The structural dynamics of US output and inflation: what explains the changes?

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Abstract

We examine the dynamics of US output and inflation using a structural time varying coefficients VAR. There are changes in the volatility of both variables and in the persistence of inflation, but variations are statistically insignificant. Technology shocks explain changes in output volatility; real demand disturbances variations in the persistence and volatility of inflation. We detect important time variations in the transmission of technology shocks to output and demand shocks to inflation and significant changes in the variance of technology and of monetary policy shocks.

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1 Introduction

There is considerable evidence suggesting that the US economy has fundamentally changed over the last 35 years. For example, Blanchard and Simon (2000), McConnell and Perez Quiroz (2001), Sargent and Cogley (2001) and Stock and Watson (2003) have reported a marked decline in the volatility of real activity and inflation since the early 1980s and a reduction in the persistence of inflation over time. What has caused these changes? One possibility is that the features and intensity of the shocks hitting the economy have changed. Another is that structural characteristics, such as the preferences of policymakers or the behavior of consumers and firms, have changed over time. The recent literature has paid particular attention to variations in policymakers' preferences, see for example, Clarida, Gali and Gertler (2000), Cogley and Sargent (2001) and (2005), Boivin and Giannoni (2002), and Lubik and Schorfheide (2004). However, the US economy has also witnessed a number of important changes that may have altered the way consumers and firms responded to economic disturbances. For example, the labor productivity boom of the 1990s was different from previous ones (see e.g. Gordon (2003)) and the goal of fiscal policy in the 1990s (balanced budget) was different than the one of 1970s or the early 1980s. Hence, studying the time profile of the dynamics of output and inflation induced by a variety of disturbances may help to clarify which structural feature has changed and to what extent observed variations reflect alterations in the propagation mechanism or in the volatility of the exogenous shocks.

This paper investigates the contribution of technology, real demand and monetary disturbances to the changes in the volatility and in the persistence of output and inflation in the US. We employ a time varying coefficients VAR (TVC-VAR), where the coefficients evolve according to a nonlinear transition equation and the variance of the forecast errors changes over time. As in Cogley and Sargent (2001), (2005) we use Markov Chain Monte Carlo (MCMC) methods to estimate the posterior distribution of the quantities of interest but, in contrast to these authors, and as in Canova and Gambetti (2004), we recursively analyze the time evolution of the *structural* relationships. The structural disturbances we construct may display different features at different points in time. In fact, we permit temporal changes in their characteristics, in their variances and in their transmission to the economy.

Structural disturbances are identified using robust sign restrictions obtained from a DSGE model featuring monopolistically competitive firms, distorting taxes, utility yielding government expenditure, and rules describing fiscal and monetary policy actions, which encompasses RBC style and New-Keynesian style models as special cases. We construct robust restrictions allowing time variations in the parameters within a range which is consistent with statistical evidence and economic considerations. The methodology we employ to link the theory and the empirical analysis uses a subset of these robust restrictions, requires much weaker assumptions than those needed to perform direct structural estimation of the model, it is computationally simple and works even when the theoretical model is misspecified in some dimensions.

Because time variations in the coefficients induce important non-linearities, standard response analysis, which assumes that coefficients are fixed over the horizon of the exercise, is inappropriate. To trace out the evolution of the economy when perturbed by structural shocks, we define impulse responses as the difference between two conditional expectations, differing in the arguments of their conditioning sets. Such a definition reduces to the standard one when coefficients are constant and, more importantly, allows us to condition on the history of the data and of the parameters.

Our results are as follows. First, while there is visual evidence of structural variations in both the volatility of output and inflation and in the persistence of inflation, these changes are a-posteriori insignificant. Second, the three structural shocks we identify explain between 50 and 65 percent of the variability of output and inflation on average in each period. Third, time variations in inflation persistence and volatility are primarily due to a decline in the contribution of real demand shocks while output volatility changes primarily because the contribution of technology shocks varies over time. Fourth, we detect variations in the transmission of demand shocks to inflation and technology shocks to output and changes in the variances of technology and monetary policy shocks.

Hence, in agreement with McConnell and Perez Quiroz (2001) and Gordon (2003), our analysis attributes to variations in the magnitude and the transmission of technology shocks an important role in explaining changes in output volatility. It also suggests that variations in the magnitude of both technology and monetary shocks and in the transmission of real demand shocks are important in explaining changes in the volatility and in the persistence of inflation. Therefore, it complements the work of Sims and Zha (2004), Primiceri (2005) and Gambetti and Canova (2004), who only examined the role of policy disturbances.

The rest of the paper is organized as follows. Section 2 describes the empirical model. Section 3 presents the identification restrictions. Section 4 deals with estimation - the technical details are in the appendix. Section 5 presents the results and section 6 concludes.

2 The empirical model

Let y_t be a 5×1 vector of time series including real output, hours, inflation, the federal funds rate and M1 with the representation

$$y_t = A_{0,t} + A_{1,t}t + A_{2,t}y_{t-1} + A_{3,t}y_{t-2} + ... + A_{p+1,t}y_{t-p} + \varepsilon_t \quad (1)$$

where $A_{0,t}, A_{1,t}$ are a 5×1 vectors; $A_{i,t}$, are 5×5 matrices, $i = 2, ..., p+1$, and ε_t is a 5×1 Gaussian white noise process with zero mean and covariance Σ_t. Letting $A_t = [A_{0,t}, A_{1,t}, A_{2,t}...A_{p+1,t}]$, $x'_t = [1_5, 1_5 * t, y'_{t-1}...y'_{t-p}]$, where 1_5 is a row vector of ones of length 5, $vec(\cdot)$ denotes the stacking column operator and $\theta_t = vec(A'_t)$, we rewrite (1) as

$$y_t = X'_t\theta_t + \varepsilon_t \quad (2)$$

where $X'_t = (I_5 \otimes x'_t)$ is a $5 \times 5(5p+2)$ matrix, I_5 is a 5×5 identity matrix, and θ_t is a $5(5p+2) \times 1$ vector. We assume that θ_t evolves according to $p(\theta_t|\theta_{t-1}, \Omega_t) \propto \mathcal{I}(\theta_t)f(\theta_t|\theta_{t-1}, \Omega_t)$, where $\mathcal{I}(\theta_t)$ discards explosive paths of y_t, and $f(\theta_t|\theta_{t-1}, \Omega_t)$ is represented as

$$\theta_t = \theta_{t-1} + u_t \quad (3)$$

where u_t is a $5(5p+2) \times 1$ Gaussian white noise process with zero mean and covariance Ω_t. We select the simple specification in (3) because more general AR and/or mean reverting structures were always discarded in out-of-sample model selection exercises. We assume that $corr(u_t, \varepsilon_t) = 0$, and that Ω_t is diagonal. The first assumption implies conditional linear responses to changes in ε_t, the second is made for computational ease - structural coefficients are allowed to change in a correlated fashion. Our model implies that forecast errors are non-normal and heteroschedastic even when $\Sigma_t = \Sigma$ and $\Omega_t = \Omega$. In fact, substituting (3) into (2) we have that $y_t = X'_t\theta_{t-1} + v_t$, where $v_t = \varepsilon_t + X'_t u_t$. Such a structure is appealing since whatever alters coefficients also imparts heteroschedastic movements to the variance of the forecasts errors. Since also Ω_t is allowed to vary over time, the model permits various forms of stochastic volatility in the forecast errors of the model (see Sims and Zha (2004) and Cogley and Sargent (2005) for alternative specifications).

Let S_t be such that $\Sigma_t = S_t D_t S'_t$, where D_t is diagonal and let H_t be an orthonormal matrix, independent of ε_t, such that $H_t H'_t = I$ and set $J_t^{-1} = H'_t S_t^{-1}$. J_t is a particular decomposition of Σ_t which transforms (2) in two ways: it produces uncorrelated innovations (via the matrix S_t) and it gives a structural interpretation to the equations of the system (via the matrix H_t). Premultiplying (1) by J_t^{-1} we obtain

$$J_t^{-1}y_t = J_t^{-1}A_{0,t} + J_t^{-1}A_{1,t}t + \sum_j J_t^{-1}A_{j+1,t}y_{t-j} + e_t \quad (4)$$

where $e_t = J_t^{-1}\varepsilon_t$ satisfies $E(e_t) = 0$, $E(e_t e_t') = H_t D_t H_t'$. Equation (4) represents the class of "structural" representations of interest: for example, a Choleski system is obtained choosing $S_t = S$ to be a lower triangular matrix and $H_t = I_5$, and more general patterns, with non-recursive zero restrictions, are obtained if $S_t = S$ is non-triangular and $H_t = I_5$.

In this paper, since S_t is an arbitrary square root matrix, identifying structural shocks is equivalent to choosing H_t. We select it so that the sign of the responses at $t+k, k = 1, 2, \ldots, K_1,\ K_1$ fixed, matches the robust model-based sign restrictions presented in the next section. We choose sign restrictions to identify structural shocks for three reasons. First, magnitude restrictions typically depend on the parameterization of the model while the sign restrictions we employ are less prone to such problem. Second, our model fails to deliver the full set of zero restrictions one would need to identify the three shocks of interest with more conventional approaches. Third, as it will be clear from the next section, the link between the theory and the empirical analysis is more direct.

