4. FORECASTING[1]

[1]This part is based on the Hamilton textbook.

1 Principles of forecasting

1.1 Forecast based on conditional expectations

• Suppose we are interested in forecasting the value of Y_{t+1} based on a set of variables X_t.

• Let $Y_{t+1|t}$ denote such a forecast.

• To evaluate the usefulness of the forecast we need to specify a *loss* function. Here we specify a quadratic loss function. A quadratic loss function means that $Y_{t+1|t}$ is chose to minimize $E(Y_{t+1} - Y_{t+1|t})^2$.

• $E(Y_{t+1} - Y_{t+1|t})^2$ is known as the *mean squared error* associated with the forecast $Y_{t+1|t}$ denoted

$$MSE(Y_{t+1|t}) = E(Y_{t+1} - Y_{t+1|t})^2$$

• Fundamental result: the forecast with the smallest MSE is the expectation of $Y_{t+1|t}$ conditional on X_t that is

$$Y_{t+1|t} = E(Y_{t+1}|X_t)$$

We now verify the claim. Let $g(X_t)$ be any other function and let $Y_{t+1|t} = g(X_t)$. The associated MSE is

$$\begin{aligned}
E\left[Y_{t+1} - g(X_t)\right]^2 &= E\left[Y_{t+1} - E(Y_{t+1}|X_t) + E(Y_{t+1}|X_t) - g(X_t)\right]^2 \\
&= E\left[Y_{t+1} - E(Y_{t+1}|X_t)\right]^2 + \\
&\quad +2E\left\{\left[Y_{t+1} - E(Y_{t+1}|X_t)\right]\left[E(Y_{t+1}|X_t) - g(X_t)\right]\right\} + \\
&\quad +E\{[E(Y_{t+1}|X_t) - g(X_t)\}^2
\end{aligned} \tag{2}$$

Define

$$\eta_{t+1} \equiv \left[Y_{t+1} - E(Y_{t+1}|X_t)\right]\left[E(Y_{t+1}|X_t) - g(X_t)\right] \tag{3}$$

The conditional expectation is

$$\begin{aligned}
E(\eta_{t+1}|X_t) &= E\left\{\left[Y_{t+1} - E(Y_{t+1}|X_t)\right]\left[E(Y_{t+1}|X_t) - g(X_t)\right]|X_t\right\} \\
&= \left[E(Y_{t+1}|X_t) - g(X_t)\right] E\left\{\left[Y_{t+1} - E(Y_{t+1}|X_t)\right]|X_t\right\} \\
&= \left[E(Y_{t+1}|X_t) - g(X_t)\right]\left[E(Y_{t+1}|X_t) - E(Y_{t+1}|X_t)\right] \\
&= 0
\end{aligned}$$

Therefore by law of iterated expectations

$$E(\eta_{t+1}) = E(E(\eta_{t+1}|X_t)) = 0$$

This means that

$$E\left[Y_{t+1} - g(X_t)\right]^2 \;=\; E\left[Y_{t+1} - E(Y_{t+1}|X_t)\right]^2 + E([E(Y_{t+1}|X_t) - g(X_t)])^2$$

Therefore the function that minimizes the MSE is

$$g(X_t) = E(Y_{t+1}|X_t)$$

$E(Y_{t+1}|X_t)$ is the optimal forecast of Y_{t+1} conditional of X_t under a quadratic loss function. The MSE of this optimal forecast is

$$E[Y_{t+1} - g(X_t)]^2 = E[Y_{t+1} - E(Y_{t+1}|X_t)]^2$$

1.2 Forecast based on linear projections

We now restrict the class of forecasts we consider to be linear function of X_t:

$$Y_{t+1|t} = \alpha' X_t$$

Suppose α' is such that the resulting forecast error is uncorrelated with X_t

$$E[(Y_{t+1} - \alpha' X_t) X_t'] = 0' \tag{4}$$

If (4) holds the we call $\alpha' X_t$ the linear projection of Y_{t+1} on X_t.

• The linear projection produces the smallest forecast error among the class of linear forecasting rules. To verify this let $g' X_t$ be any arbitrary forecasting rule.

$$\begin{aligned} E\left(Y_{t+1} - g' X_t\right)^2 &= E\left(Y_{t+1} - \alpha' X_t + \alpha' X_t - g' X_t\right)^2 \\ &= E\left(Y_{t+1} - \alpha' X_t\right)^2 + \\ &\quad +2E\left[\left(Y_{t+1} - \alpha' X_t\right)\left(\alpha' X_t - g' X_t\right)\right] + \\ &\quad +E\left(\alpha' X_t - g' X_t\right)^2 \end{aligned}$$

the middle term

$$\begin{aligned} E[(Y_{t+1} - \alpha' X_t)(\alpha' X_t - g' X_t)] &= E[(Y_{t+1} - \alpha' X_t) X_t' [\alpha - g]) \\ &= E[(Y_{t+1} - \alpha' X_t) X_t'] [\alpha - g] \\ &= 0' \end{aligned}$$

by definition of linear projection. Thus

$$E\left(Y_{t+1}-g'X_t\right)^2 \;=\; E\left(Y_{t+1}-\alpha'X_t\right)^2+E\left(\alpha'X_t-g'X_t\right)^2$$

The optimal linear forecast is the value $g'X_t=\alpha'X_t$. We use the notation

$$P(Y_{t+1}|X_t)=\alpha'X_t$$

to indicate the linear projection of Y_{t+1} on X_t.

Notice that

$$MSE[P(Y_{t+1}|X_t)]\geq MSE\left[E(Y_{t+1}|X_t)\right]$$

The projection coefficient α can be calculated in terms of moments of Y_{t+1} and X_t.

$$E(Y_{t+1}X_t')=\alpha'E(X_tX_t')$$

$$\alpha'=[E(X_tX_t')]^{-1}E(Y_{t+1}X_t')$$

Here we denote $\hat{Y}_{t+s|t}=P(Y_{t+s}|X_t,1)$ the best linear forecast of Y_{t+s} conditional on X_t.

