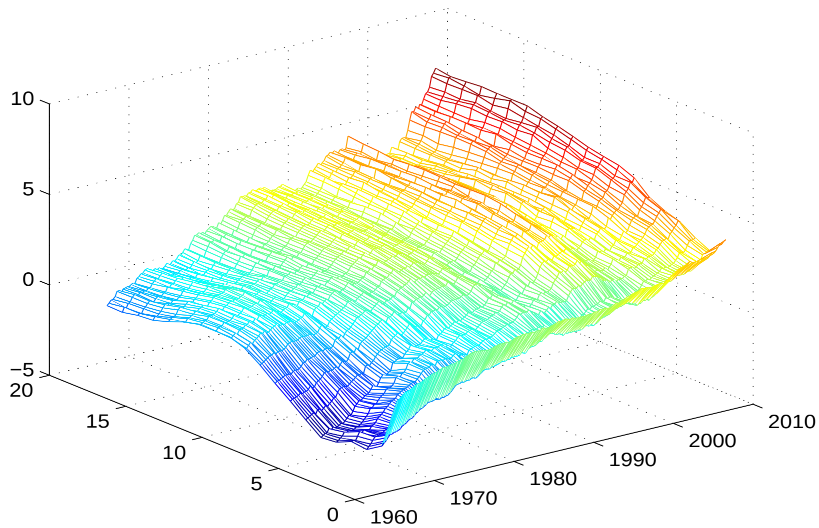



Figure 8

Estimated Responses of q minus qF to Monetary Policy Shocks: Earnings-Based Estimates

