

# Does tax evasion modify the redistributive effect of tax progressivity?\*

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## Abstract

This paper shows how tax evasion modifies the redistributive effect of tax progressivity. The analysis, which is conducted by means of the standard tax evasion model, suggests a possible reduction of the redistributive effect of progressive income taxation. This issue is important in order to empirically evaluate and compare the redistributive effect of tax systems in economies with different tax evasion levels.

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## *I Introduction*

Tax evasion is a phenomenon inherent to the existing tax systems since individuals dislike paying taxes. Hence, economic consequences of tax systems can then be driven by this problem. In most countries, tax system are progressive. Although there are several definitions and alternative measures of progressivity, the one most commonly used is that requiring that the average tax rate increases with income. In this context, according to the Lorenz criterion, the after-tax income is more equally distributed than the before-tax income. In this paper, we analyze the alteration that the existence of tax evasion brings about in the redistributive effect of tax progressivity.

Some authors, such as Yitzhaki (1987) and Goerke (2003), have examined the relationship between tax evasion and progressivity, and they found that a higher level of progressivity tends to discourage the level of evasion. However, only a few studies have analyzed the effect of more evasion on both income distribution and tax progressivity. We can mention the theoretical analysis made by Kakwani (1980) who, taking the model proposed by Allingham and Sandmo (1972) with a proportional tax schedule, briefly analyzes the behavior of both the (unobservable) actual income distribution and the (observable) reported income distribution. His results show that, according to the Lorenz criterion, the pre-tax actual income is more equally distributed than the pre-tax declared income, when the relative risk aversion is an increasing function of income. Using a similar approach Persson and Wissén (1984) conclude that the effects of non compliance on vertical equity are indeterminate on theoretical grounds. Assuming that the utility function displays constant relative risk aversion, these authors show the conditions under which the actual income is more unevenly distributed than the reported income. On the other hand, the empirical studies carried out by Bishop et al. (1994a) and, more recently, Bishop et al. (2000) have investigated the redistributive effects of non-compliance and tax evasion, using data from the Taxpayer Compliance Measurement Program. They find that, although the vertical equity effects of non-compliance are very small, a considerable amount of horizontal inequity is generated by tax evasion and non-compliance. Therefore, their results indicate that greater fiscal compliance by the taxpayers improves the redistributive role of the tax system. Following this empirical approach, Bloomquist (2003) examines the relationship between income inequality and U.S. wage and salary underreporting using time-series data for the period 1947-2000. The author finds a statistically significant correlation between income inequality and U.S. wage and salary underreporting in an extended expected utility model.

In this sense we try to analytically answer two main related questions. First, what is, in expected terms, the impact that tax evasion has on the redistributive effect of tax progressivity? Second, is it accurate to use declared income data, instead of the (unobservable) actual data, to empirically evaluate the effect that progressivity has on the inequality of the after-tax distributions? To answer both questions we will distinguish between the *actual* and the *observed distribution effect* of the tax progressivity. While both effects coincide in the absence of tax evasion, they may be different with tax evasion. Therefore, we will analyze the impact that tax evasion has on both redistributive effects. For that purpose, by using the Lorenz curve framework, we will compare the distribution of the after-tax income without evasion, the distribution of the expected income with evasion and the distribution of the after-tax declared income.

Regarding the first question, we find that the distribution of the after-tax income without evasion is more, equally or less equitable than the distribution of the expected after-tax income with evasion when the coefficient of relative risk aversion is decreasing, constant or increasing with income, respectively. Regarding the second question, we find alternative sufficient conditions for the distribution of the reported after-tax income with evasion to be less equitable than the distribution of the actual after-tax income with evasion. We also show the cases in which the impact that tax evasion has on the *observed distribution effect* of the tax progressivity is not the same as the impact on the *actual distribution effect* of the tax progressivity. This implies that the use of reported income data to empirically evaluate the effect that progressivity has on the inequality of the net income distributions, yields biased results.

## *II Tax evasion and income distribution*

In order to evaluate how tax evasion modifies the redistributive effect of tax progressivity, we have to analyze the change that tax evasion produces on the after-tax income distribution. Nevertheless, in an economy with tax evasion, two different net income distributions exist: the reported and the (unobservable) actual after-tax income distributions. Therefore, the redistributive effect of tax progressivity that one can observe may also differ from the actual redistributive effect.<sup>1</sup> Clearly, if these two redistributive effects coincide, then we could use the available declared data to empirically study the redistributive effect of the tax system, even if tax evasion is present. Otherwise, the use of reported data in an economy with tax evasion will give us an inaccurate measure of the redistributive effect of

tax progressivity.