Let $C_t = [J_t^{-1}A_{0t}, \ldots, J_t^{-1}A_{p+1t}]$, and let $\gamma_t = vec(C_t')$. As in fixed coefficient VARs, there is a mapping between the structural coefficients γ_t and the reduced form coefficients θ_t since $\gamma_t = (J_t^{-1} \otimes I_{5(p+2)})\theta_t$. Whenever $\mathcal{I}(\theta_t) = 1$, we have

$$\gamma_t = (J_t^{-1} \otimes I_{5(p+2)})(J_{t-1}^{-1} \otimes I_{5(p+2)})^{-1}\gamma_{t-1} + \eta_t \tag{5}$$

where $\eta_t = (J_t^{-1} \otimes I_{5(p+2)})u_t \sim (0, E((J_t^{-1} \otimes I_{5(p+2)})u_t u_t'(J_t^{-1} \otimes I_{5(p+2)})'))$. Since each element of γ_t depends on several u_{it} via J_t, shocks to structural parameters are no longer independent. Note that (4)-(5) contain two types of structural shocks: VAR disturbances, e_t, and structural parameter disturbances, η_t. This latter type of shock will not be dealt with here and is analyzed in detail in Canova and Gambetti (2004).

To study the transmission of disturbances in a standard VAR one typically employs impulse responses. Impulse responses are generally computed as the difference between two realizations of $y_{i,t+k}$ which are identical up to time t, but differ afterward because a shock in the j-th component of e_{t+k} occurs at time t in one realization but not in the other.

In a TVC model, responses computed this way disregard the fact that structural coefficients may also change. Hence, meaningful response functions ought to measure the effects of a shock in e_{jt} on y_{it+k}, allowing future shocks to the structural coefficients to be non-zero. To do so, let y^t be a history for y_t; θ^t be a trajectory for the coefficients up to t, $y_{t+1}^{t+k} = [y_{t+1}', ...y_{t+k}']'$ a collection of future observations and $\theta_{t+1}^{t+k} = [\theta_{t+1}', ...\theta_{t+k}']'$ a collection of future trajectories for θ_t. Let $V_t = (\Sigma_t, \Omega_t)$; recall that $\xi_t' = [e_t', \eta_t']$. Let $\xi_{j,t+1}^{\delta}$ be a realization of $\xi_{j,t+1}$ of size δ and let $\mathcal{F}_t^1 = \{y^t, \theta^t, V_t, J_t, \xi_{j,t}^{\delta}, \xi_{-j,t}, \xi_{t+1}^{t+\tau}\}$ and $\mathcal{F}_t^2 = \{y^t, \theta^t, V_t, J_t, \xi_t, \xi_{t+1}^{t+\tau}\}$ be two conditioning sets, where $\xi_{-j,t}$ indicates all shocks, excluding the one in the j-th component. Then a response vector to $\xi^{\delta}_{i,t}$, $i = 1, \ldots, 5$ is:[1]

$$IR_i(t,k) = E(y_{t+k}|\mathcal{F}^1_t) - E(y_{t+k}|\mathcal{F}^2_t) \qquad k = 1, 2, \ldots \tag{6}$$

While (6) resembles the impulse response function suggested by Gallant et al. (1996), Koop et al. (1996) and Koop (1996), three important differences need to be noted. First, our responses are history but not state dependent. Second, the size and the sign of η shocks matter for the dynamics of the system but not the size and the sign of e_t. Third, since θ_{t+1} is a random variable, $IR_i(t,k)$ is also a random variable.

When $\xi^{\delta}_{i,t} = e^{\delta}_{i,t}$, the case considered in the paper, responses are given by:

$$\begin{aligned} IR_i(t,1) &= J_t^{-1,i} e_{i,t} \\ IR_i(t,k) &= \Psi^h_{t+k,k-1} e_{i,t} \qquad k = 2, 3, \ldots \end{aligned} \tag{7}$$

where $\Psi_{t+k,k-1} = \mathcal{S}_{n,n}[(\prod_{h=0}^{k-1} \mathbf{A}_{t+k-h}) \times J_{t+1}]$, $\mathbf{A}_t$ is the companion matrix of the VAR at time t; $\mathcal{S}_{n,n}$ is a selection matrix which extracts the first $n \times n$ block of $[(\prod_{h=0}^{k-1} \mathbf{A}_{t+k-h}) \times J_{t+1}]$ and $\Psi^i_{t+k,k-1}$ is the column of $\Psi_{t+k,k-1}$ corresponding to the $i-$th shock.

When the coefficients are constant, $\prod_h \mathbf{A}_{t+k-h} = \mathbf{A}^k$ and $\Psi_{t+k,k-1} = \mathcal{S}_{n,n}(\mathbf{A}^k \times J)$ for all k, and (7) collapses to the traditional impulse response function to unitary structural shocks. In general, $IR_i(t,k)$ depends on the identifying matrix J_t, the history of the data and the dynamics of reduced form coefficients up to time t.

3 The identification restrictions

The restrictions we use to identify the shocks come from a general equilibrium model that encompasses flexible price RBC and New-Keynesian sticky price setups as special cases. The restrictions we consider are robust, in the sense that they hold for variations in the parameters within some meaningful range and for alternative specifications of the policy rules. We use a subset of the model's predictions and, as in Canova (2002), we focus only on qualitative (sign) restrictions, as opposed to quantitative (magnitude) restrictions, to identify shocks. We briefly sketch the features of the model and directly describe the restrictions it implies on the responses of the variables to shocks. Details concerning the model and the selection of the range for the parameters are in Gambetti et. al. (2005).

The economy features a representative household, a continuum of firms, a monetary authority, and a fiscal authority consuming goods that may yield utility for the household.

[1] One could alternatively average out future shocks. Our definition is preferable for two reasons: it produces numerically more stable distributions and responses are similar to those generated by constant coefficient impulse responses when shocks to the measurement equations are considered. Since future shocks are not averaged out, our impulse responses will display larger variability.

The household maximizes $E_0 \sum_{t=0}^{\infty} \beta^t \frac{[(aC_t^{p\frac{\varsigma-1}{\varsigma}} + (1-a)C_t^{g\frac{\varsigma-1}{\varsigma}})^{\frac{\theta_n \varsigma}{\varsigma-1}}(1-N_t)^{1-\theta_n}]^{1-\sigma}-1}{1-\sigma} + \frac{1}{1-\theta_M}(\frac{M_t}{P_t})^{1-\theta_M}$ choosing sequences for private consumption (C_t^p), hours (N_t), capital (K_{t+1}), nominal state-contingent bonds (D_{t+1}), nominal balances (M_{t+1}) and government bonds (B_{t+1}) subject to the sequence of budget constraints $P_t(C_t^p + I_t) + E_t\{Q_{t,t+1}D_{t+1}\} + R_t^{-1}B_{t+1} + M_{t+1} \leq (1-\tau^l)P_t w_t N_t + [r_t - \tau^k(r_t - \delta)]P_t K_t + D_t + B_t - T_t P_t + M_t + \Xi_t$, where $(1-\tau^l)P_t w_t N_t$, is the after tax nominal labor income, $[r_t - \tau^k(r_t-\delta)]P_t K_t$ is the after tax nominal capital income (allowing for depreciation), Ξ_t is nominal profits distributed by firms, $T_t P_t$ is nominal lump-sum taxes, $Q_{t,t+1}$ is period-t price of state contingent bonds and R_t is the gross return on a one period government bond. Here $0 < \beta < 1$ and the degree of substitutability between private and public consumption is regulated by $0 < \varsigma \leq \infty$; $0 < a \leq 1$ controls the share of public and private goods in consumption; $\vartheta_M > 0$ regulates the elasticity of money demand. Household time is normalized to one at each t. We assume $C_t^p = \left[\int_0^1 C_{it}^p(i)^{\frac{\lambda-1}{\lambda}} di\right]^{\frac{\lambda}{\lambda-1}}$, $C_t^g = \left[\int_0^1 C_{it}^g(i)^{\frac{\lambda-1}{\lambda}} di\right]^{\frac{\lambda}{\lambda-1}}$, where $\lambda > 0$ measures the elasticity of substitution between types of goods. Capital accumulates according to $K_{t+1} = I_t + (1-\delta)K_t - \frac{b}{2}\left[\frac{K_{t+1}-(1-\delta)K_t}{K_t} - \delta\right]^2$, where $0 < \delta < 1$ is a constant depreciation rate, and $b \geq 0$ an adjustment cost parameter.

