



Inflation, tax evasion, and the distribution of consumption

Jordi Caballé^{a,*}, Judith Panadés^b

^a *Unitat de Fonaments de l'Anàlisi Econòmica and CODE, Universitat Autònoma de Barcelona, Edifici B, Barcelona 08193, Spain*

^b *Unitat de Fonaments de l'Anàlisi Econòmica, Universitat Autònoma de Barcelona, Barcelona 08193, Spain*

Received 16 September 2002; accepted 27 June 2003

Available online 14 October 2004

Abstract

We analyze the effects of inflation both on tax compliance and on the amount of government revenues in the framework of a monetary economy where households face a cash-in-advance constraint on consumption purchases. Since households are exposed to random audits from the tax enforcement agency, the stationary equilibrium exhibits a non-degenerate distribution of consumption. Our main results include a non-monotonic characterization of the relationship between the rate of monetary expansion and government revenue. This is in contrast to the standard cash-in-advance model with no evasion, where that relationship is monotonic. In our model, as government creates inflation, the penalty imposed on evaded taxes becomes smaller in real terms. This stimulates tax evasion and, hence, aggregate revenue turns out to be decreasing in the rate of monetary expansion when inflation is sufficiently high. Even if inflation raises the variance of the distribution of consumption, we show that high inflation rates end up being welfare enhancing.

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JEL classification numbers: H26; E31; E51

Keywords: Tax evasion; Cash-in-advance; Inflation

* Corresponding author. Tel.: +34 935 812 367; fax: +34 935 812 012.
E-mail address: jordi.caballe@uab.es (J. Caballé).

1. Introduction

All the tax codes around the world contain a series of provisions concerning tax enforcement. Without these provisions no rational, selfish taxpayer would voluntarily report his true taxable income. Tax authorities conduct random inspections of the reports submitted by taxpayers and, if a taxpayer is caught evading, he has to pay a fine proportional to the amount of evaded taxes. Usually, the tax inspection occurs some months or years after taxpayers have submitted their income reports. In this scenario, inflation modifies the real payoffs of the risky investment implicit in tax evading activities. In particular, the nominal fine paid by caught tax evaders could be substantially reduced in real terms when the economy is experiencing a hyperinflationary process. Inflation could then result in both less voluntarily paid real taxes and less real revenue accruing from the fines imposed on audited taxpayers. This negative effect of inflation on the real fiscal revenue is dubbed the *Tanzi–Olivera effect* (see *Tanzi, 1977; Olivera, 1967*). *Fishlow and Friedman (1994)* pointed out that one of the consequences of the *Tanzi–Olivera effect* is that governments facing a large amount of evasion due to inflation will increase the rate of monetary growth in order to get additional inflationary financing. Therefore, the *Tanzi–Olivera effect* adds extra difficulties to the stabilization efforts of countries experiencing inflation. The empirical estimation conducted by *Fishlow and Friedman* for the cases of *Argentina, Brazil and Chile* seems to confirm the relevance of the aforementioned effect.

The aim of this paper is to analyze the effects of inflation on tax compliance and on the revenue raised by the government. The novelty of our approach is that we undertake the analysis within a dynamic general equilibrium framework where inflation and tax evasion are phenomena arising as by-products of monetary policy. In order to make explicit the relationship between the rate of monetary expansion and inflation, we introduce a cash-in-advance (or liquidity) constraint on the purchases of consumption goods. Therefore, money will have positive value in equilibrium. Moreover, since the government has the monopoly of issuing money, there will be room for seignorage.

One feature of the standard monetary model with liquidity constraints when tax evasion is absent is that government can always raise its revenue by increasing the rate of monetary growth. In fact, the resources absorbed by the government can become arbitrarily close to the total resources of the economy by selecting a sufficiently high rate of monetary growth. In this paper we will show that this monotonic relationship does not longer hold when tax evasion is present. Following the aforementioned contributions of *Tanzi and Olivera*, we show that, due to both the delay in tax auditing and the lack of indexation of fines, taxpayers will decrease the amount of reported income and, thus, the real fiscal revenue from taxes and fines could also decrease with inflation. Moreover, we will show that the plot of the total revenue raised by the government, accruing from both tax collection and seignorage, against the inflation rate exhibits an inverted U-shape. This means in particular that a given level of feasible government revenue (or government spending under balanced budget), can be financed by two different rates of monetary growth. At the low-inflation equilibrium government spending is locally increasing in the rate of monetary

growth and the converse occurs at the high-inflation equilibrium. This non-monotonic association between government spending and inflation implies in turn that the decrease in regular tax collection due to inflation could not be outweighed by the additional revenue accruing from seignorage.

Other authors provide different explanations for the aforementioned positive relationship between tax evasion and inflation. For instance, Fishburn (1981) and Crane and Nourzad (1985, 1986) basically argue that inflation reduces the real value of taxpayers' future disposable income. Therefore, taxpayers find optimal to increase their levels of tax evasion in order to restore their future purchasing power. The paper of Fishburn shows in a static framework that the amount of evaded income increases with the future price level whenever individual utilities display an increasing index of relative risk aversion. The papers of Crane and Nourzad confirm empirically the previous positive effect of inflation on the amount of unreported income.

Finally, another group of authors consider the opposite causality from evasion to inflation. For instance, Nicolini (1998) and Al-Marhubi (2000) take as given the existence of some amount of underground markets. Since cash is used in these markets, the government finds optimal to create inflation in order to tax the transactions in these illegal markets and obtain thus the corresponding seignorage. Here, the existence of corruption and/or tax evading activities results in more inflation.

Concerning the relationship between monetary growth and government revenue, we should mention the paper of Palivos and Yip (1995), where in the context of an endogenous growth model with liquidity constraints, distortionary income taxation is compared with inflationary financing in terms of growth and welfare for a given level of useless government spending. In our paper we take instead as given the tax rate and then we allow for untruthful income reports. Thus, we just look at the effect of the rate of monetary growth on the government revenue through the induced change in the reporting strategies of taxpayers.

Our model shares several features with other monetary models exhibiting cash-in-advance constraints. Households will accumulate monetary holdings before knowing the state of the nature and, thus, before knowing their future consumption. Therefore, the demand for money will arise because of transactions, precautionary, and store-of-value motives as in Svensson (1985). More precisely, households will enjoy identical income in each period but they will face idiosyncratic shocks, since some households will be audited and others will not. Therefore, the economy exhibits heterogeneity both in monetary balances and in consumption as these variables will depend on the whole history of audit shocks experienced by each household. In this respect, the model resembles that of Lucas (1980), where individuals differ in their money holdings depending on the realization of an idiosyncratic preference shock. Finally, as in Hodrick et al. (1991) and Dotsey and Sarte (2000), we will assume for tractability reasons that the cash-in-advance constraint is strictly binding, which means that the simple quantity equation will hold in equilibrium.¹

¹ Svensson (1985) and Palivos et al. (1993) allow for non-binding liquidity constraints in some states so that the quantity equation does not generally hold. These authors consider a model with macroeconomic shocks in order to analyze the response of the velocity of money to aggregate shocks.

Finally, our model makes also explicit the relationship between the return to the risky (and illegal) activity of evading taxes and the inflation rate. We are thus modelling tax evasion using the portfolio approach made famous by Allingham and Sandmo (1972) and Yitzhaki (1974). By viewing tax evasion as a particular kind of investment, our model is related to those of Bohn (1991) and Nakibullah (1992), where the effects of inflation on asset returns was also analyzed.

Our paper also contains some welfare considerations. Since we assume that government spending is unproductive and does not affect the households' utility, the rate of monetary growth maximizing the government revenue will be the one that minimizes the average consumption of taxpayers. However, the inflation rate also affects other moments of the distribution of individual consumption and, therefore, in order to evaluate the effects of inflation on the resulting expected utility of taxpayers, we should also take into account these other changes in the distribution of consumption. When inflation is very small, fines on evaded taxes are very high in real terms and, hence, no taxpayer finds optimal to misreport his true income. In this case, the distribution of disposable income and consumption is degenerate as no penalties are imposed on audited households. When inflation increases, the variance of consumption rises, since more heterogeneity among consumers is introduced by the random auditing process. However, we show that, for sufficiently high values of the rate of monetary expansion, the shift in the distribution of consumption due to inflation implies an improvement in terms of first order stochastic dominance. Therefore, higher inflation rates could be welfare improving.

The paper is organized as follows. Section 2 introduces the tax evasion phenomenon in a model of inflationary finance with liquidity constraints and characterizes the policy functions of households for both monetary holdings and income reports. Section 3 characterizes the resulting distribution of consumption at a stationary equilibrium. Section 4 analyzes the relationship between inflation and the different components of the government revenue. The welfare effects of inflation are discussed in Section 5. Section 6 contains some concluding remarks and discusses briefly some extensions. All the proofs appear in Appendix A.

2. Tax evasion and inflation

Let us consider a pure exchange monetary economy where time is discrete and the time horizon is infinite. There is a continuum of households distributed uniformly on the interval $[0, 1]$. Each household consists of a worker-shopper pair. The shopper purchases consumption goods and faces a cash-in-advance constraint. Thus, the shopper cannot purchase an amount of goods having a monetary value greater than the nominal cash holdings of the household in each period. The worker produces a constant amount of a non-storable good per period. This output is sold by the worker to the shoppers of the other households in exchange for money. Moreover, the government collects a fraction of the output produced by workers using two financing methods. On the one hand, there is a flat-rate tax on the amount of output. On the other hand, the government introduces money in the economy in exchange

for goods. Therefore, the government gets both fiscal and seignorage revenues. These revenues are devoted to a completely unproductive and useless spending. We assume that households could underreport their true income before paying their taxes. However, since tax evasion is an illegal activity, taxpayers will be audited by the government and, if a household is caught evading, then it will have to pay a fine proportional to the monetary amount of evaded taxes.