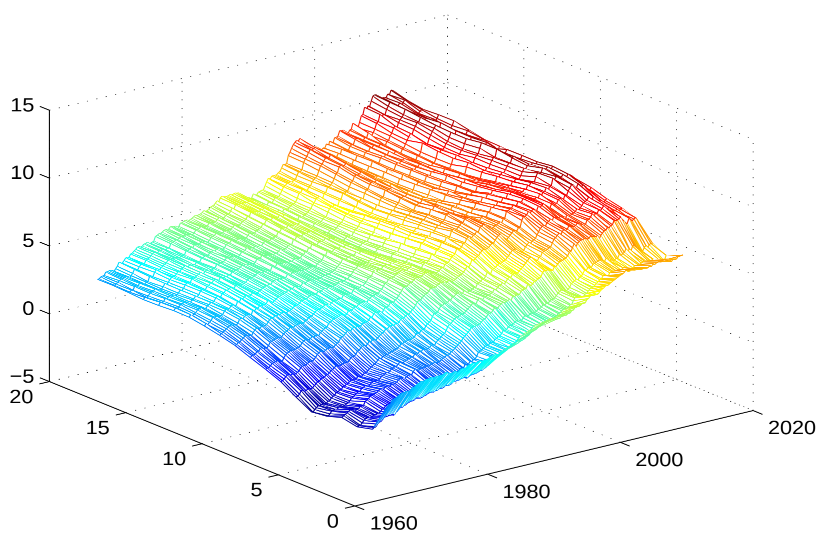


Figure 9

Estimated Response of Long Term Rate to Monetary Policy Shocks

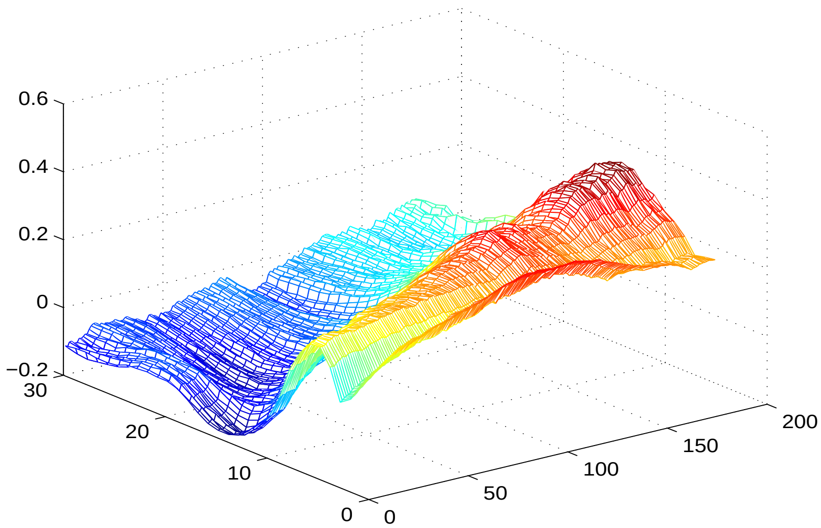
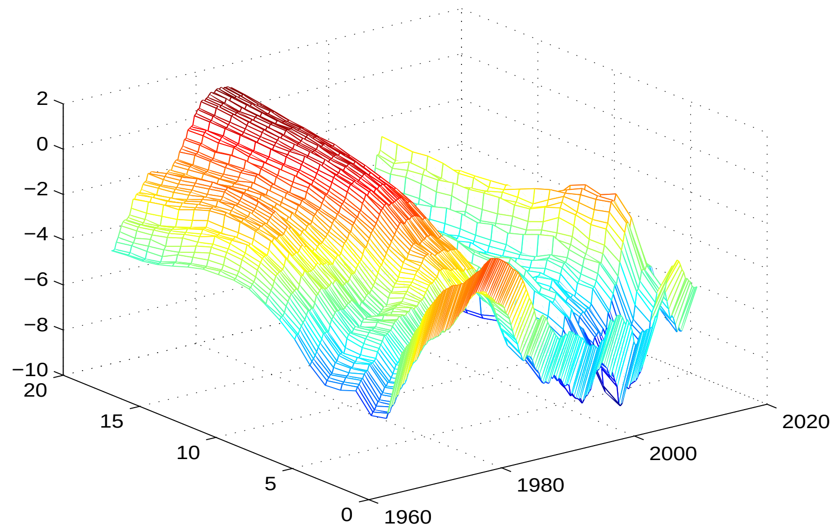
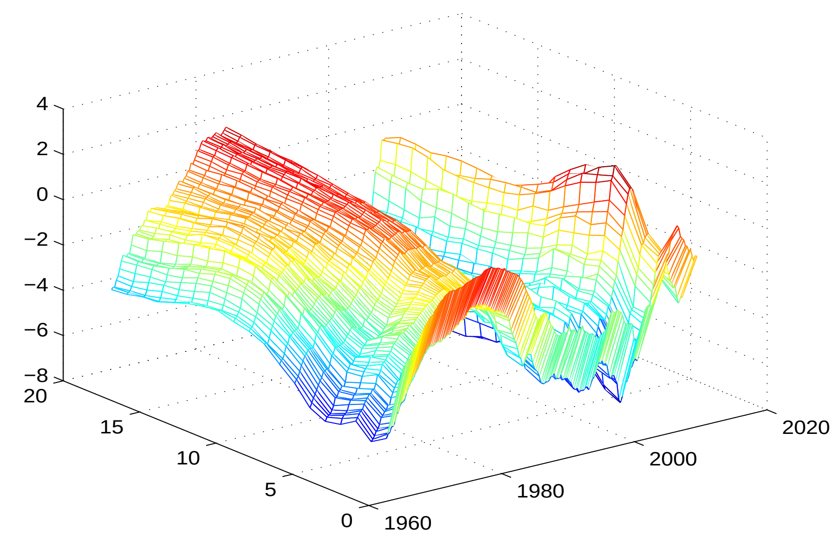


