Sufficient Information in Structural VARs*

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Abstract. Necessary and sufficient conditions under which a VAR contains *sufficient information* to estimate the structural shocks are derived. On the basis of this theoretical result we propose two simple tests to detect informational deficiency and a procedure to amend a deficient VAR. A simulation based on a DSGE model with fiscal foresight suggests that our method correctly identifies and fixes the informational problem. In an empirical application, we show that a bivariate VAR including unemployment and labor productivity is informationally deficient. Once the relevant information is included into the model, technology shocks appear to be contractionary.

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1 Introduction

Since Sims (1980)'s seminal paper, Structural Vector Autoregression (SVAR) models have become extremely popular for structural and policy analysis. The idea behind these models is that structural economic shocks can be found as linear combinations of the residuals of the linear projection of a vector of variables onto their past values, i.e. are innovations with respect to the econometrician's information set. Therefore, an obvious requirement for the analysis to be meaningful is that the variables used in the VAR convey all of the relevant information. Such *informational sufficiency* is implicitly assumed in any SVAR application.

But is this assumption always sensible? Unfortunately the answer is no. The basic problem is that, while agents typically have access to rich information, VAR techniques allow a limited number of variables to be handled. If the econometrician's information set does not span that of the agents, the structural shocks are non-fundamental and cannot be obtained from a VAR (Hansen and Sargent, 1991, Lippi and Reichlin, 1993, 1994, Chari, Kehoe and Mcgrattan, 2008).

Fernandez-Villaverde, Rubio-Ramirez, Sargent and Watson (2007), and Ravenna (2007), show theoretical cases in which VAR techniques fail. Fiscal foresight and news shocks are prominent examples (Yang, 2008, Forni, Gambetti and Sala, 2011, Leeper, Walker and Yang, 2013).[1] No identification scheme (zero restrictions, sign restrictions, etc.) can provide the correct structural shocks and impulse response functions if the VAR is informationally deficient. We are referring to standard identification schemes, involving contemporaneous combinations of the VAR residuals. Dynamic rotations as in Lippi and Reichlin (1994), Mertens and Ravn (2010), Forni, Gambetti, Lippi and Sala (2013a, 2013b) can in principle solve the problem.

To date, there is no generally accepted and systematic procedure to verify whether a specific VAR suffers from this informational problem. In this paper we provide a testing procedure which is relatively easy to implement and valid under fairly general conditions. Moreover, we propose a strategy to amend the VAR when informational sufficiency is rejected.

Our main theoretical result is a necessary and sufficient condition for informational sufficiency, which is derived under the assumption that the economy admits a state space representation. The condition is that there are no state variables that Granger cause the variables included in the VAR. The intuition is that the state variables convey all of the relevant information; therefore, if they do not help to predict a vector, such a vector must contain the same information. Based on this result, we propose the following testing procedure. First, estimate the state variables of the economy by using the

[1] See also Forni and Gambetti (2010).

principal components of a large dataset, containing all available macroeconomic information. Second, test whether the estimated principal components Granger cause the variables included in the VAR. The variables are informationally sufficient if and only if the null hypothesis of no Granger causality is not rejected.

If a set of variables is not sufficient, we propose to estimate either a factor model, or a Factor Augmented VAR model (FAVAR), where the original set of variables is enlarged with the principal components above. Our test can be applied recursively to the FAVAR in order to determine how many factors to retain. The number of factors is the minimum number such that the extended vector is informationally sufficient.

In addition, we show that, even if the VAR does not contain enough information to get all of the structural shocks, it can in principle be informationally sufficient for a single shock (or a subset of shocks). In order for an estimated shock to be a structural shock, a necessary condition is orthogonality to the past of the state variables. Under suitable assumptions, orthogonality is also a sufficient condition, provided that the identification scheme is correct. Hence, once a shock has been identified and estimated, we can test for its "structuralness" by testing for orthogonality with respect to the lags of the principal components. If orthogonality is rejected, the VAR can be amended by adding principal components until orthogonality is reached.

We show our procedures at work with both artificial and real data. In the Monte Carlo exercise, we generate series for capital and the tax rate following the fiscal foresight DSGE model of Leeper, Walker and Yang, 2013. We consider two special cases of the model. In the former one, the tax shock is not anticipated at all, so that fiscal foresight is not there and the VAR including capital and the tax rate is informationally sufficient. In the latter case, there is a two period foresight, i.e. agents can see at time t the shock which will affect the tax rate at period $t+2$. The tax rate series, being not affected by the current tax shock, does not deliver information about it, and the VAR is informationally deficient.

To perform our testing procedures, we generate a panel of auxiliary series, which can be interpreted as survey variables providing some information on the current structural shocks. Then we compute the principal components and perform the Granger causality as well as the orthogonality test on both models, with and without fiscal foresight. Results show that both tests correctly fail to reject informational sufficiency for the model with no fiscal foresight, with no harmful size distortions. Moreover, both tests correctly identify informational deficiency in the fiscal foresight case in almost all of the 1000 replications. Finally, both tests perform well in identifying the correct number of principal components to amend the VAR model. Impulse response function estimates from the misspecified VAR are dramatically misleading, whereas the amended VAR provides very good results.

In the empirical application, we focus on technology shocks in the US. We test whether a small-scale VAR model, such as those typically used to study the effects of technology shocks, is informationally sufficient. Following Barnichon (2010), we study a bivariate VAR specification with labor productivity and the unemployment rate. We find that informational sufficiency is strongly rejected by both the Granger causality and the orthogonality test. Then we apply our recursive procedure to determine the number of factors needed to amend the VAR. Finally, we identify the technology shock as the only one driving productivity in the long run, in both the original and the augmented VAR. Differences in the results of the two models are dramatic. While in the original specification technology shocks significantly reduce the unemployment rate, in the augmented VAR the result is reversed. Consistently with the test outcome, adding further factors does not change results any more.

Finally, we apply the "structuralness" test to the technology shock estimated with richer VAR specifications, including forward-looking variables: a four-variable VAR including a consumer confidence indicator and a five-variable VAR including stock prices. Both specifications pass the test and lead to impulse response functions similar to the ones found with the FAVAR model.

Our work is related to several papers. Fernandez-Villaverde *et al.* (2007) derive a necessary and sufficient condition for fundamentalness. Such condition, however, requires knowledge of the underlying DSGE model. However, if one is confident in the economic model, there is little reason to estimate a VAR. The appeal of VARs is precisely that they do not require one to take a stand on the nature of the theoretical model.

Forni and Reichlin (1996) and Giannone and Reichlin (2006) derive a necessary condition essentially equivalent to Proposition 2 below; Giannone and Reichlin (2006) proposes a Granger causality test based on it. The problem of this test is that, being based on a necessary condition, it is not conclusive if the null is not rejected. Moreover, its general applicability is limited by the fact that there is no indication about which variables to use. The crucial novelties with respect to the above works are the sufficiency results in Propositions 3 and 4 and the related identification of a set of regressors for the Granger causality test and the orthogonality test.

Forni *et al.* (2009) propose an informal way to check for fundamentalness by looking at the roots of the determinant of the matrix of impulse-response functions obtained by estimating a factor model. The shortcoming of this method is that it checks for sufficiency of the common components of the variables, rather than the variables themselves; hence results are reliable only if the idiosyncratic component is small.

As for the FAVAR literature originated by Bernanke *et al.* (2005) our contribution is twofold. On the one hand, we provide a clear theoretical motivation for the use of FAVAR models. On the other hand, we provide a practical method to verify whether a FAVAR is really necessary or not and to determine the number of factors.

The remainder of the paper is organized as follows. Section 2 presents theoretical results, as well as our proposed testing procedures. Section 3 presents our Monte Carlo exercise. Section 4 is devoted to the empirical application. Section 5 concludes. The online appendix reports the proofs, a few additional Monte Carlo results and detailed information about the data used in the empirical application.

2 Theory

The building block of our theoretical framework is a representation of the macroeconomy where q mutually orthogonal structural shocks affect macroeconomic variables through square-summable impulse-response functions.