1.3 Linear projections and OLS regression

There is a close relationship between OLS estimator and the linear projection coefficient. If Y_{t+1} and X_t are stationary processes and also ergodic for the second moments then

$$(1/T)\sum_{t=1}^{T} X_t X_t' \xrightarrow{p} E(X_t X_t')$$

$$(1/T)\sum_{t=1}^{T} X_t Y_{t+1} \xrightarrow{p} E(X_t Y_{t+1})$$

implying

$$\hat{\beta} \xrightarrow{p} \alpha$$

The OLS regression yields a consistent estimate of the linear projection coefficient.

2 Forecasting an AR(1)

For the zero-mean covariance-stationary $AR(1)$ we have

$$\hat{Y}_{t+s|t} = P(Y_{t+s}|Y_t, Y_{t-1}, ...) \;=\; \phi^s Y_t$$

The forecast decays geometrically toward zero as the forecast horizon increases.

The forecast error is

$$Y_{t+s} - \hat{Y}_{t+s|t} = \varepsilon_{t+s} + \phi\varepsilon_{t+s-1} + \phi^2\varepsilon_{t+s-2} + ... + \phi^{s-1}\varepsilon_{t+1}$$

The associated MSE will be

$$\begin{aligned} E(Y_{t+s} - \hat{Y}_{t+s|t})^2 &= E(\varepsilon_{t+s} + \phi\varepsilon_{t+s-1} + \phi^2\varepsilon_{t+s-2} + ... + \phi^{s-1}\varepsilon_{t+1})^2 \\ &= (1 + \phi^2 + \phi^4 + ... + \phi^{2(s-1)})\sigma^2 \\ &= \frac{1-\phi^{2s}}{1-\phi^2}\sigma^2 \end{aligned}$$

Notice that

$$\lim_{s\to\infty} \hat{Y}_{t+s|t} = 0$$

(in general it will converge to the mean of the process) and

$$\lim_{s\to\infty} E(Y_{t+s} - \hat{Y}_{t+s|t})^2 = \frac{\sigma^2}{1-\phi^2}$$

which is the variance of the process.

3 Forecasting an AR(p)

Now consider an $AR(p)$. Recall the $AR(p)$ can be written as

$$Z_t = FZ_{t-1} + \epsilon_t$$

where $Z_t = [Y_t, Y_{t-1}, ..., Y_{t-p+1}]'$, ${}_t = [\varepsilon_t, 0, ..., 0]'$ and

$$F = \begin{pmatrix} \phi_1 & \phi_2 & \dots & \phi_{p-1} & \phi_p \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{pmatrix}$$

Therefore

$$Y_{t+s} = f_{11}^{(s)} Y_t + f_{12}^{(s)} Y_{t-1} + ... + f_{1p}^{(s)} Y_{t-p+1} + \varepsilon_{t+s} + f_{11}^{1} \varepsilon_{t+s-1} + f_{11}^{2} \varepsilon_{t+s-2} + ... + f_{11}^{s-1} \varepsilon_{t+1}$$

where f_{mn}^j denotes the (m, n) element of F^j.

The optimal s-step ahead forecast is thus

$$\hat{Y}_{t+s|t} \;=\; f_{11}^{(s)} Y_t + f_{12}^{(s)} Y_{t-1} + ... + f_{1p}^{(s)} Y_{t-p+1} \tag{5}$$

and the associated forecast error

$$Y_{t+s} - \hat{Y}_{t+s|t} \;=\; \varepsilon_{t+s} + f_{11}^{1} \varepsilon_{t+s-1} + f_{11}^{2} \varepsilon_{t+s-2} + ... + f_{11}^{s-1} \varepsilon_{t+1} \tag{6}$$

The forecast can be computed recursively. Let $\hat{Y}_{t+1|t}$ be one period step-ahead forecast of Y_{t+1} we have

$$\hat{Y}_{t+1|t} = \phi_1 Y_t + \phi_2 Y_{t-1} + ... + \phi_p Y_{t-p+1} \tag{7}$$

In general the j-step ahead forecast $\hat{Y}_{t+j|t}$ can be computed using the recursion

$$\hat{Y}_{t+j|t} = \phi_1 \hat{Y}_{t+j-1|t} + \phi_2 \hat{Y}_{t+j-2|t} + ... + \phi_p \hat{Y}_{t+j-p|t} \tag{8}$$

An easy way to see this is to use the AR(1) process

$$Z_t = F Z_{t-1} + \epsilon_t$$

We have

$$\begin{aligned}
\hat{Z}_{t+1|t} &= F Z_t \\
\hat{Z}_{t+2|t} &= F^2 Z_t \\
&\vdots \\
\hat{Z}_{t+s|t} &= F^s Z_t
\end{aligned}$$

which means

$$\hat{Z}_{t+s|t} = F \hat{Z}_{t+s-1|t}$$

The forecast $\hat{Y}_{t+s|t}$ will be the first element of $\hat{Z}_{t+s|t}$

The associated forecast errors will be

$$
\begin{aligned}
Z_{t+1} - \hat{Z}_{t+1|t} &= \epsilon_{t+1} \\
Z_{t+2} - \hat{Z}_{t+2|t} &= F\epsilon_{t+1} + \epsilon_{t+2} \\
&\vdots \\
Z_{t+s} - \hat{Z}_{t+s|t} &= F^{s-1}\epsilon_{t+1} + F^{s-2}\epsilon_{t+2} + ... + F\epsilon_{t+s-1} + \epsilon_{t+s}
\end{aligned}
$$

Let $E(\epsilon_t \epsilon_t') = \Sigma$. The mean squared errors will be

$$
\begin{aligned}
MSE(\hat{Z}_{t+1|t}) &= \Sigma \\
MSE(\hat{Z}_{t+2|t}) &= F\Sigma F' + \Sigma \\
&\vdots \\
MSE(\hat{Z}_{t+s|t}) &= F^{s-1}\Sigma F^{s-1\prime} + F^{s-2}\Sigma F^{s-2\prime} + ... + F\Sigma F' + \Sigma \\
&= \sum_{j=0}^{s-1} F^j \Sigma F^{j\prime}
\end{aligned}
$$

4 Direct forecast

An alternative is to compute directly the linear projection. To see this consider two variables, x_t and y_t. We want to forecast x_{t+h} given the information available at time t.