We propose to use the Lorenz curve framework to evaluate how tax evasion affects both the observed and actual redistributive effects of tax progressivity. The concept of the Lorenz curve has been extended to study the relationship between the distributions of some economic variables.<sup>2</sup> The Lorenz curve (in particular the dominance of one Lorenz curve over another) is considered the most basic indicator of income inequality. Therefore, we will compare the distribution of net income without evasion with two different distributions when tax evasion is present. The first comparison will show us how tax evasion impacts the actual redistributive effect of a progressive tax system. The second exercise will state whether tax evasion modifies the observed redistributive effect of a progressive tax system.

To this end, we consider the standard model of tax evasion proposed by Allingham and Sandmo (1972).<sup>3</sup> This model assumes that individuals declare the amount of income  $x \in [0, y]$  that maximizes their expected utility, where  $y$  denotes the true income earned by the taxpayer. Note that this pre-tax income is fixed and exogenous. The tax function is denoted by  $T(\cdot)$ , where  $T(0) = 0$ ,  $0 < T' < 1$  and  $T'' > 0$ , which implies that the tax function is progressive. Taxpayers are audited by the tax authorities with probability  $p \in (0, 1)$ . The inspection allows the tax agency to find out the actual income  $y$  of an audited taxpayer. Therefore, a rational taxpayer reduces the amount of taxes to pay by  $T(y) - T(x)$  taking into account that with probability  $(1 - p)$  he will not be detected as an evader. Now then, if an individual is audited, he must pay a proportional penalty rate  $\pi > 1$  on the amount of evaded taxes (see Yitzhaki, 1974). Let us assume that the value of the probability of being audited,  $p$ , and the penalty,  $\pi$ , are exogenous.<sup>4</sup>

Therefore, a taxpayer chooses the amount  $x$  of income to declare by maximizing his expected utility,

$$(1 - p)U[y - T(x)] + pU[y - T(x) - \pi(T(y) - T(x))], \quad (1)$$

where the utility function,  $U$ , is twice continuously differentiable and strictly concave in after-tax income, i.e.,  $U' > 0$  and  $U'' < 0$ .

Since we want to analyze the impact of tax evasion on the after-tax income distribution, we rewrite the model in terms of evaded taxes. Following Yitzhaki (1987), expression (1) can be rewritten as

$$(1 - p)U(c + e) + pU(c - fe), \quad (2)$$

where  $e = T(y) - T(x)$  is the amount of evaded taxes,  $c = y - T(y)$  is the after-tax income

without evasion, and  $f = \pi - 1$ . Since (1) is concave, the conditions to guarantee that the declared income  $x$  is positive and strictly smaller than  $y$  are  $pU'(y - \pi T(y))(\pi - 1) > (1 - p)U'(y)$ , and  $p\pi < 1$ ; [or equivalently  $(1 - p - pf) < 1$ ]. From now on, we assume that these conditions hold.

*(i) The actual redistributive effect*

Let us first evaluate how tax evasion affects the actual redistributive effect of the tax system. To do that, we will compare the Lorenz curve of the distribution of the after-tax income when evasion does not exist,  $c$ , and the Lorenz curve of the distribution of the expected after-tax income with evasion,  $z$ , where<sup>5</sup>

$$z(c) = (1 - p)(c + e) + p(c - fe) = c + e(1 - p - pf). \quad (3)$$

Following Kakwani (1977, 1980), to compare the two Lorenz curves we have to compute the elasticity of  $z$  with respect to  $c$ . Therefore, to know the behavior of the expected net income,  $z$ , when the after-tax income without evasion,  $c$ , changes, we have to compute the derivative of (3) with respect to  $c$ . Thus we obtain:

$$\frac{dz}{dc} = 1 + (1 - p - pf)\frac{de}{dc}. \quad (4)$$

Since the sign of  $\frac{dz}{dc}$  depends on the sign of  $\frac{de}{dc}$ , we now have to compute the value of this latter derivative from the first order conditions of the maximization problem of (2). In the Appendix we prove that the effect of an increase in  $c$  on the amount  $e$  of evaded taxes only depends on the attitude of the individual toward absolute risk aversion:

**Proposition 1.**

- (a) *If the absolute risk aversion index is decreasing (DARA), then  $\frac{de}{dc} > 0$*
- (b) *If the absolute risk aversion index is constant (CARA), then  $\frac{de}{dc} = 0$*
- (c) *If the absolute risk aversion index is increasing (IARA), then  $\frac{de}{dc} < 0$  and  $|\frac{de}{dc}| < 1$ .*

The intuition behind Proposition 1 is trivial. Let us suppose that the taxpayer's true income increases. Under the assumption of DARA, the taxpayer will be less risk averse than before and, therefore, would now be willing to invest a larger amount of income in risky assets. In our context this implies that the taxpayer would now be willing to evade more taxes, which is a risky activity.