A firm j produces output according to $Y_{tj} = Z_t(N_{tj})^{1-\alpha}(K_{tj})^{\alpha}$ where K_{tj} and N_{tj} are capital and labor inputs and Z_t is a technology shock. With perfectly competitive input markets, cost minimization implies $\frac{K_{tj}}{N_{tj}} = \frac{\alpha}{(1-\alpha)}\frac{w_t}{r_t}, \forall j$. In the goods market firms are monopolistic competitors. When prices are sticky, each producer is allowed to reset her price with a constant probability, $(1-\gamma)$, independently of the time elapsed since the last adjustment. When a producer receives a signal, she chooses her new price, P_{tj}^*, to maximize $E_t \sum_{k=0}^{\infty} \gamma^k Q_{t+k,t+k+1}(P_{tj}^* - MC_{t+kj})Y_{t+kj}$ subject to the demand curve $Y_{t+kj} = (\frac{P_{tj}^*}{P_{t+k}})^{-\lambda} Y_{t+k}$. Optimization implies $\sum_{k=0}^{\infty} \gamma^k E_t\{Q_{t+k,t+k+1}Y_{t+kj}(P_{tj}^* - \frac{\lambda}{\lambda-1}\frac{1}{1-\tau^\lambda}MC_{t+k})\} = 0$ where $\tau^\lambda = -(\lambda-1)^{-1}$ is a subsidy that, in equilibrium, eliminates the monopolistic competitive distortion . Given the pricing assumption, the aggregate price index is $P_t = [\gamma P_{t-1}^{1-\lambda} + (1-\gamma)P_t^{*1-\lambda}]^{\frac{1}{1-\lambda}}$. When prices are flexible, $P_t = \frac{\lambda}{\lambda-1}\frac{1}{1-\tau^\lambda}MC_t, \forall t$.

Government's income consists of seigniorage, tax revenues minus subsidies to the firms and proceeds from new debt issue. The government budget constraint is $P_t C_t^g + \tau^\lambda P_t Y_t - \tau^l w_t P_t N_t - \tau^k(r_t - \delta^p)P_t K_t - P_t T_t + B_t + M_t = R_t^{-1}B_{t+1} + M_{t+1}$. We treat tax rates on labor and capital income parametrically; assume that the government takes market prices, hours and capital as given, and that B_t endogenously adjusts to satisfy the budget constraint. To guarantee a non-explosive solution for debt (see e.g., Leeper (1991)), we assume a tax rule of the form $\frac{T_t}{T^{ss}} = [(\frac{B_t}{Y_t})/(\frac{B^{ss}}{Y^{ss}}))]^{\phi_b}$, where the superscript ss indicates steady states.

Finally, an independent monetary authority sets the nominal interest rate according to $\frac{R_t}{R^{ss}} = (\frac{\pi_t}{\pi^{ss}})^{\phi_\pi} u_t$, where π_t is current inflation, and u_t is a policy shock. Given this rule, the authority stands ready to supply nominal balances that the private sector demands.

We assume that the three exogenous processes $S_t = [Z_t, C_t^g, u_t]'$ evolve according to $\log(S_t) = (I_3 - \boldsymbol{\varrho}) \log(\overline{S}) + \boldsymbol{\varrho} \log(S_{t-1}) + V_t$, where I_3 is a 3×3 identity matrix, $\boldsymbol{\varrho}$ is a 3×3 diagonal matrix with all the roots less than one in modulus, $\overline{S}$ is the mean of S and the 3×1 innovation vector V_t is a zero-mean, white noise process [2].

Table 1: Initial values and ranges for the parameters

	σ	$1-a$	ς	θ_n	b	δ	α	τ^l	τ^k	$(\frac{C^g}{Y})^{ss}$	ϕ_π	ϕ_b	γ	λ	θ_M	ρ_Z	ρ_{C_g}	ρ_u
$\mathcal{A}_0$	2.0	0.05	1.0	1.0	3	0.025	0.3	0.2	0.05	0.10	1.7	1.5	0.6	10	2.0	0.8	0.8	0.8
$\mathcal{A}_u$	3.0	0.1	3.0	1.5	10	0.05	0.4	0.3	0.12	0.07	2.5	2.4	0.85	8.0	4.0	0.95	0.95	0.9
$\mathcal{A}_l$	1.3	0.0	0.5	0.5	0.1	0.013	0.2	0.0	0.02	0.15	1.1	1.1	0.4	12.0	1.0	0.6	0.6	0.6

Let $\mathcal{A}$ represent the vector of parameters of the model, excluding β and $\frac{B^{ss}}{Y^{ss}}$ which are set to 0.99 and 1.2, respectively. We assume that $\mathcal{A}_t = \mathcal{A}_{t-1} + e_t$, where e_t is drawn from a truncated normal distribution so that at each t, $\mathcal{A}_t \in [\mathcal{A}_l, \mathcal{A}_u]$, where $\mathcal{A}_l$ and $\mathcal{A}_u$ are selected i) to cover the range of existing estimates, ii) because of stability considerations (see table 1 for these ranges together with the initial conditions). For each $t = 1, \ldots, 170$ we solve and simulate the model using $\mathcal{A}_t$ and compute responses to S_t. It turns out that in at least 68 percent of the time periods, disturbances that expand output produce the sign restrictions of table 2 at horizons ranging from contemporaneous up to, at least, 10 quarters.

Table 2: Identification restrictions

	Output	Inflation	Interest rate	Money
Technology	≥ 0	≤ 0	≤ 0	≤ 0
Government	≥ 0	≥ 0	≥ 0	≥ 0
Monetary	≥ 0	≥ 0	≤ 0	≥ 0

There are many ways of implementing these sign restrictions in the estimated VAR. The results we present are obtained using an acceptance sampling scheme where draws that jointly satisfy all the restrictions are kept and draws that do not are discarded. Following a suggestion by Tim Cogley we have also examined several importance sampling alternatives. The results we present are qualitatively independent of the sampling scheme used (more information on these schemes is in Gambetti, et. al. (2005)).

Since the restrictions in table 2 hold for several horizons, we are free to choose how many responses to restrict for identification purposes. In general, the smaller is the number of

[2] In a previous version we also allowed for government investment and government employment disturbances. Since the sign restrictions we emphasize are the same for the three types of government shocks, these two type of disturbances have been omitted.

restrictions, the larger is the number of draws satisfying them but, potentially, the weaker is the link between the model and the empirical analysis. As the number of restricted responses increases, we tight up the empirical analysis to the model more firmly, but the number of draws satisfying the restrictions may drop dramatically, making estimates of standard errors inaccurate. Since the trade-off is highly nonlinear, there is no straightforward way to optimize it. We present results obtained restricting horizons 0 and 1, since this choice seems to account for both concerns.

4 Estimation

The model (4)-(5) is estimated using Bayesian methods. We specify prior distributions for $\theta_0, \Sigma_0, \Omega_0$, and H_0 and use data up to t to compute posterior estimates of the structural parameters. Since our sample goes from 1960:1 to 2003:2, we initially estimate the model for the period 1960:1-1970:2 and then reestimate it 33 times moving the terminal date by one year up to 2003:2. Our estimation approach proceeds in two steps. First, we characterize the (truncated) posterior distribution of the reduced form parameters. Second, given the identification restrictions, we construct posteriors for the structural parameters. Since posterior distributions for the structural parameters are not available in a closed form, MCMC methods are used to simulate posterior sequences consistent with the information available up to time t. Construction of the truncated posterior for reduced form parameters is relatively standard (see e.g. Cogley and Sargent (2005)): it requires treating the parameters which are time varying as a block in a Gibbs sampler algorithm. Hence, at each t and in each Gibbs sampler cycle, one runs the Kalman filter and the simulation smoother, conditional on the draw of the other time invariant parameters, and discards paths for the coefficients generating explosive time series for the endogenous variables. The results we present are based on 20,000 draws for each t: 10,000 draws are used for burn-in, and we keep one out of five of the remaining draws to compute the posterior for reduced form parameters. After the non-explosive and the identification filters are used, about 200 remain for structural inference. Details on the methodology are in the appendix. The data comes from the FREDII data base of the Federal Reserve Bank of St. Louis and consists of GDP (GDPC1), GDP deflator inflation (ΔGDPDEF), the Federal funds rate (FEDFUNDS), hours of all persons in the non-farm business sector (HOANBS) and M1 (M1SL) - the mnemonics used by FREDII are in parenthesis. Four lags of each variable are used in the estimation.

5 The Results

5.1 The dynamics of structural volatility and persistence

Figure 1 presents the median and the 68% central posterior band for structural persistence (first row) and for structural volatility (second row) of inflation and output. Persistence is measured by the height of the spectrum at frequency zero; volatility by the value of the cumulative spectrum.

