Tax Avoidance and the Laffer Curve*

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Abstract

This paper analyzes the impact on tax revenue of a change in the tax rate when the individuals have at their disposal two alternative methods for reducing their tax liabilities: tax evasion and tax avoidance. We find that this impact takes the shape of a Laffer curve when the fixed cost associated with tax avoidance is small enough. We also examine more accurately the case of a isoelastic utility function in order to obtain stronger results on the existence of such a Laffer curve. Finally, we explore the role played by the distribution of income by considering a discrete distribution.

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1. Introduction

To pay taxes is viewed as an unpleasant obligation and, consequently, many taxpayers try to reduce their tax liabilities. Most of the literature has concentrated on tax evasion as a way of achieving such a reduction. However, this method implies that the taxpayers bear some risk since, if they are discovered underreporting their true income, they are punished with a penalty. The growing complexity of the tax code has given rise to tax avoidance as an alternative method of reducing the payment of taxes. We mean by tax avoidance the set of actions which allow taxpayers to enjoy a legal reduction in their tax liabilities. In general these actions are riskless but costly.[1] Usual practices of taking full advantage of the tax code include, for instance, income splitting, postponement of taxes, and tax arbitrage across income facing different tax treatments.[2] On the other hand, the emergence of certain countries, called *tax heavens,* where the income is lower-taxed or non-taxed at all, has encouraged to a great extent tax avoidance activities. For example, it is well known that some taxpayers have their fiscal residence located in a tax heaven in order to pay less taxes while their productive activities are located in a country exhibiting much higher tax rates.[3] In this paper, we will concentrate on the analysis of tax avoidance associated with the existence of tax heavens. Therefore, we will assume that tax avoidance is linked to the source of income so that, when an individual becomes avoider, he shelters all his income from that source.

The behavior of the taxpayer geared towards minimizing their tax bill has evident implications for the government revenues. Even if the exact measurement of both tax evasion and tax avoidance activities is, by definition, difficult to achieve, some estimations have been performed. For instance, the US Internal Revenue Service has estimated in 1994 an overall compliance rate of 87%, which may seem quite high in relative terms but, if we translated this rate into dollar terms, we would see that each 1% increase in compliance translates into USD 7-10 billion in terms of government revenues (see Lyons, 1996).

The aim of this paper is to analyze the relationship between the tax rate and the tax revenue raised by the government when agents choose between evading and avoiding as two alternatives ways of reducing their tax liabilities. If the tax rate affects the attitude of individuals towards tax compliance, then any change in the tax rate affects the tax revenue in two ways. On the one hand, for a given

[1] In some cases these actions encompass some uncertainty. This happens when the avoidance strategies depend on uncertain events or when the hoopholes in the tax law are exploited until the margin.

[2] For more details see Stiglitz (1985).

[3] This is a very common practice for firms in Netherlands, since the Netherlands Antilles is considered a tax heaven jurisdiction. Other tax heavens, located in Europe, are for example Andorra and Monaco, which usually shelter a few rich individuals.

level of tax compliance, collected taxes increase with the tax rate. On the other hand, an increase in the tax rate may increase the set of taxpayers who avoid their taxes, and this diminishes the tax revenue obtained by the government. When the former effect is stronger for low tax rates and weaker for high tax rates, we will be facing the typical Laffer curve relating the government revenue with the tax rate, that is, this relationship will exhibit an inverted U shape.

The government can achieve a larger tax revenue by increasing the tax rate until its largest feasible value, when the revenue-tax rate relationship is increasing. However, when this relationship has the shape of a Laffer curve, as a consequence of tax evasion and tax avoidance activities, the maximum tax revenue is achieved by selecting a tax rate strictly lower than 100%. Therefore, governments seeking to maximize its revenue will never find optimal to extract all the income from the private sector by setting arbitrarily large tax rates even if the income of each individual is unaffected by the tax rate.

Some authors have analyzed taxpayer behavior using a representative agent approach when both avoidance and evasion are jointly chosen. These authors study the implications that this choice has on some traditional results of the tax evasion literature but disregard the analysis of the behavior of the government revenue as a function of the tax rate. In fact, the comparative statics is immediate in the previous analysis when the fines are proportional to evaded income: revenues increase with the tax rate as there is less evaded income.[4] It should be noticed that these authors emphasize the aspects of complementarity between these two activities. We consider instead a framework where the individuals only choose one of the two alternatives. Therefore, we are treating tax evasion and tax avoidance as substitutive strategies and, thus, we are somewhat following the idea of polarization between evaders and shelters proposed by Cowell (1990). According to this author, by choosing to shelter income, an individual draws attention to himself and therefore becomes a prime target for investigation and, thus, it is inadvisable to try evading and avoiding at the same time. The presence of legal tax shelters induces thus a polarization between evaders and avoiders.

In our work, we dispense with the assumption of a representative agent, since we consider that all the individuals have a different exogenous income even though their preferences are represented by a common utility function. This means that not all the individuals select the same method for paying less taxes. We will assume that the income distribution is uniform. We will also explore the robustness of our results in an example with a discrete income distribution. Such a discrete distribution will allow us to discuss the implications of income polarization concerning the characterization of the revenue-tax rate relationship.[5] Notice that

[4]See, for example Cross and Shaw (1982) and Alm (1988) who analyze both the individual and government behavior when avoidance and evasion are jointly selected, under the assumption that avoidance is not a risky activity. See also Alm and Mccallin (1990) who carry out a mean-variance analysis when both evasion and avoidance are considered as risky assets.

[5]Waud (1988) has studied the existence of a Laffer curve for the government revenue in a context where both tax evasion and tax avoidance take place. This author establishes the

in our model individuals are indexed by different levels of income but both the aggregate income and the distribution of income among individuals will remain unaltered when the tax rate changes. Thus, our analysis focuses exclusively on the contribution of the tax code and the tax enforcement policy to the revenue-tax rate relationship and then it disregards the effects of tax compliance on labor supply and other macroeconomic variables.

We show in this paper that the Laffer curve could emerge in all the scenarios we consider. In order to get such an existence result we need to assume that the cost of avoidance is sufficiently low so as to allow almost all the individuals to avoid their income for high values of the tax rate. The robustness of this result indicates that in our context a policy of high marginal tax rates might not be the appropriate strategy when the objective of the government is to maximize its revenue.

The paper is organized as follows. Section 2 presents a description of the individual behavior both when the taxpayer is an evader and when he is an avoider. The tax revenue function of the government is presented in Section 3. Section 4 provides the analysis of the tax rate-tax revenue relationship both when the non-avoiders are honest and when they evade. In Section 5 we carry out a more detailed analysis on the existence of the Laffer curve when a isoelastic utility function is assumed. Section 6 characterizes the Laffer curve when the income distribution function is not uniform but discrete. Finally, the main conclusions and extensions of the paper are presented in Section 7. Most of the formal proofs are relegated to the Appendix.

2. Individual behavior

Let us consider an economy with a continuum of agents who are indexed by their income y. Each individual receives an idiosyncratic exogenous income y drawn from an uniform distribution on $\left[0, \overline{Y}\right]$. The preferences of agents are represented by a common Von Neumann-Morgenstein utility function $U(\cdot)$ defined on after-tax income I. The first and second derivatives U' and U'' exist and are continuous with $U' > 0$ and $U'' < 0$. Therefore, agents are risk averse. Moreover, we assume that the utility function $U(\cdot)$ satisfies the Inada condition: $\lim_{I \to 0} U'(I) = \infty$.

In this economy agents are supposed to pay proportional taxes with a flat-rate tax $\tau \in [0, 1]$. However, agents have two alternative ways of reducing their tax bill. On the one hand, they can misreport part of their income (tax evasion) and, on the other hand, they can avoid the tax payments using legal methods (tax avoidance). Tax evasion is a risky activity because if the individual is inspected, he has to pay the evaded taxes plus a penalty. Although we will assume that tax

existence a Laffer curve for the expected tax revenue in a partial equilibrium context with a representative agent. See also Waud (1986) who studied the appearance of the Laffer curve of the tax revenue when tax evasion and tax avoidance appear simultaneously in a classical supply-side model.

avoidance is a riskless action, there are positive costs associated with it. In our model, taxpayers can be classified on one of two categories: evaders or avoiders. The case of honest taxpayers also arises as a corner solution of the evaders problem and will also be studied.

2.1. Tax evaders

Let us consider the standard Allingham and Sadmo (1972) model based on a portfolio approach. In what follows, we will adopt the variation of such a model presented by Yitzhaki (1974). The individual declares an amount of income equal to x and he will be audited by the tax authorities with probability p. The true income y is always discovered by the inspection. Therefore, agents reduce their tax payments by $\tau(y-x)$ whenever inspection does not occur. If an individual is caught evading, he has to pay a proportional fine $s>0$ on evaded taxes. We assume that overreporting is never rewarded. Therefore we can restrict the declared income x to be no greater than y.[6]. On the other hand, x is no lower than zero since the tax code does not feature a loss-offset, that is, negative declared income is viewed by the tax authorities as equivalent to zero declared income. We define the evaded income as $e=y-x$ where $e\in[0,y]$. Then, the random net income of an evader with true income y and evaded income e is equal to[7]

$$y-\tau y+\tau e,\quad \text{if the individual is not inspected,}$$

and

$$y-\tau y-s\tau e,\quad \text{if the individual is inspected.}$$

We assume that the parameters defining the tax inspection policy, p and s, are given exogenously throughout the paper.

Each taxpayer chooses the amount e of evaded income in order to maximize his expected utility

$$E\left[U\left(I\right)\right]=(1-p)U(y-\tau y+\tau e)+pU(y-\tau y-s\tau e)\equiv u(e). \tag{2.1}$$

The first order condition for an interior solution of the maximization of (2.1) is

$$(1-p)U'\left(y-\tau y+\tau e\right)\tau-pU'\left(y-\tau y-s\tau e\right)s\tau=0. \tag{2.2}$$

The corresponding second-order condition is automatically satisfied because of the assumption of concavity of the utility function.

The following lemma shows the solutions for the maximization problem (2.1):

Lemma 2.1. *The optimal evaded income $e\left(\tau,y\right)$ as a function of the tax rate τ and the individual income y is:*

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[6]When an individual declares more than his true income, he will only receive the excess tax contribution if he is audited. Therefore, $s\tau=1$ whenever $x>y$.

[7]Note that $y-\tau y+\tau e=y-\tau x$.

(a) if $\tau(1+s) < 1$,

$$e\left(\tau, y\right) = \begin{cases} 0 & \textit{for} \quad s \geq \frac{1-p}{p} \\ \\ \widehat{e}(\tau, y) & \textit{for} \quad \left(\frac{1-p}{p}\right) \frac{U'(y)}{U'(y - \tau y - s\tau y)} < s < \frac{1-p}{p} \\ \\ y & \textit{for} \quad s \leq \left(\frac{1-p}{p}\right) \frac{U'(y)}{U'(y - \tau y - s\tau y)}, \end{cases}$$

(b) if $\tau(1+s) \geq 1$,

$$e\left(\tau, y\right) = \begin{cases} 0 & \textit{for} \quad s \geq \frac{1-p}{p} \\ \\ \widehat{e}(\tau, y) & \textit{for} \quad s < \frac{1-p}{p}, \end{cases}$$

where $\widehat{e}(\tau, y)$ *is the solution to equation* (2.2) *for the evaded income* e.

Proof. See the Appendix.

Lemma 2.1 states the intuitive result that an evader taxpayer will declare less than his actual income if the penalty s is small enough, whereas the amount declared is positive whenever s is large enough. Note that when $s \geq \frac{1-p}{p}$ taxpayers become honest.

Finally, substituting $e\left(\tau, y\right)$ into the expected utility we have

$$V(\tau, y) = (1-p)U\left(y - \tau y + \tau e(\tau, y)\right) + pU\left(y - \tau y - s\tau e(\tau, y)\right), \tag{2.3}$$

which is the maximum value achieved by the expected utility, i.e., the indirect utility function.

2.2. Tax avoiders

The structure of the tax system may offer some ways of reducing the tax bill of the agents. This implies that they will pay less taxes since the avoided income will be taxed at a lower tax rate. To simplify the analysis, let us assume that the avoided income is taxed at a zero tax rate.[8] Moreover, we consider that tax avoidance takes the form of sheltering a source of income in a tax heaven. Since we assume that individuals have only a source of income (typically associated with a single economic activity), when an individual decides to become an avoider, he shelters his total income.