From Proposition 1, and taking into account that  $0 < (1 - p - pf) < 1$ , it follows from (4) that  $\frac{dz}{dc} > 0$  for all  $c > 0$ . Therefore, if evasion is present, an increase in the taxpayer's after-tax actual income is always translated into an increase in his expected income. In this case, previous results obtained by Kakwani (1977, 1980) clearly indicate that the concentration curve of the function  $z$ , which depends on  $c$ , coincides with its Lorenz curve.

Finally, in order to compare the distributions of  $c$  and  $z$ , we have to calculate whether the elasticity of  $z$  with respect to  $c$  is less (greater) than unity for all  $c > 0$ . The Appendix shows this comparison and proves that the following result holds:

**Proposition 2.**

- (a) *If the relative risk aversion index is decreasing (DRRA), then the distribution of  $c$  is Lorenz superior to the distribution of  $z$ .*
- (b) *If the relative risk aversion index is constant (CRRA), then the Lorenz curves of  $c$  and  $z$  coincide.*
- (c) *If the relative risk aversion index is increasing (IRRA), then the distribution of  $z$  is Lorenz superior to the distribution of  $c$ .*

Proposition 2 asserts that the existence of evasion generates a positive, negative or null distortion between the distributions of both  $c$  and  $z$  depending on the behavior of the relative risk aversion index. In particular, under the assumption of CRRA, Proposition 2 states that tax evasion does not alter the actual redistributive effect of the tax progressivity, since the Lorenz curve of after-tax income without evasion coincides with the Lorenz curve of expected net income. The intuition is quite simple. Under the assumption of CRRA, the following condition holds:

$$\frac{de}{dc} = \frac{e}{c} \implies \frac{de}{dc} \frac{c}{e} = 1. \quad (5)$$

Note that (5) tells us that the elasticity of  $e$  with respect to  $c$  is equal to 1. This means that evasion increases in the same proportion as net income without evasion when the CRRA assumption holds. Additionally, from expression (3) it is straightforward to see that if  $c$  and  $e$  rise in the same proportion,  $z$  also increases in the same proportion.

By other hand, the existence of tax evasion under IRRA leads the actual redistributive effect to be larger than the one associated to the case without tax evasion. The intuition of this result is also quite simple. Observe that in this case the proportion of evaded taxes decreases with the income level.<sup>6</sup> Therefore, tax evasion drives the actual redistributive effect of the tax system up when the individuals exhibit IRRA.

(ii) *The observed redistributive effect*

Since most empirical studies that analyze the redistributive effect of tax progressivity are based on reported income data, instead of actual income data, these studies are measuring an apparent progressivity. Therefore, the use of declared income data to compute the redistributive effect of the tax system may generate biased results. This question was already analyzed by Persson and Wissén (1984). These authors, considering the CRRA utility function, found the conditions to ensure the existence of a bias generated by taking voluntarily paid taxes instead of the taxes corresponding to the true income. We generalize their analysis by not assuming a priori any restriction on the attitude towards risk. Moreover, the strategy we use is different.<sup>7</sup> Thus, we not only study how tax evasion affects the observed redistributive effect of the tax system, but also we compare this result with the distortions generated on the actual redistributive effect.

Therefore, the next step would be to analyze how tax evasion alters the observed redistributive effect of the tax system. In order to do this, let us compare the Lorenz curve of the after-tax income without evasion and the Lorenz curve of the after-tax declared income (before inspection). Note that the after-tax income when evasion does not exist,  $c(y)$ , is:

$$c(y) = y - T(y),$$

Now, let us define the after-tax declared income  $h(y)$  as

$$h(y) = x(y) - T(x(y)),$$

where  $x(y)$  is the solution for the problem maximization of (1). As the Appendix explains, to compare the Lorenz curve of the functions  $c(y)$  and  $h(y)$  we have to calculate the elasticity of  $c(y)$  and  $h(y)$  with respect to  $y$ , and see which of them is larger. The Appendix proves that the Lorenz curve of  $c(y)$  lies above the Lorenz curve of  $h(y)$ , under some conditions. More precisely, Proposition 3 gives the conditions ensuring that the after-tax income without evasion is more equally distributed than the after-tax declared income:

**Proposition 3.** *The distribution of  $c$  is Lorenz superior to the distribution of  $h$  if one of the following statements holds:*

- (a) *The absolute risk aversion index is increasing,*
- (b) *The absolute risk aversion index is constant,*
- (c) *The relative risk aversion index is constant.*

Proposition 3 asserts that, under the assumptions of IARA, CARA or CRRA, the after-tax declared income is more unequally distributed than the after-tax income without evasion. This means that in these cases the observed redistributive effect of the tax system is smaller than the one associated to the case without tax evasion.

Observe that Proposition 2 states that tax evasion can increase, maintain or reduce the actual redistributive effect of the tax system depending on whether the coefficient of relative risk aversion is increasing, constant or decreasing with income, respectively. Therefore, Propositions 2 and 3 together suggest that a bias can emerge when we measure the redistributive effect of tax progressivity using declared income data instead of actual income data. This result comes from the fact that, under the assumptions of IARA, CARA and IRRA, tax evasion modifies the actual and the observed distribution effects of the tax progressivity with opposite signs. Proposition 2 shows that under IRRA, tax evasion drives the actual redistributive effect of the tax system up. However, Proposition 3 tells us that under IARA and CARA, the existence of tax evasion forces the observed distributive effect down. On the other hand, under CRRA, Proposition 2 states that tax evasion does not modify the actual distributive effect of the tax system. However, Proposition 3 shows that tax evasion reduces the observed distributive effect of the tax system. Unfortunately, we do not have conclusive results on this relationship between actual and observed distributive effects when the individuals exhibit DARA.

Before closing this section, we want to remark that an alternative explanation emerges when we compare Proposition 2 with Proposition 3. Observe that the after-tax income distributions  $c$ ,  $z$  and  $h$  are different either because the distribution generated by the tax function is different, as is the case when we compare  $c$  with  $z$ , or because the initial distribution of the before-tax income is different, as is the case when we compare  $c$  with  $h$ .<sup>8</sup> For this reason, the obtained results also suggest that tax evasion can distort the redistributive effect of tax progressivity through two mechanisms. The first one modifies the distribution generated by the tax function. Part (b) of Proposition 2 ensures that under CRRA the first channel is not present, instead under DRRA and IRRA it is present. The second mechanism acts by modifying the declared income distribution. Proposition 3 ensures that this mechanism is active even under CRRA.

### *III Discussion*

In Section 2, we have shown that our results are fairly sensitive to the taxpayers' attitude

toward risk. Therefore, in order to extend the scope of our results, we will discuss some potential scenarios concerning the behaviour of taxpayers' risk aversion. In two seminal papers, Pratt (1964) and Arrow (1965) derived the measure of absolute and relative risk aversion and established the commonly accepted hypothesis that absolute risk aversion is decreasing with respect to income. However, to this day, there is no consensus about which should be the most accurate hypothesis about the relative risk aversion's behaviour. The empirical literature has tried to find a pattern for the conduct of relative risk aversion in respect to wealth. A great variety of data sources have been used to perform these estimates such as: consumer expenditures [Weber (1975), Ogaki and Zhang (2001)], demand for risky assets [Friend and Blume (1975), Morin and Fernandez (1983)], option data [Ait-Sahalia and Lo (2000), Kang and Kim (2006)] and experimental data [Haim (1994) Wik et al. (2004)]. Although the empirical evidence does not yield a conclusive result, it generally tends to support the constant relative risk aversion hypothesis as well as the decreasing relative risk aversion one.

Let us first consider the context where taxpayers exhibit constant relative risk aversion (CRRA). Under this assumption, Proposition 2 asserts that the alteration in the redistributive effect of tax progressivity due to tax evasion is null. This means that if data of paid fines are included into the analysis, no bias emerges from the existence of tax evasion. In consequence, if the assumption of CRRA is supported by the majority of countries, the evaluations of the redistributive effect of the tax system either across countries or over time, can be compared. Nevertheless, the main drawback associated with the measurement of the redistributive effect of tax system is the availability of data. Usually, fiscal data do not include the fines paid by the taxpayers who have been caught. Therefore, researchers use declared data instead of true data. Proposition 3 shows that in this case, tax evasion introduces a negative bias in the truthful redistributive effect of tax progressivity.