Assumption 1 (MA representation). *The n-dimensional vector x_t of stationary macroeconomic time series satisfies*

$$x_t = F(L)u_t, \tag{1}$$

where u_t is a q-dimensional white noise vector of structural macroeconomic shocks and $F(L) = \sum_{k=0}^{\infty} F_k L^k$ is an $n \times q$ matrix of square-summable linear filters in the non-negative powers of the lag operator L.

Representation (1) can be thought of as representing the equilibrium of a DSGE model. Consider for instance the state-space representation studied in Fernandez-Villaverde *et al.* (2007) i.e.

$$s_t = As_{t-1} + Bu_t \tag{2}$$

$$x_t = Cs_{t-1} + Du_t \tag{3}$$

where s_t is an m-dimensional vector of stationary "state" variables, $q \leq m \leq n$, A, B, C and D are conformable matrices of parameters, B has a left inverse B^{-1} such that $B^{-1}B = I_q$. It is seen from (2) and (3) that x_t admits representation (1) with

$$F(L) = \left(DB^{-1} + (C - DB^{-1}A)L\right)(I - AL)^{-1}B. \tag{4}$$

2.1 Sufficient information and fundamentalness

A key step of our analysis is the definition of the information sets of the econometrician and the VAR, and the concept of sufficient information. To begin, we assume that the econometrician observes x_t, possibly with error. Allowing for a measurement error (which can be zero), besides being an interesting generalization *per se*, will enable us to establish a link between the VAR model and the factor model introduced below, and extend our results to FAVAR models. Precisely:

Assumption 2. (Econometrician's information set) *The econometrician information set $\mathcal{X}_t^*$ is given by the closed linear space spanned by present and past values of the variables in x_t^*, i.e. $\mathcal{X}_t^* = \overline{\text{span}}(x_{1t-k}^*, \ldots, x_{nt-k}^*, k = 1, \ldots, \infty)$, where*

$$x_t^* = x_t + \xi_t = F(L)u_t + \xi_t, \tag{5}$$

ξ_t being a vector white noise of measurement errors, mutually orthogonal and orthogonal to u_{jt-k}, $j = 1, \ldots, q$, any k.

In practice the number of observable variables n is very large, so that the econometrician needs to reduce it in order to estimate a VAR. The VAR information set is then spanned by an s-dimensional sub-vector of x_t^*, or, more generally, an s-dimensional linear combination of x_t^*, say $z_t^* = Wx_t^*$ (with s not necessarily equal to q). Considering also linear combinations will enable us to apply our results to the principal components of the variables and therefore to the FAVAR model. The vector z_t^* is not necessarily affected by all of the structural shocks hitting the economy, i.e. z_t^* may be driven by a sub-vector u_t^z of u_t. We consider explicitly this case for reasons that will be clear below.

Assumption 3 (VAR information set). *The information set of the VAR is $\mathcal{Z}_t^* = \overline{\text{span}}(z_{1t-k}^*, \ldots, z_{st-k}^*, k \geq 0)$, where $z_t^* = Wx_t^*$, W being $s \times n$; z_t^* is driven by a sub-vector u_t^z of u_t, of dimension $q_z \leq q$:*

$$z_t^* = WF(L)u_{t-k} + W\xi_t = B(L)u_{t-k}^z + W\xi_t, \tag{6}$$

where $B(L) = \sum_{k=0}^{\infty} B_k L^k$ has rank q_z.

Now, consider the theoretical projection equation of z_t^* on its past history, i.e.

$$z_t^* = P(z_t^* | \mathcal{Z}_{t-1}^*) + \epsilon_t. \tag{7}$$

The SVAR methodology consists in estimating a VAR to get ϵ_t, the VAR innovation and then attempting to get the structural shocks driving z_t^* as linear combinations of the estimated entries of ϵ_t. Hence a key property of z_t^* and the related information set is that the entries of ϵ_t span the structural shocks, i.e. the information in the history of z_t^* is sufficient to estimate the shocks. We call such property "sufficient information".

Definition 1 (Sufficient information). *Let v_t be any sub-vector of u_t^z. We say that z_t^* and the related VAR is "informationally sufficient for v_t" if and only if there exist a matrix M such that $v_t = M\epsilon_t$. We say that z_t^* is "globally sufficient" if it is informationally sufficient for u_t^z.*

Observe that, for global sufficiency to hold, we do not require that z_t^* is sufficient for all of the structural shocks, but only the shocks driving it. This gives a fair chance to a small VAR with $s < q$ to be globally sufficient. Observe also that sufficiency, defined in this way, is related only to the variables in z_t^* and has nothing to do with the choice of a proper identification scheme. The correct identification of M is a further problem, which in general does make sense only if sufficiency holds true.

Informational sufficiency is closely related to "fundamentalness". Let us clarify the relation between the two concepts.

Definition 2 (Fundamentalness). *Let $w_t = Hx_t$ be driven by u_t^w, a q_w-dimensional sub-vector of u_t, $q_w \leq q$. We say that u_t^w is fundamental for w_t, and the MA representation $w_t = HF(L)u_t = A(L)u_t^w$ is fundamental, if and only if $u_{jt}^w \in \mathcal{W}_t = \overline{\text{span}}(w_{1t-k}, \dots, w_{mt-k},$
$k \geq 0)$, for $j = 1, \dots, q_w$.*

The following proposition formally establishes the relation between fundamentalness and sufficiency.

Proposition 1. *Under Assumptions 1, 2 and 3, the information in z_t^* is sufficient for u_t^z if and only if there is a matrix R such that (a) $\tilde{z}_t = Rz_t^* = Rz_t$ and (b) u_t^z is fundamental for $\tilde{z}_t$.*

For the proof see Appendix A. Proposition 1 says that, for z_t^* to be sufficient, there must be a linear transformation of z_t^* which is free of measurement errors and have a fundamental representation in the structural shocks. Therefore, informational sufficiency is almost equivalent to fundamentalness plus absence of errors. If errors are small, informational sufficiency and fundamentalness essentially coincide. If, on the contrary, a VAR includes variables with large errors, information may be insufficient even if fundamentalness of z_t is met.

To conclude this subsection, let us observe that, in the particular case of $F(L)$ being a matrix of rational functions, fundamentalness of u_t^w for w_t, along with fundamentalness of the associated MA representation $w_t = A(L)u_t^w$ is equivalent to the following condition (see e.g. Rozanov, 1967, Ch. 2).

Condition R. *The rank of $A(z)$ is q_w for all complex numbers z such that $|z| < 1$.*

When $A(L)$ is a square matrix the above condition reduces to the well known condition that the determinant of $A(z)$ has no roots smaller than one in modulus.[2] For instance, considering equation (4) and the case $w_t = x_t$, condition R is satisfied if D is invertible and the eigenvalues of $A - BD^{-1}C$ are strictly less than one in modulus, which is the Poor Man's Condition 1 of Fernandez-Villaverde *et al.*, 2007.

[2] Fundamentalness is slightly different from invertibility, since invertibility rules out also roots with modulus equal to 1. Hence invertibility implies fundamentalness, whereas the converse is not true.

2.2 Necessary and sufficient conditions for sufficient information

In this subsection we derive testable implications of sufficient information. A first relevant result is the following.

Proposition 2. *Under Assumptions 1, 2 and 3, if x_t^* Granger causes z_t^*, then z_t^* is not globally sufficient.*

For the proof see Appendix A. The intuition is that, if a set of variables is globally sufficient, then it contains all of the information which is useful to predict it, so that no other variable or set of variables can Granger cause it. Proposition 2 can be useful in practice. In particular, if the econometrician believes that a given variable in x_t^*, say y_t, conveys relevant information, he can check whether y_t Granger causes z_t^* as a vector. If y_t Granger causes z_t^*, the VAR with z_t^* is misspecified.[3] However, Proposition 2 has an important limitation in that, being only a necessary condition, it can be used to reject sufficiency but not to validate it. Clearly, testing all of the variables in x_t^* would be close to a validation, but unfortunately this is not feasible, since in practice x_t^* is of high dimension. On the one hand, we cannot use all of the variables simultaneously; on the other hand, testing each one of them separately would yield, with very high probability, to reject sufficiency even if z_t^* is informationally sufficient, owing to Type I error.