There exist positive costs associated with the tax avoidance activity given by the function $C(y)$. These costs include, for example, the cost of obtaining the relevant information about the tax clauses, or the payment to a tax advisor, or the necessary amount to create a firm in order to declare the personal income

[8] This analysis can easily be modified to accommodate a positive tax rate for avoided income.

as corporate income, which in some countries is lower-taxed. We assume that $C(y) = cy + k$, where k is a positive fixed cost and $c \in (0,1)$ is a proportional cost per unit of avoided income. Note that the existence of a fixed cost will prevent the poorest individuals from avoiding since they can not afford it. The utility achieved by an avoider is thus $U(y - cy - k)$. Observe that if there is an individual who wants to be an avoider then the following two conditions must hold simultaneously: (i) $\tau > c$ and (ii) $k \leq (1-c)\overline{Y}$. Condition (i) eliminates the values of the tax rate for which tax avoidance never takes place, since for $\tau \leq c$ an individual obtains more utility being honest than being an avoider.[9] Condition (ii) ensures that at least the richest individual will be an avoider for $\tau > c$. In order to simplify the analysis, we will assume that condition (ii) holds in what follows.

3. The Government

The government obtains resources from the taxes paid by the individuals who are not avoiders and from the fines that individuals must pay if they are caught evading. Since the government inspects each individual with probability p, a fraction p of individuals is inspected in this large economy. For the sake of simplicity we assume that there are no costs associated with tax inspection.[10] Note that the density function $f(y)$ of a uniform distribution of income on $\left[0, \overline{Y}\right]$ is

$$f(y) = \begin{cases} \frac{1}{\overline{Y}} & \text{for} \quad 0 \leq y \leq \overline{Y} \\ 0 & \text{elsewhere} \end{cases}.$$

Therefore, the total government tax revenue per capita is given by the following Lebesgue integral

$$G(\tau) = \int_{A(\tau)} \left[(1-p)\tau\left(y - e(\tau,y)\right) + p\tau\left(y + se(\tau,y)\right)\right] \tfrac{1}{\overline{Y}} dy, \tag{3.1}$$

where $A(\tau)$ is the Lebesgue measurable subset of incomes in $\left[0, \overline{Y}\right]$ for which the individuals are non-avoiders when the tax rate is τ. Obviously, $G(\tau)$ is a continuous function on $[0,1]$. Note that avoiders do not pay taxes so that evaders are the only individuals who pay taxes, although possibly less than what they

[9] The utlity achieved by a honest tax payer is $U(y - \tau y)$. Since the utility function is monotonically increasing and $\tau \leq c$, we have

$$U(y - \tau y) \geq U(y - cy) > U(y - cy - k).$$

[10] Note that the assumption of a zero inspection cost does not affect the present analysis if the government is unable to identify ex-ante who is an evader and who is an avoider. In this case, under a constant probability p of inspection, the total inspection cost will be unaffected by the proportion of evaders and avoiders and, thus, it will be independent of the tax rate.

should. Note also that the tax rate τ affects $G(\tau)$ through its effects both on $e(\tau, y)$ and on the set $A(\tau)$. Then, our objective will be to analyze the behavior of the government tax revenue $G(\tau)$ when the tax rate changes. The next section provides the corresponding analysis.

4. The Laffer curve

In this section we will analyze whether the relationship between the tax rate and the government revenue can be described by a Laffer curve. There exist several definitions of the Laffer curve. In particular we will consider only two of them. The weakest one involves a non-monotone relationship between $G(\tau)$ and τ. A stronger definition of the Laffer curve requires the existence of a single maximum for $G(\tau)$ in the open interval $(0, 1)$. In this case the Laffer curve displays the typical inverted U shape. Laffer curves conforming the different previous two definitions will be found throughout the paper.

To carry out the analysis we need to separate the case where non-avoiders are honest from the case where they are strict evaders. In the former case uncertainty vanishes since individuals declare their true income, while in the latter case individuals hide part of their true income and they face up to a potential inspection which will reduce their net income.

4.1. Honest behavior

In this case the individuals who are not avoiders report their true income so that $e(\tau, y) = 0$. Following Lemma 2.1 this occurs when $s \geq \frac{1-p}{p}$. To study how the total tax revenue is modified when τ increases, we need to know which individuals become avoiders. This decision depends on the difference between the cost of being honest and the cost of being an avoider. For example, if either the costs associated with tax avoidance are high or the tax rate is low, poor individuals will prefer paying their corresponding taxes. Thus, for an individual with income y, avoiding is optimal whenever[11]

$$U(y - \tau y) \leq U(y - cy - k). \tag{4.1}$$

Solving (4.1) for τ, we obtain that

$$\tau(y) \geq c + \frac{k}{y}.$$

Therefore, for $\tau < c + \frac{k}{y}$ the individual with income y will prefer being honest rather than avoider. In particular, we can find the tax rate that leaves the richest individual indifferent between avoiding and being honest, that is,

$$\overline{\tau}(\overline{Y}) = c + \frac{k}{\overline{Y}}. \tag{4.2}$$

[11] We adopt the innocuous convention that when an individual is indifferent between being honest and avoiding, he is honest.

Observe that $\overline{\tau}(\overline{Y})$ is greater than the proportional cost of avoiding c if the fixed cost k is strictly positive. Hence, for $\tau < \overline{\tau}\left(\overline{Y}\right)$ all the individuals will pay their corresponding taxes. Obviously, $\tau(\overline{Y}) < 1$ for $k < (1-c)\overline{Y}$.

Now we need to know how many individuals will be honest for each possible value of the tax rate above $\overline{\tau}\left(\overline{Y}\right)$. It turns out that, for $\tau \in \left[\overline{\tau}(\overline{Y}), 1\right]$ there exists a threshold level of income $y_H^*(\tau) \in \left(0, \overline{Y}\right)$ making an individual indifferent between being honest and avoiding. In particular, $y_H^*(\tau)$ is the value of y satisfying (4.1) with equality. Formally,

$$y_H^*(\tau) = \frac{k}{\tau - c}. \tag{4.3}$$

Therefore, the individuals having incomes lower than $y_H^*(\tau)$ will tell the truth, while the individuals with higher incomes will be avoiders. Note that when $k = 0$, $y_H^*(\tau) = 0$. Hence, all the individuals become either honest or avoiders. Note also that $y_H^*(\tau)$ decreases with τ so that, if the tax rate increases, the lower income individuals will also benefit from avoidance.[12] Hence, for $\overline{\tau}\left(\overline{Y}\right) < \tau \leq 1$ only the individuals who can afford the fixed cost will be avoiders. In this case, the tax revenue is given by the total taxes paid for the individuals who are honest.

The following Lemma gives us the expression for the total tax revenue function:

Lemma 4.1. *Let $s \geq \frac{1-p}{p}$. Then, the total tax revenue will be given by the following two-part function:*

$$G(\tau) = \begin{cases} \frac{\tau\overline{Y}}{2} & \text{if } \; 0 \leq \tau \leq \overline{\tau}\left(\overline{Y}\right) \\ \\ \frac{\tau}{2\overline{Y}}\left(\frac{k}{\tau - c}\right)^2 & \text{if } \; \overline{\tau}\left(\overline{Y}\right) < \tau \leq 1 \end{cases}$$

Proof. See the Appendix.

The next proposition characterizes the behavior of $G(\tau)$ with respect to the tax rate.

Proposition 4.2. *Assume that $s \geq \frac{1-p}{p}$. Then, the total tax revenue $G(\tau)$ is a continuous function on $[0,1]$ that achieves its unique maximum value when $\tau = \overline{\tau}\left(\overline{Y}\right)$.*

Proof. See the Appendix.

The intuition of this result is quite obvious. When the tax rate of the economy is lower than $\overline{\tau}\left(\overline{Y}\right)$, all the individuals pay their corresponding taxes and the

[12]Observe that $y_H^*(\overline{\tau}\left(\overline{Y}\right)) = \overline{Y}$ and, thus, the threshold level $y_H^*(\tau)$ will be lower than $\overline{Y}$ for $\tau \in \left(\overline{\tau}\left(\overline{Y}\right), 1\right]$, since $y_H^*(\tau)$ decreases with τ.

tax revenue is increasing in the tax rate. On the other hand, when the tax rate is higher than $\overline{\tau}\left(\overline{Y}\right)$, two different effects take place. The first one is the same as before since higher tax rates have a positive effect on the tax revenue. However, there exists now an additional effect since an increase in the tax rate implies that some individuals (the richest ones) prefer now becoming avoiders and, in consequence, they do not pay taxes any longer. This effect has a clear negative impact on the tax revenue. As the Proposition 4.2 shows, for $\tau > \overline{\tau}\left(\overline{Y}\right)$ the negative effect outweighs the positive one and the tax revenue becomes decreasing in the tax rate. Note that the Laffer curve obtained in this subsection conforms with the stronger definition of such a curve, that is, the curve has a unique maximum on $(0, 1)$.

As an illustration of Proposition 4.2, Figure 1 displays the total tax revenue as a function of the tax rate when $c = 0.1$, $\overline{Y} = 100$, $k = 1$. In this case we obtain a standard Laffer curve with a maximum at $\overline{\tau}\left(\overline{Y}\right) = 0.11$

[Insert Figure 1 about here]

4.2. Tax Evasion

When $s < \frac{1-p}{p}$. program (2.1) yields a strictly positive amount of evaded income, $e(\tau, y) > 0$. The decision of being either an avoider or an evader will depend on the individual income and the values of the parameters of the model. For $\tau \leq c$ all the individuals will prefer evading, since the avoidance activity is only attractive for tax rates strictly greater than c. Therefore, for $0 < \tau \leq c$ the government obtains some positive revenue at least from the fines collected by the inspection.[13]

For $\tau > c$, tax avoidance can appear. Thus, an individual prefers avoiding to evading when $V(\tau, y) < U(y - cy - k)$, and vice versa. Hence, given p, s, c, and k, we can define $y^*(\tau)$ as the income that leaves an individual indifferent between evading and avoiding when the tax rate is τ, that is,

$$V(\tau, y^*(\tau)) = U(y^*(\tau) - cy^*(\tau) - k), \tag{4.4}$$

where the function $V(\tau, y)$ is defined in (2.3). The following lemma establishes the existence of such a threshold income level $y^*(\tau)$:

Lemma 4.3. *Assume that $s < \frac{1-p}{p}$, and $(1 + s)\tau > 1 + sc$. Then there exists an income $y^*(\tau) > 0$ such that all the individuals having an income lower than $y^*(\tau)$ are evaders, while all those with an income higher than $y^*(\tau)$ are avoiders.*

Proof. See the Appendix.

[13]The government does not know if the individuals will prefer evading all their income or evading only a part of it. Note that the fines collected include the amount of evaded taxes plus the penalty paid as a punishment to evasion.

As we expected, the previous results tell us that the richest individuals avoid the payment of their taxes while the poorest ones choose the evasion as a way of reducing their tax liabilities. This is so because the poorest individuals can not afford the fixed cost k of avoiding. Notice that assuming that $(1+s)\,\tau > 1 + sc$, implies that $(1+s)\,\tau > 1$, which ensures that the problem (2.1) has an interior solution.[14] In other words, Lemma 4.3 only applies for $0 < e(\tau, y) < y$. This fact does not constitute a very restrictive assumption since the available data about tax evasion shows that tax evasion takes place in a partial way.

The next proposition summarizes the main result concerning the existence of the Laffer curve in this context:

Proposition 4.4. *Assume $s < \frac{(1-p)}{p}$. If the fixed cost k is sufficiently small, then the total tax revenue $G(\tau)$ is non-monotonic in τ, for $\tau \in [0,1]$.*

Proof. See the Appendix.

Note that, when tax evasion occurs and the fixed cost of avoidance is low, we are only able to make a characterization of the Laffer curve in the weakest sense. To ensure that almost all individuals are avoiders when the tax rate is converging to one, we need to assume that the fixed cost is small enough. Therefore, the tax revenue will take a low value because only the poorest individuals pay taxes. On the contrary, in the absence of tax evasion (see subsection 4.1), we were able to make a stronger characterization which embodied the existence of a single maximum of the revenue function without any additional assumption about the fixed cost.

The importance of Proposition 4.4 is evident since a higher tax rate does not mean a higher tax revenue if individuals can avoid their income taxes. For instance, if individuals did not have the possibility of avoiding, an increase in the tax rate implies an increase in the tax revenue under decreasing absolute risk aversion.[15] This is so because, when the fine is proportional to the amount of evaded taxes, an increase in the tax rate translates into a rise in the amount that the taxpayer has to pay as a penalty. This induces less evasion since the substitution effect has been eliminated as a consequence of imposing penalties on evaded taxes and not on evaded income. Therefore, the final outcome will be an increase in the tax revenue.[16]

[14] See Yitzhaki (1974).

[15] A sufficient condition to ensure that nobody wants to avoid is $k > (1-c)\overline{Y}$.