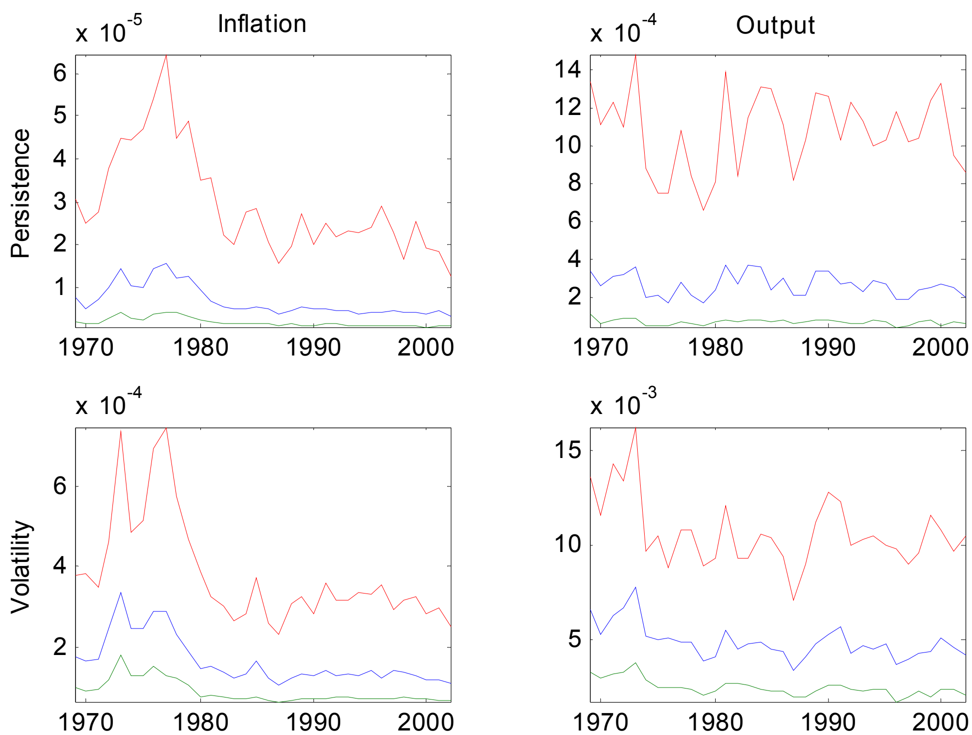

Figure 1: Dynamics of output and inflation, median and posterior 68% band

The figure displays a few interesting features. First, the median of inflation persistence shows a marked hump-shaped pattern: it displays a three-fold increase in 1973-1974 and then again in 1977-1978, it drops dramatically after that date, and since 1982 the posterior distribution of inflation persistence has displayed only marginal variations. The size of the drop is economically large: the median persistence in the mid 1985s is about 66 percent below its peak value. Second, variations over time in the posterior distribution of output persistence are random around a constant mean value. Third, the dynamics of the posterior structural inflation volatility replicate those of the posterior of structural inflation persistence, suggesting that the spectrum of inflation is relatively stable over time apart from the frequency zero. Fourth, the median of the posterior distribution of output volatility declines by roughly 25 percent from the 1973 peak to the end of the sample. Finally, since posterior standard errors are large, even remarkable changes, like those displayed by the median of inflation persistence, turn out to be a-posteriori insignificant.

While this outcome is consistent with the univariate, reduced form evidence presented by Pivetta and Reis (2004) and their classical statistical analysis, one may wonder why posterior standard errors are large. We have singled out three possibilities. First, it could be that some parameter draws are more consistent than others with the sign restrictions. If these draws imply larger volatility in the coefficients, it could be that the estimated variance of the error in the law of motion of the coefficients is larger for the accepted than the rejected draws. This turns out not to be the case: the two variances are statistically indistinguishable. Moreover, since posterior standard errors obtained with a non-structural Choleski decomposition are similar, our identification approach is not responsible for this outcome. Second, figure 1 is constructed using recursive analysis. Therefore, our estimates contain less information than those produced using, e.g., the full sample at each t. Although standard errors are somewhat reduced when smoothed estimates are considered, the changes are still statistically insignificant. Third, since our estimates are constructed allowing future coefficients to be random, this uncertainty could be responsible for the large standard errors we report. We have therefore repeated the computations averaging out future shocks to the coefficients and found that posterior standard errors are smaller, but by only 25 percent. Hence, even changing a number of features in our estimation approach, the observed changes in output and inflation persistence and volatility do not appear to be a-posteriori large.

Hence, while there is visual evidence of a decline in the median estimates of output and inflation volatility, the case for evolving volatility is considerably weakened once posterior standard errors are taken into account. This evidence should be contrasted with that obtained with univariate, in-sample, reduced form methods, for example by McConnell and Perez Quiroz (2001), or by Stock and Watson (2003), who overwhelmingly suggest the presence of a significant structural break in the variability of the two series. Consistent with the evidence contained in Cogley and Sargent (2001) and (2005), the posterior median of inflation persistence shows a declining trend. However, when structural, recursive, multivariate analysis is used, the case for evolving posterior distributions of persistence measures is also far weaker. Finally, the changes in output and inflation dynamics do not appear to be synchronized. Therefore, it is unlikely that a single explanation can account for the observed variations in output and inflation dynamics.

5.2 What drives variations in structural volatility and in persistence?

Recall that our structural model has implications for three types of disturbances, roughly speaking, technology, real demand and monetary shocks. Therefore, we can identify at most three of the five structural shocks driving the VAR and there will be a residual capturing unexplained variations in output and inflation volatility and persistence, which can be used to gauge how successful our identification exercise is.

Our three structural shocks explain between 50 and 65 percent of the variability of output and inflation on average at each t. We believe this magnitude is remarkable, given our analysis has disregarded, e.g. labor supply or investment specific shocks, which Chang and Schorfheide (2004) and Fisher (2006) have shown to be important in explaining output (and potentially inflation) fluctuations. In line with recent evidence (see Gali (1999)), the contribution of technology shocks to output fluctuations is the largest of the three but relatively low (25% on average), while monetary shocks explain a small portion of the fluctuations of both variables (14% of inflation and 12% of output fluctuations at medium-long term horizons).

Given that the spectrum at frequency ω is uncorrelated with the spectrum at frequency ω', when ω and ω' are Fourier frequencies, it is easy to compute the relative contribution of each of the three structural shocks to changes in the volatility and in the persistence of output and inflation. In fact, disregarding the constant and the trend, the (time varying) structural MA representation is $y_{it} = \sum_{j=1}^{5} \mathcal{B}_{jt}(\ell) e_{jt}$, where e_{it} is orthogonal to $e_{i't}$, $i' \neq i, i = 1, \dots, 5$. Since structural shocks are independent, the (local) spectrum of y_{it} at frequency ω is $S_{y_i}(\omega)(t) = \sum_{j=1}^{5} |\mathcal{B}_{jt}(\omega)|^2 S_{e_j}(\omega)(t)$. Therefore, the persistence in y_{it} due to structural shock j at time t is $S^j_{y_i}(\omega = 0)(t) = |\mathcal{B}_{jt}(\omega = 0)|^2 S_{e_j}(\omega = 0)(t)$ and the volatility in y_{it} due to structural shock j is $\sum_{\omega} S^j_{y_i}(\omega)(t)$. Intuitively, these measures are similar to historical decomposition numbers: they tell us what the time path of these statistics would have been if only one type of structural shock was present.

We divide the discussion into two parts. First, we examine the contribution of monetary policy shocks to the variations presented in figure 1. The large number of papers studying this issue and the discussion following the original contribution of Clarida, Gali and Gertler (2000) justify our focus. Second, we assess the role of the other two structural shocks in accounting for the observed changes.

It is useful to recall that, if the conventional wisdom is correct, the swings observed in the median of output and inflation volatility and inflation persistence should be accompanied by a significant increase and a decline in the values of these statistics produced by monetary shocks. Figure 2, which reports the median value of the evolution of the persistence and