We can provide a sufficient condition by assuming the state space representation above, i.e. by replacing Assumption 1 with the more restrictive Assumption 1′:

Assumption 1′ (ABCD representation). *The vector x_t satisfies equations (2) and (3), where u_t is a q-dimensional white noise vector.*

It is easily seen from equations (2), (3) and (5) that x_t^* follows the factor model

$$x_t^* = Gf_t + \xi_t, \tag{8}$$

where $G = (DB^{-1} \quad C - DB^{-1}A)$ and $f_t = \begin{pmatrix} s_t' & s_{t-1}' \end{pmatrix}'$. Notice that the factors contain the same information as the states. Clearly, when the states (or the factors) are observable without error and are included in z_t^*, there are no informational problems (see equation 2). In this sense, informational deficiency arises from the fact that some states are unobservable or missing.

In addition to the above assumption, we need a condition ensuring that z_t^* is predictable to some extent. Precisely,

Assumption 4. *There exists a summable sequence $\{c_k\}_{k=1}^{\infty}$ such that $\sum_{k=1}^{\infty} c_k B_k$ has rank q_z.*

[3]Observe that, according to Proposition 2, identification is not required to perform the test, consistently with the fact that sufficient information, as observed above, is independent of the identification scheme.

Assumption 4 ensures that the number of variables s is at least as large as the number of shocks q_z driving them, which of course is necessary for sufficiency. In addition, it rules out a few "perverse" cases. For instance, that the entries of z_t^* are contemporaneous linear combinations of the entries of u_t^z plus a measurement error: in this case z_t^* is not Granger caused by the factors since it is unpredictable, but is not informationally sufficient because of the measurement error.

The following proposition establishes a necessary and sufficient condition for informational sufficiency.

Proposition 3. *Let K be any non-singular $r \times r$ matrix, r being the dimension of f_t, and $g_t = Kf_t$. Under Assumptions 1′, 2, 3 and 4, z_t^* is globally sufficient if and only if $g_t = Kf_t$ does not Granger cause z_t^*.*

For the proof see Appendix A. The intuition for sufficiency is that, under Assumption 1′, the factors are informationally sufficient; therefore they Granger cause every predictable vector, unless such a vector contains the same information.

Proposition 3 is useful in that, besides providing a sufficient condition, it allows us to summarize the signals in the large dimensional vector x_t into a relatively small number of factors (the entries of f_t, or, equivalently, the entries of $g_t = Kf_t$). Such factors are unobservable, but, under suitable assumptions, can be consistently estimated by the principal components $\hat{g}_t$, as both the number of variables and the number of time observations go to infinity (Stock and Watson, 2002; Forni *et al.* 2009).

2.3 The proposed testing procedure

Proposition 3 provides the theoretical basis for the following testing procedure.

1. Take a large data set x_t^*, capturing all of the relevant macroeconomic information.

2. Set a maximum number of factors P and compute the first P principal components of x_t^*.

3. Perform a Granger causality test to see whether such principal components Granger cause z_t^*. If the null of no Granger causality is not rejected, z_t^* is informationally sufficient. Otherwise, sufficiency is rejected.

Ideally, the maximum number of principal components should be picked to be at least as large as the number of factors driving the panel, i.e. $P \geq r$, since clearly $P < r$ may lead to acceptance of the null when it is false, whereas P larger than r should not be harmful. The simulation results in the following section are in line with this prescription. Of course, P should not be chosen disproportionately large, since, as P increases, the estimates of the factors deteriorates and the Granger causality test loses power. With the data set of Section 4, values of P between 4 and 10 provide fairly consistent results. In principle, existing criteria to determine the number of factors (Bai and Ng, 2002, Onatski, 2010, Ahn and Horenstein, 2013) may provide some guidance; however, they prove useless with the data set of Section 4 (see below). This can be seen as an additional motivation for the method proposed in the following subsection.[4]

2.4 Achieving information sufficiency

What should the econometrician do if sufficient information is rejected? Assumption 1′ guarantees that the factors are informationally sufficient. Hence a possible solution is to forget the VAR and estimate the factor model (8) along the lines of Forni *et al.*, 2009. But perhaps the most natural solution in the present context is to extend the vector of variables appearing in the original VAR by adding principal components; that is, to estimate a FAVAR.

To this end, a crucial problem is to establish the number of factors to retain. Notice that here the problem is not to determine r, i.e. the number of factors driving the whole macroeconomy, but the number of factors p needed to achieve informational sufficiency, when taking as a starting point the original VAR specification. The two problems are conceptually distinct. Clearly r is an upper bound for p, but sufficiency can in principle be reached with a number of factors $p < r$. This is for two reasons: first, potentially useful information is already provided by the variables in z_t^*; second, we do not want to reach sufficiency with respect to all of the shocks driving the macroeconomy (the entries of u_t), but only the shocks driving the variables of interest z_t^* (the entries of u_t^z).

Since by Assumption 3 also linear combinations of the x^*'s can be included in the vector z_t^*, our testing procedure can be applied to a FAVAR specification to see whether it is informationally sufficient or not. Hence the idea is to add the principal components one at a time in decreasing order, apply recursively the Granger causality test and stop when informational sufficiency is no longer rejected. Precisely, we propose the following procedure.

1. Take $w_t^h = (z_t^{*\prime} \;\; \hat{g}_{1t} \;\; \cdots \;\; \hat{g}_{ht})'$ and test for sufficiency of w_t^h as explained above, for $h = 1, \ldots, P-1$.

[4] We recommend using a rich dynamic specification or some information criterion to choose the number of lags, since a too short truncation lag in the VAR specification may produce misleading results both in the Ganger causality test and the orthogonality test proposed below.

2. If sufficiency is rejected for all h, retain $p = P$ principal components. Otherwise, choose p as the smallest h such that w_t^h is informationally sufficient.

It should be noticed that the above procedure might overestimate the number of principal components to retain. This is because Granger causality tests are over-sized when the null is sufficiency of w_t^h with $h > 0$. The distortion arises from the fact that the principal components are only imperfect estimates of the factors, and is not there when the null is sufficiency of z_t^*. As shown below, overestimation of the number of factors is not particularly harmful; nonetheless, parsimonious specifications are preferable. Using out-of-sample Granger-causality tests mitigates substantially the problem, since out-of-sample tests favor parsimony. The online appendix discusses this issue in more detail and shows results supporting the use of out-sample tests.

2.5 "Structuralness" of an estimated shock

If Granger causality is rejected, the VAR cannot deliver all of the structural shocks driving z_t^*. But the econometrician is often interested in identifying just a single shock and the related impulse response functions. To this end, in the present subsection we propose a less demanding test.

The following example shows that, even if global sufficiency is rejected, z_t^* can still be sufficient for a single shock, or a subset of shocks. Consider the model

$$z_{1t}^* = u_{1t} + u_{2t-1} \tag{9}$$

$$z_{2t}^* = u_{1t} - u_{2t-1}. \tag{10}$$

In this case the determinant of the MA matrix is $-2L$, which vanishes for $L = 0$, so that the MA representation is non-fundamental by Condition R. Hence z_t^* is not sufficient for u_t by Proposition 1. Indeed, it is easily seen that u_{2t} cannot be recovered from the present and past values of z_t^*, since it is contained only in the future of z_t^*. Nevertheless, z_t^* is sufficient for u_{1t}, because $u_{1t} = (z_{1t}^* + z_{2t}^*)/2$.

We do not have a method to verify *a priori* whether a given VAR specification is informationally sufficient for a particular shock. However, after having identified and estimated the relevant shock, we can verify whether it can be a structural shock by exploiting the following result.

Proposition 4. *Let $v_t = \alpha' \epsilon_t$, $\alpha \in \mathbb{R}^s$. (A) Under Assumptions 1 to 3, if v_t is a structural shock, then it is orthogonal to $x_{j,t-k}^*$, $k > 0$, for any j, and, under Assumption $1'$, 2 and 3, to f_{t-k}, $k > 0$. (B) Under Assumption $1'$, 2 and 3, if z_t^* is free of measurement error, i.e. $z_t^* = z_t$, and v_t is orthogonal to f_{t-k}, $k > 0$, then v_t is a linear combination of the structural shocks u_{jt}^z, $j = 1, \ldots, q_z$.*

For the proof see Appendix A. Statement (A) says that the structural shocks are orthogonal to the past of all variables. Statement (B) says that, under suitable conditions, if a shock recovered from z_t^* is orthogonal to the factors, then it is a linear combination of the structural shocks. Obviously, orthogonality does not guarantee that such linear combination is the desired shock; it will, only if identification is correct.