Figure 10

Estimated Responses to Monetary Policy Shocks: Alternative Identification



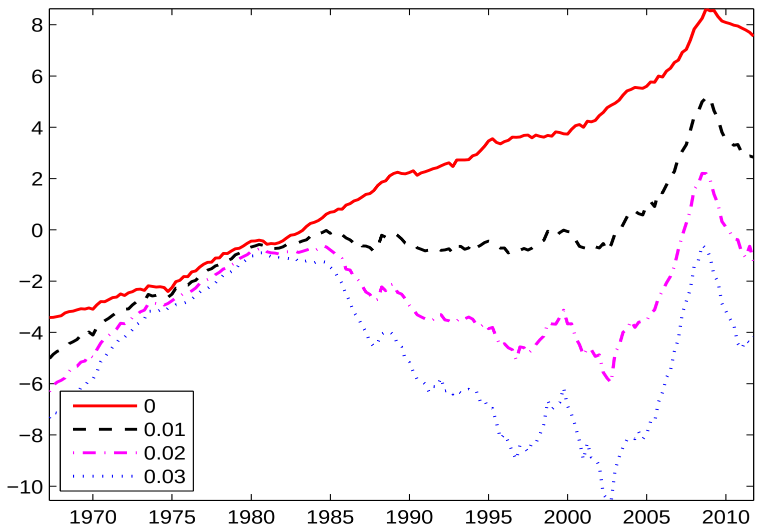

a. Stock prices




b. Prices minus fundamental

Figure 11

Estimated Response of Stock Prices at a One-Year Horizon: Alternative Calibrations of Endogenous Policy Response



# The Effects of Monetary Policy on Stock Market Bubbles: Some Evidence

Jordi Gali* Luca Gambetti†

ONLINE APPENDIX

The appendix describes the estimation of the time-varying coefficients VAR model. The model is estimated using the Gibbs sampling algorithm along the lines described in Primiceri (2005). Each iteration of the algorithm is composed of seven steps where a draw for a set of parameters is made conditional on the value of the remaining parameters. To clarify the notation, let $\mathbf{w}_t$ be a generic column vector. We denote $\mathbf{w}^T = [w_1', ..., w_T']'$. Below we report the conditional distributions used in the seven steps of the algorithm:[1]

1. $p(\boldsymbol{\sigma}^T|\mathbf{x}^T, \boldsymbol{\theta}^T, \boldsymbol{\phi}^T, \boldsymbol{\Omega}, \boldsymbol{\Xi}, \boldsymbol{\Psi}, \mathbf{s}^T)$
2. $p(\boldsymbol{\phi}^T|\mathbf{x}^T, \boldsymbol{\theta}^T, \boldsymbol{\sigma}^T, \boldsymbol{\Omega}, \boldsymbol{\Xi}, \boldsymbol{\Psi})$
3. $p(\boldsymbol{\theta}^T|\mathbf{x}^T, \boldsymbol{\sigma}^T, \boldsymbol{\phi}^T, \boldsymbol{\Omega}, \boldsymbol{\Xi}, \boldsymbol{\Psi})$
4. $p(\boldsymbol{\Omega}|\mathbf{x}^T, \boldsymbol{\theta}^T, \boldsymbol{\sigma}^T, \boldsymbol{\phi}^T, \boldsymbol{\Xi}, \boldsymbol{\Psi})$
5. $p(\boldsymbol{\Xi}|\mathbf{x}^T, \boldsymbol{\theta}^T, \boldsymbol{\sigma}^T, \boldsymbol{\phi}^T, \boldsymbol{\Omega}, \boldsymbol{\Psi})$
6. $p(\boldsymbol{\Psi}_i|\mathbf{x}^T, \boldsymbol{\theta}^T, \boldsymbol{\sigma}^T, \boldsymbol{\phi}^T, \boldsymbol{\Omega}, \boldsymbol{\Xi})$, $i = 1, 2, 3, 4$
7. $p(\mathbf{s}^T|\mathbf{x}^T, \boldsymbol{\theta}^T, \boldsymbol{\sigma}^T, \boldsymbol{\phi}^T, \boldsymbol{\Omega}, \boldsymbol{\Xi}, \boldsymbol{\Psi})$ [2]

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[1]Notice that the following ordering is not subject to the problem discussed in Del Negro and Primiceri (2012).