In order to numerically illustrate the magnitude of this negative bias introduced by means of tax evasion, we use an example for the Spanish case. Since the only available data are reported income  $x$  and paid taxes  $T(x)$ , we have to simulate an approximation of the true income distribution under CRRA. Therefore, the following analysis must be interpreted with caution in quantitative terms. Assuming the following values:  $\sigma = 2$ ,  $\pi = 2$ , and  $p = 0.4$ , we have simulated for each interval of income, what would be the corresponding distributions for the after-tax income without evasion,  $c(y)$  and the post-tax reported income,  $h(x)$ .<sup>9</sup> Next, we have calculated the Lorenz curves and computed the

corresponding Gini index. The results show that the Gini index for the distributions of  $c(x)$ , and  $h(x)$ , are 0.128, and 0.151, respectively. Therefore, we can state that the distribution of post-tax declared income is a 18% less egalitarian than the distribution of the post-tax income without evasion.

Let us now assume that taxpayers exhibit decreasing relative risk aversion (DRRA). Unfortunately, under this assumption we can only conclude that even if data of paid fines are available, a negative effect arises as a consequence of tax evasion phenomenon. However, our results allow us to conclude that under both CRRA and DRRA assumptions the bias introduced by tax evasion is negative. This means that the true redistributive effect is being underestimated.

At this stage, an important question arises. What can governments do to reduce this negative bias? The answer is straightforward: governments can act by means of the tax enforcement policy. It is easy to see that the implementation of a tax enforcement policy that makes  $p\pi$  closer to 1 results in a lower level of tax evasion and, therefore, the magnitude of the negative bias decreases. In the previous numerical exercise, by replacing the value of the audit probability for the larger value of  $p = 0.45$ , we obtain a result showing that the post-tax reported income distribution is only a 5% less egalitarian than the true post-tax distribution. Therefore, the values of the inspection policy parameters can act as a signal for the unobservable magnitude of tax evasion. When researchers analyze the redistributive effect of the tax system using reported income instead of the true one, they can predict the magnitude of the bias observing the value of both the probability of inspection and the fine.

#### *IV Final remarks*

In this paper, we have analytically studied whether the existence of tax evasion modifies the redistributive effect of a progressive income tax. In particular, we have proved that the accuracy of the empirical studies that evaluate the progressivity of a tax system depend on the effect that tax evasion has on both the actual and the observed redistributive effect. Our results predict that tax evasion increases, maintains or reduces the (unobservable) actual redistributive effect of the tax system when taxpayers exhibit increasing, constant or decreasing relative risk aversion respect to income, respectively. Moreover, we have also shown that under IARA, CARA and CRRA, tax evasion always reduces the observed redistributive effect of the tax system.

The empirical problem posed by tax evasion is relevant when we want to evaluate the

redistributive effect of taxes, either over time or after some fiscal reforms in a particular country. On the one hand, since the dishonest behavior of taxpayers could change over time, the analysis of the reported net income distributions generates biased results on the redistributive effect of taxes. On the other hand, since a tax reform can evidently alter the evasion behavior of individuals, we will misinterpret the redistributive effect of this reform when we compare the reported net income distributions generated before and after the tax reform. A similar situation could occur if we compare the redistributive effect of the tax system across countries. In this case, it is difficult to obtain robust conclusions because the existence of tax evasion will again modify the results derived from this comparison. Since one may expect differences in the evasion behavior across countries, biased results would emerge from the international comparison of the redistributive effects of taxes.

Summarizing, we claim that the analysis of the redistribution effect of tax progressivity is not independent of the analysis of tax evasion behavior. Therefore, to minimize the alteration that tax evasion generates, researchers should compare countries which have similar tax evasion patterns or use the information about tax inspection policy as a proxy of the level of tax evasion phenomenon.

Future research on this topic should consider a model which makes endogenous the taxpayer's true income by adding labour supply.<sup>10</sup> In this case, it should be considered a model where the taxpayer has to choose both the optimal declared income and the optimal amount of hours of work. It could be interesting to analyze the effects of an increase in tax progressivity on the labour supply and on the amount of evaded income, since taxpayers can try to reduce their tax burden by means of tax evasion. These new effects should be taken into account in analyzing how tax evasion modifies the distributive effect of tax progressivity.