The direct forecast works as follows:

1. Estimate the projection equation

$$x_t = a + \sum_{i=0}^{p-1} \phi_i x_{t-h-i} + \sum_{i=0}^{p-1} \psi_i y_{t-h-i} + \varepsilon_t$$

2. Using the estimated coefficients, the predictor $x_{t+h|t}$ is obtain as

$$\hat{x}_{t+h|t} = \hat{a} + \sum_{i=0}^{p-1} \hat{\phi}_i x_{t-i} + \sum_{i=0}^{p-1} \hat{\psi}_i y_{t-i}$$

5 Comparing Predictive Accuracy

Diebold and Mariano propose a procedure to formally compare the forecasting performance of two competing models. Let $\hat{Y}^1_{t+s|t}, \hat{Y}^2_{t+s|t}$ be two forecast obtained from two non-nested models. Let

$$
\begin{aligned}
w^1_{\tau+s|\tau} &= Y_{\tau+s} - \hat{Y}^1_{\tau+s|t} \\
w^2_{\tau+s|\tau} &= Y_{\tau+s} - \hat{Y}^2_{\tau+s|t}
\end{aligned}
$$

be the two forecast errors where $\tau = T_0, ...T - s$.

The accuracy of each forecast is measured by a particular loss function, say quadratic i.e. $L(w^i_{\tau+s|\tau}) = \left(w^1_{\tau+s|\tau}\right)^2$. The Diebold Mariano procedure is based on a test of the null hypothesis

$$H_0 : E(d_\tau) = 0, \quad H_1 : E(d_\tau) \neq 0$$

where $d_\tau = L(w^1_{\tau+s|\tau}) - L(w^2_{\tau+s|\tau})$. The Diebold-Mariano statistic is

$$S = \frac{\bar{d}}{\left(L\hat{R}V_{\bar{d}}\right)^{1/2}}$$

where

$$\bar{d} = (1/(T - T_0 - s)) \sum_{\tau=T_0}^{T-s} d_\tau$$

and $L\hat{R}V_{\bar{d}}$ is a consistent estimate of

$$LRV_{\bar{d}} = \gamma_0 + 2\sum_{j=1}^{\infty} \gamma_j$$

and $\gamma_j = Cov(d_t, d_{t-j})$.

Under the null

$$S \xrightarrow{L} N(0,1)$$

• DM test is for forecast comparison not model comparison.

• When using forecast obtained from models one has to be careful. In nested models the distribution of the DM statistic is non-normal.

• For nested model an alternative is the bootstrapping procedure in Clark and McCracken (Advances in Forecast Evaluation, 2013).

6 Forecast evaluation

- A key issue in forecasting is to evaluate the forecasting accuracy of a model of interest. In particular several times we will be interested in comparing the performance of competing forecasting model.

- How can we perform such a forecast evaluation?

- Answer: we can compare the mean squared errors using pseudo-out of sample forecast exercises.

6.1 Pseudo out-of-sample exercises

Suppose we have a sample of T observations. Let T_0 be the sample use for the initial estimation. A pseudo out-of-sample exercise works as follows.

1. Initialize $\tau = T_0$.

2. Use the first τ observations to estimate the parameters of the model.

3. Forecast $Y_{\tau+j}$ obtaining $\hat{Y}_{\tau+j|\tau}$ $j = 1, 2, ..., s$.

4. Compute the forecast error $w_{\tau+j|\tau} = Y_{\tau+j} - \hat{Y}_{\tau+j|\tau}$ with $j = 1, 2, ..., s$.

5. Update the sample adding one more observation $(\tau = \tau + 1)$ and repeat steps 1-3.

We repeat steps 1-4 up to the end of the sample and we compute the mean squared error

$$M\hat{S}E(\hat{Y}_{T+j|T}) = \frac{1}{T - T_0 + 1 - j} \sum_{\tau=T_0}^{T-T_0+1-j} w^2_{\tau+j|\tau}$$

or the root mean squared error

$$RM\hat{S}E(\hat{Y}_{T+j|T}) = \sqrt{\frac{1}{T - T_0 + 1 - j} \sum_{\tau=T_0}^{T-T_0+1-j} w^2_{\tau+j|\tau}}$$

where $T - T_0 + 1 - j$ is the total number of data points in the evaluation period.

5. FORECASTING: APPLICATIONS

"Why has U.S. inflation become harder to forecast?"
By Stock, J and M., Watson

7 "Why has U.S. inflation become harder to forecast?"

• The rate of price inflation in the United States has become both harder and easier to forecast.

• Easier: inflation is much less volatile than it was in the 1970s and the root mean squared error of inflation forecasts has declined sharply since the mid-1980s.

• Harder: standard multivariate forecasting models do not a better job than simple naive models. The point was made by Atkeson and Ohanian (2001) (henceforth, AO), who found that, since 1984 in the U.S., backwards-looking Phillips curve forecasts have been inferior to a forecast of twelve-month inflation by its average rate over the previous twelve months (naive or random walk forecast).

• Relevance of the topic. Change in terms of forecasting properties can signal changes in the structure of the economy. This can be taken as evidence that suggests that some relations have changed

• What relations? Structural models should be employed (next part of the course).

7.1 U.S. Inflation forecasts: facts and puzzles

7.1.1 Data

• GDP price index inflation (π).

• Robustness analysis done using personal consumption expenditure deflator for core items (PCEcore), the personal consumption expenditure deflator for all items (PCE-all), and the consumer price index (CPI, the official CPI-U).

• Real activity variables: the unemployment rate (u), log real GDP (y), the capacity utilization rate, building permits, and the Chicago Fed National Activity Index (CFNAI)

• Quarterly data. Quarterly values for monthly series are averages of the three months in the quarter.

• The full sample is from 1960:I through 2004:IV.

7.1.2 Forecasting models

- Two univariate models and one multivariate forecasting models.