On the basis of Proposition 4 we suggest testing for orthogonality of the estimated shock with respect to the lags of the principal components.[5] If orthogonality is rejected, we can try to enlarge the information set by adding either suitable variables, or the principal components themselves. The number of principal components to retain can be determined by recursive application of the orthogonality test, as explained in the previous subsection.

3 A Monte Carlo simulation: Fiscal foresight

As discussed in Leeper, Walker and Yang, (2013, LWY henceforth), informational deficiency is likely to arise when fiscal policy shocks have delayed effects on fiscal policy variables, the phenomenon known as "fiscal foresight". The intuition is that current values of fiscal variables might not provide enough information about the current fiscal shock because the latter has only delayed effects on that variables. In the paper it is shown that standard VAR analysis aimed at identifying fiscal policy shocks can provide misleading results. In this section we show our proposed method in action with artificial data generated by the LWY model.

3.1 The fiscal foresight example

To begin, let us briefly describe the model which will be our Data Generating Process in the controlled experiment and show its implications on informational sufficiency. The model is a simple RBC model with income taxes, inelastic labor supply and full capital depreciation. The log-linearized equilibrium condition for capital is

$$k_t = \alpha k_{t-1} + u_{at} - (1-\theta)\frac{\tau}{1-\tau}\sum_{k=0}^{\infty}\theta^k E_t \hat{\tau}_{t+k+1}, \qquad (11)$$

where k_t and $\hat{\tau}_t$ are capital and the tax rate expressed in log deviations from the steady state, u_{at} is an i.i.d technology shock observed by agents, τ is the steady state value of the tax rate, and the

[5] Ramey, 2011, uses an orthogonality test to show that the government spending shock obtained with a SVAR *á la* Perotti, 2008, being predicted by the forecast of public expenditure from the survey of professional forecasters, cannot be the desired structural shock.

parameters satisfy the inequalities $0 < \theta < 1$, $0 < \alpha < 1$. Fiscal foresight is modeled by assuming that agents know at time t the tax rate they will face in $t+h$, i.e. $\hat{\tau}_t = u_{\tau t-h}$, $u_{\tau t}$ being an i.i.d. tax shock. For further details see LWY, Section 2.

We shall focus on the cases $h = 0$ (no fiscal foresight, model NFF), which is an example of sufficient information, and $h = 2$ (a two-period foresight, model FF), which is an example of deficient information.

In the former case, (11) reduces to $k_t = \alpha k_{t-1} + u_{at}$, so that capital and the tax rate follow the VAR(1) model

$$\begin{pmatrix} \hat{\tau}_t \\ k_t \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & \alpha \end{pmatrix} \begin{pmatrix} \hat{\tau}_{t-1} \\ k_{t-1} \end{pmatrix} + \begin{pmatrix} u_{\tau t} \\ u_{at} \end{pmatrix}. \qquad \text{(NFF)}$$

The above equation can also be seen as the ABCD representation of the model, the tax rate and the capital being at the same time the variables of interest and the (observable) states. Here there are no information problems: assuming as in LWY that the econometrician observes $z_t^* = (\hat{\tau}_t \;\; k_t)'$, the structural shocks and the related impulse response functions can be obtained simply by estimating the above VAR.

Now let us come to the fiscal foresight case. With $h = 2$, equation (11) gives $k_t = \alpha k_{t-1} + u_{at} - \kappa(\theta u_{\tau t} + u_{\tau t-1})$, where $\kappa = \tau(1-\theta)/(1-\tau)$. The model can be written in the ABCD form (2)-(3) with states k_t, $u_{\tau t}$ and $u_{\tau t-1}$. Since the state $u_{\tau t}$ is missing, intuition suggests that z_t^* is now informationally deficient. This is in fact the case, as is seen by looking at the structural moving average representation of z_t^*:

$$\begin{pmatrix} \hat{\tau}_t \\ k_t \end{pmatrix} = \begin{pmatrix} L^2 & 0 \\ -\dfrac{\kappa(L+\theta)}{1-\alpha L} & \dfrac{1}{1-\alpha L} \end{pmatrix} \begin{pmatrix} u_{\tau t} \\ u_{at} \end{pmatrix}. \qquad \text{(FF)}$$

The determinant of the above matrix vanishes for $L = 0$; since such a root is smaller than 1 in modulus, the MA representation is non-fundamental by Condition R. Hence the variables of interest are not globally sufficient by Proposition 1. Moreover, neither $u_{\tau t}$ nor u_{at} can be recovered as linear combinations of the VAR innovations.

To complete the model, we assume that the econometrician can observe a panel of n time series x_t^*, including $z_t^* = (\hat{\tau}_t \;\; k_t)'$, as well as an $(n-2)$-dimensional vector y_t^*, providing noisy information about the current values of the structural shocks $u_{\tau t}$ and u_{at}. To make it concrete, we can think of such series as being survey variables, such as confidence indexes and/or professional forecasts, reflecting agent's information about the states of the economy. Precisely, we assume

$$y_{it}^* = b_i u_{\tau t} + c_i u_{at} + \xi_{it},$$

$i = 1, \dots n-2$, where ξ_{it} is the measurement error.

3.2 Simulation design

For the simulation exercise, we produced artificial series for capital, the tax rate and $n-2 = 30$ survey series y^*_{it} of sample size $T = 200$ according to both models NFF and FF. As in LWY, we set $\alpha = 0.36$, $\theta = 0.2673$, $\tau = 0.25$. The structural shocks $u_{\tau t}$, u_{at} were generated as unit variance, Gaussian white noises, mutually independent at all leads and lags. The measurement errors ξ_{it} were produced as Gaussian white noises mutually independent (and independent of the structural shocks) at all leads and lags, with random standard deviation σ_i uniformly distributed between 0 and 1.

We generated b_i as a Bernoulli random variable assuming value 1 with probability 0.25 and value 0 with probability 0.75; c_i was set equal to $1 - b_i$. In this way, we generated two groups of variables: the smallest one providing information about $u_{\tau t}$, the largest one providing information about u_{at}. This ensures that the first principal component captures u_{at}, which does not solve the informational problem, so that two principal components are needed to get informational sufficiency.

Having the data, we computed the first $P = 4$ principal components of the standardized y^*'s and performed our Granger causality test on $z^*_t = (\hat{\tau}_t \ k_t)'$, including such principal components as candidates causing variables. To determine the number of principal components to retain, we tested recursively whether $w^h_t = (z^{*\prime}_t \ \hat{g}_{1t} \ \cdots \ \hat{g}_{ht})'$, $h = 1, 2, 3$, was Granger caused by the remaining principal components, as described in Section 2. We used the out-of-sample "regression" test proposed by Gelper and Croux (2007), which is a multivariate generalization of the univariate test proposed by Harvey *et al.* (1998).[6]

Then we estimated, for both NFF and FF data sets, the VAR-FAVAR specifications including $0, 1, \dots, P-1$ principal components. For each specification we identified the tax shock by imposing that no other shocks affect the tax rate in the long run. The number of lags was determined with the AIC. Then we tested for orthogonality of the estimated tax shocks with respect to 2 and 4 lags of the principal components by using a standard F-test, each time including as regressors the principal components not used in the FAVAR. We performed the test for $P = 1$, $P = 2$ and $P = 4$.

The above procedure was replicated 1000 times, re-drawing each time b_i and σ_i, as well as the shocks and the measurement errors.

[6] In the online appendix we report results supporting the use of the above causality test in the present context. We set an out-of-sample period of 80 observations and used 500 replications to compute the p-values of the test. The number of lags was determined using the AIC.