[16] If tax avoidance is not allowed, then the tax revenue is given by

$$G(\tau) = \int_0^{\overline{Y}} \left[(1-p)\tau\left(y - e(\tau, y)\right) + p\tau\left(y + se(\tau, y)\right)\right] \frac{1}{\overline{Y}} dy.$$

Calculating the derivative of $G(\tau)$ respect to the tax rate we have

$$\frac{dG(\tau)}{d\tau} = \int_0^{\overline{Y}} \left[(1-p)\left(y - e(\tau, y) - \tau\frac{\partial e(\tau, y)}{\partial \tau}\right) + py + ps\left(e(\tau, y) + \tau\frac{\partial e(\tau, y)}{\partial \tau}\right)\right] \frac{1}{\overline{Y}} dy.$$

It is important to remark the important role that tax avoidance costs play in the analysis that we have just carried out. First, the fixed cost is necessary to guarantee that not everybody benefits from avoidance. Second, if the costs associated with the tax avoidance are too high, nobody wants to avoid and the tax revenue function will become increasing in the tax rate.

In the next section, we examine exhaustively an example with an isoelastic utility function which will allow us a more precise characterization of the revenue function $G(\tau)$.

5. An example: the isoelastic case

The isoelastic utility function has been widely used in several instances of economic analysis ranging from financial economics to macroeconomics. Let us only consider the interesting case where the evasion takes places (i.e. $s < \frac{1-p}{p}$) since the honest case has been fully characterized in Section 4.1 without using any specific functional form.

The optimal evasion when the utility function is $U(C) = \frac{C^{1-\gamma}}{1-\gamma}$, with $\gamma > 0$, becomes the following:

$$e(\tau, y) = \begin{cases} y & \text{if} \quad \tau \leq \tau^* \\ \left[\frac{(A-1)(1-\tau)}{\tau(1+As)}\right] y & \text{if} \quad \tau > \tau^*. \end{cases} \tag{5.1}$$

where $A = \left(\dfrac{ps}{1-p}\right)^{-\frac{1}{\gamma}}$, and $\tau^* \in (0,1)$ is the tax rate which separates full from partial evasion. Notice that $A > 1$ since $s < \frac{1-p}{p}$. The value of τ^* is

$$\tau^* = \frac{1 - \left(\frac{sp}{1-p}\right)^{\frac{1}{\gamma}}}{1+s}, \tag{5.2}$$

which is obtained rearranging the condition for $e < y$ given by Lemma 2.1. We can observe that the optimal evasion is a constant proportion $\phi(\tau) \in [0,1]$ of the true income y, where $\phi(\tau)$ can be expressed as

$$\phi(\tau) = \begin{cases} 1 & \text{if} \quad \tau \leq \tau^* \\ \frac{(A-1)(1-\tau)}{\tau(1+As)} & \text{if} \quad \tau > \tau^*. \end{cases} \tag{5.3}$$

The decision to be a full or a partial evader depends on the level of the tax rate. When the tax rate is small enough, individuals decide to evade all their income because, if they are inspected, the penalty that they must pay is not

It can be seen that $\frac{dG(\tau)}{d\tau} > 0$ since $\frac{\partial e(\tau,y)}{\partial \tau} < 0$ under the assumption of decreasing absolute risk (see Yitzhaki, 1974) and $s < \frac{(1-p)}{p}$.

very large. Nevertheless, when the tax rate increases the fine paid becomes also higher and this discourages individuals from evading all their income. Note that $\phi(1) = 0$. This means that the optimal evasion is equal to zero when $\tau = 1$ as a consequence of the assumption $\lim_{I \to 0} U'(I) = \infty$.

To see if the tax revenue displays the shape of a Laffer curve, we need to know how the individuals modify their behavior when the tax rate changes. Obviously, if individuals become avoiders, they do not pay taxes any longer and this fact has a significative impact on the tax revenue. Basically, we will consider two different values for the tax rate which imply qualitative changes in the individuals' behavior. On the one hand, we have the value τ^* of the tax rate which separates full from partial evasion. This value depends on the policy inspection parameters, s and p, and the parameter γ of the utility function. On the other hand, we have the proportional cost c of avoidance which affects the decision of becoming avoider. In this framework, we consider two possible scenarios: $\tau^* \leq c$ and $\tau^* > c$. The main difference between these two cases is that when $\tau^* \leq c$ the taxpayers are only full evaders for very low values of the tax rate $(\tau \leq \tau^*)$, while they are full evaders for a larger range larger of tax rates whenever $\tau^* > c$.

5.1. CASE 1: $\tau^* \leq c$

The first step is to calculate the different components of the tax revenue function $G(\tau)$. If $0 \leq \tau \leq \tau^*$, everybody prefers evading since $\tau^* < c$. Furthermore, as $\tau \leq \tau^*$ all the individuals will choose to be full evaders, that is, $e(\tau, y) = y$. Hence, the tax revenue will be only composed by the penalties paid by the consumers who are caught evading.

When the tax rate is higher than τ^*, evaders choose to declare only a part of their true income, that is, $e(\tau, y) < y$. On the other hand, all the individuals prefer being evaders for all $\tau \in [0, c]$. Therefore, we need to know if some individual prefers becoming avoider in the interval $\tau \in (c, 1]$. Observe that for an individual with income y we can find the value of the tax rate which leaves him indifferent between evading and avoiding. More precisely, this value is such that satisfies the following equality:

$$(1-p)\left[y - \tau y + \tau \frac{(A-1)(1-\tau)}{\tau(1+As)} y\right]^{1-\gamma} + \\ p\left[y - \tau y - s\tau \frac{(A-1)(1-\tau)}{\tau(1+As)} y\right]^{1-\gamma} = (y - cy - k)^{1-\gamma}. \tag{5.4}$$

Solving for τ we get[17]

$$\widehat{\tau}(y) = 1 - \left[\frac{(1-c)}{D} - \frac{k}{Dy}\right], \tag{5.5}$$

[17] We adopt the innocuous convention that when an individual is indifferent between evading and avoiding, he evades.

where

$$D \equiv \left[\left(\frac{1+s}{1+As} \right) \left((1-p)A^{1-\gamma} + p \right)^{\frac{1}{1-\gamma}} \right]. \tag{5.6}$$

Observe that $\widehat{\tau}(y)$ is decreasing in y and this means that for high income levels $\widehat{\tau}(y)$ will be small because the rich individuals can meet the cost of avoidance more easily. In particular, the first potential avoider is the richest individual, i.e., the one with $y = \overline{Y}$. Therefore, we can find which is the tax rate that leaves the richest individual indifferent between evading or avoiding by substituting $y = \overline{Y}$ into the expression (5.5). Thus, we get

$$\widehat{\tau}(\overline{Y}) = 1 - \left[\frac{(1-c)}{D} - \frac{k}{D\overline{Y}} \right]. \tag{5.7}$$

The following Lemma ensures that $\widehat{\tau}\left(\overline{Y}\right) \in (c, 1)$:

Lemma 5.1. *Assume* $s < \frac{1-p}{p}$, *then*
(a) $D > 1$.
(b) $\widehat{\tau}\left(\overline{Y}\right) \in (c, 1)$.

Proof. See the Appendix.

Note that the value $\widehat{\tau}\left(\overline{Y}\right)$ is greater than c because the richest individual prefers being an evader when $\tau = c$. This implies that, if $\tau^* \leq \tau \leq \widehat{\tau}\left(\overline{Y}\right)$, all the individuals prefer evading. Thus, as the evasion takes place only partially, the tax revenue comes both from the taxes voluntarily paid and from the fines paid by the inspected individuals.

Finally, when $\widehat{\tau}\left(\overline{Y}\right) < \tau \leq 1$, it is no longer true that the best option for all individuals is to become evaders. As the tax rate is growing the individuals who enjoy higher incomes will tend to prefer avoiding. Therefore, there exists an income $y_P^*(\tau)$ that makes an individual indifferent between being a partial evader and being avoider for a given tax rate. We can calculate explicitly this threshold level since $y_P^*(\tau)$ is such that (5.4) holds. The value of $y_P^*(\tau)$ is given by

$$y_P^*(\tau) = \frac{k}{(1-c) - (1-\tau)D}. \tag{5.8}$$

Note that $\tau \in \left(\overline{\tau}\left(\overline{Y}\right), 1\right]$ ensures that $y_P^*(\tau) < \overline{Y}$. Thus, in this case the tax revenue is only composed by the payments collected from the individuals who do not avoid.

The following Lemma gives us the expression of the function $G(\tau)$:

Lemma 5.2. *Let $s < \frac{1-p}{p}$ and $\tau^* \leq c$. Then, the total tax revenue will be given by the following three-part function:*

$$G(\tau) = \begin{cases} \frac{1}{2}p\tau\,(1+s)\,\overline{Y} & \text{if} \quad 0 \leq \tau \leq \tau^* \\ \\ \frac{1}{2}\tau\,(1-\phi(\tau)+p\,(1+s)\,\phi(\tau))\,\overline{Y} & \text{if} \quad \tau^* < \tau \leq \widehat{\tau}\left(\overline{Y}\right) \\ \\ \frac{1}{2\overline{Y}}\tau\,(1-\phi(\tau)+p\,(1+s)\,\phi(\tau))\,(y_P^*(\tau))^2 & \text{if} \quad \widehat{\tau}\left(\overline{Y}\right) < \tau \leq 1 \end{cases}$$

Proof. See the Appendix.

The next proposition describes the Laffer curve.

Proposition 5.3. *Assume that $\tau^* \leq c$ and $s < \frac{1-p}{p}$. Then, the total tax revenue $G(\tau)$ is strictly increasing on $\left[0, \widehat{\tau}\left(\overline{Y}\right)\right)$ and strictly decreasing on $\left(\widehat{\tau}\left(\overline{Y}\right), 1\right]$.*

Proof. See the Appendix.

We can illustrate the previous proposition by evaluating the tax rates $\widehat{\tau}(\overline{Y})$ given in expression (5.7) and τ^* given in expression (5.2) at $p = 0.1$, $s = 3$, $c = 0.15$, $\overline{Y} = 100$, $\gamma = 2$ and $k = 1$. We get $\tau^* = 0.10566$ and $\widehat{\tau}\left(\overline{Y}\right) = 0.19376$. Observe that in this case $s < \frac{1-p}{p}$, $\tau^* < c$ and $\widehat{\tau}\left(\overline{Y}\right) > c$. Figure 2 shows the tax revenue for these values.

[Insert Figure 2 about here]

Note that the function $G(\tau)$ is increasing in $\tau \in \left(0, \widehat{\tau}\left(\overline{Y}\right)\right)$, since all the individuals prefer being evaders when $\tau < \widehat{\tau}\left(\overline{Y}\right)$. This means that a higher tax rate implies a higher tax revenue as a consequence of two positive effects. First, the increase in the tax rate causes a raise of the tax revenue accruing from declared income. Second, the individuals declare a higher part of their true income since evaded income is decreasing in τ. When $\tau > \widehat{\tau}\left(\overline{Y}\right)$ not all the individuals want to be evaders. In particular, only the individuals with an income greater than $y_P^*(\tau)$ will evade. Thus, in this case we can distinguish two different effects on tax revenue. On the one hand, there is a positive effect due to the fact that a higher tax rate implies that the individuals who are evaders pay more taxes. On the other hand, we have a negative effect, since an increase of the tax rate causes a decrease on $y_P^*(\tau)$ which implies that less people pays taxes. As it has been proved, the second effect offsets the first one and the tax revenue is decreasing in τ for $\tau \in \left(\widehat{\tau}\left(\overline{Y}\right), 1\right]$. Therefore, the maximum value of the government revenue is reached at the tax rate $\widehat{\tau}\left(\overline{Y}\right)$.

Finally, let us point out that for the case $\tau^* < c$ we have been able to obtain a Laffer curve in its strongest sense under isoelastic preferences, while this characterization was not available without such a parametrization of the utility function (see Proposition 4.4). Moreover, we have dispensed with the more vague assumption of "sufficiently" low fixed avoidance cost required in Proposition 4.4.

5.2. CASE 2: $\tau^* > c$

Like in case 1 we will calculate the multi-part tax revenue function. We know that if $\tau < \tau^*$, the evader individuals are full evaders, that is, they evade all their true income. Moreover, all the individuals prefer evading rather than avoiding if $\tau \leq c$. Therefore, we need to know which will be the behavior of the richest individual when $\tau = \tau^*$. If he prefers being full evader, all the individuals will be full evaders. This situation would be similar as the one analyzed in the case 1. On the other hand, the richest individual could prefer being avoider at $\tau = \tau^*$. The following lemma establishes the condition under which such a circumstance occurs:

Lemma 5.4. *The richest individual of this economy strictly prefers being an avoider to being a full evader at $\tau = \tau^*$ if and only if*

$$k < \overline{Y}\left[(1-c) - \left[(1-p) + pA^{\gamma-1}\right]^{\frac{1}{1-\gamma}}\right]. \tag{5.9}$$

Proof. See the Appendix.