[2]See below the definition of $\mathbf{s}_t$.

## Priors Specification

We assume that the covariance matrices $\mathbf{\Omega}$, $\mathbf{\Xi}$ and $\mathbf{\Psi}$ and the initial states, $\boldsymbol{\theta}_0$, $\boldsymbol{\phi}_0$ and $\log \boldsymbol{\sigma}_0$, are independent, the prior distributions for the initial states are Normal and the prior distributions for $\mathbf{\Omega}^{-1}$, $\mathbf{\Xi}^{-1}$ and $\mathbf{\Psi}_i^{-1}$ are Wishart. More precisely

$$\begin{aligned}
\boldsymbol{\theta}_0 &\sim N(\hat{\boldsymbol{\theta}}, 4\hat{\mathbf{V}}_{\boldsymbol{\theta}}) \\
\log \boldsymbol{\sigma}_0 &\sim N(\log \hat{\boldsymbol{\sigma}}_0, \mathbf{I}_n) \\
\boldsymbol{\phi}_{i0} &\sim N(\hat{\boldsymbol{\phi}}_i, \hat{\mathbf{V}}_{\boldsymbol{\phi}_i}) \\
\mathbf{\Omega}^{-1} &\sim W(\underline{\mathbf{\Omega}}^{-1}, \underline{\rho}_1) \\
\mathbf{\Xi}^{-1} &\sim W(\underline{\mathbf{\Xi}}^{-1}, \underline{\rho}_2) \\
\mathbf{\Psi}_i^{-1} &\sim W(\underline{\mathbf{\Psi}}_i^{-1}, \underline{\rho}_{3i})
\end{aligned}$$

where $W(\mathbf{S}, d)$ denotes a Wishart distribution with scale matrix $\mathbf{S}$ and degrees of freedom $d$ and $\mathbf{I}_n$ is a $n \times n$ identity matrix where $n$ is the number of variables in the VAR.

Prior means and variances of the Normal distributions are calibrated using a time invariant VAR for $\mathbf{x}_t$ estimated using the first $\tau = 48$ observations. $\hat{\boldsymbol{\theta}}$ and $\hat{\mathbf{V}}_{\boldsymbol{\theta}}$ are set equal to the OLS estimates. Let $\hat{\mathbf{\Sigma}}$ be the covariance matrix of the residuals $\hat{\mathbf{u}}_t$ of the initial time-invariant VAR. We apply the same decomposition discussed in the text $\hat{\mathbf{\Sigma}} = \hat{\mathbf{F}}\hat{\mathbf{D}}\hat{\mathbf{F}}'$ and set $\log \hat{\boldsymbol{\sigma}}_0$ equal to the log of the diagonal elements of $\hat{\mathbf{D}}^{1/2}$. $\hat{\boldsymbol{\phi}}_i$ is set equal to the OLS estimates of the coefficients of the regression of $\hat{\mathbf{u}}_{i+1,t}$, the $i+1$-th element of $\hat{\mathbf{u}}_t$, on $-\hat{\mathbf{u}}_{1,t}, ..., -\hat{\mathbf{u}}_{i,t}$ and $\hat{\mathbf{V}}_{\phi_i}$ equal to the estimated variances.