3.3 Simulation results

Table 1 reports the percentage of rejections obtained. For model NFF (upper panel), the two-variable VAR is informationally sufficient, so that the frequency of rejections provides information about the size of the tests. The Granger causality test does not present relevant size distortions. By contrast, the orthogonality test is clearly under-sized. This depends on the fact that the shock tested is only an imperfect estimate of the true shock.[7] However, the fact that the test under-rejects the null when the null is true is not harmful.

The bottom half of the Table shows results for the fiscal foresight case (model FF). Since in this case the two-variable VAR is informationally deficient, this part of the table provides information about the power of the tests. Numbers tell that both tests are extremely powerful for this data generating process, since fundamentalness is rejected for almost all experiments even at the 1% level, with the exception of the case $P = 1$. Not surprisingly, taking a maximum number of principal components smaller than the true number of factors can lead to acceptance of the null when it is false. On the contrary, picking a maximum number of factors larger than the true one ($P = 4$) does not entail problems.

Table 2 shows the frequency distribution and the average of the number of principal components selected by the recursive procedure proposed in the previous section, using both the orthogonality test and the Granger causality test, at the 5% significance level. Both tests perform reasonably well in selecting the correct number of factors. With $P = 4$, the orthogonality tests are less powerful than the Granger-causality test in rejecting the null of just one factor (2.8% and 4.2% of failures with 2 and 4 lags, respectively, as against 0.7% of failures for the Granger causality test). As anticipated in Section 2.4, the Granger causality test is over-sized (12.8%) when the null is informational sufficiency of the 2-factor FAVAR. However, the shocks and the impulse response functions estimated with the 3-factor and the 4-factor FAVAR are almost identical to those obtained with the 2-factor FAVAR, so that overestimating the number of factors does not have serious consequences.

Figure 1 compares the impulse response functions estimated with the informationally deficient and the amended VAR, with the theoretical impulse response functions. The two upper plots show the impulse response functions of the tax shock on the tax rate and capital, respectively, estimated with

[7] The estimated tax shock has, by construction, exactly zero sample covariance with the lags of z_t^* included in the VAR. This reduces the sample covariance with the lags of the true shocks, i.e. the lags of the factors. For instance, the sample covariance with $u_{\tau t-1} = \hat{\tau}_{t-1}$ is exactly zero. Therefore the regression coefficients are smaller than those which would be obtained with the true tax shock, for which orthogonality holds in population, but does not hold exactly in the sample.

the misspecified VAR. The red solid line is the point-wise average across the experiments; the shadowed regions represent the 68% and the 90% bands; the black line is the theoretical i.r.f. The upper-right figure closely resembles Figure 1 of LWY. The error made by the econometrician is substantial: fiscal foresight is simply not there.

Lower panels show the response functions obtained with the FAVAR specification selected, for each experiment, by the Granger causality test procedure, so that the number of principal components retained varies across experiments. Clearly, the procedure has been successful, since estimates are now very close to the target.[8]

4 An empirical application: Technology shocks

In this section we apply our testing procedure to real data. We revisit the debate about the role of technology shocks as a source of economic fluctuations and their effects on labor market variables. Despite the large amount of works that have addressed this question over the last years, no consensus has been reached. In his seminal paper, Gali (1999) finds a very modest role for technology shocks as a source of economic fluctuations. On the contrary other authors (see e.g. Christiano, Eichenbaum and Vigfusson, 2003) provide evidence that technology shocks are capable of generating sizable fluctuations in macroeconomic aggregates. A common feature of most of the existing evidence is that it is obtained using small-scale VAR models which are likely to suffer from informational deficiency.

Following Barnichon, 2010, we focus on a two-variable specification including the growth rate of labor productivity and the unemployment rate. The state variables of the economy are estimated by using the principal components of a panel of 61 quarterly US macroeconomic series covering the period 1960-I to 2010-IV. Appendix C reports details about the data and their treatment.

4.1 Testing for informational sufficiency

To begin, we test for informational sufficiency of z_t^*. As in the simulation exercise of the previous section, we use the Granger causality "regression" test suggested by Gelper and Croux (2007).[9] To pick P, we first try existing criteria to determine the number of factors. Unfortunately, all Bai and Ng criteria point to 20 or more factors; too many to get accurate estimates of the parameters and the factors themselves. On the other hand, Onatski criterion and Ahn and Horenstein criteria indicate

[8]The response functions and confidence bands obtained with the FAVAR specification selected with the orthogonality tests, not shown here, are indistinguishable from those presented in the Figure.

[9]The out-of sample period includes the last 20 years. The number of lags is determined using the AIC.

just one factor, which is at odds with most of macroeconomic theory and the theoretical premises of structural VAR literature. Hence we simply try different values of P, ranging from 4 to 10.

Table 3, left panel, shows the results. The first row of the table shows the p-value of the test of the null hypothesis that the first P principal component do not Granger cause z_t^*. The hypothesis is strongly rejected for all choices of P, indicating that the two variables do not contain sufficient information to correctly recover the structural shocks. Row $h+1$ shows the p-values of the test of the null hypothesis that the VAR augmented by the first h principal component, i.e. $w_t^h = (z_t^{*\prime} \ \cdots \ \hat{g}_{ht})'$, is not Granger caused by the remaining principal components, from the $h+1$-th to the P-th. Taking the 5% significance level, our recursive procedure picks four factors with $P=4$ and $P=8$, and three factors for $P=6$ and $P=10$. We conclude that the four factor FAVAR is globally sufficient.

Even if the VAR information is not sufficient to get all of the structural shocks, it can in principle be sufficient to recover the technology shock. To check whether this is the case, we identify the technology shock by imposing the standard restriction that the other shocks do not affect productivity in the long run. Then we test whether the estimated shock is orthogonal to the past of the estimated principal components (we perform an F-test like in Section 3). The right panel in Table 3 displays the p-values of the test. Again, sufficiency of z_t^* is strongly rejected for all choices of P, suggesting that the shock obtained from the original VAR is not structural. At the 5% significance level, the recursive procedure to determine the number of factors points to 2 factors for $P=4$ and $P=6$ and 3 factors for $P=8$ and $P=10$. We conclude that three factors are sufficient to estimate the technology shock and the related impulse response functions.

4.2 Information and impulse response functions

Next we study the consequences of insufficient information in terms of impulse response functions. In particular, we investigate to what extent the effects of technology shocks change by augmenting the original VAR with the principal components. According to the results of the test, impulse response functions are expected to change when adding principal components.

Figure 2 shows the impulse response functions. The left panels plot the impulse response functions for the two variables, labor productivity and the unemployment rate, for all the eleven specifications $z_t^*, w_t^1, \dots, w_t^{10}$. The solid line with dots represents the impulse response functions estimated with z_t^*. The line with crosses represents the impulse response functions estimated with w_t^4. The remaining lines are the estimated responses of the other models. The effects are expressed in percentage terms. The right panels display, for the two variables, the impact effect (dots), the effect at 1 year (crosses), 2 years (circles) and in the long run (diamonds). The horizontal axis displays the number of principal components included in the VAR.

The VAR without principal components predicts that technology shocks reduce unemployment, i.e. the shock is expansionary. The finding is in line with the theoretical predictions of standard RBC models and the empirical findings of Christiano, Eichenbaum and Vigfusson (2003) Labor productivity reacts positively on impact and stays roughly constant afterward, with almost no delay in the diffusion process, and somewhat unlikely negative effects after one year.

The picture changes dramatically when adding the principal components. Indeed the effect of the technology shock on the unemployment rate changes sign, becoming positive. Moreover, the impact effect of productivity reduces substantially while the long run effect is roughly unchanged so that the diffusion process is much slower, in line with the S-shape view and the recent news shocks literature (Beaudry and Portier, 2006, Schmitt-Grohe and Uribe, 2008). As can be seen in the right panels of Figure 1, consistently with the results of the test, models including more than three principal components all deliver similar impulse response functions.