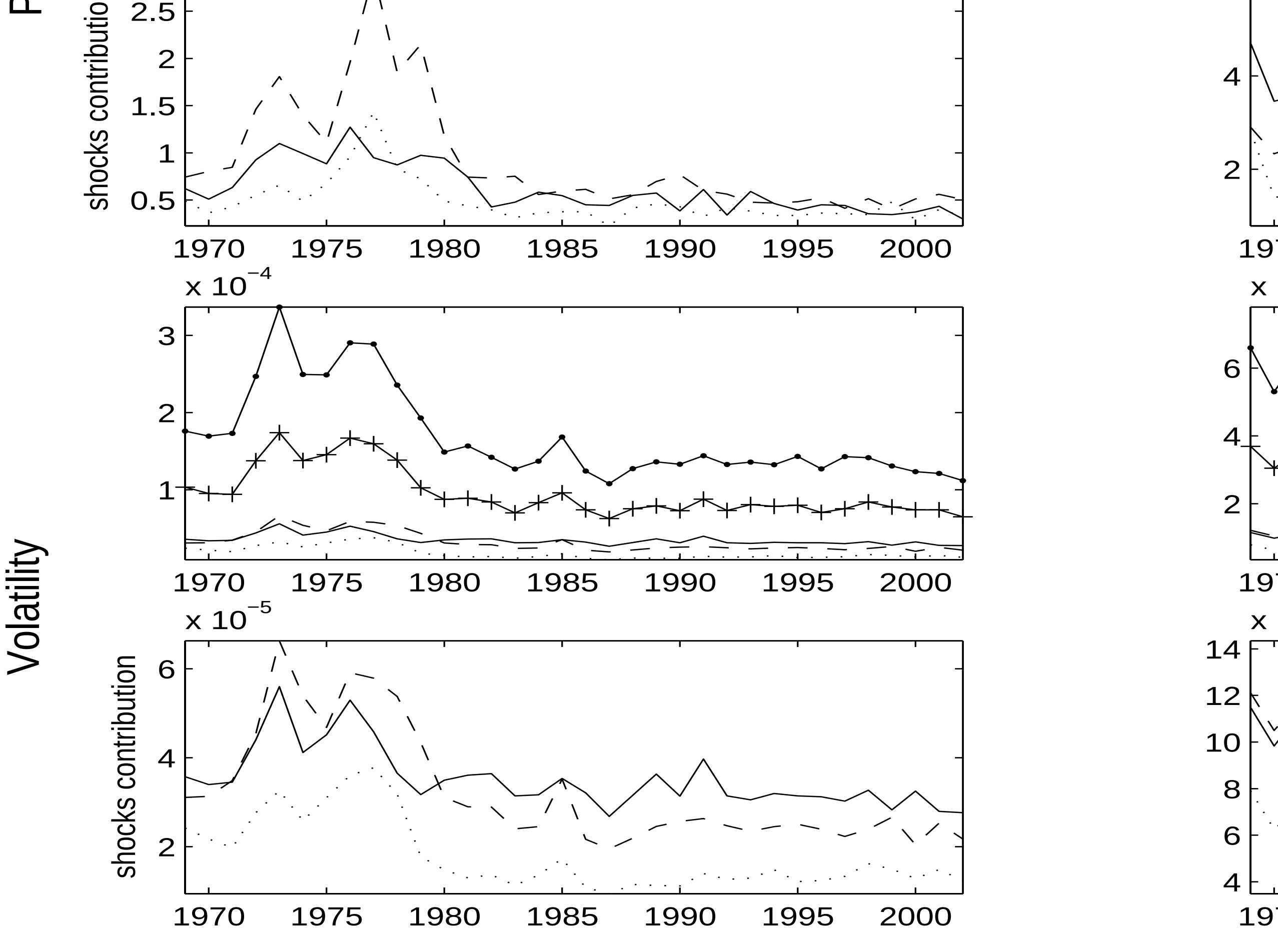

Figure 1: Figure 2: Supply (solid), real demand (dashed), monetary (dotted) shocks, sums (+), and total (circled), median values.

volatility of output and inflation and the component explained by each of the three shocks, only partially confirms this story. The time profile of the component of inflation persistence due to monetary shocks displays some variations over the sample and there are some swings in the 1970s. Since the share of persistence due to these shocks is increasing over time (median contribution in 2003 is about 30 percent larger than in the 1970s), the decline produced by other shocks must have been larger. The contribution of monetary policy shocks to output volatility shows a declining trend up to the middle of 1990s but the share of total volatility due to these shocks increases, once again, toward the end of the sample. It is only when looking at inflation volatility that the contribution of monetary shocks is statistically time varying. Morevoer, the ups and downs somewhat track the ups and downs in inflation volatility in the 1970s.

Several authors related changes in inflation persistence to changes in the stance of monetary policy (see e.g. Cogley and Sargent (2001), or Benati (2005)), or to the way monetary shocks are transmitted to the economy (see e.g. Leeper and Zha (2003), or Sims and Zha

(2004)). Contrary to the views of many policymakers, our results suggest that monetary policy could not have been the major factor behind the observed decline in inflation persistence and that other shocks may have played a larger role. Similarly, the claim that the increased stability observed in the US economy since the mid 1980s is a result of a more conservative monetary policy actions does not square well with figure 2: the decline in output volatility does not seem to be explained by monetary policy changes.

What is the role of the other two shocks? Figure 2 suggests that real demand shocks account for a considerable portion of both the increase and the decrease in inflation persistence observed in the 1970s. Furthermore, demand and supply shocks substantially contribute to the swings of inflation volatility in the 1970s and are largely responsible for the two volatility peaks. On the other hand, supply shocks drive the fluctuations over time in output persistence, while both real demand and supply shocks account for the majority of the swings in output volatility observed over the sample. Therefore, our identification scheme attributes to real demand shocks and to supply shocks both the Great Inflation of the 1970s and the subsequent disinflation and the decline in output volatility experienced in the 1970s. Interestingly, the two non-identified shocks do not contribute to any of these episodes. Hence, whatever is left out of the analysis is unimportant to explain the phenomenon of interest.

5.3 Time Varying Transmission?

Since the component of inflation and output volatility and persistence due to a structural shock may vary because the variance of structural shocks at frequency ω (i.e. $S_{e_j}(\omega)(t)$) changes, or because the transmission mechanism (i.e. $|\mathcal{B}_{jt}(\omega)|^2$) of shocks changes, it is worth disentangling the two sources of variations to understand whether it is changes in the structure or in the volatility of the shocks which is responsible for the swings reported in figure 1. Figure 3 plots the median responses of output and inflation to the three structural shocks at three dates: 1973, 1986 and 2003. Since responses obtained for dates from 1983 to 1994 are indistinguishable from those of 1986, the responses obtained in this year should be considered as representative of the dynamics present in this decade. Since the impulse is the same in every period, the evolution of these responses over time gives us an idea of the changes in the transmission in isolation from the changes in the distribution of the shocks.

Three features of figure 3 are worth discussing. First, while there are notable changes when comparing the dynamics generated in 1973 with the other two time periods, responses in 1986 and 2003 have similar shape and, roughly, the same magnitude - the structure of the US economy has changed very little in the last 20 years. Second, there are changes in the persistence of the responses but minor changes in their magnitude. For example, output responses to all shocks are much less persistent in the last two samples as are inflation responses to monetary and demand shocks but the size of the initial or of the maximal response is very similar in the three samples. Third, and relatively speaking, time variations in the structure are more evident in the inflation responses to demand shocks and in the output responses to supply shocks. Interestingly, none of the variations present in the responses of the two variables to monetary shocks is a-posteriori significant.

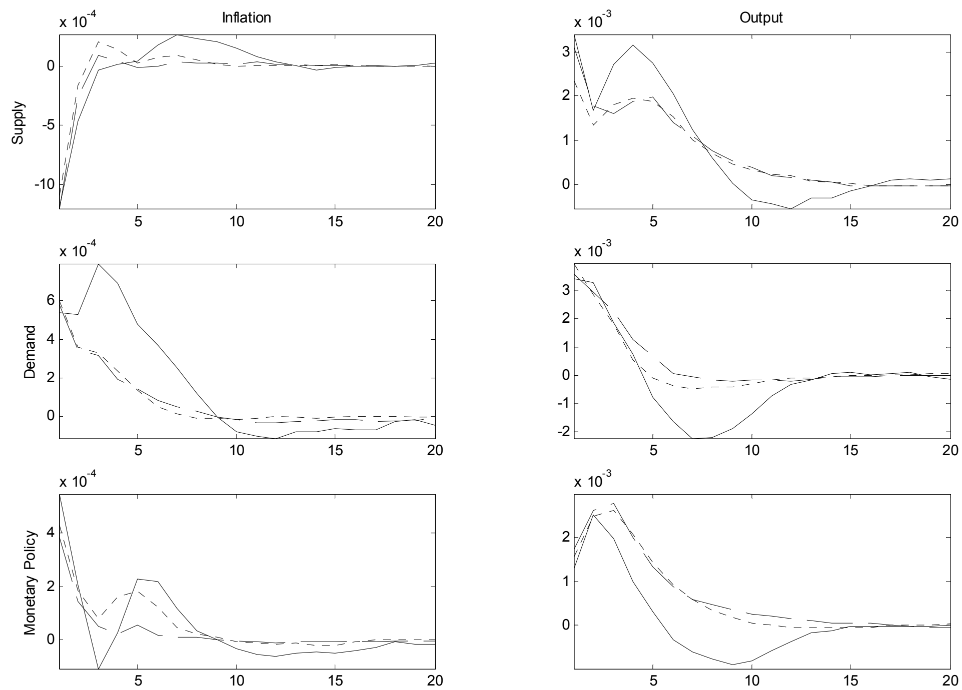

Figure 3: Impulse responses, 1973 (solid), 1986 (dotted), 2003 (dashed)

Hence, while figure 3 supports the idea that there have been changes in the way shocks have been transmitted to output and inflation, two features make it hard to reconcile with the conventional wisdom: the changes took place prior 1983 and therefore predate the major monetary policy changes the literature has emphasized; and variations in response to monetary policy shocks are minor in size and a-posteriori insignificant.