The timing of events within each period is thus the following. Each household enters period t with some amount M_t of money holdings. At the beginning of each period t the tax enforcement agency picks randomly a fraction $\pi \in (0, 1)$ of households to be audited. Therefore, the law of large numbers implies that the probability that a given household will be audited is also equal to π . The inspection takes place immediately and consists on verifying the accuracy of the income report submitted by the household to the tax authority in the previous period $t - 1$. We assume that the reports of all earlier periods are not audited. The true income is always discovered by the tax enforcement agency. Note that, even if there is no uncertainty about the output produced by a worker, an audit by the agency is necessary in order to certify indisputably the level of income of a given household. Then, the shopper leaves home and purchases c_t units of the consumption good using the money holdings of the household. Therefore, the shopper will conduct his purchases after knowing the exact amount of the penalty (if any) that the household will have to pay during the current period. Simultaneously, the worker stays at home and produces the constant amount y of a non-storable good per period. Consider now a worker of a household that has been audited by the tax enforcement agency. After he has produced the output, he pays the corresponding fine at the rate $\varphi > 1$ on the nominal amount of taxes evaded in the previous period. Let $\tau \in (0, 1)$ be the constant tax rate on the amount of output. If x_{t-1} is the real income reported by the household in period $t - 1$, then the amount of evaded taxes measured in monetary units is $p_{t-1}\tau(y - x_{t-1})$.² Note that in the vast majority of tax codes around the world the fine is imposed proportionally to that monetary amount. We are thus assuming that the penalty rate is not indexed. Therefore, an audited household has to pay the nominal amount $\varphi p_{t-1}\tau(y - x_{t-1})$ as a tax penalty in period t . Moreover, each worker sells the remaining output to the shoppers of the other households and to the government in exchange for money. The growth of aggregate nominal monetary balances is set by the government at the constant net rate $\sigma > 0$. After finishing the purchasing session, the shopper returns home. Then, the consumption of the perishable goods purchased by the shopper takes place. Finally, the household fills a tax income report and decides to declare a real income x_t (or, equivalently, a nominal income $p_t x_t$). This means that the household voluntarily pays an amount τx_t of real taxes (or $\tau p_t x_t$ in nominal terms) in period t . The amount of money carried by the household into period $t + 1$ will be the amount

² In fact, taxpayers report the nominal income $p_t x_t$ in period t . However, since the price level is known, we will consider the amount x_t of reported real income as the relevant choice variable.

collected by the worker plus the amount not spent on consumption purchases and tax payments.

The budget constraint faced by an audited household is thus

$$p_t c_t + M_{t+1} \leq M_t + p_t y - \tau p_t x_t - \phi p_{t-1} \tau (y - x_{t-1}),$$

whereas for a non-audited household is

$$p_t c_t + M_{t+1} \leq M_t + p_t y - \tau p_t x_t.$$

The previous two constraints can be compactly rewritten as

$$p_t c_t + M_{t+1} \leq M_t + p_t y - \tau p_t x_t - h_t p_{t-1} \tau (y - x_{t-1}), \tag{2.1}$$

where h_t is an i.i.d. random variable taking the value ϕ with probability π and the value zero with probability $1 - \pi$ for all t . Let us define the amount of evaded real income $e_t \equiv y - x_t$. Thus, making this change of variable and dividing (2.1) by p_t , we obtain the budget constraint in real terms,

$$c_t + (1 + f_{t+1})m_{t+1} \leq m_t + (1 - \tau)y - \left(\frac{\tau}{1 + f_t}\right)h_t e_{t-1} + \tau e_t, \tag{2.2}$$

where $f_{t+1} = \frac{p_{t+1} - p_t}{p_t}$ is the inflation rate and m_t denotes the real money balances per household at t .

Moreover, the household must satisfy the liquidity (or cash-in-advance) constraint on consumption purchases:

$$p_t c_t \leq M_t. \tag{2.3}$$

Dividing the previous constraint by the price level p_t , we obtain the following version of (2.3) in real terms:

$$c_t \leq m_t. \tag{2.4}$$

The problem of a given household is to maximize the following expected sum of discounted utilities:

$$E_t \left[\sum_{j=t}^{\infty} \rho^{j-t} u(c_j) \right], \quad 0 < \rho < 1,$$

where ρ is the discount factor and the operator E_t is the conditional expectation in period t computed immediately after the potential tax inspection has occurred, that is, after observing the realization of the random variable h_t . The instantaneous utility function u is strictly increasing and strictly concave. The two constraints of the previous problem are the budget constraint (2.2) and the cash-in-advance constraint (2.4). To solve the problem of the household we write the corresponding Lagrangian,

$$\begin{aligned} \mathcal{L} = E_t \left[\sum_{j=t}^{\infty} \rho^{j-t} \left\{ u(c_j) + \lambda_j \left[m_j + (1 - \tau)y - \left(\frac{\tau}{1 + f_j}\right)h_j e_{j-1} \right. \right. \right. \\ \left. \left. \left. + \tau e_j - c_j - (1 + f_{j+1})m_{j+1} \right] + \theta_j (m_j - c_j) \right\} \right], \end{aligned}$$

where λ_t and θ_t are the Lagrange multipliers associated with constraints (2.2) and (2.4), respectively. Differentiating \mathcal{L} with respect to c_t , e_t , m_{t+1} , λ_t and θ_t , we get the following first order conditions:

$$u'(c_t) = \lambda_t + \theta_t, \tag{2.5}$$

$$\lambda_t = \frac{\rho}{1 + f_{t+1}} E_t[\lambda_{t+1} h_{t+1}], \tag{2.6}$$

$$(1 + f_{t+1})\lambda_t = \rho E_t[\lambda_{t+1} + \theta_{t+1}], \tag{2.7}$$

and the constraints (2.2) and (2.4).

As aggregate nominal monetary balances grow at the net rate $\sigma > 0$, the market clearing equation for money is

$$M_{t+1}^* = (1 + \sigma)M_t^*, \tag{2.8}$$

where M_t^* denotes the aggregate amount of nominal balances in the economy at the beginning of period t . Let $m_t^* = M_t^*/p_t$ be the aggregate amount of real monetary balances at the beginning of period t . Note that this aggregate amount of real balances coincides with the corresponding average amount per household, since households are uniformly distributed on the interval $[0, 1]$. After dividing by p_t , the market clearing equation (2.8) becomes thus

$$(1 + f_{t+1})m_{t+1}^* = (1 + \sigma)m_t^*.$$

Since at a stationary equilibrium the distribution of monetary real balances across households must be time-invariant, we have that $m_{t+1}^* = m_t^*$. Therefore, the previous market clearing equation for money at a stationary equilibrium becomes simply

$$f = \sigma, \tag{2.9}$$

where the variables without subindex denote their corresponding stationary values. It is important to point out that the inflation rate f_t is a macroeconomic variable determined by a simple monetary rule that consists on increasing the aggregate nominal monetary balances at the constant rate σ . Therefore, the inflation rate is non-stochastic.

Note that, since the instantaneous utility function u is strictly increasing, the constraint (2.2) will hold with equality. Following Hodrick et al. (1991) and Dotsey and Sarte (2000), for the sake of tractability we will assume that the cash-in-advance constraint (2.4) also holds with equality for all households. We will provide at the end of this section the exact condition under which households effectively face a strictly binding cash-in-advance constraint.

For simplicity we are going to assume that the instantaneous utility function is logarithmic, $u(c) = \ln c$. The qualitative results of this paper will also hold under an isoelastic utility, $u(c) = \frac{c^{1-\eta}-1}{1-\eta}$, when the parameter η takes values close to one.

Since the cash-in-advance constraint (2.4) holds with equality by assumption, the budget constraint (2.2) becomes in equilibrium

$$(1 + f_{t+1})m_{t+1} = (1 - \tau)y - \left(\frac{\tau}{1 + f_t}\right)h_t e_{t-1} + \tau e_t. \quad (2.10)$$

The following proposition gives the policy functions followed by households at a stationary equilibrium.

Proposition 2.1. *Assume that the cash-in-advance constraint is binding for all the households at a stationary equilibrium. Then, the policy functions for real monetary balances m_{t+1} and for the amount e_t of evaded real income at a stationary equilibrium are*

$$m_{t+1} = \alpha + \beta h_t e_{t-1}, \quad (2.11)$$

and

$$e_t = \delta + \gamma h_t e_{t-1}, \quad (2.12)$$

where

$$\alpha = \frac{(1 - \tau)(1 - \rho\pi)\varphi y}{(1 + f)[\varphi - (1 + f)]}, \quad (2.13)$$

$$\beta = -\frac{(1 - \rho\pi)\tau}{(1 + f)^2}, \quad (2.14)$$

$$\delta = \frac{(1 - \tau)[(1 + f) - \rho\pi\varphi]y}{\tau[\varphi - (1 + f)]}, \quad (2.15)$$

$$\gamma = \frac{\rho\pi}{1 + f}. \quad (2.16)$$

In order to generate a positive demand for money for all households and, thus, a positive level of consumption, it is necessary to assume that

$$\varphi > 1 + f, \quad (2.17)$$

which ensures that the value of the coefficient α is positive. Note that the real money demand by households that are not inspected in period t is equal to α , since these are the households for which $h_t = 0$.

We are also going to assume that always some evasion takes place at the individual level. Therefore, we need to assume that $\delta > 0$, since δ is the amount of income evaded by the households that are not audited in period t . It is immediate to see that $\delta > 0$ if and only if

$$1 + f > \rho\pi\varphi, \quad (2.18)$$

whenever (2.17) holds. Note that the amount of real income evaded by households inspected in period t is larger than that of non-audited households. This is so because $\gamma > 0$ and, hence,

$$e_t = \delta + \gamma\varphi e_{t-1} > \delta > 0.$$

It should also be pointed out that audited households, which are the ones for which $h_t = \varphi$, exhibit a demand for real monetary balances that is lower than that of non-audited households. This is so because $\beta < 0$ and $e_{t-1} > 0$, which implies that

$$m_{t+1} = \alpha + \beta\varphi e_{t-1} < \alpha.$$

Finally note that, since only the income report submitted in period $t - 1$ is potentially inspected in period t , both the money demand and the amount of evaded income of a non-audited household take always constant values (α and δ , respectively) independently of the previous audit and evasion history of the household.