- Let $\pi_t = 400 \log(p_t/p_{t-1})$ where p_t is the quarterly price index and let the h-period average inflation be $\pi_t^h = (1/h) \sum_{i=0}^{h-1} \pi_{t-i}$. Let $\pi_{t+h|t}^h$ be the forecast of π_{t+h}^h using information up to date t.

7.1.3 AR(r)

• Forecasts made using a univariate autoregression with r lags. r is estimated using the Akaike Information Criterion (AIC).

• Multistep forecasts are computed by the direct method: projecting h-period ahead inflation on r lags

• The h-step ahead AR(r) forecast was computed using the model

$$\pi_{t+h}^{h} - \pi_t = \mu^h + \alpha^h(L)\Delta\pi_t + v_t^h \tag{9}$$

where

1. μ^h is a constant
2. $\alpha^h(L)$ is a polynomial in the lag operator
3. v_t^h is the h-step ahead error term

• The number of lags is chosen according to the Akaike Information Criterion (AIC) meaning that r is such that

$$AIC(p) = \log(T^{-1}\sum_{t=1}^{T}\hat{\varepsilon}_t^2) + 2r/T$$

is minimum. An alternative criterion is the Bayesian Information Criterion (BIC)

$$BIC(p) = \log(T^{-1} \sum_{t=1}^{T} \hat{\varepsilon}_t^2) + r \log(T)$$

7.1.4 AO. Atkeson-Ohanian (2001)

AO forecasted the average four-quarter rate of inflation as the average rate of inflation over the previous four quarters. The AO forecast is

$$\pi^h_{t+h|t} = \pi^4_t = \frac{1}{4}(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3})$$

7.1.5 Backwards-looking Phillips curve (PC)

The PC forecasts are computed by adding a predictor to (9) to form the autoregressive distributed lag (ADL) specification,

$$\pi^h_{t+h} - \pi_t = \mu^h + \alpha^h(L)\Delta\pi_t + \beta^h xgap_t + \delta^h(L)\Delta x_t + v^h_t \tag{10}$$

where

1. μ^h is a constant
2. $\alpha^h(L)$, $\delta^h(L)$, is a polynomial in the lag operator (lag length chosen using AIC)
3. $xgap_t$ is the gap variable (deviations from a low pass filter) based on the variable x_t
4. v^h_t is the h-step ahead error term

• The PC forecast using $u_t = xgap_t = x_t$ and $\Delta u_t = \Delta x_t$ is called $PC - u$.

• The forecasts $PC - \Delta u$, $PC - \Delta y$, $PC - \Delta CapUtil$, $PC - \Delta Permits$, $PC - CFNAI$ omit the gap variable and only include stationary predictors Δu, Δy, $\Delta CapUtil$, $\Delta Permits$, $CFNAI$.

7.2 Out-of-sample methodology

• The forecasts were computed using the pseudo out-of-sample forecast methodology: that is, for a forecast made at date t, all estimation, lag length selection, etc. was performed using only data available through date t.

• The forecasts are recursive, so that forecasts at date t are based on all the data (beginning in 1960:I) through date t.

• The period 1960-1970 was used for initial parameter estimation. The forecast period 1970:I 2004:IV was split into the two periods 1970:I 1983:IV and 1984:I 2004:IV.

7.3 Results

Table 1
Pseudo Out-of-Sample Forecasting Results for GDP Inflation

Multivariate forecasting model: $\pi^h_{t+h} - \pi_t = \mu^h + \alpha^h(\mathrm{B})\Delta\pi_t + \beta^h xgap_t + \delta^h(\mathrm{B})\Delta x_t + u^h_t$

	1970:I – 1983:IV				1984:I – 2004:IV				$\frac{RMSFE^{h=4}_{84-04}}{RMSFE^{h=4}_{70-83}}$
	h=1	h=2	h=4	h=8	h=1	h=2	h=4	h=8	
AR(AIC) RMSFE	1.72	1.75	1.89	2.38	0.78	0.68	0.62	0.73	
Relative MSFEs									
AR(AIC)	1.00	1.00	1.00	1.00	1.00	**1.00**	1.00	1.00	0.33
AO	1.95	1.57	1.06	1.00	1.22	1.10	**0.89**	**0.84**	0.30
PC-u	**0.85**	0.92	0.88	0.61	**0.95**	1.11	1.48	1.78	0.42
PC-Δu	0.87	**0.87**	0.86	0.64	1.06	1.27	1.83	2.21	0.48
PC-$ugap^{1\text{-}sided}$	0.88	0.99	0.98	0.87	1.06	1.29	1.84	2.39	0.45
PC-Δy	0.99	1.06	0.93	0.58	1.05	1.06	1.23	1.53	0.37
PC-$ygap^{1\text{-}sided}$	0.94	0.97	0.99	0.78	0.97	0.97	1.25	1.55	0.37
PC-CapUtil	0.85	0.88	**0.79**	**0.55**	0.95	1.01	1.35	1.52	0.43
PC-ΔCapUtil	1.02	1.00	0.87	0.64	1.03	1.10	1.30	1.51	0.40
PC-Permits	0.93	1.02	0.98	0.78	1.08	1.23	1.31	1.52	0.38
PC-ΔPermits	1.02	1.04	0.99	0.86	1.00	1.00	1.00	1.02	0.33
PC-CFNAI	.	.	.	.	1.11	1.27	1.86	2.25	.

• The RMSFE of forecasts of GDP inflation has declined and the magnitude of this reduction is striking. In this sense inflation has become easier to forecast

• The relative performance of the Phillips curve forecasts deteriorated substantially from the first period to the second. This deterioration of Phillips curve forecasts is found for all the activity predictors.

• The AO forecast substantially improves upon the AR(AIC) and Phillips curve forecasts at the four- and eight-quarter horizons in the 1984-2004 period, but not at shorter horizons and not in the first period.

⇒ Substantial changes in the univariate inflation process and in the bivariate process of inflation and its activity-based predictors.