4.3 Richer VAR specifications

In what follows we test whether richer VAR specifications can be successful in passing the informational sufficiency test. We consider two specifications, say VAR 1 and VAR2. VAR1 includes five variables: GDP, the GDP deflator, both in log differences, and a component of the Michigan University consumer confidence index, i.e. business conditions expected during the next 5 years (series 1, 17 and 55), in addition to labor productivity (in log differences) and the unemployment rate. VAR2 includes four variables: productivity and unemployment are complemented with the GDP deflator and the Standard & Poor's index of 500 common stocks, divided by the GDP deflator (series 52 and 17), both taken in log differences. Both specifications include a forward-looking variable, aimed at capturing agents' information about technology; stock prices have been used to identify technology shocks in Beaudry and Portier (2006); the confidence indicator, and its ability to anticipate future growth, has been extensively analyzed in Barsky and Sims (2012). Being concerned with just one shock, we do not try to reach global sufficiency, but only orthogonality of the estimated shock.

Table 4 shows results of the orthogonality test, with different choices of P and number of lags, for both VAR1 and VAR2. For comparison, we include also the corresponding results for the deficient VAR specification (VAR0). For both VAR1 and VAR2, orthogonality is never rejected at the 5% significance level.

Figure 3 compares the impulse response functions obtained with VAR1 and VAR2 (black solid lines) with the corresponding impulse response functions of the deficient VAR specification (red dashed lines) and the amended 4-factor FAVAR specification (blue dash-dotted lines). For both specifications, the unemployment rate exhibits on impact a significant positive reaction. The impulse response functions are fairly similar to each other and the FAVAR specification.

5 Conclusions

The variables included in a VAR contain enough information to identify and estimate the structural shocks if, and only if, they are not Granger caused by the common factors (the 'states') driving macroeconomic variables. Moreover, assuming that the identification scheme adopted by the econometrician is correct, an estimated shock is a consistent estimate of the desired structural shock if and only if it is orthogonal to the lags of the factors. On the basis of these results, we have proposed misspecification tests which are simple and relatively easy to implement. If a VAR specification is rejected, the tests can be used to amend it.

The methods proposed here are particularly useful when there are good reasons to suspect that a VAR specification is informationally deficient, either because the VAR is very small, or because the relevant shocks may be anticipated by economic agents, so that the structural MA representation of the macroeconomic equilibrium can be non-fundamental. Two important examples are "fiscal foresight" and "news shocks". Forni, Gambetti and Sala (2011), uses the orthogonality test proposed here to show that the VARs in Beaudry and Portier (2006) have information problems, and, when information is properly complemented, the effects of news shocks are smaller and qualitatively different, in line with the findings of Barsky and Sims (2011) whose specification passes the test. Applications to fiscal foresight seem a promising task for future research.

References

Ahn, S.C., Horenstein, A.R., 2013. Eigenvalue Ratio Test for the Number of Factors. Econometrica 81, 1203-1227.

Bai, J., Ng, S., 2002. Determining the number of factors in approximate factor models. Econometrica 70, 191-221.

Barnichon, R., 2010. Productivity and unemployment over the business cycle. Journal of Monetary Economics 57, 1013-25.

Barsky, R.B., Sims, E.R., 2011. News shocks and business cycles. Journal of Monetary Economics 58, 273-289.

Barsky, R.B., Sims, E.R., 2012. Information, Animal Spirits, and the Meaning of Innovations in Consumer Confidence. American Economic Review 102, 1343-1377.

Beaudry, P., Portier, F., 2006. Stock Prices, News, and Economic Fluctuations. American Economic Review 96, 1293-1307.

Bernanke, B. S., Boivin, J. Eliasz, P., 2005. Measuring Monetary Policy: A Factor Augmented Autoregressive (FAVAR) Approach. The Quarterly Journal of Economics 120, 387-422.

Chari, V.V., Kehoe, P.J., McGrattan, E.R., 2008. Are structural VARs with long-run restrictions useful in developing business cycle theory? Journal of Monetary Economics 55, 1337-1352.

Christiano, L., M. Eichenbaum, Vigfusson, R., 2003. What Happens After a Technology Shock? NBER Working Papers 9819.

Fernandez-Villaverde J., Rubio-Ramirez, J.F., Sargent, T.J., Watson, M.W., 2007. ABCs (and Ds) of Understanding VARs. American Economic Review 97, 1021-1026.

Forni, M. Gambetti, L., 2010. Fiscal Foresight and the Effects of Government Spending. CEPR Discussion Papers series no. 7840.

Forni, Gambetti, L., Sala, L., 2011. No News in Business Cycles. Forthcoming in Economic Journal.

Forni, M., Gambetti, L., Lippi, M., Sala, L., 2013. Noisy News in Business Cycles. CEPR Discussion Papers series no. 9601.

Forni, M., Gambetti, L., Lippi, M., Sala, L., 2013. Noise Bubbles. CEPR Discussion Papers series no. 9532.

Forni, M., Giannone, D., Lippi, M., Reichlin, L., 2009. Opening the Black Box: Structural Factor Models with Large Cross-Sections. Econometric Theory 25, 1319-1347.

Forni M., Reichlin, L., 1996. Dynamic Common Factors in Large Cross-Sections. Empirical Economics 21, 27-42.

Galí, J., 1999. Technology, Employment and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations? American Economic Review 89, 249-271.

Gelper, S., Croux, C., 2007. Multivariate out-of-sample tests for Granger causality. Computational Statistics & Data Analysis 51, 3319-3329.

Giannone, D., Reichlin, L., 2006. Does Information Help Recovering Structural Shocks from Past Observations? Journal of the European Economic Association 4, 455-465.

Hansen, L.P., Sargent, T.J., 1991. Two problems in interpreting vector autoregressions. In: Rational Expectations Econometrics, L.P. Hansen and T.J. Sargent, eds. Boulder: Westview, 77-119.

Harvey, D.I., Leybourne, S.J., Newbold, P., 1998. Tests for Forecast Encompassing. Journal of Business & Economic Statistics 16, 254-259.

Leeper, E.M., Walker, T.B., Yang, S.S., 2013. Fiscal Foresight and Information Flows. Econometrica 81, 1115-1145.

Lippi, M., Reichlin, L., 1993. The Dynamic Effects of Aggregate Demand and Supply Disturbances: Comment. American Economic Review 83, 644-652.

Lippi, M., Reichlin, L., 1994. VAR analysis, non-fundamental representation, Blaschke matrices. Journal of Econometrics 63, 307-325.

Mertens, K., Ravn, M.O., 2010. Measuring the Impact of Fiscal Policy in the Face of Anticipation: A Structural VAR Approach. Economic Journal 120, 393-413.

Onatski, A., 2010. Determining the Number of Factors from Empirical Distribution of Eigenvalues. The Review of Economics and Statistics 92, 1004-1016.

Perotti, R., 2008. In Search of the Transmission Mechanism of Fiscal Policy. NBER Macroeconomics Annual 22, 169-226.

Ramey, V.A., 2011. Identifying Government Spending Shocks: It's all in the Timing. The Quarterly Journal of Economics 126, 1-50.

Ravenna, F., 2007. Vector autoregressions and reduced form representations of DSGE models. Journal of Monetary Economics 54, 2048-2064.

Rozanov, Yu., 1967. Stationary Random processes. San Francisco: Holden Day.

Schmitt-Grohe, S., Uribe, M., 2012. What's News in Business Cycles. Econometrica 80(6):27332764.

Sims, C.A., 1980. Macroeconomics and Reality. Econometrica, 48(1), 1-48.

Stock, J.H., Watson, M.W., 2002. Forecasting Using Principal Components from a Large Number of Predictors. Journal of the American Statistical Association 97, 1167-1179.

Yang, S.S., 2008. Quantifying tax effects under policy foresight. Journal of Monetary Economics 52, 1557-1568.

Tables

Significance level	Orthogonality, 2 lags			Orthogonality, 4 lags			Causality, AIC
	$P = 1$	$P = 2$	$P = 4$	$P = 1$	$P = 2$	$P = 4$	$P = 4$
Model NFF							
1%	0.0	0.0	0.1	0.0	0.0	0.1	0.9
5%	0.0	0.0	0.4	0.4	0.0	0.9	4.8
10%	0.2	0.0	1.3	0.7	0.3	2.3	10.8
Model FF							
1%	76.6	99.9	99.8	41.8	99.8	99.8	100.0
5%	98.6	100.0	99.9	84.1	100.0	99.9	100.0
10%	100.0	100.0	100.0	96.2	100.0	99.9	100.0

Table 1: Percentage of rejections across 1000 experiments for the null of informational sufficiency of the VAR specification including the tax rate $\hat{\tau}_t$ and capital k_t, generated with model NFF (no fiscal foresight) and model FF (two-period foresight). We report results for the orthogonality test (informational sufficiency for the tax rate shock) and the Granger causality test (global sufficiency), for different significance levels, numbers of lags and numbers of principal components P.