We should check now under which circumstances the maintained assumption of a binding cash-in-advance constraint for all households holds effectively. The following lemma provides the exact parametric restriction.

Lemma 2.2. *The cash-in-advance constraint is always strictly binding for all households if and only if*

$$\pi\varphi > 1. \tag{2.19}$$

Recalling that the inflation rate satisfies $f = \sigma$ at a stationary equilibrium, we can summarize the conditions on the parameters of the model for positive consumption, positive evasion and binding cash-in-advance constraint given in (2.17)–(2.19), respectively, by means of the following chain of inequalities:

$$\varphi > 1 + \sigma > \rho\pi\varphi > \rho. \tag{2.20}$$

We will assume from now on that the parameter constellation of our model satisfies the previous condition whenever the amount of evaded taxes is assumed to be strictly positive. We will see that the limiting case with no evasion at the stationary equilibrium is obtained when the inequality (2.18) is replaced by the corresponding equality.

3. The distribution of consumption

At the stationary equilibrium of our economy, households follow the policy functions (2.11) and (2.12), the inflation rate is equal to σ , and the distributions of real balances and of evaded income across households are time-invariant.

Let us analyze first the distribution of evaded income at the stationary equilibrium. The dynamics of this distribution is governed by the policy function (2.12). The Markov process of evaded income within a household is thus

$$e_{t+1} = \begin{cases} \delta & \text{with probability } 1 - \pi, \\ \delta + \gamma\varphi e_t & \text{with probability } \pi. \end{cases}$$

The distribution of evaded income across households in each period is a probability measure defined on the measurable space $(\mathbb{R}, \mathcal{B})$ where \mathcal{B} is the σ -algebra of Borel

sets of \mathbb{R} . Let μ_t be the distribution of evaded income at period t . This distribution evolves along time according to the following equation:

$$\mu_{t+1}(B) = (1 - \pi)\mathbb{1}_B(\delta) + \pi \int_{g(B)} d\mu_t, \quad \text{for all } B \in \mathcal{B}, \tag{3.1}$$

where $\mathbb{1}_B$ is the indicator function of the Borel set B , that is,

$$\mathbb{1}_B(e) = \begin{cases} 1 & \text{if } e \in B, \\ 0 & \text{if } e \notin B, \end{cases}$$

and

$$g(B) = \{e \in \mathbb{R} \text{ such that } (\delta + \gamma\varphi e) \in B\}.$$

Proposition 3.1. *There exists a unique distribution μ^* on the measurable space $(\mathbb{R}, \mathcal{B})$ such that, for every initial distribution μ_0 , the sequence of distributions of evaded income $\{\mu_t\}_{t=0}^\infty$ defined in (3.1) satisfies*

$$\lim_{t \rightarrow \infty} |\mu_t(B) - \mu^*(B)| = 0, \quad \text{for all } B \in \mathcal{B}.$$

The time-invariant distribution μ^* of evaded income across households should satisfy

$$\mu^*(B) = (1 - \pi)\mathbb{1}_B(\delta) + \pi \int_{g(B)} d\mu^*, \quad \text{for all } B \in \mathcal{B}, \tag{3.2}$$

as dictated by (3.1). The next proposition fully characterizes this time-invariant distribution at a stationary equilibrium:

Proposition 3.2. *The unique time-invariant distribution of evaded income at a stationary equilibrium satisfies*

$$\mu^* \left(\left\{ \delta \sum_{i=0}^K (\gamma\varphi)^i \right\} \right) = (1 - \pi)\pi^K, \quad \text{for } K = 0, 1, 2, \dots \tag{3.3}$$

Now we are in the position of finding the time-invariant distribution ν^* of real monetary balances. This distribution coincides in fact with that of consumption according to the maintained assumption (2.19), which means that the cash-in-advance constraint is binding. The law of motion of real monetary balances is driven by the policy function (2.11) so that

$$m_{t+1} = \begin{cases} \alpha & \text{with probability } 1 - \pi, \\ \alpha + \beta\varphi e_{t-1} & \text{with probability } \pi. \end{cases} \tag{3.4}$$

The time-invariant distribution of real balances at a stationary equilibrium is thus a by-product of the distribution of evaded income. Combining (3.3) with the previous law of motion of real balances, we get the following:

Proposition 3.3. *There exists a unique time-invariant distribution of real monetary balances (and of consumption) at a stationary equilibrium. This distribution satisfies*

$$v^*(\alpha) = 1 - \pi,$$

and

$$v^* \left(\left\{ \alpha + \beta \delta \sum_{i=0}^{K-1} \gamma^i \varphi^{i+1} \right\} \right) = (1 - \pi) \pi^K \quad \text{for } K = 1, 2, 3, \dots \tag{3.5}$$

Moreover, for any initial distribution v_0 of real balances, the sequence of distributions $\{v_t\}_{t=0}^\infty$ of real monetary balances satisfies

$$\lim_{t \rightarrow \infty} |v_t(B) - v^*(B)| = 0, \quad \text{for all } B \in \mathcal{B}.$$

Figs. 1 and 2 depict the shape of the probability functions associated with the distributions of evaded income and real monetary balances (and consumption), respectively. It should be noted that the support of the two distributions is bounded. Concerning the distribution μ^* of evaded income, the infimum of its support is $\underline{e} = \delta$, whereas its supremum is

$$\bar{e} = \lim_{K \rightarrow \infty} \left[\delta \sum_{i=0}^K (\gamma \varphi)^i \right] = \frac{\delta}{1 - \gamma \varphi},$$

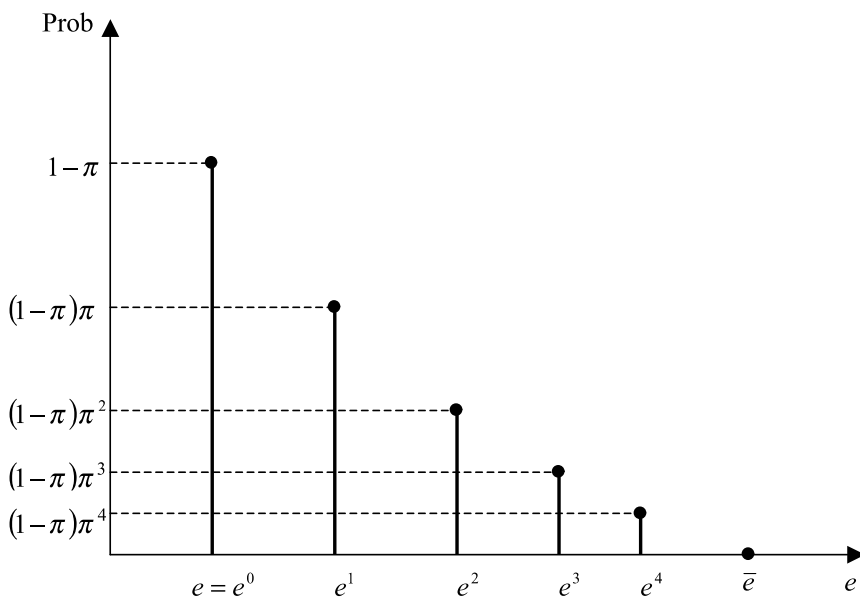


Fig. 1. Probability function of the time-invariant distribution of evaded income with $e^K = \delta \sum_{i=0}^K (\gamma \varphi)^i$, for $K = 0, 1, 2, \dots$

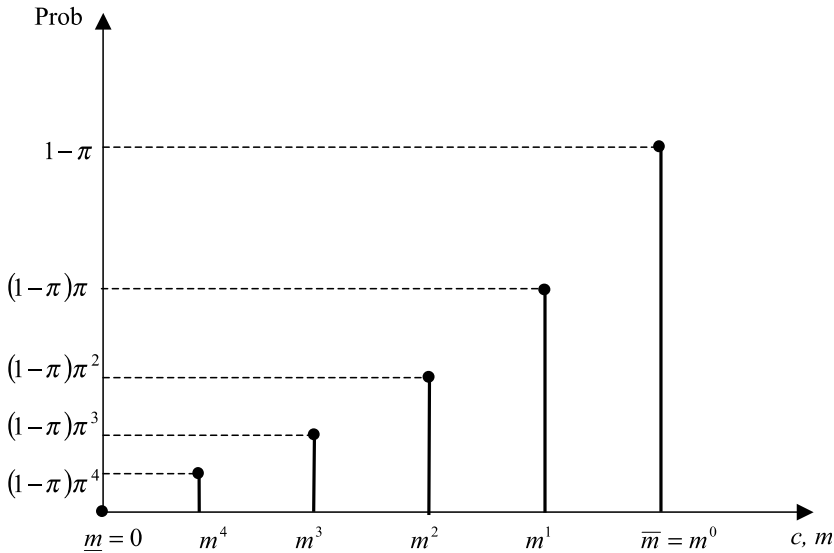


Fig. 2. Probability function of the time-invariant distribution of real monetary balances (and of consumption) with $m^0 = \alpha$ and $m^K = \alpha + \beta\delta \sum_{i=0}^{K-1} \gamma^i \varphi^{i+1}$, for $K = 1, 2, \dots$

where the second equality follows because

$$0 < \gamma\varphi = \frac{\rho\pi\varphi}{1+f} < 1,$$

as dictated by condition (2.18).

The supremum of the support of the distribution v^* of real balances is $\bar{m} = \alpha$, whereas its infimum is

$$\underline{m} = \lim_{K \rightarrow \infty} \left(\alpha + \beta\delta \sum_{i=0}^{K-1} \gamma^i \varphi^{i+1} \right) = \alpha + \frac{\beta\delta\varphi}{1-\gamma\varphi} = 0,$$

where the second equality follows again because $0 < \gamma\varphi < 1$, and the last equality is obtained by just using the equilibrium values of the coefficients α , β , δ and γ .