## APPENDIX

### *I Proof of Propositions 1 and 2*

Based on some well-known results obtained by Kakwani (1977, 1980), our strategy proof will consist on the comparison of the Lorenz curves of the different income distributions using the elasticities of their functions. Therefore, to compare these two Lorenz curves we have to compute the elasticity of  $z$  with respect to  $c$ . Since the sign of  $\frac{dz}{dc}$  depends on the sign of  $\frac{de}{dc}$ , we first have to compute the value of this latter derivative. The first and second

order conditions which ensure that the amount  $e$  of evaded taxes maximizes (2) are

$$(1 - p)U'(c + e) - pfU'(c - fe) = 0, \quad (6)$$

$$(1 - p)U''(c + e) + pU''(c - fe)(-f)^2 < 0. \quad (7)$$

To compute  $\frac{de}{dc}$ , we can apply the Implicit Function Theorem in equation (6), and we obtain the following expression:

$$\frac{de}{dc} = -\frac{(1 - p)U''(c + e) - pfU''(c - fe)}{(1 - p)U''(c + e) + pU''(c - fe)(-f)^2}. \quad (8)$$

Using the first-order condition in (6), we can rewrite (8) as:

$$\frac{de}{dc} = \frac{R_A(c - fe) - R_A(c + e)}{fR_A(c - fe) + R_A(c + e)}, \quad (9)$$

where  $R_A(I) = -\frac{U''(I)}{U'(I)}$  is the Arrow-Pratt index of absolute risk aversion. Therefore, Proposition 1 follows directly from equation (9).

We then compute and prove the continuous and strictly positiveness of the first derivatives of our functions in order to compare their elasticities as the Kakwani's results state.

From Proposition 1, and taking into account that  $0 < (1 - p - pf) < 1$ , it follows from (4) that  $\frac{dz}{dc} > 0$  for all  $c > 0$ . Therefore, if evasion is present, an increase in the taxpayer's after-tax actual income is always translated into an increase in his expected income. In this case, the existence of a continuous and strictly positive  $\frac{dz}{dc}$  ensures that the concentration curve of the function  $z$  which depends on  $c$  coincides with its Lorenz curve.

Following Kakwani (1977, 1980), to compare the distributions of  $c$  and  $z$ , we now calculate whether the elasticity of  $z$  with respect to  $c$  is less (greater) than unity for all  $c > 0$ . This elasticity is given by:

$$\eta_z = \frac{dz}{dc} \frac{c}{z} = \left(1 + (1 - p - pf)\frac{de}{dc}\right) \frac{c}{z}. \quad (10)$$

Rewriting expression (10) and using (3) we obtain:

$$\eta_z - 1 = \frac{1}{z}(1 - p - pf) \left(c \frac{de}{dc} - e\right). \quad (11)$$

Note that the sign of (11) only depends on the sign of

$$\phi = \left(c \frac{de}{dc} - e\right) \quad (12)$$

since  $1 - p - pf > 0$ . Note that  $\phi > 0$  when  $\frac{de}{dc} > \frac{e}{c}$  (and  $\phi < 0$  when  $\frac{de}{dc} < \frac{e}{c}$ ). This implies that, if  $\phi > 0$ , then the marginal evasion of taxes with respect to the after-tax income without evasion is larger than the expected evasion with respect to that income. Then Kakwani's results states that the Lorenz curve of  $c$  is above the Lorenz curve of  $z$ . This means that the after-tax income without evasion is more equally distributed than the expected income with evasion. In the same way, if  $\phi < 0$ , the after-tax income without evasion is more unequally distributed than the expected income with evasion.

Introducing (8) into (12) and using condition (6), we can obtain

$$\phi = -\frac{(1-p)U'(c+e)[R_R(c-fe) - R_R(c+e)]}{D},$$

where  $R_R(I) = -\frac{U''(I)I}{U'(I)}$  is the Arrow-Pratt index of relative risk aversion. Since  $D < 0$ , the sign of  $\phi$  will only depend on the assumption made regarding the behavior of the relative risk aversion index. Proposition 2 summarizes the results on the Lorenz curves of the distributions of  $c$  and  $z$ .