Observe that the fulfillment of this condition requires small values for both the fixed cost k and the proportional cost c since this makes easy for the richest individual to become avoider at $\tau = \tau^*$.

Assume from now that condition (5.9) holds. In consequence, we can ensure that there exists a tax rate smaller than τ^* leaving the richest individual indifferent between being full evader and avoiding. In particular, this tax rate satisfies:

$$(1-p)\left[\overline{Y}\right]^{1-\gamma} + p\left[\overline{Y} - \tau\overline{Y} - s\tau\overline{Y}\right]^{1-\gamma} = \left(\overline{Y} - c\overline{Y} - k\right)^{1-\gamma}. \tag{5.10}$$

Solving for τ we obtain

$$\widetilde{\tau}\left(\overline{Y}\right) = \frac{1 - \left[\frac{1}{p}\left(1 - c - \frac{k}{\overline{Y}}\right)^{1-\gamma} - \frac{(1-p)}{p}\right]^{\frac{1}{1-\gamma}}}{1+s}. \tag{5.11}$$

Note that condition (5.9) guarantees that $\widetilde{\tau}\left(\overline{Y}\right) \in (c, \tau^*)$. Then, for $0 \leq \tau \leq \widetilde{\tau}\left(\overline{Y}\right)$ we have that all the individuals are full evaders. Therefore, the tax revenue will be equal to the penalties paid by full evaders which have been inspected.

When $\widetilde{\tau}\left(\overline{Y}\right) \leq \tau \leq \tau^*$ it is not longer true that everybody prefers evading to avoiding. We define $y_T^*(\tau)$ as the income for which an individual is indifferent between evading all his true income and avoiding. The value of $y_T^*(\tau)$ is[18]

$$y_T^*(\tau) = \frac{k}{(1-c) - ((1-p) + p(1-\tau-\tau s)^{1-\gamma})^{\frac{1}{1-\gamma}}}.$$

Observe that $k \in \left(0, (1-c)\overline{Y}\right)$ and $\tau \in \left(\widetilde{\tau}\left(\overline{Y}\right), \tau^*\right]$ ensure that $y_T^*(\tau) \in \left(0, \overline{Y}\right)$. Hence, the tax revenue is only composed by the penalties paid by the inspected individuals who prefer being full evaders.

Finally, if $\tau^* < \tau \leq 1$, the evaded income is lower than the true income since for $\tau > \tau^*$ the full evaders become partial evaders.[19] In this case the tax revenue collected by the tax authorities comes both from the taxes voluntarily paid and from the penalties paid by the inspected partial evaders.

The following Lemma gives us the expression of the function $G(\tau)$:

Lemma 5.5. *Let $s < \frac{1-p}{p}$ and $\tau^* > c$. Then, the total tax revenue will be given by the following three-part function:*

$$G(\tau) = \begin{cases} \frac{1}{2} p\tau (1+s) \overline{Y} & \text{if} \quad 0 \leq \tau \leq \widetilde{\tau}\left(\overline{Y}\right) \\ \\ \frac{1}{2\overline{Y}} p\tau (1+s) \left(y_T^*(\tau)\right)^2 & \text{if} \quad \widetilde{\tau}\left(\overline{Y}\right) < \tau \leq \tau^* \\ \\ \frac{1}{2\overline{Y}} \tau \left(1 - \phi(\tau) + p(1+s)\phi(\tau)\right) \left(y_P^*(\tau)\right)^2 & \text{if} \quad \tau^* < \tau \leq 1. \end{cases}$$

Proof. See the Appendix.

The following proposition gives us the results obtained concerning the existence of the Laffer curve:

Proposition 5.6. *Assume $\tau^* > c$ and $s < \frac{1-p}{p}$. The total tax revenue $G(\tau)$ is non monotonic in τ , for $\tau \in [0,1]$ if $k < (1-c)\overline{Y}\sqrt{c\left(1-\phi(c) + p(1+s)\phi(c)\right)}$.*

Proof. See the Appendix. ■

In order to illustrate this result we evaluate the expressions (5.11) and (5.2) at $p = 0.1$, $s = 1.5$, $c = 0.1$, $\overline{Y} = 100$, $\gamma = 2$, $k = 1$. Thus, we have $\tau^* = 0.10566$ and $\widehat{\tau}\left(\overline{Y}\right) = 0.19376$. Figure 3 shows the tax revenue function $G(\tau)$ for the previous parameters values:

[Insert Figure 3 about here]

[18]The threshold $y_T^*(\tau)$ is obtained equating the utility from full evasion to the utility from avoiding.

[19]Note that $y_T^*(\tau) = y_P^*(\tau)$ for $\tau = \tau^*$.

When $\tau^* > c$ we only are able to make a characterization of the Laffer curve in the weakest sense like in the general case (see subsection 4.2). However, we do not invoke the assumption of "sufficiently" low fixed avoidance cost to obtain the Laffer curve since in this particular case we have found an explicit formula for the threshold fixed cost of avoidance below which a Laffer curve is obtained.

6. A discrete distribution of income

In the previous setup, we have assumed that the income distribution of the individuals of the economy was uniform. However, we can ask if the previous results about the existence of a Laffer curve relating tax revenues with tax rates would change if the income distribution is modified. Since it is impossible to address such a question with enough generality, in this section we consider a simple discrete income distribution with only three income classes y_1, y_2 and y_3, where $y_1 < y_2 < y_3$. Individuals are distributed into these three classes according to the proportions $\{\alpha_1, \alpha_2, \alpha_3\}$ respectively, where $\alpha_1 + \alpha_2 + \alpha_3 = 1$. In order to simplify the exposition we assume, without loss of generality, that $y_1 = 0$, $y_2 = m$ and $y_3 = 1$, where $0 < m < 1$. The combination of the proportions α_1, α_2, and α_3 as well as the size of m, will allow us to analyze different types of discrete income distributions. Observe that this distribution corresponds to an economy with a high degree of income polarization whereas such a polarization was absent under a uniform distribution. For simplicity we adopt the isoelastic utility function taken in Section 5. The present analysis will be divided into two parts: first we assume that $\tau^* \leq c$ holds and we then will examine the case where $c < \tau^*$.

6.1. CASE 1: $\tau^* \leq c$

Like in Section 5 we will find the different parts of the tax revenue function. Hence, for $\tau \in [0, \tau^*]$ all the individuals evade all their income, and then the tax revenue is given by the penalties paid by the inspected evaders. In this context the individuals with income $y_1 = 0$ do not contribute to the tax revenue because although they declare all their true income, the taxes paid are zero.[20]

When the current tax rate is greater than τ^*, partial evasion takes place. Moreover, for $\tau \leq c$ all the individuals will want to be evaders. However, when the tax rate is greater than c, the advantage of evasion over avoidance only comes from the fixed cost associated to avoidance. Similarly to the case with an uniform distribution of income, we can find which is the tax rate that leaves the richest individuals indifferent between avoiding and evading. Formally, we obtain this value substituting the value of y_3 into the expression (5.5). This yields

$$\widehat{\tau}_{y_3} = 1 - \left[\frac{(1-c) - k}{D} \right]. \tag{6.1}$$

[20] Note from (5.1) that the individuals with $y_1 = 0$ do not evade.

For $\tau \in (\tau^*, \widehat{\tau}_{y_3}]$ everybody is a partial evader. In this case, the tax revenue is composed both by the taxes voluntary paid and by the penalties from inspected individuals. Note that there exists a discontinuity at $\tau = \widehat{\tau}_{y_3}$ since the richest agents stop paying taxes and the tax revenue then falls. The magnitude of this jump depends on the proportion α_3, since a proportion very large of rich individuals implies that a large fraction of people does not pay their taxes any longer and this fact causes a dramatic decrease of the tax revenue. Hence, for $\tau > \widehat{\tau}_{y_3}$ only the individuals who have income y_2 continue paying taxes. However, as the tax rate raises the benefit obtained from evading becomes smaller since a high tax rate diminishes the expected utility of an evader. Then, from expression (5.5) the value of the tax rate that will leave the individuals with income y_2 indifferent between evading and avoiding is

$$\widehat{\tau}_{y_2} = 1 - \left[\frac{(1-c)}{D} - \frac{k}{Dm} \right]. \tag{6.2}$$

Then, $\widehat{\tau}_{y_2}$ is as the maximum value of the tax rate that has associated a positive value of the tax revenue since for $\tau > \widehat{\tau}_{y_2}$ the tax revenue becomes zero. Therefore, for $\tau \in (\widehat{\tau}_{y_3}, \widehat{\tau}_{y_2}]$ the tax revenue is equal to the taxes voluntarily paid by the individuals with income y_2 and to the fines paid by the inspected ones.

Observe from expression (6.2) that $\widehat{\tau}_{y_2} < 1$ if $k < (1-c)m$. This is so because when the fixed cost is large enough, the individuals with income y_2 never resort to avoidance and thus, they pay taxes until $\tau = 1$. Obviously, for $\tau > \widehat{\tau}_{y_2}$ all the individuals except the ones who have income y_1 prefer avoiding and, as a consequence, the tax revenue becomes zero.

The next Lemma gives us the expression of the function $G(\tau)$.

Lemma 6.1. *Let $s < \frac{1-p}{p}$ and $\tau^* \leq c$. Then, the total tax revenue will be given by the following four-part function:*

$$G(\tau) = \begin{cases} p\tau\,(1+s)\,(\alpha_2 m + \alpha_3) & \text{if} \quad 0 \leq \tau \leq \tau^* \\ (\alpha_2 m + \alpha_3)\,[\tau\,(1-\phi(\tau) + p\,(1+s)\,\phi(\tau))] & \text{if} \quad \tau^* < \tau \leq \widehat{\tau}_{y_3} \\ \alpha_2 m\,[\tau\,(1-\phi(\tau) + p\,(1+s)\,\phi(\tau))] & \text{if} \quad \widehat{\tau}_{y_3} < \tau \leq \widehat{\tau}_{y_2} \\ 0 & \text{if} \quad \widehat{\tau}_{y_2} < \tau \leq 1. \end{cases}$$

Proof. See the Appendix.

The following proposition summarizes the results concerning the existence of the Laffer curve when different assumptions on the proportions α_i when $i = 1, 2, 3$ are imposed.

Proposition 6.2. *Assume that $s < \frac{1-p}{p}$ and $k \in (0, m(1-c))$.*

(a) if $\alpha_3 \geq \alpha_2$, then the tax revenue function $G(\tau)$ has a single maximum at $\tau = \widehat{\tau}_{y_3}$,

(b) if the proportion α_3 of rich individuals is small enough, the tax revenue function $G(\tau)$ has a single maximum at $\tau = \widehat{\tau}_{y_2}$.

Proof. See the Appendix. ■

The intuition of result (a) relies on the fact that, if α_3 is large enough, when the tax rate is equal to $\widehat{\tau}_{y_3}$ many rich individuals stop paying taxes because they prefer avoiding their income. This constitutes a very significant reduction of the collected taxes since the great source of revenues comes from the rich individuals. Although the tax rate rises, the tax revenue can not recover the previous value achieved when $\tau = \widehat{\tau}_{y_3}$. This is so because the individuals with income y_2, which are the only ones who pay taxes, have not enough income to offset the loss that the avoidance of the rich individuals brings about. Observe that the assumption about the size of the fixed cost k allows the individuals with income y_2 to avoid his income for $\tau > \widehat{\tau}_{y_2}$, since otherwise when $k \in (m(1-c), (1-c))$ those individuals never benefit from avoidance for $\tau \in (0, 1)$.

Figure 4 shows the behavior of the tax revenue described in part (a) of Proposition 6.1. The parameters values considered are: $p = 0.1$, $s = 3$, $\gamma = 0.5$, $\alpha_1 = 0.4$, $\alpha_2 = 0.2$, $\alpha_3 = 0.4$, $m = 0.7$, $c = 0.25$, and $k = 0.25$.

[Insert Figure 4 about here] .

On the other hand, the part (b) of the previous proposition says us that when α_3 is quite small there is almost no difference between $G_2(\tau)$ and $G_3(\tau)$. Although rich individuals are avoiders, their weight on the total income is very small, and the tax revenue quickly offsets the loss caused by the avoidance of the rich individuals. Note that if $\alpha_3 < \alpha_2$ but α_3 and α_2 are close enough, we get the same results as in part (a) of Proposition 6.2.

Figure 5 illustrates, the behavior of the tax revenue described by part (b) of Proposition 6.1 when $p = 0.1$, $s = 3$, $\gamma = 0.5$, $\alpha_1 = 0.1$, $\alpha_2 = 0.8$, $\alpha_3 = 0.1$, $m = 0.7$, $c = 0.25$, and $k = 0.25$.

[Insert Figure 5 about here] .