We parametrize the scale matrices as follows $\underline{\mathbf{\Omega}} = \underline{\rho}_1(\lambda_1 \hat{\mathbf{V}}_{\boldsymbol{\theta}}^f)$, $\underline{\mathbf{\Xi}} = \underline{\rho}_2(\lambda_2 \mathbf{I}_n)$ and $\underline{\mathbf{\Psi}}_i = \underline{\rho}_{3i}(\lambda_3 \hat{\mathbf{V}}_{\boldsymbol{\phi}_i}^f)$. The degrees of freedom for the priors on the covariance matrices $\underline{\rho}_1$ and $\underline{\rho}_2$ are set equal to the number of rows $\underline{\mathbf{\Omega}}^{-1}$ and $\mathbf{I}_n$ plus one respectively while $\underline{\rho}_{3i}$ is $i+1$ for $i = 1, ..., n-1$. We assume $\lambda_1 = 0.005$, $\lambda_2 = 0.01$ and $\lambda_3 = 0.01$. Finally $\hat{\mathbf{V}}_\theta^f$ and $\hat{\mathbf{V}}_{\phi_i}^f$ are obtained as $\hat{\mathbf{V}}_\theta$ and $\hat{\mathbf{V}}_{\boldsymbol{\phi}_i}$ but using the estimates of the whole sample.

## Gibbs sampling algorithm

Let $\bar{T}$ be the total number of observations, in our case equal to 204. To draw realizations from the posterior distribution we use $T = \bar{T} - \tau/2$ observations starting from $\tau/2 + 1$.[3] The algorithm works as follows:

*Step 1*: sample $\boldsymbol{\sigma}^T$. The states $\boldsymbol{\sigma}^T$ are drawn using the algorithm of Kim, Shephard and Chib (1998, KSC hereafter). Let $\mathbf{x}_t^* \equiv \mathbf{F}_t^{-1}(\mathbf{x}_t - \mathbf{W}_t'\boldsymbol{\theta}_t) = \mathbf{D}_t^{1/2}\mathbf{u}_t$, where $\mathbf{u}_t \sim N(0, \mathbf{I}_n)$, $\mathbf{W}_t = (\mathbf{I}_n \otimes \mathbf{w}_t)$, and $\mathbf{w}_t = [1_n, \mathbf{x}_{t-1}...\mathbf{x}_{t-p}]'$. Notice that conditional on $\mathbf{x}^T, \boldsymbol{\theta}^T$, and $\boldsymbol{\phi}^T$, $\mathbf{x}_t^*$ is observable. Therefore, by squaring and taking logs, we obtain the following state-space representation

$$\mathbf{x}_t^{**} = 2\mathbf{r}_t + \boldsymbol{\upsilon}_t \tag{1}$$

$$\mathbf{r}_t = \mathbf{r}_{t-1} + \boldsymbol{\zeta}_t \tag{2}$$

where $\mathbf{x}_{i,t}^{**} = \log({\mathbf{x}_{i,t}^*}^2)$, $\boldsymbol{\upsilon}_{i,t} = \log(\mathbf{u}_{i,t}^2)$ and $\mathbf{r}_t = \log \boldsymbol{\sigma}_{i,t}$.[4] The above system is non-normal since the innovation in (1) is distributed as $\log \chi^2(1)$. Following KSC, we use a mixture of 7 Normal densities with mean $m_j - 1.2704$, and variance $v_j^2$ ($j$=1,...,7) to approximate the system with a Gaussian one (see Table A1 for the values used).

Let $\mathbf{s}_t$ be the $n \times 1$ vector whose elements indicate which of the seven Normal densities has to be used for the corresponding element of $\boldsymbol{\upsilon}_t$. Conditional on $\mathbf{s}^T$, we have that $(\boldsymbol{\upsilon}_{i,t}|\mathbf{s}_{i,t} = j) \sim N(m_j - 1.2704, v_j^2)$ and the algorithm of Carter and Kohn (1994, CK henceforth) is used to draw $\mathbf{r}_t$ from $N(\mathbf{r}_{t|t+1}, \mathbf{R}_{t|t+1})$, where $\mathbf{r}_{t|t+1} = E(\mathbf{r}_t|\mathbf{r}_{t+1}, \mathbf{x}^t, \boldsymbol{\theta}^T, \boldsymbol{\phi}^T, \boldsymbol{\Omega}, \boldsymbol{\Xi}, \boldsymbol{\Psi}, \mathbf{s}^T, )$ and $\mathbf{R}_{t|t+1} = Var(\mathbf{r}_t|\mathbf{r}_{t+1}, \mathbf{x}^t, \boldsymbol{\theta}^T, \boldsymbol{\phi}^T, \boldsymbol{\Omega}, \boldsymbol{\Xi}, \boldsymbol{\Psi}, \mathbf{s}^T)$ are the conditional mean and variance obtained from the backward recursion equations.