"Unpredictability and Macreconomic Stability"
By D'Agostino, A., D. Giannone and P. Surico

8 Unpredictability and Macreconomic Stability

• D'Agostino Giannone and Surico extend the result for inflation to other economic activity variables: the ability to predict several measures of real activity declined remarkably, relative to naive forecasts, since the mid-1980s.

• The fall in the predictive ability is a common feature of many forecasting models including those used by public and private institutions.

• The forecasts for output (and also inflation) of the Federal Reserves Greenbook and the Survey of Professional Forecasters (SPF) are significantly more accurate than a random walk only before 1985. After 1985, in contrast, the hypothesis of equal predictive ability between naive random walk forecasts and the predictions of those institutions is not rejected for all horizons but the current quarter.

• The decline in predictive accuracy is far more pronounced for institutional forecasters and methods based on large information sets than for univariate specifications.

• The fact that larger models are associated with larger historical changes suggests that the main sources of the decline in predictability are the dynamic correlations between variables rather than the autocorrelations of output and inflation.

8.1 Data

• Forecasts for nine monthly key macroeconomic series: three price indices, four measures of real activity and two interest rates:

1. The three nominal variables are Producer Price Index (PPI), Consumer Price Index (CPI) and Personal Consumption Expenditure implicit Deflator (PCED).
2. The four forecasted measures of real activity are Personal Income (PI), Industrial Production (IP) index, Unemployment Rate (UR), and EMPloyees on non-farm Payrolls (EMP).
3. the interest rates are 3 month Treasury Bills (TBILL) and 10 year Treasury Bonds (TBOND).

• The data set consists of monthly observations from 1959:1 through 2003:12 on 131 U.S.macroeconomic time series including also the nine variables of interest.

8.2 Forecasting models

The model used are the following:

1. A Naive forecast model (N or RW).
2. Univariate AR, where the forecasts are based exclusively on the own past values of the variable of interest.
3. Factor augmented AR forecast (FAAR), in which the univariate models are augmented with common factors extracted from the whole panel of series.
4. Pooling of bivariate forecasts (POOL): for each variable the forecast is defined as the average of 130 forecasts obtained by augmenting the AR model with each of the remaining 130 variables in the data set.

8.3 Out-of-sample methodology

• Pseudo out-of-sample forecasts are calculated for each variable and method over the horizons h = 1, 3, 6, and 12 months.

• The pseudo out-of-sample forecasting period begins in January 1970 and ends in December 2003. Forecasts constructed at date T are based on models that are estimated using observations dated T and earlier.

• Forecast based on rolling samples using, at each point in time, observations over the most recent 10 years.

8.4 Results: full sample

Table 1: *Relative Mean Square Forecast Errors - Full Period*

Random Walk (absolute values)

hor(m)	PPI	CPI	PCED	PI	IP	UR	EMP	TBILL	TBOND
1	0.45	0.11	0.06	45.58	75.84	0.03	9.45	0.31	0.11
3	1.83	0.59	0.32	13.93	46.23	0.14	7.25	1.29	0.47
6	4.40	1.63	0.94	7.72	35.04	0.45	6.66	2.50	0.99
12	11.87	5.02	2.90	5.03	25.30	1.38	5.75	4.74	2.20

Method AR (relative to RW)

hor(m)	PPI	CPI	PCED	PI	IP	UR	EMP	TBILL	TBOND
1	0.96	0.83***	0.83***	1.22	0.86*	0.91	0.60***	0.98	0.92
3	1.03	0.88*	0.82**	1.09	0.86	0.81*	0.53***	1.10	1.10
6	1.00	0.84	0.82	1.08	0.94	0.88	0.61***	1.05	1.05
12	1.05	0.93	1.00	1.01	0.95	0.97	0.75***	1.20	1.03

Method FAAR (relative to RW)

hor(m)	PPI	CPI	PCED	PI	IP	UR	EMP	TBILL	TBOND
1	0.94	0.76***	0.78***	1.15	0.74***	0.72***	0.50***	0.93	0.95
3	0.91	0.71***	0.77**	0.93	0.64**	0.58***	0.39***	1.06	1.19
6	0.84	0.60***	0.75	0.90	0.63*	0.55***	0.43***	0.95	1.17
12	0.84	0.60*	0.83	0.94	0.63	0.64*	0.56***	1.05	1.26

Method POOL (relative to RW)

hor(m)	PPI	CPI	PCED	PI	IP	UR	EMP	TBILL	TBOND
1	0.94	0.80***	0.80***	1.18	0.80**	0.83***	0.56***	0.94	0.91
3	0.96	0.81***	0.78**	1.02	0.76**	0.73**	0.47***	1.08	1.12
6	0.92	0.72**	0.76*	1.00	0.80*	0.76*	0.54***	0.99	1.07
12	0.92	0.73*	0.85	0.93**	0.78**	0.84***	0.65	1.12	1.07

Notes: Asterisks denote model forecasts that are statistically more accurate than the Naive at 1% ($***$), 5% ($**$) and 10% ($*$) significance levels.

• For all prices and most real activity indicators, the forecasts based on large information are significantly more accurate than the Naive forecasts.

• The factor augmented model produces the most accurate predictions.

• Univariate autoregressive forecasts significantly improve on the naive models for EMP at all horizons and for CPI and PCED at one and three month horizons only. As far as interest rates are concerned, no forecasting model performs significantly better than the naive benchmark.