Observe that, even if the results stated in Proposition 6.2 do not depend explicitly on the value of m, this value plays an important role in the analysis of the behavior of the government revenue under alternative income distributions. An increase of m, for given values of α_1, α_2, and α_3, gives rise to two opposite effects on the government revenues. On the one hand, if the value of the parameter m increases, then the individuals having an income equal to y_2 become richer and, therefore, they pay more taxes and contribute more to the revenues raised by the government. On the other hand, from inspection of (6.2) we can check that the value of the tax rate that will leave the individuals with income y_2 indifferent between evading and avoiding decreases as m increases. This implies in turn that the government is not going to get revenues for lower values of the tax rate.

In order to get a more precise intuition on the role played by the parameter m, let us consider an income distribution where α_1 and α_3 are small and α_2 is large. In this context, when m is close enough to zero one obtains an income distribution where a vast majority of individuals is poor. In this case, the government could set very high tax rates since poor individuals will keep paying taxes since they could not afford the avoidance costs. This kind of fiscal policy ends up being very regressive since the individuals that contribute to the increase in government revenues are just the ones having lower income. However, when m is close to 1 and the vast majority of individuals is thus quite rich, the government should set tax rates not very high so as to prevent individuals from avoiding. Obviously, since now the individuals enjoy a higher income, the value of the government revenue is generically higher than in the previous case displaying high tax rates with poor individuals.

Finally, observe that when the income distribution function is discrete, the existence of the Laffer curve in the weak sense is always guaranteed by the non-continuity of the function $G(\tau)$.

6.2. CASE 2: $\tau^* > c$

The results obtained in this case will depend on the behavior that individuals adopt at $\tau = \tau^*$. More specifically, we will have the following scenarios according to the values of the parameters:

Scenario A

$$0 < c < \widetilde{\tau}_{y_3} < \widetilde{\tau}_{y_2} < \tau^* < 1 \qquad \text{when} \qquad k < m\Phi$$

Scenario B

$$0 < c < \widetilde{\tau}_{y_3} < \tau^* < \widehat{\tau}_{y_2} < 1 \qquad \text{when} \qquad m\Phi < k < \Phi,$$

where

$$\Phi = \left[1 - c - \left(1 - p + pA^{\gamma-1}\right)^{\frac{1}{1-\gamma}}\right],$$

$$\widetilde{\tau}_{y_3} = \frac{1 - \left[\frac{1}{p}(1-c-k)^{1-\gamma} - \frac{(1-p)}{p}\right]^{\frac{1}{1-\gamma}}}{1+s},$$

and

$$\widetilde{\tau}_{y_2} = \frac{1 - \left[\frac{1}{p}\left(1-c-\frac{k}{m}\right)^{1-\gamma} - \frac{(1-p)}{p}\right]^{\frac{1}{1-\gamma}}}{1+s}. \tag{6.3}$$

The values of $\widetilde{\tau}_{y_3}$ and $\widetilde{\tau}_{y_2}$ have been obtained substituting $y_3 = 1$ and $y_2 = m$ in expression (5.11).

Scenario A presents a situation where the fixed cost k is low enough to induce both the richest individuals and the individuals with income m to prefer avoiding at $\tau = \tau^*$. Using the same arguments as in Sections 5 it is easy to see that for $\tau \in [0, \widetilde{\tau}_{y_3}]$ all individuals are full evaders. For $\tau \in (\widetilde{\tau}_{y_3}, \widetilde{\tau}_{y_2}]$ the richest individuals avoid their income while the rest of the individuals still are full evaders. Finally, nobody pays taxes for $\tau \in (\widetilde{\tau}_{y_2}, 1]$. Hence, the tax revenue function is given by

$$G(\tau) = \begin{cases} p\tau\,(1+s)\,(\alpha_2 m + \alpha_3) & \text{if} \quad 0 \le \tau \le \widetilde{\tau}_{y_3} \\ \\ p\tau\,(1+s)\,\alpha_2 m & \text{if} \quad \widetilde{\tau}_{y_3} < \tau \le \widetilde{\tau}_{y_2} \\ \\ 0 & \text{if} \quad \widetilde{\tau}_{y_2} < \tau \le 1. \end{cases}$$

In scenario B, the fixed cost is not low enough for allowing the individuals with income m to avoid their income when $\tau = \tau^*$, then they evade all their true income. Then, using the arguments seen in the previous sections it is easy to see that for $\tau \in [0, \widetilde{\tau}_{y_3}]$ everybody will evade. For $\tau \in (\widetilde{\tau}_{y_3}, \tau^*]$ the richest individuals (with income y_3) are avoiders and the individuals with income m are full evaders, for $\tau \in (\tau^*, \widehat{\tau}_{y_2}]$ the individuals with income m still being evaders but now they are partial evaders. Finally, the tax revenue is zero for $\tau \in (\widehat{\tau}_{y_2}, 1]$ because the only individuals who do not avoid their income (individuals with income y_1) do not even pay taxes. Summarizing, we have that the tax revenue function is the following four-part function:

$$G(\tau) = \begin{cases} p\tau\,(1+s)\,(\alpha_2 m + \alpha_3) & \text{if} \quad 0 \le \tau \le \widetilde{\tau}_{y_3} \\ \\ p\tau\,(1+s)\,\alpha_2 m & \text{if} \quad \widetilde{\tau}_{y_3} < \tau \le \tau^* \\ \\ \alpha_2 m \left[\tau\,(1 - \phi(\tau) + p\,(1+s)\,\phi(\tau))\right] & \text{if} \quad \tau^* < \tau \le \widehat{\tau}_{y_2} \\ \\ 0 & \text{if} \quad \widehat{\tau}_{y_2} < \tau \le 1. \end{cases}$$

The main result concerning the existence of the Laffer curve both in scenario A and in scenario B is that the Laffer curve always exists, and its specific form depends on the values taken by the proportions α_1, α_2 and by the income m. Next proposition summarizes the main results about the Laffer curve under the two scenarios considered above:

Proposition 6.3. *Assume that $s < \frac{1-p}{p}$. Then,*

(a) if $k < m\Phi$ and the income m is large enough, the tax revenue function achieves its unique maximum at $\tau = \widetilde{\tau}_{y_3}$,

(b) if $k < m\Phi$ and the proportion α_3 is small enough, the tax revenue function achieves its unique maximum at $\tau = \widetilde{\tau}_{y_2}$,

(c) if $m\Phi < k < \Phi$ and the proportion α_3 is large enough, the tax revenue function achieves its unique maximum at $\tau = \widetilde{\tau}_{y3}$,

(d) if $m\Phi < k < \Phi$ and the proportion α_3 is small enough, the tax revenue function achieves its unique maximum at $\tau = \widehat{\tau}_{y_2}$.

Proof. See the Appendix.

The intuition behind the statement (a) is quite clear because, if m is close to one, it means that the individuals with income y_2 are rather rich and in consequence they will resort to tax avoidance almost for the same tax rates as the richest individuals. Thus, the tax revenue becomes zero for $\tau > \widetilde{\tau}_{y_2}$ when $\widetilde{\tau}_{y_2} \in (\widetilde{\tau}_{y_3}, \widetilde{\tau}_{y_3} + \varepsilon(m))$, where ε is decreasing in m and $\lim_{m \to 1} \varepsilon(m) = 0$. Hence, for m large enough we will have that the tax revenue will be zero before the penalties paid by the inspected individuals with income y_2, can offset the loss in the tax revenue caused by the richest individuals. The intuition of the statements (b), (c) and (d) is supported by the role that the proportion α_3 of richest individuals plays. If α_3 is large enough the loss in the tax revenue can not be offset by the resources obtained from the individuals with income m. When α_3 is small enough, the previous argument works in the opposite direction since the loss due to the tax avoidance by the richest individuals is very small and it can be offset by the payments made by the individuals with income y_2.

Finally, we have to point out that the scenarios considered when $\tau^* > c$ do not constitute a good approach of the situation in the real world since the available data of tax evasion reveals that tax evasion takes place in a partial way, i.e., $e < y$.

7. Conclusions and extensions

We have shown that the possibility of choosing between avoiding and evading brings about a tax revenue function exhibiting the shape of a Laffer curve. That is, the relationship between tax rates and government revenue is non-monotonic. We have carried out the analysis in a partial equilibrium context where the individuals have the same utility function but differ in their incomes. In all the scenarios studied we have found that the tax revenue always displays a Laffer curve under some conditions. This fact has to be taken into account when the different governments design the fiscal policies because, when the tax avoidance phenomenon is present, raising the tax rate might not result in an increase of the tax revenue. Moreover, when the costs associated with tax avoidance are low, to set high tax rates and to implement a strong anti-evasion policy is not only ineffective but also regressive because all the rich individuals will avoid their incomes and will pay no taxes. Under this policy only the poorest individuals will pay taxes (and/or penalties associated with tax evasion) since they cannot afford the cost of avoidance. In fact, a government could reduce the negative impact on tax revenues accruing from the possibility of tax avoidance by implementing the kind of policies analyzed by Chu (1990). He proposes a new policy called FATOTA which allows a target group of corporations to choose between two options: to pay a fixed amount of taxes set by the tax authorities and thus to be exempted from tax inspection, or to pay only an amount chosen by the corporation but be subject to a positive probability of being audited. This type of policies would become effective provided the fixed amount to be paid by the potential avoiders were smaller than the costs associated with avoidance.

Some extensions of the present work are possible. We comment on some of them. We could introduce a progressive tax function in our framework and to analyze the effects on government tax revenue of a modification in the progressiveness of the tax function. The result one should expect is that a higher degree of progressiveness will stimulate the avoidance of rich individuals and, therefore, the decreasing part of the Laffer curve will appear at lower average tax rates.

We could also consider other continuous income distributions, like the lognormal or the Pareto ones, which could better fit the empirical income distributions. Nevertheless, such functional form would prevent us from getting explicit results and we should rely instead on simulations.

Finally, in our model all the individuals have the same utility function and they only differ in their income. It could be interesting to consider heterogenous preferences. To this end, we could assume a distribution on the relevant parameters characterizing the indexes of risk aversion of the individuals.

A. Appendix

Proof of Lemma 2.1. Since the function $u(e)$ defined in (2.1) is strictly concave, we obtain the corner solution $e = 0$ whenever $u'(0) \leq 0$. This weak inequality is in fact equivalent to $s \geq \frac{1-p}{p}$. Notice that when $\tau(1+s) \geq 1$, it is impossible to have $e = y$ since, then, the income of an inspected individual $y - \tau(1+s)y$ cannot be non-positive as follows from the Inada condition. For $\tau(1+s) < 1$, to obtain the corner solution $e = y$, we need that $u'(y) \geq 0$. This inequality becomes

$$s \leq \left(\frac{1-p}{p}\right) \frac{U'(y)}{U'(y - \tau y - s\tau y)}. \blacksquare$$

Proof of Lemma 4.1. For $0 \leq \tau \leq \overline{\tau}\left(\overline{Y}\right)$ the tax revenue is composed by the taxes that all the individuals pay. This revenue is

$$G_1(\tau) = \int_0^{\overline{Y}} \tau y \frac{1}{\overline{Y}} dy.$$

For $\overline{\tau}\left(\overline{Y}\right) < \tau \leq 1$ the tax revenue is given by the total taxes paid for the individuals who are honest. This revenue is,

$$G_2(\tau) = \int_0^{\frac{k}{\tau - c}} \tau y \frac{1}{\overline{Y}} dy.$$

Performing the integrals $G_i(\tau)$ where $i = 1, 2$, we obtain the expression appearing in the statement of the Lemma. $\blacksquare$

Proof of Proposition 4.2. Differentiating $G_1(\tau)$ respect to τ we have $\frac{dG_1}{d\tau} = \frac{\overline{Y}}{2}$ which is unambiguously positive. On the other hand, differentiating $G_2(\tau)$ respect to τ we obtain $\frac{dG_2}{d\tau} = -\frac{k^2(\tau+c)}{2\overline{Y}(\tau-c)^3}$, which is unambiguously negative. Obviously, the maximum value achieved by $G(\tau)$ is just the kink point $\tau = c + \frac{k}{\overline{Y}}$. $\blacksquare$

Proof of Lemma 4.3. The proof adapts some of the arguments in Chu (1990).

Step 1. We will prove that $V(\tau, y)$ and $U(y - cy - k)$ intersect at least once.