No. of factors	Orthogonality, 2 lags			Orthogonality, 4 lags			Causality, AIC
retained	$P = 1$	$P = 2$	$P = 4$	$P = 1$	$P = 2$	$P = 4$	$P = 4$
0	1.4	0	0.1	15.9	0	0.1	0
1	98.6	2.8	2.8	84.1	3.4	4.2	0.7
2	-	97.2	91.5	-	96.6	90.2	86.5
3-4	-	-	5.6	-	-	5.5	12.8
total	100	100	100	100	100	100	100
average	0.99	1.97	2.05	0.84	1.97	2.04	2.16

Table 2: Frequency distribution and average, across 1000 experiments, of the number of principal components retained in the FAVAR specification, according to the orthogonality test and the Granger causality test, with different numbers of lags and numbers of principal components P. The data are generated with model FF (two-period foresight).

	Granger causality, AIC				Orthogonality, 2 lags			
	$P = 4$	$P = 6$	$P = 8$	$P = 10$	$P = 4$	$P = 6$	$P = 8$	$P = 10$
z_t^*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
w_t^1	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00
w_t^2	0.00	0.03	0.01	0.03	0.10	0.44	0.03	0.03
w_t^3	0.00	0.26	0.02	0.12	0.08	0.45	0.09	0.10
w_t^4	-	0.97	0.43	0.47	-	0.84	0.39	0.11

Table 3: p-values of the out-of-sample Granger causality test (global sufficiency) and the orthogonality F-test for the estimated productivity shock. The rows correspond to different specifications, z_t^* and w_t^h for $h = 1, \ldots, 4$. P refers to the number of principal components used in the test.

	Orthogonality, 2 lags				Orthogonality, 4 lags			
	$P = 4$	$P = 6$	$P = 8$	$P = 10$	$P = 4$	$P = 6$	$P = 8$	$P = 10$
VAR0	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
VAR1	0.33	0.23	0.11	0.11	0.34	0.39	0.09	0.18
VAR2	0.82	0.67	0.33	0.16	0.94	0.52	0.12	0.16

Table 4: p-values of the orthogonality F-test, with two and four lags, for the productivity shock, estimated with different VAR specifications. P is the number of principal components used in the test. VAR0: labor productivity growth, unemployment rate. VAR1: labor productivity growth, unemployment rate, GDP growth, GDP deflator (inflation rate), business conditions 5 year confidence indicator. VAR2: labor productivity growth, unemployment rate, GDP deflator (inflation rate), deflated S&P500 index (growth rate).

Figures

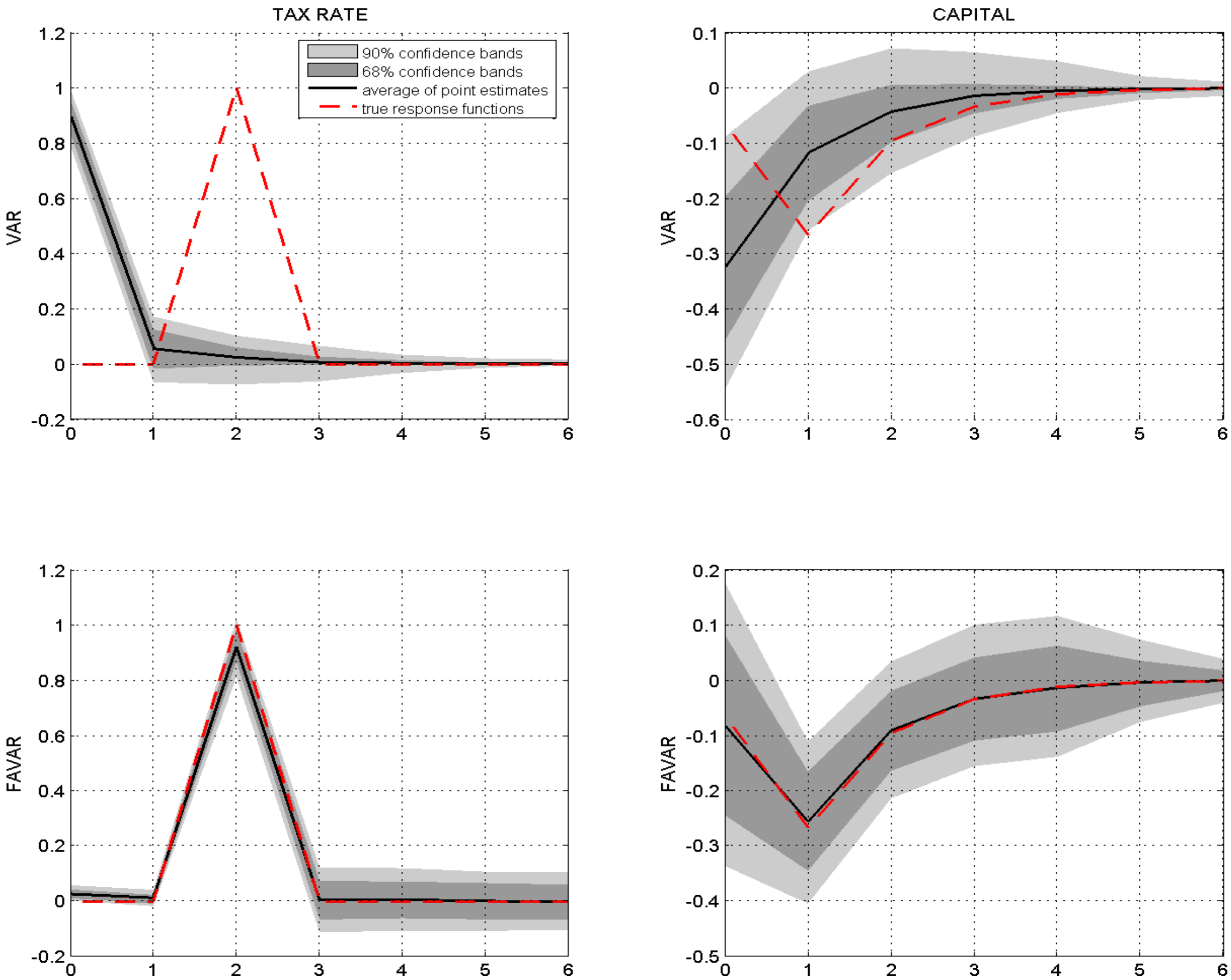

Figure 1: Impulse response functions of the tax rate and capital to a tax shock with model FF (two-period fiscal foresight). The first row (VAR) plots the true responses (dashed line) and the responses obtained using a bivariate VAR for tax rate and capital (solid line) along with the 68% and 90% confidence bands, dark and light grey area respectively. The second row (FAVAR) plots the true responses (dashed line) and the responses obtained using a estimated using a FAVAR with a number of factor equal to the number suggested by the test (solid line) along with the 68% and 90% confidence bands, dark and light grey area respectively.