5.4 Time Varying volatility of the structural shocks?

To examine whether there have been significant changes in the relative distribution of the structural shocks, we plot in figure 4 the time profile of the posterior median of their standard deviation. Real demand shocks are those associated with the first structural equation (normalized on output), supply shocks are those associated with the second structural equation (normalized on inflation) and the monetary policy shocks are those associated with the third structural equation (normalized on the nominal rate).

Overall, the volatility of supply and of the monetary policy disturbances has declined over time. However, while the decline is smoother for the former, it is much more abrupt for the latter, where a drop of about 30% in the late 1970s is evident. The standard deviation of demand shocks is higher on average than for the other two shocks and it is relatively stable over time. Interestingly, the decline in the volatility of technology and monetary policy shocks terminates by the late 1970s and since then only random variations are detected.

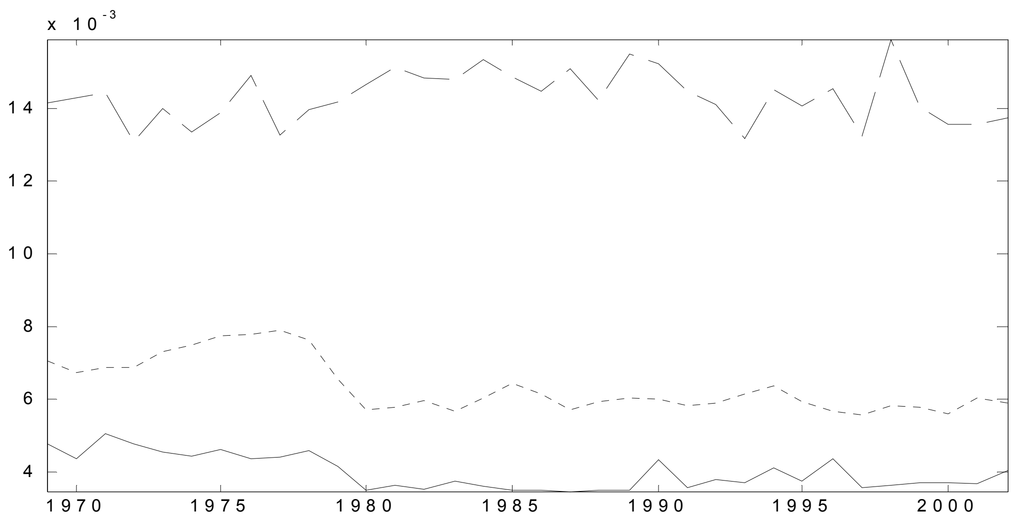

Figure 4: Standard deviations: supply (solid), real demand (dashed), monetary (dotted) shocks

The decline in the volatility of monetary policy shocks appears to precede the one found by Sims and Zha (2004). However, differences can be reconciled if one takes into account different estimation techniques and the different ways in which these volatilities are computed (recursive vs. smoothed estimates). Several authors have argued that there is very little evidence that the monetary policy rule and the transmission of monetary policy shocks have changed over time. Instead, they have suggested that drops in the volatility of monetary policy shocks could be responsible for the fall in the variability of output and inflation. Our results are consistent with this view but also suggest that the sharp increase and rapid decline in the variability of reduced form output and inflation forecast errors observed at the end of the 1970s are due, in part, to variations in the distribution from which technology shocks are drawn.

Since the volatility of demand shocks is of an order of magnitude larger than the one of the other two shocks, and since the volatility of demand shocks oscillates around a constant mean value, one must conclude that changes over time in the standard deviation of structural shocks can not fully account for changes in the dynamics of inflation persistence and volatility. On the other hand, changes in the volatility of output are explained, in part, by changes in the volatility of supply shocks.

5.5 Some counterfactuals

The previous two subsections have shown that changes in the transmission of certain shocks and in their volatility could be important for understanding the decline in the median value of the inflation persistence and of the volatility of inflation and output.

To further assess the role of the two sources of variations and quantitatively measure their contribution to the observed variations, we perform a few counterfactual exercises where, in turn, we change either the volatility of structural shocks or their transmission properties and recalculate the values of persistence and volatility which would have been obtained in these alternative scenarios. We have selected as relevant baseline dates 1973, 1986 and 2003: if the statistics of interest do not substantially vary as we input, say, the coefficients estimated in 1986 together with the variance of 1973 relative to the baseline 1973 value, then variations in the transmission of shocks can not account for the decline in inflation persistence observed from 1973 to 1986.

While these counterfactuals are meaningful only to the extent that the alternative scenarios are relevant for the historical period considered, it turns out that the posterior distributions for the coefficients and the volatility of the shocks are large enough that the point estimates are always within a two standard deviations posterior credible set obtained at the other two dates. Therefore, these counterfactuals reasonably represent variations within the same regime.

Table 3, which contains the results of our counterfactuals, confirms the previous conclusions. For example, take 1973 as baseline. The first four rows of the table suggest that imputing the estimated variance of the three structural shocks in 1986 and 2003, while maintaining the level of the coefficients in 1973 would have marginally changed the level of inflation persistence and the level of output and inflation volatility while output persistence would have somewhat declined. On the other hand, maintaining the structural variances estimated in 1973 and inputting a point estimate of the coefficients in 1986 would have reduced inflation persistence by about 65 percent, inflation volatility by about 50 percent and output volatility by about 40 percent. Inputting the point estimate of the coefficients in 2003 would have further reduced the value of inflation persistence and volatility but changed the level of output volatility very little.

Table 3: Counterfactuals

1973	**Baseline**	**Variance_1986**	**Variance_2003**	**Coefficients_1986**	**Coefficients_2003**
Persistence					
Inflation	0.000006811	0.000006450	0.000006425	0.000002439	0.000001654
Output	0.000152919	0.000134841	0.000138450	0.000156270	0.000106202
Volatility					
Inflation	0.000271981	0.000241438	0.000245108	0.000132590	0.000115473
Output	0.005685535	0.005073469	0.005141907	0.003647230	0.003427056

1986	**Baseline**	**Variance_1973**	**Variance_2003**	**Coefficients_1973**	**Coefficients_2003**
Persistence					
Inflation	0.000002088	0.000002439	0.000002151	0.000006450	0.000001444
Output	0.000131490	0.000156270	0.000135506	0.000134841	0.000089892
Volatility					
Inflation	0.000104557	0.000132590	0.000113525	0.000241438	0.000096407
Output	0.003064628	0.003647230	0.003171813	0.005073469	0.003015512

2003	**Baseline**	**Variance_1973**	**Variance_1986**	**Coefficients_1973**	**Coefficients_1986**
Persistence					
Inflation	0.000001433	0.000001654	0.000001444	0.000006425	0.000002151
Output	0.000091910	0.000106202	0.000089892	0.000138450	0.000135506
Volatility					
Inflation	0.000100914	0.000115473	0.000096407	0.000245108	0.000113525
Output	0.003024093	0.003427056	0.003015512	0.005141907	0.003171813

The other two panels of the table roughly tell the same story. While persistence and volatility of both variables would have increased somewhat if variances from 1973 were used with coefficient estimates of the other two dates, the largest increases occur when we use the estimated coefficients of 1973 with the variances of the other two dates. Therefore, while changes in the structure and in the variance of the structural shocks play a role in accounting for the dynamics of the statistics we have shown in figures 1 and 2, it is changes in the transmission of shocks that dominate quantitatively. This result together with the conclusions we have reached in section 5.3 suggest that inflation volatility and persistence declined because the way the private sector transformed demand shocks into inflation dynamics has changed - there is less persistence and a smaller lagged effect over the last 20 years - and that output volatility has subsided because of the way the economy transformed supply shocks in output fluctuations - again, the effect is less persistent and the lagged effect is much smaller.

One can think of several reasons for why these changes have occurred. For example, the slope of the Phillips curve may have changed and this may have altered the way inflation reacts to demand shocks. This, in turn, could be due to changes in the stickiness of prices, to the inflation indexation mechanism, or to variations in the labor supply elasticity. Canova (2005), recursively estimating a small scale DSGE for the US economy, finds that indeed the slope of the Phillips curve has varied substantially over time and that variations in the elasticity of labor supply are responsible for these changes. Changes in the persistence and magnitude of output to supply shocks could be the result of better inventory management, as suggested by McConnell and Perez Quiroz (2001), of changes in the underlying long run level of labor productivity (Gordon (2003)) or of changes in the risk attitude of consumers. However, to pin down which of the parameters of the agents' preferences and technologies have changed, one needs to go beyond structural VAR analysis and study time variations in the context of a microfounded structural model.

6 Conclusions

This paper examined structural sources of output and inflation volatility and persistence and attempted to draw some conclusions about the causes of the variations experienced in the US economy over the last 30 years. There has been a healthy discussion in the literature on this issue, thanks to the work of Clarida, Gali and Gertler (2000), Cogley and Sargent (2001) (2003), Boivin and Giannoni, (2002), Leeper and Zha (2003), Sims and Zha (2004), Lubik and Schorfheide (2004), Primiceri (2005) and Canova and Gambetti (2004) among others and, although opinions differ, there have been remarkable methodological improvements in the study of time variations in the structure of the economy and in the distribution of the shocks.