As when the true income tends to zero, the optimal evasion also tends to zero, we have that at least for $y < \frac{k}{\tau - c}$, $U(y - cy - k) < V(\tau, y)$, since k is positive and by assumption $\tau > c$. Then, we only have to see that there exists an income level such that $U(y - cy - k) > V(\tau, y)$ for a given τ. Consider the level evasion $e_0(\tau, y) = \frac{(1-\tau)y}{\tau s}$, which is less than y since we were assuming that $(1+s)\tau > 1$ holds. It can be seen that

$$y - \tau y + \tau e_0 = \frac{(1+s)(1-\tau)y}{s}$$

and

$$y - \tau y - \tau s e_0 = 0.$$

Therefore, we get that

$$u'(e_0) = (1-p)U'\left(\frac{(1+s)(1-\tau)y}{s}\right)\tau - pU'(0)\, s\tau.$$

Since, by assumption, $\lim_{I\to 0} U'(I) = \infty$ we have

$$\lim_{I\to 0} u'(e_0) = -\infty,$$

which implies that $\widehat{e}\,(\tau, y) < e_0\,(\tau, y)$, where $\widehat{e}\,(\tau, y)$ is the optimal evasion given in Lemma 2.1. Obviously, this means that

$$y - \tau y + \tau\widehat{e}\,(\tau, y) < y - \tau y + \tau e_0\,(\tau, y)\,. \tag{A.1}$$

Now define y_0 as the income that makes equal the net income of avoiding with the net income of evading for $e_0\,(\tau, y)$ when the inspection does not occurs. By definition we have

$$y_0 - cy_0 - k = y_0 - \tau y_0 + \tau e_0\,(\tau, y_0)\,.$$

After rearranging terms we have that the value y_0 becomes

$$y_0 = \frac{sk}{s(1-c) - (s+1)(1-\tau)}.$$

Condition $(1+s)\,\tau > 1 + sc$ guarantees that $y_0 > 0$. Then, for $y > y_0$ we get

$$y\left[(1-c) - \frac{(s+1)\,(1-\tau)}{s}\right] > y_0\left[(1-c) - \frac{(s+1)\,(1-\tau)}{s}\right] = k.$$

Hence, we have

$$y\,(1-c) - k > \frac{y\,(s+1)\,(1-\tau)}{s} = y - \tau y + \tau e_0\,(\tau, y)\,.$$

Finally by (A.1) it holds that

$$y\,(1-c) - k > y - \tau y + \tau\widehat{e}\,(\tau, y)\,.$$

Therefore for $y > y_0$ the following inequality must also hold:

$$y\,(1-c) - k > y - \tau y + \tau\widehat{e}\,(\tau, y) > V(\tau, y).$$

Summarizing, we have that the individuals with income $y > y_0$ prefer avoiding to evading while the individuals with an income y small enough prefer evading. This guarantees then, that $V(\tau, y)$ and $U(y - cy - k)$ intersect at least once.

Step 2. We will prove that $V(\tau, y)$ and $U(y - cy - k)$ intersect only once.

Let be y^* any intersection of $V(\tau, y)$ and $U(y - cy - k)$. This implies that $V(\tau, y^*) = U(y^* - cy^* - k)$ and consequently we have that

$$y^* - \tau y^* - s\tau\widehat{e}(\tau, y^*) < y^* - cy^* - k < y^* - \tau y^* + \tau\widehat{e}(\tau, y^*)$$

must hold. This inequality implies that

$$U'(y^* - cy^* - k) > U'(y^* - \tau y^* + \tau\widehat{e}(\tau, y^*)), \tag{A.2}$$

since the function $U(\cdot)$ is concave. By the envelop theorem we get that

$$\begin{aligned} \frac{\partial V(\tau, y^*)}{\partial y} &= (1-p)U'(y^* - \tau y^* + \tau\widehat{e}(\tau, y^*)) + \\ &\quad pU'(y^* - \tau y^* - s\tau\widehat{e}(\tau, y^*))(1 - \tau - \tau s), \end{aligned}$$

which can be simplified using (2.2) as

$$\frac{\partial V(\tau, y^*)}{\partial y} = \left(\frac{(1-p)(1+s)(1-\tau)}{s}\right) U'(y^* - \tau y^* + \tau\widehat{e}(\tau, y^*)). \tag{A.3}$$

The condition $(1+s)\tau > 1 + sc$, implies that $(1+s)\tau > 1$ also holds, and this is a sufficient condition for ensuring that $\left(\frac{(1-p)(1+s)(1-\tau)}{s}\right) < 1$. Thus, using (A.2) and (A.3) we get that

$$U'(y^* - cy^* - k) > V'(\tau, y^*). \tag{A.4}$$

It is immediate to check that $V(\tau, y)$ is a concave function, then the inequality (A.4) means that $U(\cdot)$ and $V(\cdot)$ intersect once at most. ■

Proof of Proposition 4.4. When the tax rate is zero the tax revenue is also zero because the government can not collect neither taxes nor fines. For $\tau \in (0, c]$ all the individuals prefer being evaders, therefore the government gets some revenue at least from the fines collected by the inspection. Moreover, for $\tau \in (0, c]$, $G(\tau)$ is increasing in τ since the optimal evasion is decreasing in τ (see Yitzhaki, 1974). In particular evaluating the tax revenue function given by (3.1) at $\tau = c$, we have that $G(c) > 0$. Nevertheless, we do not know which is the exact value of $G(c)$ because, although all the individuals are evaders, we ignore if they evade all his income or only a part of it. This decision depends on the own income, the tax rate and the values of p and s. Our proof leans on showing that $G(c) > G(1)$, because this implies that the tax revenue function can not be monotonic. Let us first investigate what happens with the optimal evasion when $\tau = 1$. From the expression (2.2), we have that

$$U'(y - \tau y - s\tau e(\tau, y)) = \frac{(1-p)}{ps} U'(y - \tau y + \tau e(\tau, y)).$$

We know that $y - \tau y + \tau e(\tau, y) > 0$ for $\tau \in (0,1)$ so that condition $\lim_{I \to 0} U'(I) = \infty$ implies that

$$y - \tau y - s\tau e(\tau, y) > 0.$$

Rearranging the last inequality we have that

$$0 \leq e(\tau, y) < \frac{(1-\tau)\, y}{\tau s}. \tag{A.5}$$

Therefore,

$$0 \leq \lim_{\tau \to 1} e(\tau, y) \leq \lim_{\tau \to 1} \frac{y(1-\tau)}{\tau s} = 0.$$

We see that when the tax rate tends to one the optimal tax evasion tends to zero or, in other words, the individuals tell the truth independently of their respective incomes. This result allows us to say that only the individuals who can not face up to the fixed cost of avoidance will pay taxes, whereas the others will be avoiders. Thus, the tax revenue when $\tau = 1$ will be

$$\begin{aligned} G(1) &= \lim_{\tau \to 1} \int_0^{y^*(\tau)} \left[(1-p)\tau \left(y - e(\tau, y) \right) + p\tau \left(y + se(\tau, y) \right) \right] \frac{1}{\overline{Y}} dy \\ &= \int_0^{\frac{k}{1-c}} y \frac{1}{\overline{Y}} dy = \frac{k^2}{2\overline{Y}(1-c)^2}, \end{aligned}$$

since $\lim_{\tau \to 1} y^*(\tau) = \lim_{\tau \to 1} y_H^*(\tau) = \dfrac{k}{(1-c)}$ when $e(\tau, y) = 0$. Note that $\lim_{\tau \to 1} y^*(\tau) \in \left(0, \overline{Y}\right)$ whenever $k \in \left(0, (1-c)\overline{Y}\right)$. As $G(c)$ does not depend on k, we can make $G(c) > G(1)$ for a small enough value of k. ■

Proof of Lemma 5.1.
(a) Fix γ and then consider D defined in (5.6) as a function of s and p, $D(s,p)$. After tedious differentiation, it can be proved that $\frac{\partial D}{\partial s} = 0$ and $\frac{\partial D}{\partial p} = 0$ only when $s = \frac{1-p}{p}$. Moreover, the Hessian of $D(s,p)$ evaluated at $s = \frac{1-p}{p}$ is

$$H(s,p) = \begin{pmatrix} \frac{p^3}{\gamma(1-p)} & \frac{p}{\gamma(1-p)} \\ \frac{p}{\gamma(1-p)} & \frac{1}{\gamma p(1-p)} \end{pmatrix},$$

which is positive semi-definite. Finally, we get that $D(s,p) = 1$ whenever $s = \frac{1-p}{p}$. We conclude thus that $D(s,p) > 1$ for all $s \neq \frac{1-p}{p}$. In particular, $D > 1$ for $ps < 1-p$.

(b) To see that $c < \widehat{\tau}\left(\overline{Y}\right) < 1$ we only have to prove that $0 < \left[\frac{(1-c)}{D} - \frac{k}{D\overline{Y}}\right] < 1$. The assumption that $k \in \left(0, (1-c)\,\overline{Y}\right)$ ensures that $\left[\frac{(1-c)}{D} - \frac{k}{D\overline{Y}}\right] > 0$, while the part (a) of this lemma implies that $\left[\frac{(1-c)}{D} - \frac{k}{D\overline{Y}}\right] < 1$. ■

Proof of Lemma 5.2. For $0 \leq \tau \leq \tau^*$ the tax revenue is only composed by the fines paid by the inspected individuals. This revenue is

$$G_1(\tau) = \int_0^{\overline{Y}} p\tau\,(1+s)\,y\frac{1}{\overline{Y}}dy$$

For $\tau^* \leq \tau \leq \hat{\tau}\left(\overline{Y}\right)$ the tax revenue is given by the taxes and penalties paid for all the individuals. This revenue is

$$\begin{aligned} G_2(\tau) &= \int_0^{\overline{Y}} \left[(1-p)\tau(1-\phi(\tau)) + p\tau\,(1+s\phi(\tau))\right] y\frac{1}{\overline{Y}}dy \\ &= \int_0^{\overline{Y}} \left[\tau(1-\phi(\tau)) + p\tau\,(1+s)\,\phi(\tau)\right] y\frac{1}{\overline{Y}}dy. \end{aligned}$$

Finally, for $\hat{\tau}\left(\overline{Y}\right) < \tau \leq 1$ the tax revenue is

$$\begin{aligned} G_3(\tau) &= \int_0^{y_P^*(\tau)} \left[(1-p)\tau(1-\phi(\tau)) + p\tau\,(1+s\phi(\tau))\right] y\frac{1}{\overline{Y}}dy \\ & \int_0^{y_P^*(\tau)} \left[\tau(1-\phi(\tau)) + p\tau\,(1+s)\,\phi(\tau)\right] y\frac{1}{\overline{Y}}dy, \end{aligned}$$

since the tax authorities can only collect payments from the individuals who do not avoid.

Performing the integrals $G_i\,(\tau)$ where $i = 1,2,3$ we obtain the expression appearing in the statement of the Lemma. ■

Proof of Proposition 5.3. We prove the proposition by stating two claims.

Claim 1: $\frac{dG_1(\tau)}{d\tau} > 0$ for $(0,\tau^*]$ and $\frac{dG_2(\tau)}{d\tau} > 0$ for $\left(\tau^*, \hat{\tau}\left(\overline{Y}\right)\right]$.

Calculating $\frac{dG_1(\tau)}{d\tau}$ we have

$$\frac{dG_1(\tau)}{d\tau} = \frac{p(1+s)\overline{Y}}{2} > 0,$$

which is unambiguously positive. In a similar way, we obtain

$$\frac{dG_2(\tau)}{d\tau} = \frac{\overline{Y}}{2}\left[(1-\phi(\tau) + p\,(1+s)\,\phi(\tau)) + \tau\,(p\,(1+s)-1)\,\frac{d\phi(\tau)}{d\tau}\right]. \tag{A.6}$$

From expression (5.3) we have $\frac{d\phi(\tau)}{d\tau} = -\alpha\frac{1}{\tau^2} < 0$, where $\alpha > 0$ is given by

$$\alpha = \frac{(A-1)}{(1+As)}. \tag{A.7}$$

Note that the first term inside the square brackets of (A.6) is positive since the parameters satisfy the interior condition $s < \frac{1-p}{p}$. On the other hand, the second term inside the square brackets is also positive since $\frac{d\phi(\tau)}{d\tau} < 0$ and $(p\,(1+s)-1) < 0$. Then, $G_2(\tau)$ is also increasing in the tax rate.

Claim 2: $\frac{dG_3(\tau)}{d\tau} < 0$ for $\left(\widehat{\tau}\left(\overline{Y}\right), 1\right]$.