## II Proof of Proposition 3

Calculating the first derivative of  $h(y)$ , we obtain that

$$h'(y) = [1 - T'(x(y))] \frac{dx}{dy}. \quad (13)$$

Since the first order condition for the maximization of (1) is

$$-(1-p)U'[y - T(x)]T'(x) + pU'[y - T(x) - \pi(T(y) - T(x))]T'(x)(\pi - 1) = 0, \quad (14)$$

we can apply the Implicit Function Theorem in equation (14) to compute  $\frac{dx}{dy}$ . Thus, we obtain that

$$\frac{dx}{dy} = \quad (15)$$

$$\frac{-(1-p)T'(\hat{x})U''[y - T(\hat{x})] + pT'(\hat{x})(\pi - 1)U''[y - T(\hat{x}) - \pi(T(y) - T(\hat{x}))]}{(1-p)U''[y - T(\hat{x})][T'(\hat{x})]^2 + pU''[y - T(\hat{x}) - \pi(T(y) - T(\hat{x}))][T'(\hat{x})]^2(\pi - 1)^2},$$

where  $\hat{x} = x(y)$ . Substituting (14) into (15) and after some simplifications we obtain

$$\frac{dx}{dy} = \frac{\pi T'(y)R_A(Z) - [R_A(Z) - R_A(Y)]}{\pi T'(x)R_A(Z) - T'(x)[R_A(Z) - R_A(Y)]}, \quad (16)$$

where  $R_A(\cdot)$  is the Arrow-Pratt index of absolute risk aversion,  $Y = y - T(\hat{x})$ , and  $Z = y - T(\hat{x}) - \pi(T(y) - T(\hat{x}))$ . Note that the sign of  $\frac{dx}{dy}$  will depend on the attitude of individuals towards risk aversion. In particular, it is clear that if individuals exhibit constant absolute risk aversion (CARA) or increasing absolute risk aversion (IARA) we have

$$\frac{dx}{dy} > 1,$$

since  $T'(y) > T'(\hat{x})$ , because of the assumption  $T''(\cdot) > 0$ . Note that this result means that, when true income increases, declared income also increases, and more than proportionally. Hence, under the constant relative risk aversion assumption (CRRA) expression (16) becomes

$$\frac{dx}{dy} = \frac{\sigma \left[ \pi T'(y) \frac{1}{Z} - \left( \frac{Y-Z}{ZY} \right) \right]}{\sigma T'(\hat{x}) \left[ \frac{\pi}{Z} - \left( \frac{Y-Z}{ZY} \right) \right]},$$

where  $\sigma$  is the constant relative risk aversion parameter. After some simplifications we obtain

$$\frac{dx}{dy} = \frac{T'(y)y - T(y) + T(\hat{x})[1 - T'(y)]}{T'(\hat{x})[y - T(y)]}. \quad (17)$$

The assumption  $T''(\cdot) > 0$  implies that  $T'(y)y - T(y) > 0$ , so that  $\frac{dx}{dy} > 0$  and, therefore,  $h'(y) > 0$ . Once we have proved that  $c'(y) > 0$  and  $h'(y) > 0$  under the CARA, IARA and CRRA assumptions, the Kakwani's results allow us to compare the Lorenz curve of the functions  $c(y)$  and  $h(y)$ . In order to do this, we have to calculate the elasticity of  $c(y)$  and  $h(y)$  with respect to  $y$ , and see which of them is larger. The elasticity of  $c(y)$  with respect to  $y$  is

$$\eta_c = \frac{y[1 - T'(y)]}{y - T(y)}, \quad (18)$$

whereas the elasticity of  $h(y)$  with respect to  $y$  is given by

$$\eta_h = \frac{y[1 - T'(\hat{x})]}{\hat{x} - T(\hat{x})} \frac{dx}{dy}. \quad (19)$$

Firstly, since the tax function is convex, the relation  $1 - T'(x) > 1 - T'(y)$  holds. Then, if we compute  $c(y) - h(y)$  we obtain

$$c(y) - h(y) = y - T(y) - \hat{x} + T(\hat{x}) = y - \hat{x} - [T(y) - T(\hat{x})].$$

Clearly,  $c(y) > h(y) > 0$  since the amount of hidden income is relatively larger than the amount of evaded taxes. Consequently, the after-tax income without evasion is larger than the after-tax declared income, i.e.,  $c(y) > h(y)$ . If we compare (18) and (19) and we use the

inequalities  $1 - T'(\hat{x}) > 1 - T'(y)$ ,  $c(y) > h(y)$  and  $\frac{dx}{dy} > 1$ , it is obvious that for the CARA and IARA assumptions it holds that  $\eta_h > \eta_c$ .