We can now compute the aggregate amounts of evaded income and of real monetary balances (and consumption). Note that these aggregate amounts coincide in fact with the corresponding average and expected values of the distributions μ^* and v^* , since households are uniformly distributed on the interval $[0, 1]$.

Corollary 3.4. *The aggregate amount of evaded income in the economy at the stationary equilibrium is*

$$e^* = \frac{(1-\tau)(1+\sigma)[(1+\sigma) - \rho\pi\varphi]y}{\tau[\varphi - (1+\sigma)][(1+\sigma) - \rho\pi^2\varphi]}, \tag{3.6}$$

and the aggregate amounts of consumption and of real monetary balances in the economy at the stationary equilibrium are

$$c^* = m^* = \frac{(1 - \tau)(1 - \pi)(1 - \rho\pi)\varphi y}{[\varphi - (1 + \sigma)][(1 + \sigma) - \rho\pi^2\varphi]}. \tag{3.7}$$

The previous expressions for the aggregate amounts of evaded income, consumption, and real monetary balances play a crucial role in order to derive the effects of inflation on government revenue, as we will see in the next section.

It should be noticed that the standard model of inflationary finance with a cash-in-advance constraint on consumption purchases is in fact the limiting case of the previous model when evasion is absent. In order to remove the evasion phenomenon from the economy, we should set the inspection policy parameters π and φ so that they satisfy

$$\pi\varphi = \frac{1 + \sigma}{\rho}. \tag{3.8}$$

The previous condition means that the inspection policy is no longer exogenous and is chosen instead as a function of the rate of monetary growth. Condition (3.8) means that the condition (2.18) holds now with equality. In this case, it is obvious that the inequalities (2.17) and (2.19) hold, since $\rho \in (0, 1)$, $\pi \in (0, 1)$, and $\sigma = f > 0$. Therefore, real monetary balances are strictly positive and the cash-in-advance constraint is always binding. Moreover, when (3.8) is satisfied, we have that

$$\alpha = \frac{(1 - \tau)y}{1 + f} = \frac{(1 - \tau)y}{1 + \sigma},$$

and

$$\delta = 0,$$

as follows from (2.13) and (2.15). Finally, it is immediate to see from Propositions 3.2 and 3.3 that in this case the distributions of evaded income and real monetary balances (and of consumption) are degenerate at a stationary equilibrium. The stationary value of evaded income by each household is in fact $e = 0$, whereas the stationary amount of consumption and real monetary balances of each household is

$$c = m = \frac{(1 - \tau)y}{1 + \sigma}. \tag{3.9}$$

It is worth noting that this amount of monetary balances is also obtained under a general (not necessarily logarithmic) increasing and concave utility function u .

4. Inflation and seignorage

In order to discuss the relationship between the inflation rate and the revenue raised by the government from the private sector, let us first write the budget

constraint of the government when tax evasion activities are present. This constraint in monetary terms at a stationary equilibrium is

$$p_t g = p_t \tau (y - e^*) + \pi \varphi \tau p_{t-1} e^* + \underbrace{(p_{t+1} m_{t+1}^* - p_t m_t^*)}_{M_{t+1}^*},$$

where g_t is the government revenue per household. This revenue is devoted entirely to finance the consumption of the government. Recall that we assume that government consumption is unproductive and does not enter in the utility function of households. The first term of the RHS of the previous equation is the amount of taxes voluntarily paid in period t , the second term is the revenue accruing from the fines imposed on the fraction π of audited taxpayers, and the third term is the revenue accruing from seignorage, that is, from the purchases of goods made by the government in exchange for money.

By noticing that at a stationary equilibrium $f = \sigma$ and $m_t^* = m^*$, for all t , we can divide the previous budget constraint by the price level p_t to obtain the following government budget constraint in real terms at a stationary equilibrium:

$$g = \tau y + \tau \left(\frac{\pi \varphi}{1 + \sigma} - 1 \right) e^* + \sigma m^*. \quad (4.1)$$

Using the equilibrium values of e^* and m^* , we will obtain the exact relationship between the rate σ of monetary expansion and the government revenue.

Let us first consider the scenario where tax evasion is absent, which means that the inspection policy satisfies condition (3.8). In this case, $e = 0$ for all households, the distribution of real monetary balances is degenerate at the value m given by (3.9), and the government budget constraint in real terms at a stationary equilibrium (4.1) becomes simply

$$g = \tau y + \sigma m. \quad (4.2)$$

Therefore, plugging the stationary value of real monetary balances given in (3.9) into (4.2), we obtain the total revenue collected by the government at a stationary equilibrium,

$$g = \frac{(\tau + \sigma)y}{1 + \sigma}.$$

By differentiating the revenue with respect to the rate of monetary growth, we easily obtain

$$\frac{dg}{d\sigma} = \frac{(1 - \tau)y}{(1 + \sigma)^2} > 0.$$

Moreover, $g = \tau y$ when $\sigma = 0$, while g tends to y as σ approaches infinity. Therefore, by increasing the rate of monetary expansion (i.e., by creating inflation) the government can raise its revenue. In fact, by letting the rate of monetary growth be arbitrarily large, the government can extract from the private sector an amount of resources arbitrarily close to the total resources available in the econ-

omy. We will next see that this monotonic relation between the government revenue and the inflation rate does not longer hold if we allow for the possibility of tax evasion.

It is also clear from (3.9) that the stationary amount of consumption (and of real monetary balances) is monotonically decreasing in the rate of monetary growth when there is no evasion. Since the government can raise its spending by injecting money in the economy and the total output per period is fixed, the amount of resources left for private consumption decreases and, thus, inflation ends up reducing the welfare of households at the stationary equilibrium. We will also see that the negative effect of inflation on welfare does not necessarily hold when evasion is present, i.e., when condition (2.18) is satisfied.

When evasion is strictly positive, inflation (or, equivalently, the rate of growth of nominal balances) affects the government revenue through two channels. First, it affects the regular revenue accruing from the tax system. The two first terms of the RHS of (4.1) constitute the fiscal revenue in real terms,

$$r \equiv \tau y + \tau \left(\frac{\pi\varphi}{1 + \sigma} - 1 \right) e^*.$$

As σ increases, the real value of the penalties paid by the audited taxpayers decreases in a direct way. This effect is captured by the term $\frac{\pi\varphi}{1+\sigma}$ in the previous expression. Moreover, inflation affects the incentives of taxpayers to report their true income. This effect is captured by the dependence of e^* on σ (see (3.6)). Second, the rate of monetary growth σ obviously affects the real amount of seignorage, which is the last term of expression (4.1),

$$s \equiv \sigma m^*.$$

The following proposition provides the comparative statics of the different components of the government revenue with respect to the rate σ of monetary growth. In our analysis we restrict the rate σ to lie in the interval $(\rho\pi\varphi - 1, \varphi - 1)$ in order to satisfy the condition (2.20). Part (d) of the proposition constitutes in fact the main result of our analysis.

Proposition 4.1

- (a) $\frac{de^*}{d\sigma} > 0$ for all $\sigma \in (\rho\pi\varphi - 1, \varphi - 1)$.
- (b) *There exists a $\hat{\sigma} \in (\rho\pi\varphi - 1, \varphi - 1)$ such that $\frac{ds}{d\sigma} > 0$ for $\sigma \in (\rho\pi\varphi - 1, \hat{\sigma})$, and $\frac{ds}{d\sigma} < 0$ for $\sigma \in (\hat{\sigma}, \varphi - 1)$.*
- (c) *If $\rho\pi^2\varphi > 1$ and*

$$[(\varphi - 1)(\rho\pi^2\varphi - 1)]^{\frac{1}{2}} > \rho\pi\varphi - 1, \tag{4.3}$$
then there exists a $\hat{\sigma} \in (\rho\pi\varphi - 1, \varphi - 1)$ such that $\frac{ds}{d\sigma} < 0$ for $\sigma \in (\rho\pi\varphi - 1, \hat{\sigma})$, and $\frac{ds}{d\sigma} > 0$ for $\sigma \in (\hat{\sigma}, \varphi - 1)$. Otherwise, $\frac{ds}{d\sigma} > 0$ for all $\sigma \in (\rho\pi\varphi - 1, \varphi - 1)$.
- (d) *There exists a $\sigma^* \in (\rho\pi\varphi - 1, \varphi - 1)$ such that $\frac{dg}{d\sigma} > 0$ for $\sigma \in (\rho\pi\varphi - 1, \sigma^*)$, and $\frac{dg}{d\sigma} < 0$ for $\sigma \in (\sigma^*, \varphi - 1)$. Moreover,*

$$\frac{dm^*}{d\sigma} = \frac{dc^*}{d\sigma} = -\frac{dg}{d\sigma}.$$

Part (a) of the previous proposition tells us that, as inflation increases, the real penalty rate $\frac{\phi}{1+\sigma}$ on evaded taxes decreases and, therefore, individuals decide optimally to evade more income. Part (b) reflects the trade-off between the decrease in regular revenue due to the fact that individuals are willing to pay less taxes voluntarily, as follows from part (a), and the increase of revenue accruing from the total penalty imposed on audited taxpayers who evade a larger amount of taxes. For low values of the inflation rate the second effect outweighs the first, and the opposite occurs for high values of the inflation rate.

Part (c) also shows some ambiguity concerning the relationship between seignorage and inflation. For the case $\rho\pi^2\phi > 1$ such ambiguity could appear as a consequence of the non-monotonic behavior of real monetary balances (see part (d)). However, when $\rho\pi^2\phi \leq 1$ such an ambiguity vanishes and seignorage revenue is monotonically increasing in σ , as occurs in the model without tax evasion. It should also be pointed out that the inequality $\rho\pi^2\phi \leq 1$ holds for any reasonable calibration of the model, so that seignorage is generally increasing in the inflation rate.