We have the following derivative:

$$\frac{dG_3(\tau)}{d\tau} = \frac{1}{\overline{\overline{Y}}}\left[\left(\tau\,(1-\phi(\tau)) + \tau p\,(1+s)\,\phi(\tau)\right)\frac{dy_P^*(\tau)}{d\tau}y^*(\tau)\right] +$$

$$\frac{1}{\overline{\overline{Y}}}\left[\frac{1}{2}\,(y^*(\tau))^2\left(1-\phi(\tau)+p\,(1+s)\,\phi(\tau)+\tau\,(p\,(1+s)-1)\,\frac{d\phi(\tau)}{d\tau}\right)\right]. \tag{A.8}$$

To evaluate the sign of the previous expression we need to calculate $\frac{dy^*(\tau)}{d\tau}$. Using expression (5.8), we get

$$\frac{dy_P^*(\tau)}{d\tau} = -\frac{D}{E}y_P^*(\tau), \tag{A.9}$$

where $E = [(1-c)-(1-\tau)D]$. Since $y_P^*(\tau) > 0$, $D > 0$ and $E > 0$ for $\tau \geq \widehat{\tau}\left(\overline{Y}\right)$, we can conclude that $\frac{dy_P^*(\tau)}{d\tau} < 0$. Then, plugging (A.9) in (A.8) and rearranging terms, we have

$$\frac{dG_3(\tau)}{d\tau} = \frac{(y_P^*(\tau))^2}{\overline{Y}}\left[\left[\tau\,(1+\alpha\,(1-p\,(1+s)))-\alpha\,(1-p\,(1+s))\right]\left(\frac{-D}{E}\right)\right] +$$

$$\frac{(y_P^*(\tau))^2}{\overline{Y}}\left[\frac{1}{2}\,(1+\alpha\,(1-p\,(1+s)))\right].$$

Note that the sign of the previous derivative depends only on the sign of the expression into the brackets. Define

$$F \equiv \left(\tau\,(1+\alpha\,(1-p\,(1+s)))-\alpha\,(1-p\,(1+s))\right)\left(\frac{-D}{E}\right)+\frac{1}{2}\,(1+\alpha\,(1-p\,(1+s))),$$

which can be rewritten as

$$F = \frac{-D\left[1-\alpha(1-ps)+\tau\,(1+\alpha\,(1-p\,(1+s)))\right]+(1-c)\,(1+\alpha\,(1-p\,(1+s)))}{2E}$$

The sign of F is the same as that of its numerator since $E > 0$ for $\tau \geq \widehat{\tau}\left(\overline{Y}\right)$. Therefore if our objective is to prove that $\frac{dG_3(\tau)}{d\tau} < 0$, we have to prove that the following inequality holds:

$$D\left[1-\alpha\,(1-p\,(1+s))+\tau\,(1+\alpha\,(1-p\,(1+s)))\right] >$$

$$(1-c)\,(1+\alpha\,(1-p\,(1+s))). \tag{A.10}$$

Note that the tax revenue $G_3(\tau)$ is obtained when the tax rate moves between $\widehat{\tau}\left(\overline{Y}\right) < \tau \leq 1$. Then, if

$$D\left[1-\alpha(1-p\left(1+s\right))+\widehat{\tau}\left(1+\alpha\left(1-p\left(1+s\right)\right)\right)\right] > (1-c)\left(1+\alpha\left(1-p\left(1+s\right)\right)\right), \tag{A.11}$$

we can guarantee that inequality (A.10) also holds for $\tau \in \left[\widehat{\tau}\left(\overline{Y}\right), 1\right]$ since the left term of inequality (A.10) is increasing in τ. Thus, substituting

$$\widehat{\tau}\left(\overline{Y}\right) = 1 + \frac{k}{D\overline{Y}} - \frac{(1-c)}{D}$$

into (A.11) we get

$$D\left[1-\alpha\left(1-p\left(1+s\right)\right)+\left(1+\frac{k}{D\overline{Y}}-\frac{(1-c)}{D}\right)\left(1+\alpha\left(1-p\left(1+s\right)\right)\right)\right] > (1-c)\left(1+\alpha\left(1-p\left(1+s\right)\right)\right).$$

Rearranging and simplifying, we obtain

$$2D + \frac{k}{\overline{Y}}\left(1+\alpha\left(1-p\left(1+s\right)\right)\right) > 2(1-c)\left(1+\alpha\left(1-p\left(1+s\right)\right)\right). \tag{A.12}$$

According to lemma 5.1, $D > 1$. Hence,

$$(1-c)\left(1+\alpha(1-ps)\left(1-p\left(1+s\right)\right)\right) < 1, \tag{A.13}$$

becomes a sufficient condition for (A.12). Rearranging the inequality (A.13) we have

$$\alpha(1-c) - \alpha p\left(1+s\right)(1-c) - c < 0. \tag{A.14}$$

It is easy to see that inequality (A.14) holds if

$$c > \frac{\alpha}{1+\alpha}. \tag{A.15}$$

Using (5.2) and (A.7) and rearranging terms, we get that the sufficient condition (A.15) becomes simply $c > \tau^*$, and that is always true by assumption. Hence, it follows that $\frac{dG_3(\tau)}{d\tau} < 0$. ■

Proof of Lemma 5.4. The richest individual wants to be avoider if the utility from avoidance is greater than the expected indirect utility from full evasion at $\tau = \tau^*$. This is

$$(1-p)\left[\overline{Y}\right]^{1-\gamma} + p\left[\overline{Y} - \tau^*\overline{Y} - s\tau^*\overline{Y}\right]^{1-\gamma} < \left(\overline{Y} - c\overline{Y} - k\right)^{1-\gamma}. \tag{A.16}$$

Rearranging the previous inequality we obtain that $\frac{k}{\overline{Y}} < (1-c) - \left[(1-p) + pA^{\gamma-1}\right]^{\frac{1}{1-\gamma}}$ has to hold for getting (A.16) . ■

Proof of Lemma 5.5. For $0 \leq \tau \leq \widetilde{\tau}\left(\overline{Y}\right)$ the tax revenue is only composed by the fines paid by the inspected individuals. This revenue is

$$G_1(\tau) = \int_0^{\overline{Y}} p\tau\,(1+s)\,y\frac{1}{\overline{Y}}dy.$$

For $\widetilde{\tau}\left(\overline{Y}\right) \leq \tau \leq \tau^*$ the tax revenue is given by the penalties paid for the individuals which prefer being full evaders rather avoiders. This revenue is

$$G_2(\tau) = \int_0^{y_T^*(\tau)} p\,(1+s)\,y\frac{1}{\overline{Y}}dy.$$

Finally, for $\tau^* < \tau \leq 1$ the tax revenue collected by the tax authorities comes both from the taxes voluntarily paid and from the penalties paid by the inspected partial evaders, that is,

$$\begin{aligned} G_3(\tau) &= \int_0^{y_P^*(\tau)} \left[(1-p)\tau(1-\phi(\tau)) + p\tau\,(1+s\phi(\tau))\right] y\frac{1}{\overline{Y}}dy \\ & \int_0^{y_P^*(\tau)} \left[\tau(1-\phi(\tau)) + p\tau\,(1+s)\,\phi(\tau)\right] y\frac{1}{\overline{Y}}dy. \end{aligned}$$

Performing the integrals $G_i(\tau)$ where $i = 1, 2, 3$ we obtain the expression appearing in the statement of the Lemma. ■

Proof of Proposition 5.6. We know that $G(0) = 0$ and then, we only have to prove that $G(1)$ is less than some positive value taken by the function $G(\tau)$ on $(0, 1)$. Computing the value of $G(1)$ we obtain

$$G(1) = \frac{1}{2\overline{Y}}\frac{k^2}{(1-c)^2} > 0,$$

that can be as small as we want, taking values of k low enough. Similarly, evaluating the tax revenue function $G(\tau)$ at $\tau = c$, we have

$$G(c) = \frac{1}{2}c\,(1-\phi(c) + p\,(1+s)\,\phi(c))\,\overline{Y} > 0.$$

Thus, it is easy to check that $G(c) > G(1)$, for $k < (1-c)\overline{Y}\sqrt{c\,(1-\phi(c) + p\,(1+s)\,\phi(c))}$. ■

Proof of Lemma 6.1. For $\tau \in [0, \tau^*]$ all the individuals evade all their income, and then the tax revenue is given by the penalties paid by the evaders inspected. Formally, this revenue is given by

$$G_1(\tau) = \alpha_1 p\tau\,(1+s)\,y_1 + \alpha_2 p\tau\,(1+s)\,y_2 + \alpha_3 p\tau\,(1+s)\,y_3.$$

Simplifying the last expression we have

$$G_1(\tau) = p\tau\,(1+s)\,(\alpha_2 m + \alpha_3)\,.$$

For $\tau \in (\tau^*, \widehat{\tau}_{y3}]$ everybody is a partial evader. In this case, the tax revenue is composed both by the taxes voluntary paid and by the penalties from inspected individuals. After rearranging the terms the tax revenue is given by

$$G_2(\tau) = (\alpha_2 m + \alpha_3)\,[\tau\,(1 - \phi(\tau) + p\,(1+s)\,\phi(\tau))]\,.$$

Finally for $\tau \in (\widehat{\tau}_{y3}, \widehat{\tau}_{y2}]$ the tax revenue is equal to the taxes voluntarily paid by the individuals with income y_2 and to the fines paid by the inspected ones. This revenue is

$$G_3(\tau) = \alpha_2 m\,[\tau\,(1 - \phi(\tau) + p\,(1+s)\,\phi(\tau))]\,.$$

■

Proof of Proposition 6.2

(a) We need to prove that $G(\widehat{\tau}_{y3})$ is the maximum value that the tax revenue function $G(\tau)$ can achieve. Computing $\frac{dG_1(\tau)}{d\tau}$ we have

$$\frac{dG_1(\tau)}{d\tau} = p\,(1+s)\,(\alpha_2 m + \alpha_3) > 0.$$

Similarly differentiating $G_2(\tau)$, we obtain after rearranging the terms

$$\frac{dG_2(\tau)}{d\tau} = (\alpha_2 m + \alpha_3)\left[1 + \frac{(A-1)\,(1 - p\,(1+s))}{1+sA}\right],$$

which is unambiguously positive. Then, for $\tau \in [0, \widehat{\tau}_{y3})$, we have that $G_2(\widehat{\tau}_{y3})$ is the highest value of the tax revenue.

On the other hand, if we compute $\frac{dG_3(\tau)}{d\tau}$ we obtain that

$$\frac{dG_3(\tau)}{d\tau} = \alpha_2 m\left[1 + \frac{(A-1)\,(1 - p\,(1+s))}{1+sA}\right] > 0.$$

Then, we only have to prove that $G_2(\widehat{\tau}_{y3}) > G_3(\widehat{\tau}_{y2})$. Evaluating these expressions, we get

$$G_2(\widehat{\tau}_{y3}) = (\alpha_2 m + \alpha_3)\,\Psi\,(\widehat{\tau}_{y3})\,,$$

and

$$G_3(\widehat{\tau}_{y2}) = \alpha_2 m \Psi\,(\widehat{\tau}_{y2})\,,$$

where $\Psi\,(\tau) = [\tau\,(1 - \phi\,(\tau)) + p\tau\,(1+s)\,\phi\,(\tau)]$. Comparing these two expressions is straightforward to see that, if $\alpha_3 \geq \alpha_2$ we only have to prove that

$$\left[\frac{m\,(\Psi\,(\widehat{\tau}_{y2}) - \Psi\,(\widehat{\tau}_{y3}))}{\Psi\,(\widehat{\tau}_{y3})}\right] < 1, \tag{A.17}$$

to show that $G_2(\widehat{\tau}_{y3}) > G_3(\widehat{\tau}_{y2})$. Then, taking expressions (6.1) and (6.2) and plugging then into $\Psi(\tau)$, the inequality (A.17) becomes

$$\frac{D - B(1-c) + kBm}{D} > 0, \tag{A.18}$$

where $D = \left[\left(\frac{1+s}{1+As}\right)\left((1-p)A^{1-\gamma}+p\right)^{\frac{1}{1-\gamma}}\right]$, and $B = 1+\alpha\left(1-p\left(1+s\right)\right)$. Lemma 5.1 and condition (A.13) ensure that $D - B(1-c) > 0$ so that the inequality (A.18) holds.

(b) Following the proof of (a), it is straightforward to see that the proof of (b) reduces to check when

$$\alpha_3 < \alpha_2 \left[\frac{m\left(\Psi\left(\widehat{\tau}_{y2}\right) - \Psi\left(\widehat{\tau}_{y3}\right)\right)}{\Psi\left(\widehat{\tau}_{y3}\right)}\right], \tag{A.19}$$

holds. Since inequality (A.17) holds, we need a small enough value of α_3 to ensure the fulfillment of (A.19). ■

Proof of Proposition 6.3.
(a) It is immediate to see that $\frac{dG_1(\tau)}{d\tau} > 0$ and $\frac{dG_2(\tau)}{d\tau} > 0$, where $G_1(\tau) = p\tau\left(1+s\right)\left(\alpha_2 m+\alpha_3\right)$ and $G_2(\tau) = p\tau\left(1+s\right)\alpha_2 m$. Then, we only have to prove that $G_1\left(\widetilde{\tau}_{y3}\right) > G_2\left(\widetilde{\tau}_{y2}\right)$, which is equivalent to prove that

$$\alpha_3 < \alpha_2 m \left[\frac{\widetilde{\tau}_{y2} - \widetilde{\tau}_{y3}}{\widetilde{\tau}_{y3}}\right], \tag{A.20}$$

after substituting the corresponding values of the tax rate. The expression (6.3) tells us how $\widetilde{\tau}_{y2}$ depends on m. Then, we get

$$\lim_{m\to 1}\left[\alpha_2 m\left(\frac{\widetilde{\tau}_{y2} - \widetilde{\tau}_{y3}}{\widetilde{\tau}_{y3}}\right)\right] = 0.$$

In consequence we can conclude that for a m sufficiently close to one, the inequality (A.20) will hold.