[3]We start the sample from $\tau/2 + 1$ instead of $\tau$ in order to not to lose too many data points.

[4]We do not use any offsetting constant since given that the variables are in logs multiplied by 100, we do not have numerical problems.

Table A1

| $j$ | $q_j$ | $m_j$ | $v_j^2$ |
|---|---|---|---|
| 1.0000 | 0.0073 | -10.1300 | 5.7960 |
| 2.0000 | 0.1056 | -3.9728 | 2.6137 |
| 3.0000 | 0.0000 | -8.5669 | 5.1795 |
| 4.0000 | 0.0440 | 2.7779 | 0.1674 |
| 5.0000 | 0.3400 | 0.6194 | 0.6401 |
| 6.0000 | 0.2457 | 1.7952 | 0.3402 |
| 7.0000 | 0.2575 | -1.0882 | 1.2626 |

*Step 2*: sample $\boldsymbol{\phi}^T$. Let $\hat{\mathbf{x}}_t = \mathbf{x}_t - \mathbf{W}_t'\boldsymbol{\theta}_t$. The $i+1$-th $(i = 1, ..., n-1)$ equation of the system $\mathbf{F}_t^{-1}\hat{\mathbf{x}}_t = \mathbf{D}_t^{1/2}\mathbf{u}_t$ can be written as

$$\hat{\mathbf{x}}_{i+1,t} \quad = \quad -\hat{\mathbf{x}}_{[1,i],t}\boldsymbol{\phi}_{i,t} + \sigma_{i,t}\mathbf{u}_{i+1,t} \quad i = 2, ..., n \tag{3}$$

where $\boldsymbol{\sigma}_{i,t}$ and $\mathbf{u}_{i,t}$ are the $i$th elements of $\boldsymbol{\sigma}_t$ and $u_t$ respectively and $\hat{\mathbf{x}}_{[1,i],t} = [\hat{\mathbf{x}}_{1,t}, ..., \hat{\mathbf{x}}_{i,t}]$. Conditional on $\boldsymbol{\theta}^T$ and $\boldsymbol{\sigma}^T$, equation (3) is the observable equation of a state-space model where the states are $\boldsymbol{\phi}_{i,t}$. Moreover, since $\boldsymbol{\phi}_{i,t}$ and $\boldsymbol{\phi}_{j,t}$ are independent for $i \neq j$, the algorithm of CK can be applied equation by equation to draw $\boldsymbol{\phi}_{i,t}$ from a $N(\boldsymbol{\phi}_{i,t|t+1}, \boldsymbol{\Phi}_{i,t|t+1})$, where $\boldsymbol{\phi}_{i,t|t+1} = E(\boldsymbol{\phi}_{i,t}|\boldsymbol{\phi}_{i,t+1}, \mathbf{x}^t, \boldsymbol{\theta}^T, \boldsymbol{\sigma}^T, \boldsymbol{\Omega}, \boldsymbol{\Xi}, \boldsymbol{\Psi})$ and $\boldsymbol{\Phi}_{i,t|t+1} = Var(\boldsymbol{\phi}_{i,t}|\boldsymbol{\phi}_{i,t+1}, \mathbf{x}^t, \boldsymbol{\theta}^T, \boldsymbol{\sigma}^T, \boldsymbol{\Omega}, \boldsymbol{\Xi}, \boldsymbol{\Psi})$.