Finally, part (d) establishes the main difference with the model without tax evasion. Now the government cannot increase monotonically the revenue extracted from the private sector by arbitrarily raising the rate of inflation. On the one hand, a higher inflation rate makes individuals to evade more taxes, since penalties become smaller in real terms. On the other hand, money creation is generally a source of revenue for the government. The first effect dominates for sufficiently large values of σ , whereas the second one is the dominating for small values of σ . Fig. 3 depicts the set of admissible values of the parameters σ and ϕ according to condition (2.20) and the subsets for which the government revenue either increases or decreases with the rate σ of monetary growth. We see thus that the maximum level of the government revenue is achieved for an inflation rate equal to $\sigma^* = \frac{1}{2}\phi(1 + \rho\pi^2) - 1$ (see (4.12) in Appendix A). Moreover, for each revenue level smaller than that maximum, there exist two rates of monetary growth and, thus, two rates of inflation that support it. The relation between government revenue and inflation is locally increasing at the low-inflation equilibrium, whereas the converse holds at the high-inflation equilibrium. Obviously, at the latter equilibrium, the government cannot increase its revenue by means of increasing further the rate of monetary growth.

The inverted U-shaped relationship between government revenue g and the rate σ of monetary growth implies that the relationship between aggregate private consumption c^* and σ is U-shaped, since the total output is kept constant at the level y . Finally, the effect of σ on aggregate real monetary balances m^* is the same as that on consumption since, under our parametric assumptions, the cash-in-advance is strictly binding for all households so that m^* is always equal to c^* .

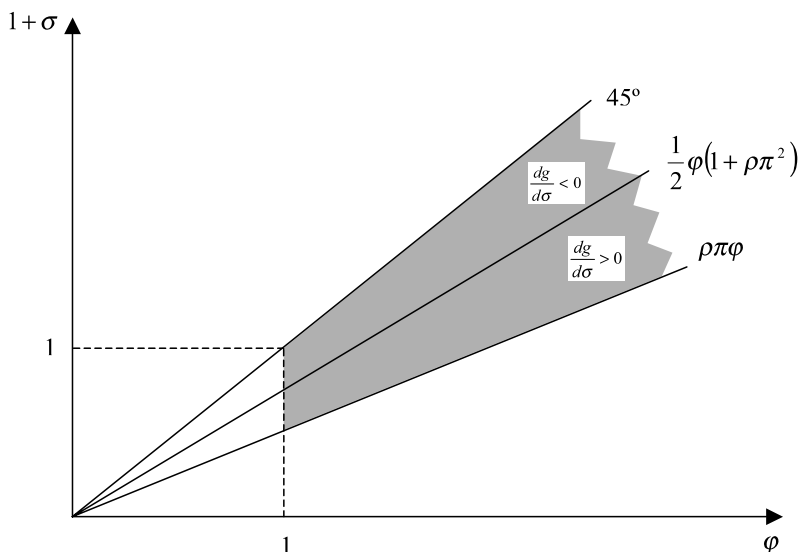


Fig. 3. Regions for which $\frac{dg}{d\sigma} < 0$ and $\frac{dg}{d\sigma} > 0$. The admissible values of ϕ and σ are in the shaded region.

5. Distributional and welfare implications of inflation under tax evasion

When government spending does not provide utility to the agents of the economy, the objective of revenue maximization by the government implies the minimization of average private consumption. In principle, this results in a reduction of the ex ante welfare of households. However, in order to calculate the overall impact of inflation on welfare, we need to take into account all the effects of changes in the rate σ of monetary growth on the time-invariant distribution v^* of consumption. In fact, the expected utility of a household at a stationary equilibrium will be the result of the interaction between the utility function u and the time-invariant distribution of consumption.

As a first step towards discussing distributional issues, we have the following result concerning the distribution μ^* of evaded income.

Proposition 5.1

- (a) The variance $Var(e)$ of the distribution of evaded income at a stationary equilibrium is increasing in the inflation rate.
- (b) Let $F_{\mu^*}(e; \sigma)$ be the cumulative distribution function of evaded income at the stationary equilibrium when the rate of inflation is σ . Then, $\frac{dF_{\mu^*}(e; \sigma)}{d\sigma} \leq 0$ for all $e \in [e, \bar{e}]$, with strict inequality for at least one $e \in [e, \bar{e}]$.

Part (a) tells us that the distribution of evaded income displays more variance as inflation rises. This is a consequence of the fact that the amount of evaded taxes associated with a given evasion and audit history rises when the tax fine becomes smaller

in real terms. Part (b) tells us that, as inflation increases, the cumulative distribution function shifts down uniformly and, therefore, an increase of inflation generates a new distribution of evaded income that dominates the previous one in the sense of first order stochastic dominance (see Hadar and Russell, 1969).

The next proposition characterizes the effects of inflation on the time-invariant distribution v^* of consumption (and of real monetary balances):

Proposition 5.2

- (a) *The variance $Var(c)$ of the distribution of consumption at a stationary equilibrium satisfies $\lim_{\sigma \rightarrow \rho\pi\varphi - 1} Var(c) = 0$ and $\lim_{\sigma \rightarrow \varphi - 1} Var(c) = \infty$.*
- (b) *Assume that $\rho\pi < 1/2$ and let $F_{v^*}(c; \sigma)$ be the cumulative distribution function of consumption at the stationary equilibrium when the rate of inflation is σ . Then, $\frac{dF_{v^*}(c; \sigma)}{d\sigma} \geq 0$ for all $c \in [\underline{m}, \bar{m}]$, with strict inequality for at least one $c \in [\underline{m}, \bar{m}]$, whenever $\sigma \in (\rho\pi\varphi - 1, \frac{\varphi}{2} - 1)$; and $\lim_{\sigma \rightarrow \varphi - 1} \frac{dF_{v^*}(c; \sigma)}{d\sigma} \leq 0$ for all $c \in [\underline{m}, \bar{m}]$, with strict inequality for at least one $c \in [\underline{m}, \bar{m}]$.*

The previous proposition shows in part (a) that, when the inflation rate σ approaches $\rho\pi\varphi - 1$, the distribution of consumption tends to become degenerate, since all the mass of v^* is concentrated around the value α . This can be easily seen by noticing that $\lim_{\sigma \rightarrow \rho\pi\varphi - 1} \delta = 0$, so that the points $\{m^K\}_{K=0}^{\infty}$ in the support of v^* satisfy $\lim_{\sigma \rightarrow \rho\pi\varphi - 1} m^K = \alpha$ for $K = 0, 1, 2, \dots$ (see (3.5)). The reason for this relies on the fact that when $\pi\varphi = \frac{1+\sigma}{\rho}$ taxpayers decide to make truthful reports of their income and, hence, regardless of whether they are audited or not, they do not have to pay any penalty. This means that consumption is not longer stochastic. However, the variance of consumption becomes arbitrarily large when σ approaches the upper bound $\varphi - 1$.

From part (a) of the last proposition we know that, when the inflation rate is high, the dispersion (as measured by the variance) of the distribution of consumption increases with inflation. Moreover, we know that the expected (or average) consumption is increasing in the inflation rate for high values of σ , namely, when $\sigma > \sigma^*$ (see part (d) of Proposition 4.1). Therefore, the expected utility of households is exposed in this case to two effects of apparent opposite sign. However, part (b) of the last proposition shows that, if the rate σ of monetary growth (and, thus, the inflation) is close to the upper bound $\varphi - 1$, then the distribution $F_{v^*}(c; \sigma)$ uniformly shifts downwards as σ increases. In this case, all the individuals with increasing utility functions will be better off after that inflation increase (see Hadar and Russell, 1969). We have thus resolved the potential ambiguity of the welfare effect brought about by having simultaneously a higher expectation and a higher variance for consumption.

On the contrary, when inflation is sufficiently small, it is clear that the two aforementioned effects go in the same direction. Indeed, when inflation is small, expected consumption decreases with σ , according to part (d) of Proposition 4.1, and the variance of consumption increases according to part (a) of Proposition 5.2. Therefore, we could expect that the ex ante welfare of households be decreasing in σ for low values of inflation. This presumption is confirmed by part (b) of Proposition 5.2.

Since the distribution $F_{v^*}(c; \sigma)$ uniformly shifts upwards as inflation increases for values of σ lying on the interval $(\rho\pi\varphi - 1, \frac{\varphi}{2} - 1)$, we can conclude that inflation induces a deterioration of the distribution v^* of consumption in terms of first order stochastic dominance. Therefore, in this case all the individuals with increasing utility functions will suffer a decrease in their expected utilities. Finally, note that the assumption $\rho\pi < 1/2$ appearing in part (b) of Proposition 5.2 is very reasonable in empirical terms.

Figs. 4 and 5 show the changes in the probability function of the distribution of consumption when inflation increases for low and high values of the inflation rate, respectively.

It is well known that the first two moments of a distribution do not contain, in most of the cases, enough information to assess the desirability of a random variable from the perspective of an agent who wants to maximize his expected utility. The analysis based on stochastic dominance relations, when it is possible, allows to rank unambiguously distributions in terms of the expected utility they provide. In our model, we have been able to characterize the welfare effects of inflation by just looking at the first order stochastic dominance ordering.

We conclude this section with one example that illustrates the result of part (b) of Proposition 5.2. Recall that we have assumed throughout this paper that the utility function is logarithmic and, thus, strictly increasing. As follows from Proposition 3.3, the unconditional expected sum of discounted utilities of a household at a stationary equilibrium is

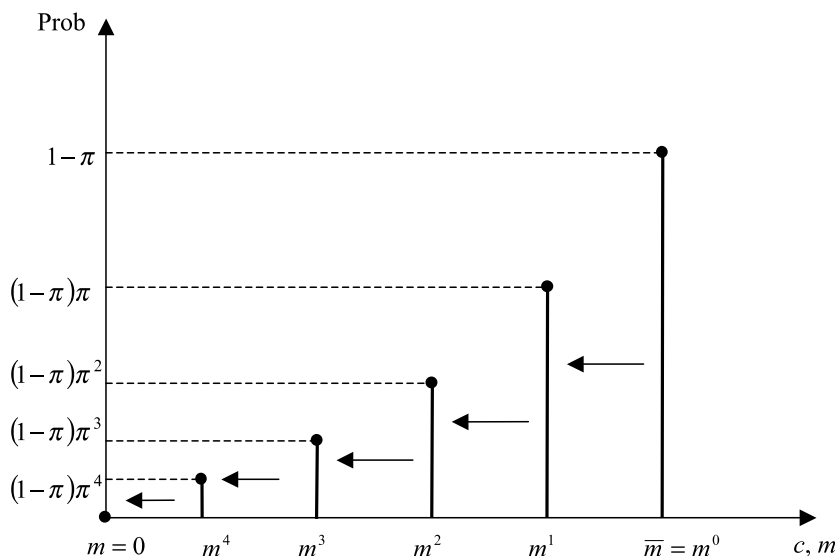


Fig. 4. Shift in the probability function of the distribution of consumption (and of real monetary balances) when inflation rises for σ lying on the interval $(\rho\pi\varphi - 1, \frac{\varphi}{2} - 1)$.