For the case of CRRA, we want to know when  $\eta_h$  will be larger than  $\eta_c$ . Using (17), (18) and (19) the inequality  $\eta_h > \eta_c$  becomes

$$\frac{y [1 - T'(\hat{x})]}{\hat{x} - T(\hat{x})} \left[ \frac{T'(y)y - T(y) + T(\hat{x}) [1 - T'(y)]}{T'(\hat{x}) [y - T(y)]} \right] > \frac{y [1 - T'(y)]}{y - T(y)}.$$

Rearranging the terms of the last inequality, and after some simplifications, we get

$$\frac{T'(y)y - T(y)}{1 - T'(y)} > \frac{T'(\hat{x})\hat{x} - T(\hat{x})}{1 - T'(\hat{x})}. \quad (20)$$

It is easy to see that inequality (20) will be true if the function  $F(s) = \frac{T'(s)s - T(s)}{1 - T'(s)}$  is monotonically increasing. Calculating  $F'(s)$  we obtain

$$F'(s) = \frac{[1 - T'(s)] T''(s)s + T''(s) [T'(s)s - T(s)]}{[1 - T'(s)]^2},$$

which is positive because of the assumption  $T''(\cdot) > 0$ .

Finally, according to Kakwani (1977, 1980) we can state that, since the first derivatives of the functions  $c(y)$  and  $h(y)$  exist, and they are continuous, if we prove that they are strictly positive for all  $y$ , then  $c(y)$  is Lorenz superior (inferior) to  $h(y)$  if  $\eta_c(y)$  is less (greater) than  $\eta_h(y)$ , for all  $y \geq 0$ . Proposition 3 summarizes the obtained results that directly follows.

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## FOOTNOTES

1. It is obvious that the observed and the actual redistributive effects are the same in absence of tax evasion.
2. See Kakwani (1977), Bishop et al. (1994b) and Kakwani and Lambert (1998) among others.
3. The Allingham and Sandmo framework has been extended in several directions over the past thirty years and it is still present in the current literature [see for instance, Bernasconi (1998), Borck (2004), Glen Ueng and Yang (2004), Richter and Boadway (2005), Fortin et al. (2007)].
4. See Reinganum and Wilde (1985) and (1986) and Caballé and Panadés (2005), for different approaches that relax the assumption of a constant audit probability.
5.  $z$  can be interpreted as the post-auditing income when tax authorities inspect a proportion  $p$  of taxpayers.
6. Note that IARA implies IRRA. Proposition 1 states that, under IARA, the evaded taxes decreases when the income level increases. CARA also implies IRRA. In this case, Proposition 1 show that  $\frac{de}{dc}=0$ .
7. In particular, these authors consider two-type individuals distribution. We instead do not impose any restriction on the distribution of individuals, since we follow an analysis based on a Lorenz curve framework.
8. Remember that after-tax income without evasion is given by  $c = y - T(y)$ , where  $y$  is the true income and  $T(\cdot)$  is the tax function. Similarly, the after-tax reported income is given by  $h = \hat{x} - T(\hat{x})$ , where  $\hat{x} = x(y)$  is the optimal declared income. Finally, the after-tax expected income can be written as  $z = y - E(\hat{T}(y))$ , where  $E(\hat{T}(y)) = (T(y) - T(x))(1 - p - pf) - T(y)$

is an expected tax function that takes into account the existence of evasion and the probability of auditing with the corresponding penalty.

9. We assume that  $T(y)$  is given by the polynomial specification:  $T(y) = \alpha + \beta y + \gamma y^2 + \delta y^3$ . Using reported data from the Spanish Tax Administration Report (2000), the parameter's estimates (by OLS) are:

$$T_t = \underset{(-12.27)}{-1692.835} + \underset{(20.28)}{0.1571104}x_t + \underset{(20.81)}{2.32(E-06)}x_t^2 - \underset{(-16.37)}{7.04(E-12)}x_t^3 + \hat{\varepsilon}_t,$$

where t-statistics are between brackets. Under the CRRA assumption, equation (A.1) becomes equal to:  $y - T(x) = A [y - T(x) - \pi (T(y) - T(x))]$ , where  $A = \left( \frac{p(\pi-1)}{1-p} \right)^{-\frac{1}{\sigma}}$ . Replacing the estimated values of  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ , and the values of  $\sigma$ ,  $\pi$  and  $p$ , we obtain the approximation of the distribution of true income. The reader can obtain the details of this exercise upon request to the authors.

10. See for instance Pencavel (1979), Sandmo (1981), Cowell (1985) or Pestieau and Posse (1991).