In this paper, we contribute to advance the technical frontiers by estimating a structural time varying coefficient VAR model; by identifying a number of structural shocks using sign restrictions derived from a general DSGE model; by providing recursive analysis, consistent with information available at each point in time; and by using frequency domain tools to address questions concerning time variations in persistence and volatility. In our opinion, the paper also enhances our understanding of the causes of the observed variations in output and inflation dynamics. In particular, we show that while there are time variations in both the volatility of output and inflation and in the persistence of inflation, the differences are a-posteriori insignificant. Standard errors are larger than in other studies for two reasons: our recursive analysis makes them depend on the information available at each t; and shocks to future parameters are not averaged out.

We show that time variations in inflation persistence are primarily due to a decline in the relative contribution of real demand shocks while output and inflation volatility change primarily because the contribution of real demand and supply shocks varies over time. Furthermore, we detect variations in the transmission of demand disturbances to inflation and supply shocks to output and some changes in the variances of technology and monetary policy shocks. Overall, consistent with the work of Sims and Zha (2004), Canova and Gambetti (2004) and Primiceri (2005), our analysis attributes only a small role to monetary policy in the evolution of the persistence and volatility of inflation and output over time and, consistent with the work of McConnell and Perez Quiroz (2001) and Gordon (2003), suggests that changes in the way the private sector responds to supply and real demand shocks, together with changes in the variability of structural shocks, may be a key to understanding the nature of the changes we have observed over the last 30 years

To put our results in proper perspective, a few words of caution are needed. First, by construction, our analysis excludes the possibility that in one period of history the monetary policy rule produced indeterminate equilibria. Therefore, our analysis differs from the one of Lubik and Schorfheide (2004). One interpretation of our results is that a large portion of the observed variations can be accounted for without any need to resort to sunspot explanations. Second, while the decline in the volatility of some of the shocks is consistent with exogenous explanations of the changes in output and inflation dynamics, such a pattern is also consistent with explanations which give policy actions some role. For example, if monetary policy had a better control of inflation expectations over the last 20 years and no measure of inflation expectations was included in the VAR, such an effect may show up as a reduction in the variance of the shocks.

Clearly, much work still needs to be done. We think it would be particularly useful to study the structural shocks we have extracted in details, to look at how they correlate with what economists think technological sources of disturbances are and whether they proxy for missing variables or shocks. The model has implications for a number of variables which are excluded from the empirical analysis. Enlarging the size of our VAR could provide additional evidence on the reasonableness of the structural disturbances we have extracted. There are many studies using US data, but very few exercises have looked at other countries, or compared sources and causes of variations in output and inflation volatility and persistence across countries. Finally, understanding which of the parameters describing the behavior of the private sector has changed may help to tie up the empirical evidence we have uncovered.

Appendix

Priors

We choose prior densities which give us analytic expressions for the conditional posteriors of subvectors of the unknowns. Let T be the end of the estimation sample and let K_1 be the number of periods for which the identifying restrictions must be satisfied. Let $H_T = \rho(\varphi_T)$ be a rotation matrix whose columns represents orthogonal points in the hypersphere and let φ_T be a vector in R^6 whose elements are $U[0,1]$ random variables. Let $\mathcal{M}_T$ be the set of impulse response functions at time T satisfying the restrictions and let $F(\mathcal{M}_T)$ be an indicator function which is one if the identifying restrictions are satisfied, that is, if $(\Psi^i_{T+1,1}, ..., \Psi^i_{T+K_1,K_1}) \in \mathcal{M}_T$, and zero otherwise. Let the joint prior for θ^{T+K_1}, Σ_T, Ω_T and H_T be

$$p(\theta^{T+K_1}, \Sigma_T, \Omega_T, \omega_T) = p(\theta^{T+K}|\Sigma_T, \Omega_T)p(\Sigma_T, \Omega_T)F(\mathcal{M}_T)p(H_T) \tag{8}$$

Assume that $p(\theta^{T+K}|\Sigma_T, \Omega_T) \propto I(\theta^{T+K})f(\theta^{T+K}|\Sigma_T, \Omega_T)$ where $f(\theta^{T+K}|\Sigma_T, \Omega_T) = f(\theta_0)$ $\prod_{t=1}^{T+K} f(\theta_t|\theta_{t-1}, \Sigma_t, \Omega_t)$ and $I(\theta^{T+K}) = \prod_{t=0}^{T+K} I(\theta_t)$. Since $f(\theta^{T+K}|\Sigma_T, \Omega_T)$, is normal $p(\theta^{T+K}|\Sigma, \Omega_T)$ is truncated normal.

We assume that Σ_0 and Ω_0 have independent inverse Wishart distributions with scale matrices Σ_0^{-1}, Ω_0^{-1} and degrees of freedom ν_{01} and ν_{02}, and assume that $\Sigma_t = \alpha_1\Sigma_{t-1} + \alpha_2\Sigma_0$ and $\Omega_t = \alpha_3\Omega_{t-1} + \alpha_4\Omega_0$, $\forall t$, where $\alpha_i, i = 1, 2, 3, 4$ are fixed. We also assume that the prior for θ_0 is truncated Gaussian independent of Σ_T and Ω_T, i.e. $f(\theta_0) \propto I(\theta_0)N(\bar{\theta}, \bar{P})$. Finally we assume a uniform prior $p(H_T)$, since all rotation matrices are a-priori equally likely. Collecting pieces, the joint prior is:

$$p(\theta^{T+K_1}, \Sigma_T, \Omega_T, \omega_T) \propto I(\theta^{T+K})F(\mathcal{M}_T)[f(\theta_0) \prod_{t=1}^{T+K} f(\theta_t|\theta_{t-1}, \Sigma_t, \Omega_t)]p(\Sigma_t)p(\Omega_t) \tag{9}$$

We "calibrate" prior parameters by estimating a fixed coefficients VAR using data from 1960:1 up to 1969:1. We set $\bar{\theta}$ equal to the point estimates of the coefficients and $\bar{P}$ to the estimated covariance matrix. Σ_0 is equal to the estimated covariance matrix of VAR innovations, $\Omega_0 = \varrho\bar{P}$ and $\nu_{10} = \nu_{20} = 4$ (so as to make the prior close to non-informative). After some experimentation we select $\alpha_2 = \alpha_2 = 0, \alpha_2 = \alpha_4 = 1$. The parameter ϱ measures how much time variation is allowed in coefficients. Although as T grows the likelihood dominates, the choice of ϱ matters in finite samples. We choose ϱ as a function of T i.e. for the sample 1969:1-1981:2, $\varrho = 0.0025$; for 1969:1-1983:2, $\varrho = 0.003$; for 1969:1-1987:2, $\varrho = 0.0035$; for 1969:1-1989:2, $\varrho = 0.004$; for 1969:1-1995:4, $\varrho = 0.007$; for 1969:1-1999:1, $\varrho = 0.008$, and for 1969:1-2003:2, $\varrho = 0.01$. This range of values implies quite conservative prior coefficient variations: in fact, time variation accounts between 0.35 and a 1 percent of the total coefficients' standard deviations.

Since impulse response functions depend on $\Phi_{T+k,k}$, S and H_T, we first characterize the posterior of $\theta^{T+K}, \Sigma_T, \Omega_T$, which are used to construct $\Phi_{T+k,k}$ and S, and then describe an approach to sample from them.

Posteriors

To draw posterior sequences we need $p(H_T, \theta_{T+1}^{T+K}, \theta^T, \Sigma_T, \Omega_T|y^T)$, which is analytically intractable. However, note that

$$\begin{aligned} p(H_T, \theta_{T+1}^{T+K}, \theta^T, \Sigma_T, \Omega_T|y^T) &\equiv p(H_T, \theta^{T+K}, \Sigma_T, \Omega_T|y^T) \\ &\propto p(y^T|H_T, \theta^{T+K}, \Sigma_T, \Omega_T)p(H_T, \theta^{T+K}, \Sigma, \Omega_T) \end{aligned} \quad (10)$$

Second, since the likelihood is invariant to any orthogonal rotation $p(y^T|H_T, \theta^{T+K}, \Sigma_T, \Omega_T) = p(y^T|\theta^{T+K}, \Sigma_T, \Omega_T)$. Third, $p(H_T, \theta^{T+K}, \Sigma_T, \Omega_T) = p(\theta^{T+K}, \Sigma_T, \Omega_T)F(\mathcal{M}_T)p(H_T)$. Thus

$$p(H_T, \theta^{T+K}, \Sigma_T, \Omega_T|y^T) \propto p(\theta^{T+K}, \Sigma_T, \Omega_T|y^T)F(\mathcal{M}_T)p(H_T) \quad (11)$$

where $p(\theta^{T+K}, \Sigma_T, \Omega_T|y^T)$ is the posterior distribution for the reduced form parameters, which, in turn can be factored as

$$p(\theta^{T+K}, \Sigma_T, \Omega_T|y^T) = p(\theta_{T+1}^{T+K}|y^T, \theta^T, \Sigma_T, \Omega_T)p(\theta^T, \Sigma_T, \Omega_T|y^T) \quad (12)$$