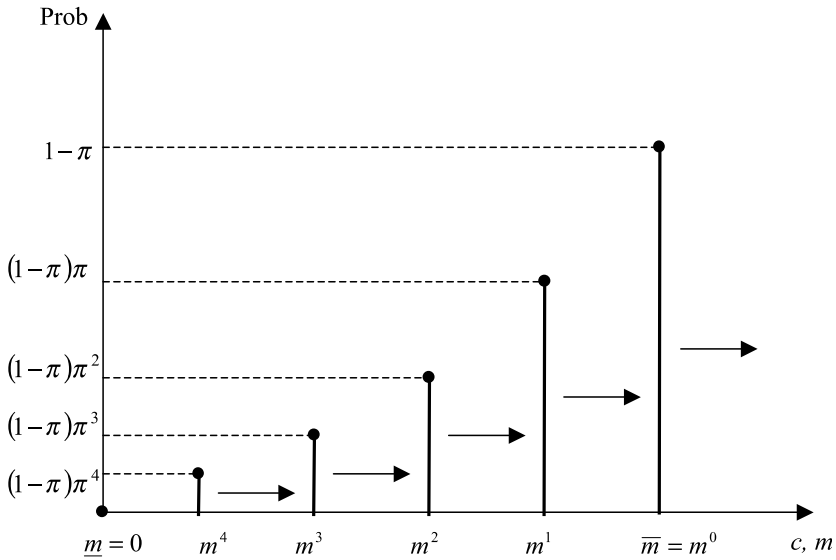


Fig. 5. Shift in the probability function of the distribution of consumption (and of real monetary balances) when inflation rises for σ close to $\varphi - 1$.

$$\begin{aligned}
 W &\equiv E \left[\sum_{j=t}^{\infty} \rho^{j-t} u(c_j) \right] \\
 &= \frac{1}{1-\rho} \left[(1-\pi) \ln \alpha + \sum_{K=1}^{\infty} \left\{ (1-\pi) \pi^K \ln \left[\alpha + \beta \delta \left(\sum_{i=0}^{K-1} \gamma^i \varphi^{i+1} \right) \right] \right\} \right].
 \end{aligned}$$

Fig. 6 displays the plot of W against the rate of inflation rate for the following reasonable parameter configuration $\rho = 0.9$, $\pi = 0.4$, $\varphi = 2.5$, $\tau = 0.2$ and $y = 1$. In this

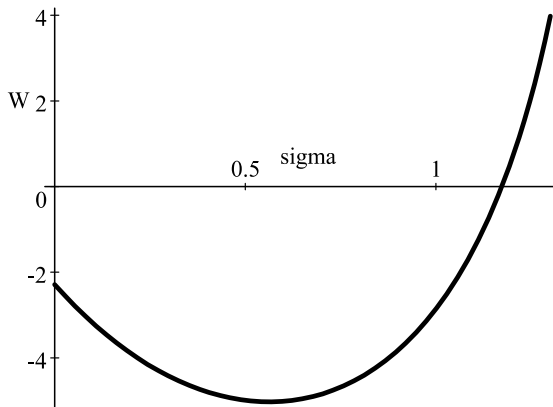


Fig. 6. The unconditional expected sum of discounted utilities at a stationary equilibrium as a function of the rate σ of monetary growth for $\rho = 0.9$, $\pi = 0.4$, $\varphi = 2.5$, $\tau = 0.2$ and $y = 1$.

example the unconditional expected sum of discounted utilities reaches its minimum at $\sigma = 0.625$. However, expected consumption is minimized at $\sigma = \frac{1}{2}\varphi(1 + \rho\pi^2) - 1 = 0.43$. The discrepancy between these two values comes from the effect of higher moments of the consumption distribution on the households' ex ante welfare.

We see thus that the relationship between ex ante welfare and inflation is U-shaped, as dictated by part (b) of Proposition 5.2. In particular, higher rates of inflation could enhance the ex ante welfare of households.

6. Conclusion

In this paper we have analyzed the effects of inflation on the reporting strategies of taxpayers, on the distribution of consumption, and on the government revenue. Our analysis has been carried out within the framework of a monetary economy where households face a cash-in-advance constraint on their consumption purchases. Moreover, households face a probability of being audited by the tax enforcement agency and, thus, their decision about the level of income to be reported resembles that of portfolio selection.

Our main results include a non-monotonic relationship between the rate of growth of money and the government revenue. As government creates inflation, the real penalty imposed on evaded taxes goes down and this stimulates tax evasion. While this reduction in regular tax collection is outweighed by seignorage for small levels of inflation, the aggregate revenue turns out to be decreasing in the rate of monetary expansion when inflation is sufficiently high. This means that high values of inflation could leave more resources on the hands of the private sector than more moderate values. Even if inflation raises the variance of the distribution of consumption, the overall welfare effect of raising the inflation rate could be positive.

The economic environment we have considered is simple enough to enable us to obtain explicit solutions for all the variables we wanted to characterize. One of the simplifying assumption of our analysis is that individuals have the same ex ante income. This assumption allows us to emphasize the heterogeneity that arises from the fact that in every period only a subset of taxpayers is audited. This brings about heterogeneous consumption and reporting patterns that give rise in turn to a non-degenerate distribution of consumption across households.

Another assumption that can be relaxed (at a non-trivial complexity cost) is that households cannot save part of their income. The introduction of capital markets and a process of capital accumulation could allow us to analyze the effects of inflation either on the growth rate and on the convergence rate of the economy. In absence of tax evasion this analysis has been carried out by Abel (1985), Jones and Manuelli (1995), and Mino (1997). Moreover, the analysis of the optimal mix of financing policies by the government in a growing economy has been undertaken also by Jones and Manuelli (1995), Palivos and Yip (1995), Smith (1996) and Pecorino (1997). To add the possibility of tax evasion in a capital accumulation economy exhibiting a liquidity constraint is thus left for future research.

Acknowledgment

Financial support from the Spanish Ministry of Science and Technology through grants SEC2003-00306 and HI2001-0039, the Generalitat of Catalonia through CREA and grant SGR2001-00162, and the Instituto de Estudios Fiscales (Spanish Ministry of Finance) is gratefully acknowledged. This paper has benefited from the comments of an anonymous referee and the Editor of this Journal. They should not bear any responsibility for the remaining errors.

Appendix A

Proof of Proposition 2.1. Let us conjecture that the two policy functions have the functional forms given in (2.11) and (2.12). Combining (2.5) and (2.7) we obtain

$$(1 + f_{t+1})\lambda_t = \rho E_t[u'(c_{t+1})]. \quad (\text{A.1})$$

Since $c_{t+1} = m_{t+1}$ when the cash-in-advance constraint is binding, we can use the policy function (2.11) and the logarithmic specification for u to solve for the Lagrange multiplier in the previous equation:

$$\lambda_t = \frac{\rho}{1 + f_{t+1}} E_t \left(\frac{1}{\alpha + \beta h_t e_{t-1}} \right). \quad (\text{A.2})$$

Therefore, plugging (A.2) into (2.6) we get

$$\frac{\rho}{1 + f_{t+1}} E_t \left(\frac{1}{\alpha + \beta h_t e_{t-1}} \right) = \frac{\rho}{(1 + f_{t+1})} E_t \left[\left(\frac{\rho}{1 + f_{t+2}} E_{t+1} \left(\frac{1}{\alpha + \beta h_{t+1} e_t} \right) \right) h_{t+1} \right].$$

As the inflation rate is non-stochastic under a deterministic monetary policy rule, and h_t , e_{t-1} and e_t are all known at period t after the potential inspection has taken place, a straightforward application of the law of iterated expectation to the previous equation yields

$$\frac{1}{\alpha + \beta h_t e_{t-1}} = \frac{\rho}{1 + f_{t+2}} E_t \left(\frac{h_{t+1}}{\alpha + \beta h_{t+1} e_t} \right). \quad (\text{A.3})$$

Since h_t is an i.i.d. random variable, we can easily compute the conditional expectation appearing in the previous expression,

$$E_t \left(\frac{h_{t+1}}{\alpha + \beta h_{t+1} e_t} \right) = \frac{\pi \varphi}{\alpha + \beta \varphi e_t}.$$

Moreover, the inflation rate f_t is constant at a stationary equilibrium because the government increases the nominal monetary balances at a constant rate. Therefore, (A.3) becomes at a stationary equilibrium

$$\frac{1}{\alpha + \beta h_t e_{t-1}} = \frac{\rho \pi \varphi}{(1 + f)(\alpha + \beta \varphi e_t)}.$$

Using the conjectured policy function (2.12), the previous equation becomes

$$\frac{1}{\alpha + \beta h_t e_{t-1}} = \frac{\rho \pi \varphi}{(1 + f)[\alpha + \beta \varphi(\delta + \gamma h_t e_{t-1})]},$$

which, after collecting terms, can be rewritten as

$$\rho \pi \varphi \alpha - (1 + f)(\alpha + \beta \delta \varphi) + [\rho \pi \varphi \beta - (1 + f)\beta \gamma \varphi] h_t e_{t-1} = 0.$$