*Step 3:* sample $\boldsymbol{\theta}^T$. Consider the state-space representation

$$\mathbf{x}_t \quad = \quad \mathbf{W}_t'\boldsymbol{\theta}_t + \boldsymbol{\varepsilon}_t \tag{4}$$

$$\boldsymbol{\theta}_t \quad = \quad \boldsymbol{\theta}_{t-1} + \boldsymbol{\omega}_t. \tag{5}$$

$\boldsymbol{\theta}_t$ is drawn from a $N(\boldsymbol{\theta}_{t|t+1}, \mathbf{P}_{t|t+1})$, where $\boldsymbol{\theta}_{t|t+1} = E(\boldsymbol{\theta}_t|\boldsymbol{\theta}_{t+1}, \mathbf{x}^t, \boldsymbol{\sigma}^T, \boldsymbol{\phi}^T, \boldsymbol{\Omega}, \boldsymbol{\Xi}, \boldsymbol{\Psi})$ and $\mathbf{P}_{t|t+1} = Var(\boldsymbol{\theta}_t|\boldsymbol{\theta}_{t+1}, \mathbf{x}^t, \boldsymbol{\sigma}^T, \boldsymbol{\phi}^T, \boldsymbol{\Omega}, \boldsymbol{\Xi}, \boldsymbol{\Psi})$ are obtained using the CK algorithm.

*Step 4:* sample $\mathbf{\Omega}$. A draw is obtained as follows: $\mathbf{\Omega} = (\mathbf{M}\mathbf{M}')^{-1}$ where $\mathbf{M}$ is an $(n^2p + n) \times \overline{\rho}_1$ matrix whose columns are independent draws from a $N(0, \overline{\mathbf{\Omega}}^{-1})$ where $\overline{\mathbf{\Omega}} = \underline{\mathbf{\Omega}} + \sum_{t=1}^{T} \Delta\boldsymbol{\theta}_t(\Delta\boldsymbol{\theta}_t)'$ (see Gelman et. al., 1995).

*Step 5*: sample $\mathbf{\Xi}$. As above, $\mathbf{\Xi} = (\mathbf{M}\mathbf{M}')^{-1}$ where $\mathbf{M}$ is an $n \times \overline{\rho}_2$ matrix whose columns are independent draws from a $N(0, \overline{\mathbf{\Xi}}^{-1})$ where $\overline{\mathbf{\Xi}} = \underline{\mathbf{\Xi}} + \sum_{t=1}^{T} \Delta \log \boldsymbol{\sigma}_t(\Delta \log \boldsymbol{\sigma}_t)'$.

*Step 6:* sample $\mathbf{\Psi}_i$ $i = 1, ..., 5$. As above, $\mathbf{\Psi}_i = (\mathbf{M}\mathbf{M}')^{-1}$ where $\mathbf{M}$ is an $i \times \overline{\rho}_{3i}$ matrix whose columns are independent draws from a $N(0, \overline{\mathbf{\Psi}}_i^{-1})$ where $\overline{\mathbf{\Psi}}_i = \underline{\mathbf{\Psi}}_i + \sum_{t=1}^{T} \Delta\boldsymbol{\phi}_{i,t}(\Delta\boldsymbol{\phi}_{i,t})'$.

*Step 7:* sample $\mathbf{s}^T$. Each $\mathbf{s}_{i,t}$ is independently sampled from the discrete density defined by $Pr(\mathbf{s}_{i,t} = j|\mathbf{x}^{**}_{i,t}, \mathbf{r}_{i,t}) \propto q_j f_N(\mathbf{x}^{**}_{i,t}|2\mathbf{r}_{i,t} + m_j - 1.2704, v_j^2)$, where $f_N(x|\mu, \sigma^2)$ denotes the Normal pdf with mean $\mu$ and variance $\sigma^2$, and $q_j$ is the probability reported in Table A1 associated to the $j$-th density.

We make 22000 draws discarding the first 20000 and collecting one out of two of the remaining 2000 draws. The results presented in the paper are therefore based on 1000 draws from the posterior distribution. Parameters convergence is assessed using trace plots.