9 Results: sub samples - inflation

Table 2: *Relative MSFEs across Sub-Periods - Inflation*

PERIOD I: sub-sample 1971:1 - 1984:12					PERIOD II: sub-sample 1985:1 - 2002:12					CHANGE
Series: Producer Price Index										
hor	RW	AR	FAAR	POOL	hor	RW	AR	FAAR	POOL	Average
1	0.55	1.03	1.01	0.99	1	0.37	0.89*	0.87*	0.88***	7%
3	2.23	1.05	0.85	0.94	3	1.51	1.01	0.98	0.99**	20%
6	5.79	0.95	0.67	0.82**	6	3.31	1.08	1.08	1.07	34%
12	17.95	1.02	0.65	0.84	12	7.12	1.13	1.20	1.09	33%
Series: Consumer Price Index										
hor	RW	AR	FAAR	POOL	hor	RW	AR	FAAR	POOL	Average
1	0.17	0.83***	0.75***	0.78***	1	0.07	0.85*	0.77**	0.83***	5%
3	0.94	0.84*	0.61***	0.74***	3	0.31	0.99	0.93	0.96**	38%
6	2.85	0.78*	0.46***	0.65***	6	0.68	1.04	1.05	0.98*	83%
12	9.43	0.87	0.44***	0.64**	12	1.57	1.22	1.32	1.16	118%
Series: Personal Consumption Expenditure Deflator										
hor	RW	AR	FAAR	POOL	hor	RW	AR	FAAR	POOL	Average
1	0.08	0.73***	0.71***	0.71***	1	0.05	0.96	0.88**	0.93***	9%
3	0.50	0.72***	0.67**	0.68***	3	0.18	1.04	0.98	1.01	29%
6	1.63	0.72**	0.66*	0.66**	6	0.40	1.13	1.05	1.08	48%
12	5.52	0.92	0.75	0.77	12	0.85	1.37	1.27	1.27	59%

Notes: The column 'change' reads the percentage historical decline in predictability averaged across methods (excluding Naive). Asterisks denote model forecasts that are statistically more accurate than the Naive at 1% ($***$), 5% ($**$) and 10% ($*$) significance levels.

- For all lags except the first, result of AO confirmed, deterioration of the forecasting performance of inflation.

9.1 Results: sub samples - real activity

Table 3: *Relative MSFEs across Sub-Periods - Real Activity*

PERIOD I: sub-sample 1971:1 - 1984:12					PERIOD II: sub-sample 1985:1 - 2002:12						CHANGE
Series: Real Personal Income											
hor	RW	AR	FAAR	POOL	hor	RW	AR	FAAR	POOL		Average
1	38.54	1.02	0.95	0.98	1	51.09	1.33	1.27	1.30		21%
3	17.15	1.01	0.86	0.94	3	11.41	1.19	1.01	1.12		14%
6	10.41	1.05	0.83	0.96	6	5.62	1.12	1.01	1.05		2%
12	6.92	0.97	0.84	0.87*	12	3.55	1.07	1.09	1.02		3%
Series: Industrial Production											
hor	RW	AR	FAAR	POOL	hor	RW	AR	FAAR	POOL		Average
1	124.01	0.81*	0.65***	0.75**	1	38.14	0.97	0.95	0.92		14%
3	81.48	0.85	0.55**	0.73**	3	18.64	0.92	0.98	0.88		16%
6	61.42	0.94	0.49*	0.76*	6	14.41	0.97	1.11	0.95		34%
12	43.24	0.95	0.43**	0.72**	12	11.27	0.98	1.22	0.97		62%
Series: Unemployment Rate											
hor	RW	AR	FAAR	POOL	hor	RW	AR	FAAR	POOL		Average
1	0.05	0.86	0.63***	0.78**	1	0.02	0.99	0.88*	0.94***		21%
3	0.25	0.79	0.52***	0.69**	3	0.06	0.91	0.79*	0.84**		18%
6	0.80	0.88	0.49***	0.75	6	0.17	0.85	0.75	0.80*		22%
12	2.42	0.99	0.56**	0.82**	12	0.56	0.93	0.90	0.89		41%
Series: Employees on Nonfarm Payrolls											
hor	RW	AR	FAAR	POOL	hor	RW	AR	FAAR	POOL		Average
1	16.37	0.65***	0.51***	0.60***	1	4.04	0.42***	0.45***	0.40***		4%
3	12.39	0.60**	0.41***	0.53***	3	3.23	0.31***	0.34***	0.29***		-1%
6	11.16	0.70**	0.42***	0.60**	6	3.14	0.37**	0.44*	0.36*		-3%
12	9.21	0.82***	0.49***	0.69***	12	3.05	0.58**	0.72	0.56		8%

Notes: see Table 2.

- Little change in the structure of univariate models for real activity.

- The relative MSFEs of FAAR and POOL suggest that important changes have occurred in the relationship between output and other macroeconomic variables.

- The decline in predictability does not seem to extend to the labor market, especially at short horizons. The forecasts of the employees on nonfarm payrolls are associated with the smallest percentage changes across subsamples.

9.2 Results: sub samples - interest rates

Table 4: *Relative MSFEs across Sub-Periods - Interest Rates*

PERIOD I: sub-sample 1971:1 - 1984:12					PERIOD II: sub-sample 1985:1 - 2002:12					CHANGE
Series: 3 Months Treasury Bills										
hor	RW	AR	FAAR	POOL	hor	RW	AR	FAAR	POOL	Average
1	0.64	1.00	0.94	0.95	1	0.05	0.84	0.87	0.81***	-10%
3	2.59	1.12	1.05	1.10	3	0.27	0.98	1.16	0.94**	-5%
6	4.63	1.06	0.88	0.98	6	0.83	1.03	1.25	1.01	11%
12	7.63	1.27	0.93	1.14	12	2.47	1.04	1.34	1.06	8%
Series: 10 Years Treasury Bonds										
hor	RW	AR	FAAR	POOL	hor	RW	AR	FAAR	POOL	Average
1	0.17	0.95	0.96	0.94	1	0.07	0.88**	0.92	0.87***	-9%
3	0.68	1.17	1.21	1.18	3	0.31	1.00	1.15	1.02	-11%
6	1.28	1.07	1.12	1.09	6	0.77	1.02	1.23	1.05	3%
12	2.57	1.04	1.12	1.06	12	1.91	1.01	1.42	1.09	7%

Notes: see Table 2.

• In the second sample the AR, FAAR and P OOL methods produce more accurate forecasts than the RW at one month horizon.

• Possible interpretation: increased predictability of the FED due to a better communication strategy.

9.3 Results: private and institutional forecasters

• The predictions for output and its deflator from two large forecasters representing the private sector and the policy institutions are considered.