Therefore, the coefficients of the policy functions (2.11) and (2.12) must satisfy

$$\rho \pi \varphi \alpha - (1 + f)(\alpha + \beta \delta \varphi) = 0, \tag{A.4}$$

$$\rho \pi \varphi \beta - (1 + f)\beta \gamma \varphi = 0. \tag{A.5}$$

Using again the policy functions (2.11) and (2.12), the budget constraint at a stationary equilibrium when the cash-in-advance constraint is binding is (see (2.10))

$$(1 + f)(\alpha + \beta h_t e_{t-1}) = (1 - \tau)y - \left(\frac{\tau}{1 + f}\right) h_t e_{t-1} + \tau(\delta + \gamma h_t e_{t-1}).$$

After collecting terms, the previous equation becomes

$$(1 + f)\alpha - \tau \delta - (1 - \tau)y + \left[(1 + f)\beta - \tau \gamma + \left(\frac{\tau}{1 + f}\right) \right] h_t e_{t-1} = 0,$$

so that the coefficients of the policy functions (2.11) and (2.12) must also satisfy

$$(1 + f)\alpha - \tau \delta - (1 - \tau)y = 0, \tag{A.6}$$

$$(1 + f)\beta - \tau \gamma + \left(\frac{\tau}{1 + f}\right) = 0. \tag{A.7}$$

Solving the system of equations (A.4)–(A.7) for the unknown coefficients α , β , δ , and γ , we get after some tedious algebra the values given in the statement of the proposition. \square

Proof of Lemma 2.2. From the first order condition (2.5), we have that $\theta_t > 0$ if and only if $u'(c_t) > \lambda_t$. Therefore, from (A.1) we have that $\theta_t > 0$ if and only if

$$u'(c_t) > \frac{\rho E_t[u'(c_{t+1})]}{1 + f_{t+1}}.$$

Evaluating the previous inequality at a stationary equilibrium where the cash-in-advance constraint is binding, we get after some rearranging,

$$\frac{1 + f}{\rho} > \frac{E_t[u'(m_{t+1})]}{u'(m_t)}.$$

Since u is logarithmic and real balances are governed by the policy function (2.11), the previous inequality becomes

$$\frac{1 + f}{\rho} > \frac{\alpha + \beta h_{t-1} e_{t-2}}{\alpha + \beta h_t e_{t-1}} = \frac{\alpha + \beta h_{t-1} e_{t-2}}{\alpha + \beta \delta h_t + \beta \gamma h_t h_{t-1} e_{t-2}}, \tag{A.8}$$

where the equality comes from the policy function (2.12) for e_{t-1} . The last term in (A.8) is clearly increasing in h_t since $\beta\delta < 0$ and $\beta\gamma < 0$. Therefore, for a given value of h_{t-1} , the largest value of the last term in (A.8) is reached when $h_t = \varphi$. Therefore, the Lagrange multiplier θ_t is strictly positive for all households whenever

$$\frac{1+f}{\rho} > \frac{\alpha + \beta h_{t-1} e_{t-2}}{\alpha + \beta\delta\varphi + \beta\gamma\varphi h_{t-1} e_{t-2}}, \tag{A.9}$$

for both $h_{t-1} = 0$ and $h_{t-1} = \varphi$. However, the RHS of (A.9) takes the same value regardless of whether $h_{t-1} = \varphi$ or $h_{t-1} = 0$. To see this, we only have to show that the following inequality holds:

$$\frac{\alpha}{\alpha + \beta\delta\varphi} = \frac{\alpha + \beta\varphi e_{t-2}}{\alpha + \beta\delta\varphi + \beta\gamma\varphi^2 e_{t-2}}.$$

The previous equality simplifies to

$$\alpha\gamma\varphi = \alpha + \beta\delta\varphi.$$

It can be checked that the previous equality holds by simply substituting the equilibrium values of α , β , δ , and γ given in (2.13)–(2.16).

Therefore, the cash-in-advance constraint is strictly binding ($\theta_t > 0$) for all households if and only if

$$\frac{1+f}{\rho} > \frac{\alpha}{\alpha + \beta\delta\varphi} = \frac{1+f}{\rho\pi\varphi},$$

where the equality comes from using again the equilibrium values of α , β , and δ . It is immediate to see that the previous inequality is satisfied if and only if (2.19) holds. \square

Proof of Proposition 3.1. Let $T(e, B)$ be the transition function of the Markov process of evaded income. This transition function gives the probability that a household evading the amount e of income in a given period, evades an amount of income lying in the Borel set B in the next period. Therefore,

$$T(e, B) = (1 - \pi)\mathbb{1}_B(\delta) + \pi\mathbb{1}_B(\delta + \gamma\varphi e). \tag{A.10}$$

Let B^c be the complementary of the Borel set B in \mathbb{R} . It is obvious from (A.10) that, for every $B \in \mathcal{B}$, $T(e, B) \geq 1 - \pi$ if $\delta \in B$, for all $e \in \mathcal{B}$. Moreover, $T(e, B^c) \geq 1 - \pi$ if $\delta \in B^c$, for all $e \in \mathcal{B}$. This means that condition M in Section 11.4 of Stokey and Lucas (1989) holds, and from their Theorems 11.2 and 11.6 the desired convergence is obtained. \square

Proof of Proposition 3.2. We just have to check that the distribution given in (3.3) satisfies the functional equation (3.2). Note that the support of μ^* is discrete,

$$\text{supp}(\mu^*) = \{e^K\}_{K=0}^\infty,$$

where $e^K = \delta \sum_{i=0}^K (\gamma\varphi)^i$. Since $\delta + \gamma\varphi e^K = e^{K+1}$, we have that $g(\{e^{K+1}\}) = e^K$. Therefore, if $e^{K+1} > \delta$, Eq. (3.2) becomes

$$\mu^*({e^{K+1}}) = (1 - \pi)\mathbb{1}_{\{e^{K+1}\}}(\delta) + \pi \int_{\{e^K\}} d\mu^* = 0 + \pi\mu^*({e^K}) = (1 - \pi)\pi^{K+1},$$

as $\mu^*({e^K}) = (1 - \pi)\pi^K$. For the case $e^{K+1} = \delta$, Eq. (3.2) becomes

$$\mu^*(\delta) = (1 - \pi)\mathbb{1}_{\{\delta\}}(\delta) + \pi \int_{g(\{\delta\})} d\mu^* = (1 - \pi).$$

The last equality comes from the fact that the set $g(\delta)$ is empty and has, thus, zero measure. The emptiness of $g(\{\delta\})$ follows because $\delta + \gamma\varphi e > \delta$ as $\gamma\varphi > 0$ and $e > 0$ under condition (2.18). \square

Proof of Proposition 3.3. Obvious from Propositions 3.1 and 3.2, since the sequence of distributions $\{\mu_t\}_{t=0}^\infty$ of evaded income converges to μ^* and the distribution v_{t+1} is entirely determined by the distribution μ_{t-1} through the law of motion (3.4). \square

Proof of Corollary 3.4. Since there is a continuum of households uniformly distributed on the interval $[0, 1]$ and the random variable h_t is identically and independently distributed across all households, the strong law of large numbers implies that $e^* = E(e)$ and $c^* = m^* = E(m)$. Then, we just have to compute

$$E(e) = \sum_{K=0}^\infty (1 - \pi)\pi^K \delta \left(\sum_{i=0}^K (\gamma\varphi)^i \right) = \frac{\delta}{1 - \gamma\pi\varphi}, \tag{A.11}$$

where the first equality is implied by (3.3), and the second equality comes after some algebra.³ The expression (3.6) is finally obtained by just using the equilibrium values of δ and γ , and the stationary money market equilibrium condition $f = \sigma$. Similarly,

$$E(m) = (1 - \pi)\alpha + \sum_{K=1}^\infty (1 - \pi)\pi^K \left(\alpha + \beta\delta \sum_{i=0}^{K-1} \gamma^i \varphi^{i+1} \right) = \alpha + \frac{\beta\delta\pi\varphi}{1 - \gamma\pi\varphi},$$

where the first equality is a consequence of (3.5) and the second arises after some simple computation.⁴ We arrive at the expression (3.7) by using the equilibrium values of α , β , δ and γ , and the fact that $f = \sigma$ at the stationary equilibrium. \square

³ A much simple computation of $E(e)$ is obtained by taking the unconditional expectation on both sides of the policy function (2.12),

$$E(e) = \delta + \gamma\pi\varphi E(e),$$

and then solving for $E(e)$.

⁴ Again a more direct computation is obtained by just taking the unconditional expectation on both sides of the policy function (2.11),

$$E(m) = \alpha + \beta\pi\varphi E(e),$$

and then using (4.11).

Proof of Proposition 4.1. (a) Compute the derivative of (3.6) with respect to σ , and equate that derivative to zero. The solutions for σ to the resulting (messy) equation turn out to be the following conjugate roots:

$$\frac{\rho\pi[1 + \pi(\varphi - 1)] - 1 \pm \rho\pi\varphi\sqrt{-\pi\varphi(1 - \pi)(1 - \rho\pi)}}{1 - \rho\pi(1 - \pi)}.$$

Since the term inside the square root is negative, we conclude that the aggregate amount e^* of evaded income is strictly monotonic in σ . It just remains to check numerically that the sign of $\frac{de^*}{d\sigma}$ is strictly positive.

(b) We compute the derivative of r with respect to σ ,

$$\frac{dr}{d\sigma} = -\frac{(1 - \tau)(1 - \pi)(1 - \rho\pi)[(1 + \sigma)^2 - \rho\pi^2\varphi^2]\varphi y}{[(1 + \sigma) - \varphi]^2[(1 + \sigma) - \rho\pi^2\varphi]^2}.$$

Clearly, $\frac{dr}{d\sigma} \geq 0$ whenever $\sigma \leq \pi\varphi\rho^{1/2} - 1 \equiv \hat{\sigma}$. Note that $\hat{\sigma} \equiv \pi\varphi\rho^{1/2} - 1 > \rho\pi\varphi - 1$ as $\rho \in (0, 1)$. Moreover, $\varphi - 1 > \pi\varphi\rho^{1/2} - 1$ since $\pi\rho^{1/2} < 1$.