(b) Following the proof of (a), it is straightforward to see that the proof of (b) reduces to check when

$$\alpha_3 < \alpha_2 m \left[\frac{\widetilde{\tau}_{y2} - \widetilde{\tau}_{y3}}{\widetilde{\tau}_{y3}}\right], \tag{A.21}$$

holds. Obviously, a value of α_3 small enough ensures that (A.21) holds.

(c) It is immediate to see that $\frac{dG_1(\tau)}{d\tau} > 0$, $\frac{dG_2(\tau)}{d\tau} > 0$ and $\frac{dG_3(\tau)}{d\tau} > 0$, where $G_1(\tau) = p\tau\left(1+s\right)\left(\alpha_2 m+\alpha_3\right)$, $G_2(\tau) = p\tau\left(1+s\right)\alpha_2 m$ and $G_3(\tau) = \alpha_2 m\Psi\left(\tau\right)$

with $\Psi(\tau) = [\tau(1-\phi(\tau)) + p\tau(1+s)\phi(\tau)]$. Then, we only have to prove that $G_1(\widetilde{\tau}_{y3}) > G_3(\widehat{\tau}_{y2})$ since $G_2(\tau)$ and $G_3(\tau)$ are continuous on their respective domains. The last inequality is equivalent to

$$\alpha_3 > \alpha_2 m \left[\frac{\widehat{\tau}_{y2} - \widetilde{\tau}_{y3}}{\widetilde{\tau}_{y3}}\right]. \tag{A.22}$$

Thus, when the proportion α_3 is large enough, inequality (A.22) holds.

(d) Following the proof of (c), it is straightforward to see that the proof of (d) reduces to check when the following inequality is satisfied:

$$\alpha_3 < \alpha_2 m \left[\frac{\widehat{\tau}_{y2} - \widetilde{\tau}_{y3}}{\widetilde{\tau}_{y3}}\right].$$

Obviously, such an inequality holds for a small enough value of α_3. ■

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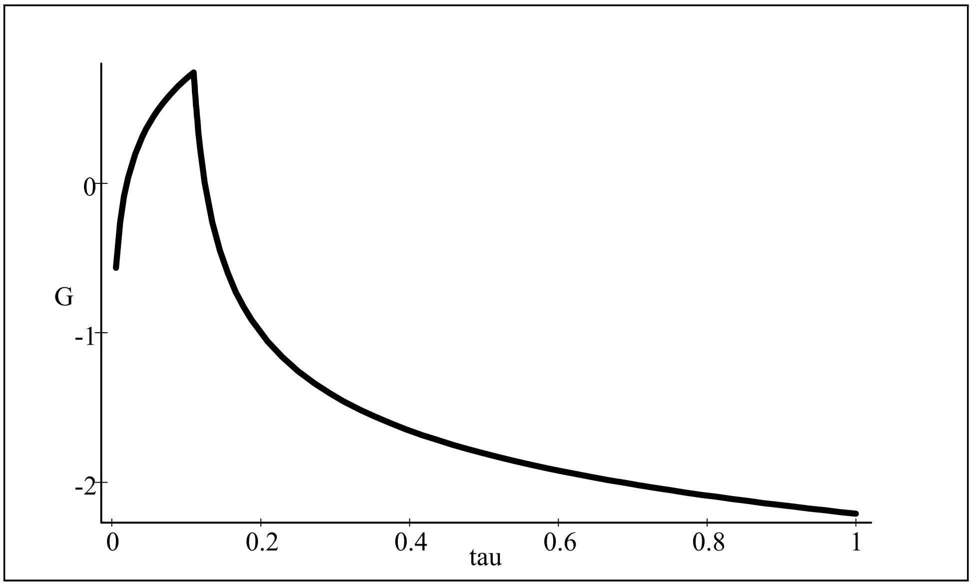

Figure 1. The function $G(\tau)$ when the individuals are honest, using logarithmic scale ($c = 0.1$, $\overline{Y} = 100$, $k = 1$).

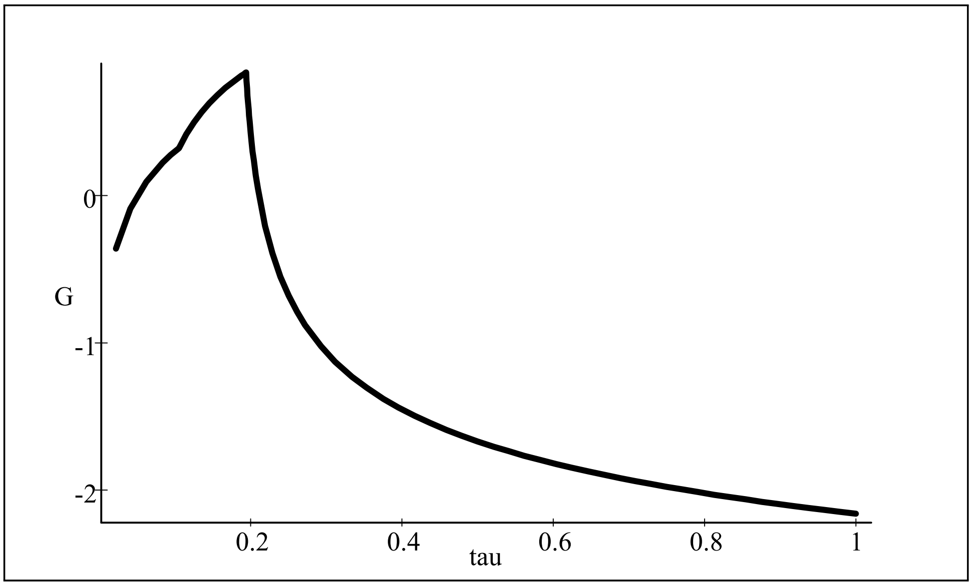

Figure 2. The function $G(\tau)$ when the evasion takes place and $\tau^* \leq c$, using logarithmic scale ($p = 0.1$, $s = 3$, $c = 0.15$, $\overline{Y} = 100$, $k = 1$, $\gamma = 2$).

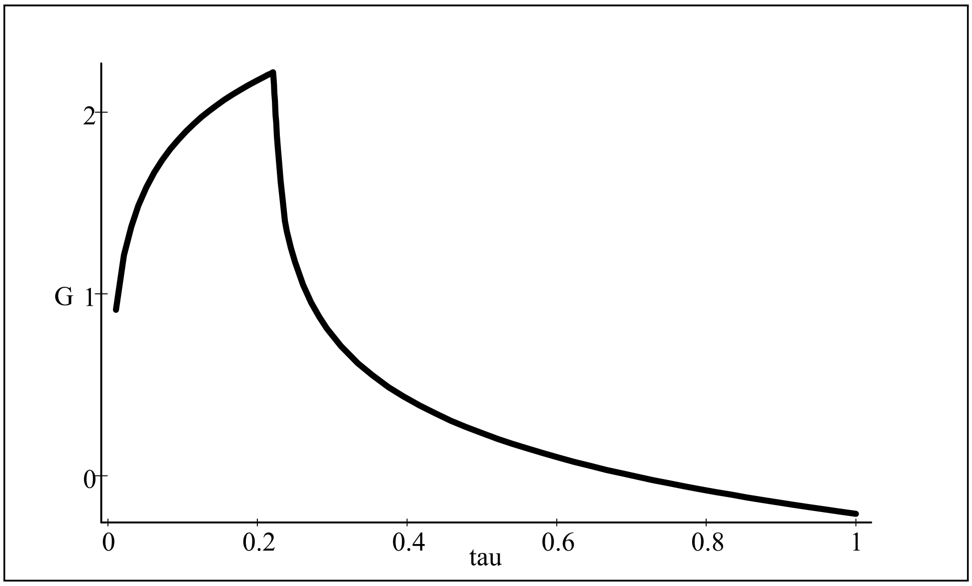

Figure 3. The function $G(\tau)$ when the evasion takes place and $\tau^* > c$, using logarithmic scale ($p = 0.1$, $s = 1.5$, $c = 0.1$, $\overline{Y} = 100$, $k = 1$, $\gamma = 2$).

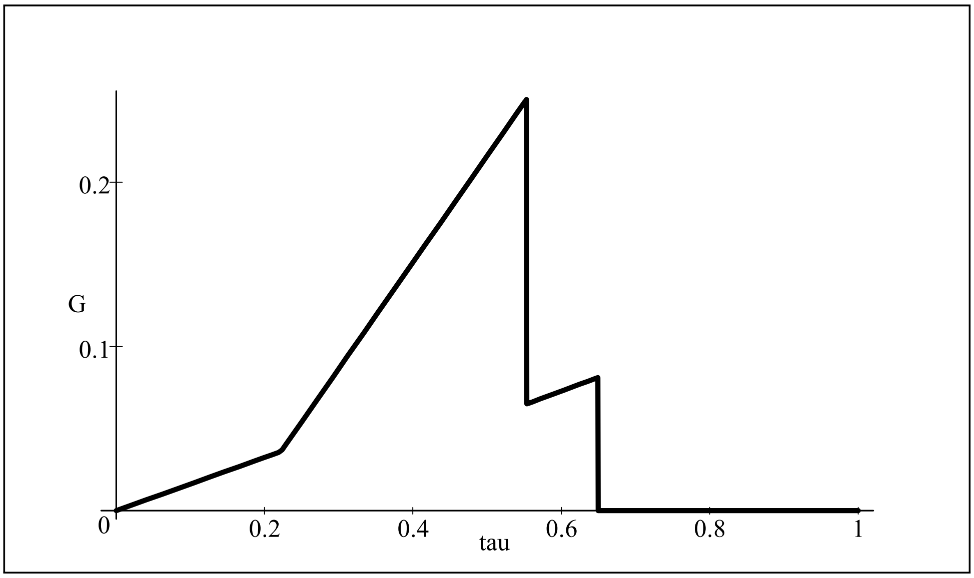

Figure 4. The function $G(\tau)$ when the income distribution is discrete and $\alpha_3 > \alpha_2$ ($p = 0.1$, $s = 3$, $c = 0.25$, $k = 0.25$, $\gamma = 0.5$, $\alpha_1 = 0.4$, $\alpha_2 = 0.2$, $\alpha_3 = 0.4$, $m = 0.7$).

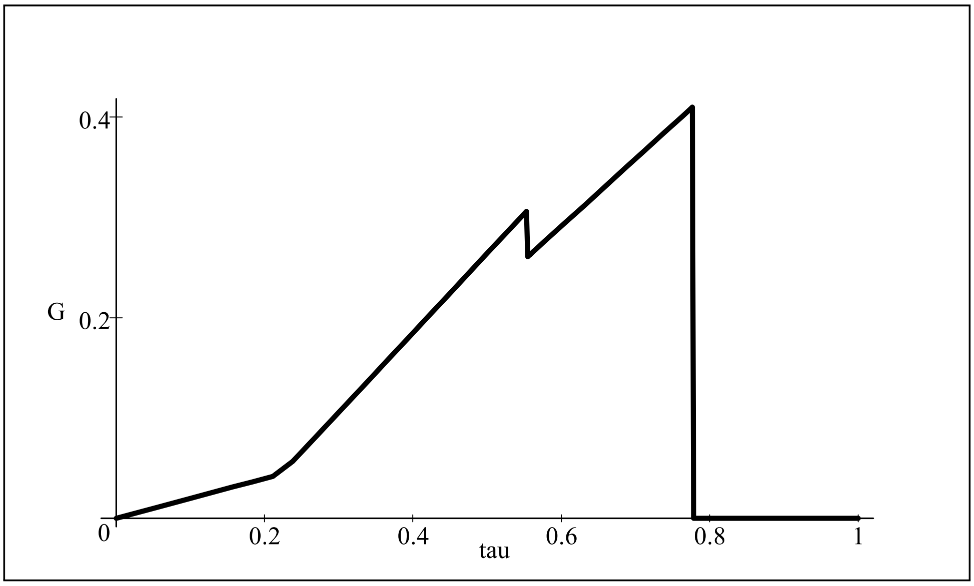

Figure 5. The function $G(\tau)$ when the income distribution is discrete and $\alpha_3 < \alpha_2$ ($p = 0.1$, $s = 3$, $c = 0.25$, $k = 0.25$, $\gamma = 0.5$, $\alpha_1 = 0.1$, $\alpha_2 = 0.8$, $\alpha_3 = 0.1$, $m = 0.7$).