The first term on the right hand side of (12) represents beliefs about the future and the second term the posterior density for states and hyperparameters. Note that $p(\theta_{T+1}^{T+K}|y^T, \theta^T, \Sigma_T, \Omega_T) = p(\theta_{T+1}^{T+K}|\theta^T, \Sigma_T, \Omega_T) = \prod_{k=1}^{K} p(\theta_{T+k}|\theta_{T+k-1}, \Sigma_T, \Omega_T)$ because the states are Markov. Finally, since θ_{T+k} is conditionally truncated normal with mean $\theta_{T+k-1}$and variance Ω_T,

$$\begin{aligned} p(\theta_{T+1}^{T+K}|\theta^T, \Sigma_T, \Omega_T) &= I(\theta_{T+1}^{T+K}) \prod_{k=1}^{K} f(\theta_{T+k}|\theta_{T+k-1}, \Sigma_T, \Omega_T) \\ &= I(\theta_{T+1}^{T+K}) f(\theta_{T+1}^{T+K}|\theta_T, \Sigma_T, \Omega_T) \end{aligned} \quad (13)$$

The second term in (12) can be factored as

$$p(\theta^T, \Sigma_T, \Omega_T|y^T) \propto p(y^T|\theta^T, \Sigma_T, \Omega_T)p(\theta^T, \Sigma_T, \Omega_T) \quad (14)$$

The first term in (14) is the likelihood function which, given the states, has a Gaussian shape so that $p(y^T|\theta^T, \Sigma_T, \Omega_T) = f(y^T|\theta^T, \Sigma_T, \Omega_T)$. The second term is the joint posterior for states and hyperparameters. Hence:

$$p(\theta^T, \Sigma_T, \Omega_T|y^T) \propto f(y^T|\theta^T, \Sigma_T, \Omega_T)p(\theta^T|\Sigma_T, \Omega_T)p(\Sigma_T, \Omega_T) \quad (15)$$

Furthermore, since $p(\theta^T|\Sigma_T,\Omega_T) \propto I(\theta^T)f(\theta^T|\Sigma_T,\Omega_T)$ where $f(\theta^T|\Sigma_T,\Omega_T) = f(\theta_0|\Sigma_T,\Omega_0)$ $\prod_{t=1}^T f(\theta_t|\theta_{t-1},\Sigma_t,\Omega_t)$and $I(\theta^T) = \prod_{t=0}^T I(\theta_t)$, we have

$$p(\theta^T,\Sigma,\Omega_T|y^T) \propto I(\theta^T)f(y^T|\theta^T,\Sigma_T,\Omega_T)f(\theta^T|\Sigma_T,\Omega_T)p(\Sigma_T,\Omega_T) = I(\theta^T)p_u(\theta^T,\Sigma_T,\Omega_T|y^T) \tag{16}$$

where $p_u(\theta_T,\Sigma_T,\Omega_T|y^T) \equiv f(y^T|\theta^T,\Sigma_T,\Omega_T)f(\theta^T|\Sigma_T,\Omega_T)p(\Sigma_T,\Omega_T)$ is the posterior density obtained if no restrictions are imposed. Collecting pieces, we finally have

$$p(H_T,\theta_{T+1}^{T+K},\theta^T,\Sigma_T,\Omega_T|y^T) \quad \propto \quad \left[\prod_{t=0}^T I(\theta_t)f(\theta_{T+1}^{T+K}|\theta^T,\Sigma_T,\Omega_T)I(\theta^T)p_u(\theta^T,\Sigma_T,\Omega_T|y^T)\right] F(\mathcal{M}_T)p(H_T) \tag{17}$$

Drawing structural parameters

Given (17) draws for the structural parameters can be obtained as follows

1. Draw $(\theta^T,\Sigma_T,\Omega_T)$ from the unrestricted posterior $p_u(\theta^T,\sigma_T,\Omega_T|y^T)$ via the Gibbs sampler (see below). Apply the filter $I(\theta^T)$.
2. Given $(\theta^T,\Sigma_T,\Omega_T)$, draw future states θ_{T+1}^{T+K}, i.e. obtain draws of u_{T+k} from $N(0,\Omega_T)$ and iterate in $\theta_{T+k} = \theta_{T+k-1} + u_{T+k}$, K times. Apply the filter $I(\theta^{T+K})$.
3. Draw $\varphi_{i,T}$ for $i = 1,...,6$ from a $U[0,1]$. Draw $H_T = \rho(\varphi_T)$.
4. Given Σ, find the matrix S_T, such that $\Sigma_T = S_T S_T'$. Construct J_T^{-1}.
5. Compute $(\Psi_{T+1,1}^{i,\ell},...,\Psi_{T+K,K}^{i,\ell})$ for each replication ℓ. Apply the filter $F(\mathcal{M}_T)^\ell$ and keep the draw if the identification restrictions are satisfied.

Drawing reduced form parameters

The Gibbs sampler we use to compute the posterior for the reduced form parameters iterates on two steps. The implementation is identical to Cogley and Sargent (2001).

• Step 1: States given hyperparameters
Conditional on (y^T,Σ_T,Ω_T), the unrestricted posterior of the states is normal and $p_u(\theta^T|y^T,\Sigma_T,\Omega_T) = f(\theta_T|y^T,\Sigma_T,\Omega_T)\prod_{t=1}^{T-1} f(\theta_t|\theta_{t+1},y^t,\Sigma_t,\Omega_t)$. All densities on the right hand side are Gaussian and their conditional means and variances can be computed using a simulation smoother. Let $\theta_{t|t} \equiv E(\theta_t|y^t,\Sigma_t,\Omega_t)$; $P_{t|t-1} \equiv Var(\theta_t|y^{t-1},\Sigma_t,\Omega_t)$; $P_{t|t} \equiv Var(\theta_t|y^t,\Sigma_t,\Omega_t)$. Given

$P_{0|0}$, $\theta_{0|0}$, Ω_0 and Σ_0, we compute Kalman filter recursions

$$\begin{aligned} P_{t|t-1} &= P_{t-1|t-1} + \Sigma_t \\ \mathcal{K}_t &= (P_{t|t-1} X_t)(X_t' P_{t|t-1} X_t + \Omega_t)^{-1} \\ \theta_{t|t} &= \theta_{t-1|t-1} + \mathcal{K}_t (y_t - X_t' \theta_{t-1|t-1}) \\ P_{t|t} &= P_{t|t-1} - \mathcal{K}_t (X_t' P_{t|t-1}) \end{aligned} \tag{18}$$

The last iteration gives $\theta_{T|T}$ and $P_{T|T}$ which are the conditional means and variance of $f(\theta_t|y^T, \Sigma, \Omega_T)$. Hence $f(\theta_T|y^T, \Sigma, \Omega_T) = N(\theta_{T|T}, P_{T|T})$.

• Step 2: Hyperparameters given states

Conditional on the states and the data ε_t and u_t are observable and Gaussian. Combining a Gaussian likelihood with an inverse-Wishart prior results in an inverse-Wishart posterior, so that $p(\Sigma_t|\theta^T, y^T) = IW(\Sigma_{1t}^{-1}, \nu_{11}); p(\Omega_t|\theta^T, y^T) = IW(\Omega_{1t}^{-1}, \nu_{12})$ where $\Sigma_{1t} = \Sigma_0 + \Sigma_T$, $\Omega_{1t} = \Omega_0 + \Omega_T$, $\nu_{11} = \nu_{01} + T$, $\nu_{12} = \nu_{02} + T$ and Σ_T and Ω_T are proportional to the covariance estimator $\frac{1}{T}\Sigma_T = \frac{1}{T}\sum_{t=1}^{T} \varepsilon_t \varepsilon_t'; \frac{1}{T}\Omega_T = \frac{1}{T}\sum_{t=1}^{T} u_t u_t'$. Under regularity conditions and after a burn-in period, iterations on these two steps produce draw from $p_u(\theta^T, \Sigma, \Omega|y^T)$.

In our exercises T varies from 1970:2 to 2003:2. For each of these T, 20000 iterations of the Gibbs sampler are made. CUMSUM graphs are used to check for convergence and we found that the chain had converged roughly after 2000 draws for each date in the sample. The densities for the parameters obtained with the remaining draws are well behaved and none is multimodal.

Computing structural impulse responses and spectra

Given a draw from the posterior of the structural parameters, calculation of impulse responses to VAR shocks is straightforward. In fact, given a draw for $(\theta^{T+K}, \Sigma, \Omega_T, H_{T+1})$ we calculate $\Psi_{T+k,k}$, compute the posterior median and the 68% central credible set at each horizon k across draws. Then, spectra are computed as described in section 5.2.

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