(c) We compute the derivative of s with respect to σ ,

$$\frac{ds}{d\sigma} = \frac{(1 - \tau)(1 - \pi)(1 - \rho\pi)[\sigma^2 + (\varphi - 1)(1 - \rho\pi^2\varphi)]\varphi y}{[\varphi - (1 + \sigma)]^2[(1 + \sigma) - \rho\pi^2\varphi]^2}.$$

Then, we solve the equation $\frac{ds}{d\sigma} = 0$, to obtain the unique positive root.

$$\hat{\sigma} = [(\varphi - 1)(\rho\pi^2\varphi - 1)]^{\frac{1}{2}},$$

which is clearly strictly smaller than the upper bound $\varphi - 1$. If $\rho\pi^2\varphi > 1$ and (4.3) holds, then $\hat{\sigma}$ is real and greater than the lower bound $\rho\pi\varphi - 1$. Obviously, in this case we have $\frac{ds}{d\sigma} \leq 0$ for $\sigma \leq \hat{\sigma}$. Conversely, if either $\rho\pi^2\varphi \leq 1$ or (4.3) does not hold, then $\frac{ds}{d\sigma} > 0$ for all $\sigma \in (\rho\pi\varphi - 1, \varphi - 1)$.

(d) From Walras' law, equilibrium in the money market implies equilibrium in the good market, that is,

$$y = c^* + g.$$

Since the amount of output y is constant, and c^* is equal to m^* , we have that

$$\frac{dg}{d\sigma} = -\frac{dc^*}{d\sigma} = -\frac{dm^*}{d\sigma}.$$

Using the expression (3.7), we can compute the following derivative:

$$\frac{dc^*}{d\sigma} = -\frac{(1 - \tau)(1 - \pi)(1 - \rho\pi)[(1 + \rho\pi^2)\varphi - 2(1 + \sigma)]\varphi y}{[\varphi - (1 + \sigma)]^2[(1 + \sigma) - \rho\pi^2\varphi]^2}.$$

Therefore, $\frac{dg^*}{d\sigma} = -\frac{dc^*}{d\sigma} \geq 0$ whenever

$$\sigma \leq \frac{1}{2}\varphi(1 + \rho\pi^2) - 1 \equiv \sigma^*. \tag{A.12}$$

Note that $\sigma^* < \varphi - 1$, since $1 + \rho\pi^2 < 2$. Moreover, to verify that $\sigma^* > \rho\pi\varphi - 1$, we only have to check that

$$\frac{1}{2}(1 + \rho\pi^2) > \rho\pi,$$

which can be written as

$$\rho\pi^2 - 2\rho\pi + 1 > 0.$$

The previous inequality holds since

$$\rho\pi^2 - 2\rho\pi + 1 > \rho^2\pi^2 - 2\rho\pi + 1 = (\rho\pi - 1)^2 > 0. \quad \square$$

Proof of Proposition 5.1. (a) From the policy function (2.12) and the independence between h_t and e_{t-1} , it is clear that

$$\begin{aligned} \text{Var}(e_t) &= \gamma^2 \text{Var}(h_t e_{t-1}) = \gamma^2 [E(h_t^2 e_{t-1}^2) - [E(h_t e_{t-1})]^2] \\ &= \gamma^2 [E(h_t^2) E(e_{t-1}^2) - [E(h_t)]^2 [E(e_{t-1})]^2] \\ &= \gamma^2 [\pi\varphi^2 E(e_{t-1}^2) - \pi^2 \varphi^2 [E(e_{t-1})]^2] \\ &= \gamma^2 [\pi\varphi^2 \text{Var}(e_{t-1}) + [E(e_{t-1})]^2] - \pi^2 \varphi^2 [E(e_{t-1})]^2 \\ &= \gamma^2 \pi\varphi^2 [\text{Var}(e_{t-1}) + (1 - \pi)[E(e_{t-1})]^2]. \end{aligned} \tag{A.13}$$

Therefore, at a stationary equilibrium we can suppress the time subindex,

$$\text{Var}(e) = \gamma^2 \pi\varphi^2 [\text{Var}(e) + (1 - \pi)[E(e)]^2].$$

Solving for $\text{Var}(e)$ in the previous expression we get

$$\text{Var}(e) = \frac{\gamma^2 \pi(1 - \pi)\varphi^2}{1 - \gamma^2 \pi\varphi^2} [E(e)]^2.$$

Taking into account that $E(e)$ coincides with the aggregate amount e^* of evaded income given in (3.6), and using the equilibrium value of γ given in (2.16), we obtain that

$$\text{Var}(e) = \frac{(1 - \tau)^2 (1 + \sigma)^2 (1 - \pi) [(1 + \sigma) - \rho\pi\varphi] \pi^3 \rho^2 \varphi^2 y^2}{\tau^2 [\varphi - (1 + \sigma)]^2 [(1 + \sigma) - \rho\pi^2 \varphi]^2 [(1 + \sigma)^2 - \rho^2 \pi^3 \varphi^2]}. \tag{A.14}$$

Next, we compute the derivative $\frac{d\text{Var}(e)}{d\sigma}$ and equate it to zero. The conjugate solutions to the resulting equation are

$$\frac{-1 + \rho\pi(1 + \pi(\varphi - 1)) \pm \rho\pi\varphi \sqrt{-\pi(1 - \pi)(1 - \rho\pi)}}{1 - \rho\pi(1 - \pi)}.$$

As the term inside the square root is negative, both solutions are imaginary. We only have then to check numerically that the derivative $\frac{d\text{Var}(e)}{d\sigma}$ is strictly positive.

(b) To prove this part, we will show that all the values of the discrete support of the distribution μ^* increase with σ . Recall that, from Proposition 3.2, the support of μ^* is the sequence $\{e^K\}_{K=0}^\infty$ with $e^K = \delta \sum_{i=0}^K (\gamma\varphi)^i$ for $K = 0, 1, 2, \dots$. Observe that

$e^0 = \delta$ is clearly increasing in σ (see (2.15)). For e^K , with $K > 0$, we only have to check whether the product $\delta\gamma$ is increasing in σ . To this end, we compute

$$\delta\gamma = \frac{(1 - \tau)[(1 + \sigma) - \rho\pi\varphi]\rho\pi y}{\tau[\varphi - (1 + \sigma)](1 + \sigma)},$$

so that

$$\frac{d(\delta\gamma)}{d\sigma} = \frac{(1 - \tau)[(1 + \sigma)^2 + \rho\pi\varphi[\varphi - 2(1 + \sigma)]]\rho\pi y}{\tau[\varphi - (1 + \sigma)]^2(1 + \sigma)^2}.$$

The solutions for σ to the equation $\frac{d(\delta\gamma)}{d\sigma} = 0$ are

$$-1 + \rho\pi\varphi \pm \varphi\sqrt{-\rho\pi(1 - \rho\pi)},$$

which are clearly imaginary. Therefore, since the derivative $\frac{d(\delta\gamma)}{d\sigma}$ does not change its sign, we only have to check numerically that $\frac{d(\delta\gamma)}{d\sigma} > 0$. \square

Proof of Proposition 5.2. (a) From the policy function (2.11), and taking into account that $c_t = m_t$, we have

$$Var(c_{t+1}) = Var(m_{t+1}) = \beta^2 Var(h_t e_{t-1}) = \beta^2 \pi \varphi^2 [Var(e_{t-1}) + (1 - \pi)[E(e_{t-1})]^2],$$

where the third equality follows from (4.13). Suppressing the time subindexes, we get

$$Var(c) = \beta^2 \pi \varphi^2 [Var(e) + (1 - \pi)[E(e)]^2].$$

Using the values of $Var(e)$ and $E(e)$ in (4.14) and (3.6), respectively, we obtain

$$Var(c) = \frac{(1 - \pi)(1 - \rho\pi)^2(1 - \tau)^2[(1 + \sigma) - \rho\pi\varphi]^2 \pi \varphi^2 y^2}{[\varphi - (1 + \sigma)]^2 [(1 + \sigma) - \rho\pi^2 \varphi]^2 [(1 + \sigma)^2 - \rho^2 \pi^3 \varphi^2]}.$$

It is then obvious that $\lim_{\sigma \rightarrow \rho\pi\varphi - 1} Var(c) = 0$ and $\lim_{\sigma \rightarrow \varphi - 1} Var(c) = \infty$.

(b) For this part, we will prove that all the values of the discrete support of the distribution v^* are decreasing in σ for $\sigma \in (\rho\pi\varphi - 1, \frac{\varphi}{2} - 1)$, while all these values increase with σ when σ is sufficiently close to $\varphi - 1$. Recall from Proposition 3.3 that the support of v^* is given by the sequence $\{m^K\}_{K=0}^\infty$ with $m^0 = \alpha$, and $m^K = \alpha + \beta\delta \sum_{i=0}^{K-1} \gamma^i \varphi^{i+1}$ for $K = 1, 2, \dots$. After some tedious algebra it can be proved that $\frac{dm^K}{d\sigma} = 0$ when

$$\sigma = \left(\frac{K + 1}{K + 2}\right)\varphi - 1 \equiv \sigma^K, \quad \text{for } K = 0, 1, 2, \dots$$

Moreover, m^K reaches its global minimum at σ^K . On the one hand, if $\sigma \leq \sigma^0$, where $\sigma^0 = \frac{\varphi}{2} - 1$, then $\frac{dm^0}{d\sigma} = 0$ and $\frac{dm^K}{d\sigma} < 0$ for all $K > 0$, since $\sigma^K > \sigma^0$ for all $K > 0$. On the other hand, $\lim_{\sigma \rightarrow \varphi - 1} \frac{dm^K}{d\sigma} > 0$, since $\sigma^K < \varphi - 1$ for $K = 0, 1, 2, \dots$ \square

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