• The survey was introduced by the American Statistical Association and the National Bureau of Economic Research and is currently maintained by the Philadelphia Fed. The SPF refers to quarterly measures and is conducted in the middle of the second month of each quarter (here the median of the individual forecasts is considered)

• The forecasts of the Greenbook are prepared by the Board of Governors at the Federal Reserve for the meetings of the Federal Open Market Committee (FOCM), which takes place roughly every six weeks.

• Four forecast horizons ranging from 1 to 4 quarters.

• The measure of output is Gross National Product (GNP) until 1991 and Gross Domestic Product (GDP) from 1992 onwards.

• The evaluation sample begins in 1975 (prior to this date the Greenbook forecasts were not always available up to the fourth quarter horizon).

9.4 Results: private and institutional forecasters - inflation

Table 5: *Relative MSFEs of Institutional Forecasters - Inflation*

FULL SAMPLE: 1975:1 1999:4

hor(q)	RW	Fed's Green Book(GB)/RW	Survey of Professional Forecasters(SPF)/RW
1	0.26	0.35***	0.37***
2	0.79	0.30**	0.36**
3	1.57	0.29*	0.37
4	2.51	0.32	0.46

PERIOD I: sub-sample 1975:1 - 1984:4

hor(q)	RW	Fed's Green Book(GB)/RW	Survey of Professional Forecasters(SPF)/RW
1	0.54	0.30***	0.27***
2	1.72	0.21**	0.24**
3	3.51	0.21**	0.25*
4	5.69	0.23*	0.32*

PERIOD II: sub-sample 1985:1 - 1999:4

hor(q)	RW	Fed's Green Book(GB)/RW	Survey of Professional Forecasters(SPF)/RW
1	0.08	0.58**	0.82
2	0.17	0.93	1.15
3	0.28	0.97	1.39
4	0.39	1.18	1.82

Notes: Asterisks denote rejection of the null hypothesis of equal predictive accuracy between each model and the RW at 1% ($***$), 5% ($**$) and 10% ($*$) significance levels.

9.5 Results: private and institutional forecasters - real activity

Table 6: *Relative MSFEs of Institutional Forecasters - Output*

FULL SAMPLE: 1975:1 1999:4

hor(q)	RW	Fed's Green Book(GB)/RW	Survey of Professional Forecasters(SPF)/RW
1	12.59	0.44**	0.51**
2	9.11	0.49**	0.46**
3	7.45	0.48**	0.50***
4	6.49	0.51**	0.51***

PERIOD I: sub-sample 1975:1 - 1984:4

hor(q)	RW	Fed's Green Book(GB)/RW	Survey of Professional Forecasters(SPF)/RW
1	25.82	0.37**	0.45**
2	19.01	0.44**	0.41**
3	15.39	0.40***	0.45***
4	13.18	0.42***	0.46***

PERIOD II: sub-sample 1985:1 - 1999:4

hor(q)	RW	Fed's Green Book(GB)/RW	Survey of Professional Forecasters(SPF)/RW
1	3.77	0.73	0.77
2	2.51	0.77	0.70
3	2.15	0.85	0.73
4	2.03	0.89	0.74

Notes: see Table 5.

”The Return of the Wage Phillips Curve”

10 The Return of the Wage Phillips Curve

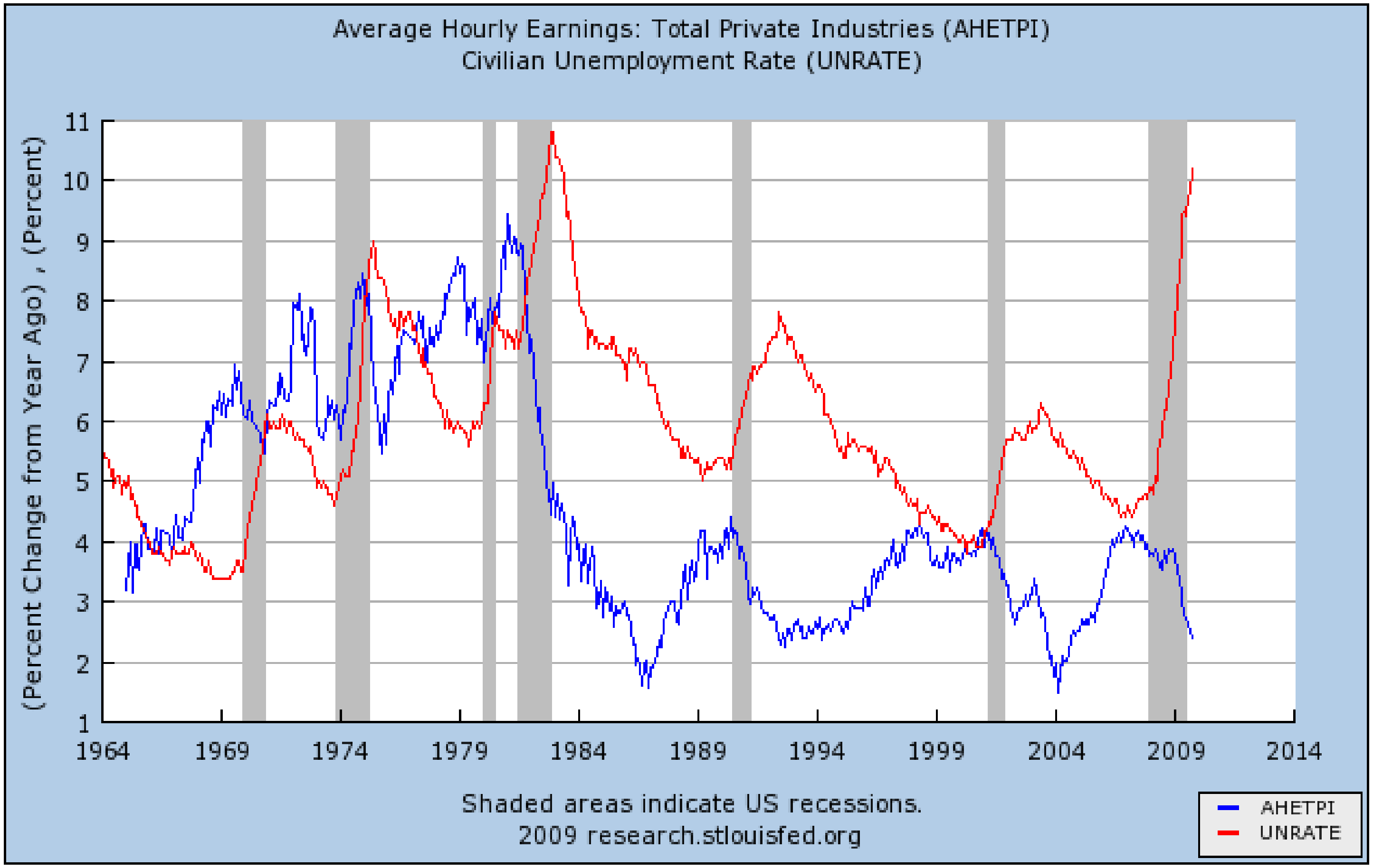
- Previous evidence has been taken as a motivation to dismiss the Phillips curve as a theoretical concept.