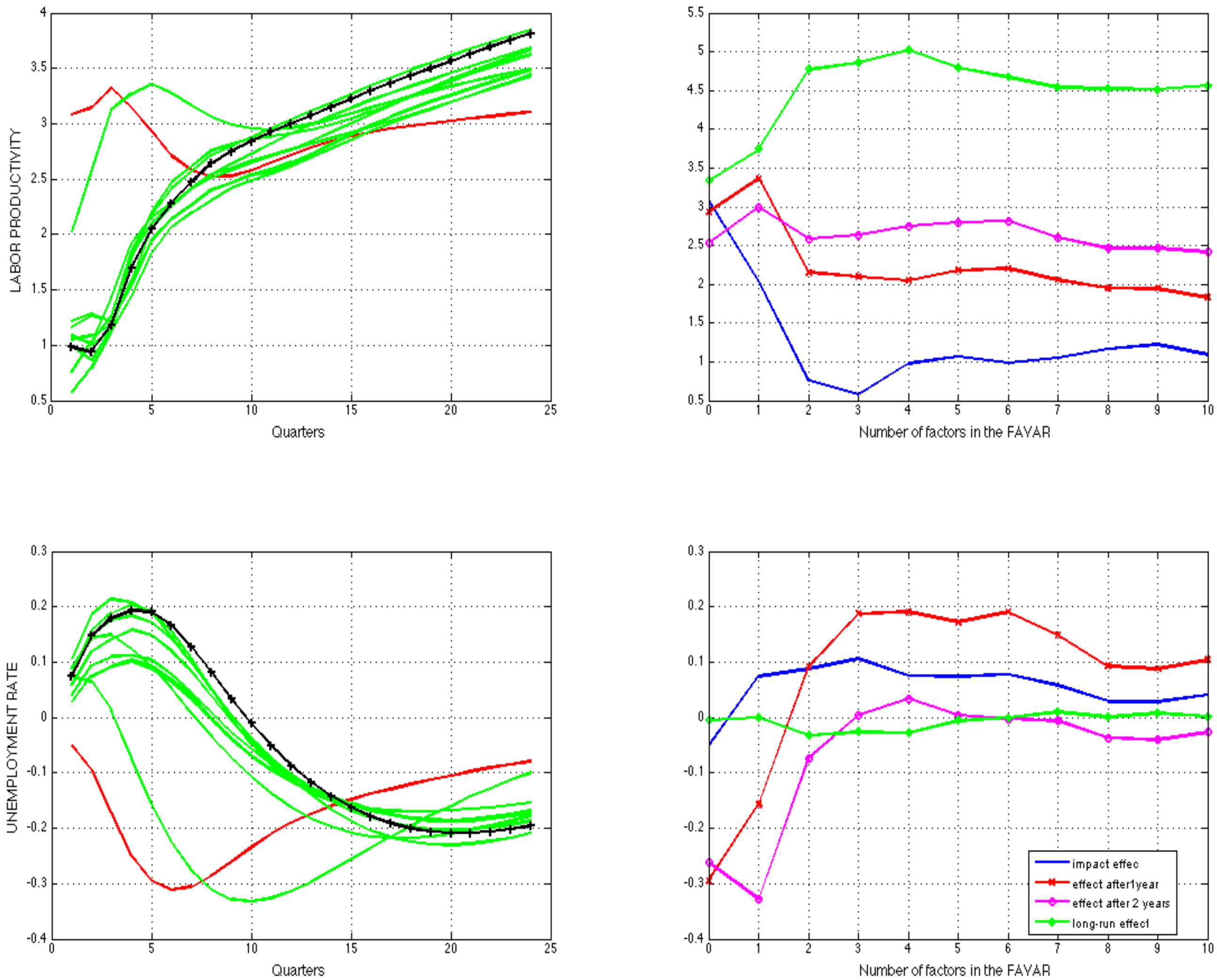

Figure 2: Impulse response functions to a technology shock. The left panels plot the impulse response functions for the two variables, labor productivity and the unemployment rate, for all the specifications $z_t^*, w_t^1, \ldots, w_t^{10}$. The starred line represents the impulse response functions estimated with z_t^*. The line with crosses represents the impulse response functions estimated with w_t^4. The remaining lines are the estimated responses of the other models. The effects are expressed in percentage terms. The right panels display the impact effect (stars), the effect at 1 year (crosses), 2 years (circles) and in the long run (diamonds). The horizontal axis displays the number of principal components included in the VAR.

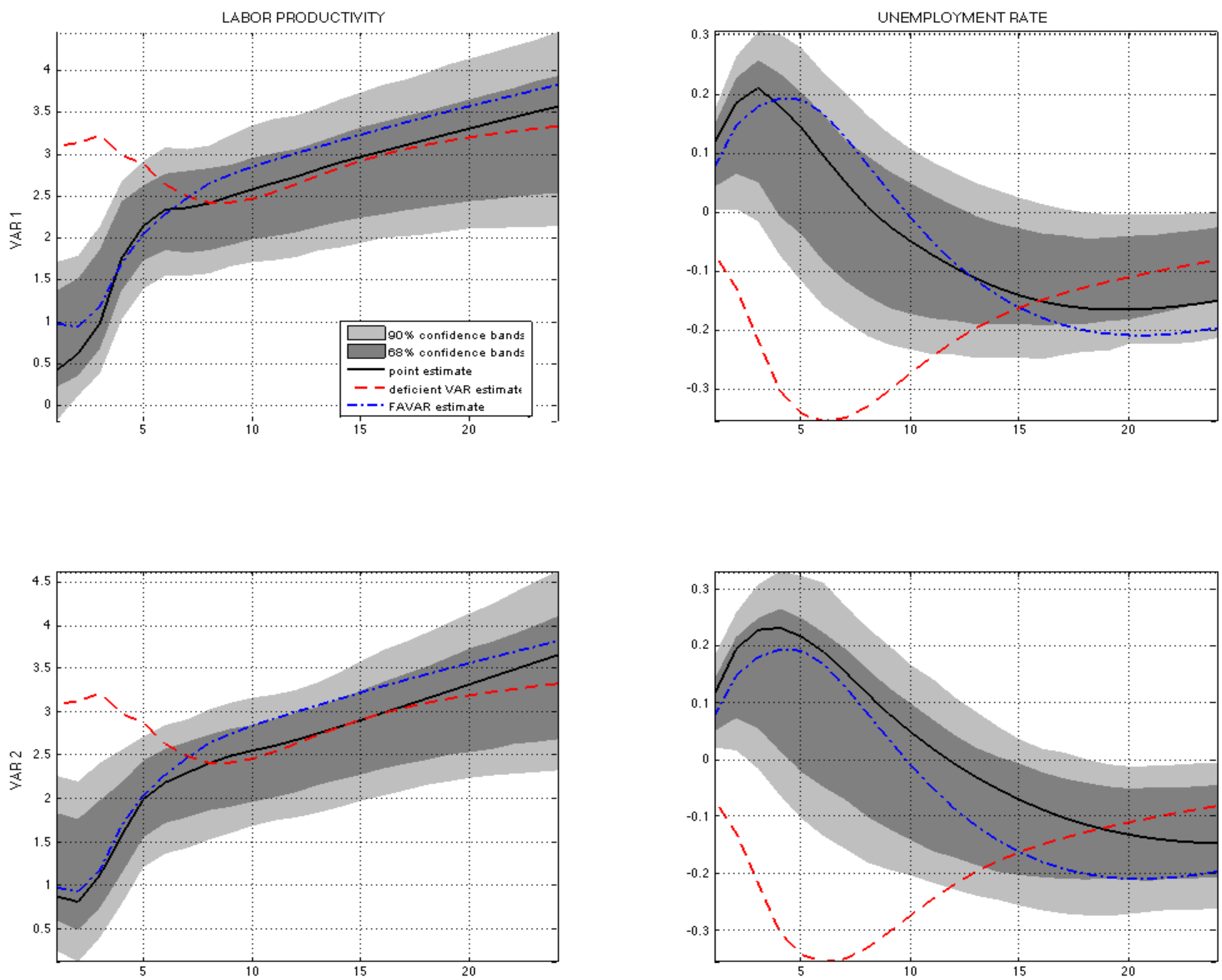

Figure 3: Impulse response functions to a technology shock of labor productivity and the unemployment rate in VAR 1 and VAR 2 specifications. VAR 1: labor productivity growth, unemployment rate, GDP growth, GDP deflator (inflation rate), business conditions 5 year confidence indicator. VAR 2: labor productivity growth, unemployment rate, GDP deflator (inflation rate), deflated S&P500 index (growth rate). The solid line is the point estimate, the dark-grey area represents the 68% confidence bands, the light-grey area represents the 90% confidence bands, the dashed line is the point estimate of the informationally deficient VAR (labor productivity growth and unemployment rate), the dot-dashed line is the point estimate of the 4-factor FAVAR.