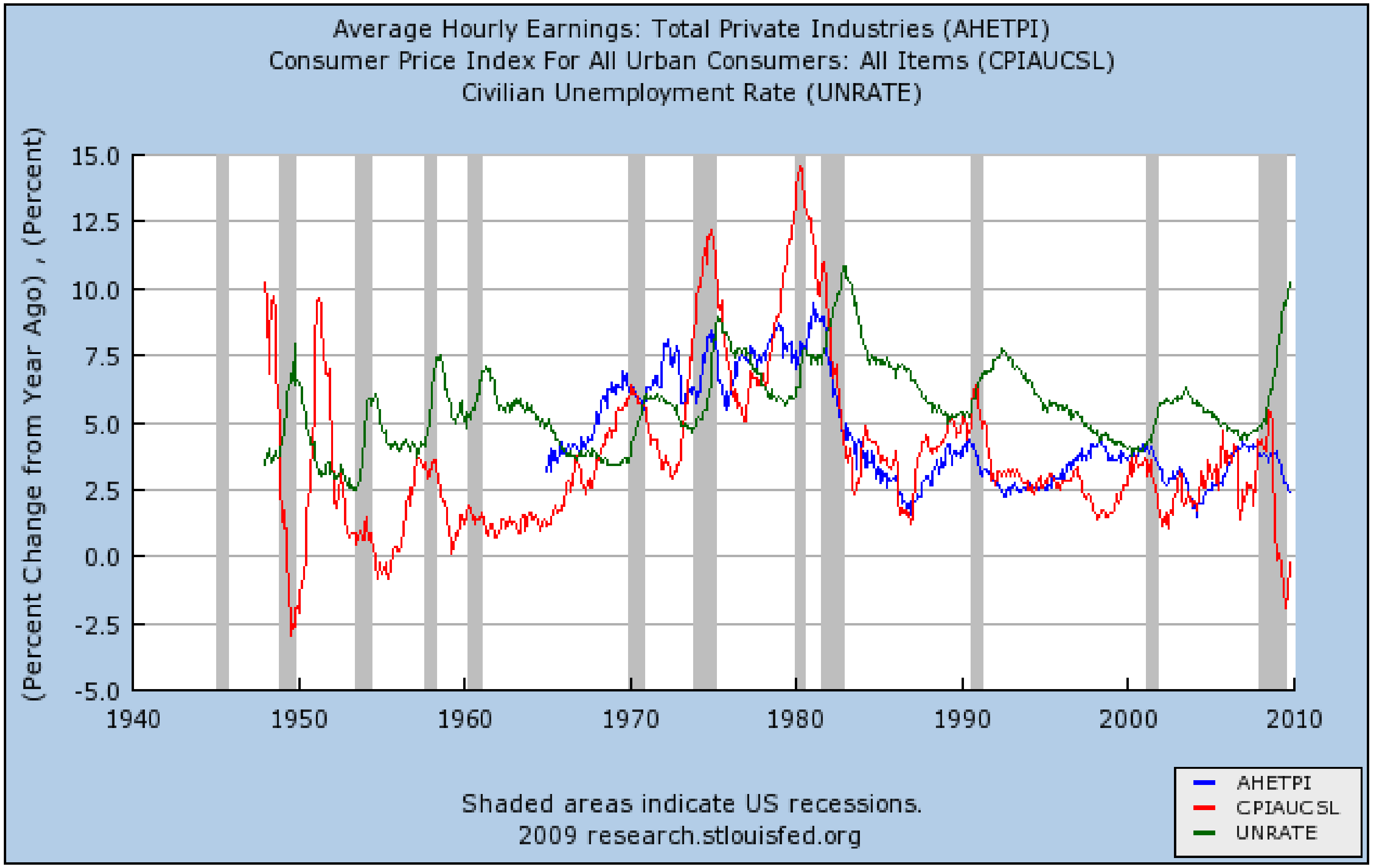
- Danger with that interpretation.

- In 1958 Phillips uncovered an inverse relation between wage rate inflation and unemployment.

- The focus however in recent years has been shifted to price inflation

10.1 Back to the origins




Average Hourly Earnings: Total Private Industries (AHETPI)
Consumer Price Index For All Urban Consumers: All Items (CPIAUCSL)
Civilian Unemployment Rate (UNRATE)
(Percent Change from Year Ago) , (Percent)
15.0
12.5
10.0
7.5
5.0
2.5
0.0
-2.5
-5.0
1940
1950
1960
1970
1980
1990
2000
2010
Shaded areas indicate US recessions.
2009 research.stlouisfed.org
AHETPI
CPIAUCSL
UNRATE

10.2 Results

forecast horizon	RMSE VAR/RW	RMSE AR/RW	% gain using var
1	0.2252	0.2048	9.976
4	0.3642	0.3976	-8.4208
8	0.4892	0.6110	-19.9406
12	0.544	0.6646	-18.1371
16	0.5356	0.6157	-13.0069
18	0.5259	0.5914	-11.0704

- Phillips curve still (now more than then) characterize dynamics of wage growth and unemployment.

- Crucial question: what has changed in the relation between prices and wages?