International Tax and Public Finance, 12, 239–263, 2005

Cost Uncertainty and Taxpayer Compliance

JORDI CABALLÉ Jordi.Caballe@uab.es
Unitat de Fonaments de l'Anàlisi Econòmica and CODE, Departament d'Economia i d'Història Econòmica, Universitat Autònoma de Barcelona, Edifici B, 08193 Bellaterra, Barcelona, Spain

JUDITH PANADÉS Judith.Panades.uab.es
Unitat de Fonaments de l'Anàlisi Econòmica, Universitat Autònoma de Barcelona, Spain

Abstract

The existence of a private cost borne by audited taxpayers affects the tax enforcement policy. This is so because tax auditors will face now two sources of uncertainty, namely, the typical one associated with taxpayers' income and that associated with the taxpayers' idiosyncratic attitude towards tax compliance. Moreover, the inspection policy can be exposed to some randomness from the taxpayers' viewpoint due to the uncertainty about the audit cost borne by the tax authority. In this paper we provide an unified framework to analyze the effects of all these sources of uncertainty in a model of tax compliance with strategic interaction between auditors and taxpayers. We show that more variance in the distribution of the taxpayers' private cost of evading raises both tax compliance and the ex-ante welfare of taxpayers. The effects of the uncertainty about the audit cost faced by the tax authority are generally ambiguous. We also discuss the implications of our model for the regressive (or progressive) bias of the effective tax system.

Keywords: tax evasion, audit cost

JEL Code: H26

1. Introduction

The existence of a private cost borne by audited taxpayers affects the tax enforcement policy. This is so because tax auditors will face now two sources of uncertainty, namely, the typical one associated with taxpayers' income and that associated with the taxpayers idiosyncratic attitude towards tax compliance. Moreover, the inspection policy can be exposed to some randomness from the taxpayers' viewpoint due to the uncertainty concerning the audit cost faced by the tax enforcement agency. The aim of this paper is to provide an unified framework to analyze the effects of all these sources of uncertainty in a model of tax compliance where there is strategic interaction between auditors and taxpayers and, moreover, the tax enforcement agency does not commit to follow a given audit policy.

The first models that analyzed the phenomenon of tax evasion through a portfolio selection approach (like those of Allingham and Sandmo, 1972; Yitzhaki, 1974) assumed that all taxpayers were facing a constant and identical probability of being audited by the tax

enforcement agency. However, consider a tax auditor who observes the amount of income reported by a taxpayer before conducting the corresponding audit. If this auditor wants to maximize the expected revenue from each taxpayer, there is no apparent reason why he should commit to an audit policy independent of the report he observes. An auditor using optimally all the relevant information at his disposal should make both the probability of inspection and the effort applied to a taxpayer contingent on the corresponding amount of reported income. One of the first attempts to analyze those contingent policies was made by Reinganum and Wilde (1985), who considered a principal-agent model where the tax enforcement agency commits to follow a cut-off audit policy. According to this policy, taxpayers reporting less income than a given level are inspected, whereas the other taxpayers are not inspected. In a very influential paper the same authors (Reinganum and Wilde, 1986) considered an alternative scenario where a revenue-maximizing tax authority does not even commit to an audit rule but selects an optimal policy given the realization of the taxpayers' reports. Moreover, in this new framework the probability of inspection is allowed to take all the possible values in the interval [0, 1]. After the taxpayer has submitted his tax report, the tax agency will choose the optimal probability which that taxpayer is audited with. This probability ends up being a decreasing function of the reported amount of individual income. Taking as given the optimal audit probability function of the tax agency, taxpayers choose their optimal reports in order to maximize their disposable income. Optimal reports turn out to be increasing functions of the true income.[1]

The interaction between taxpayers and tax auditors is usually exposed to several sources of randomness. For instance, each taxpayer faces an idiosyncratic cost when he suffers a tax inspection. Since this cost is private information of taxpayers, tax reports are not just a function of the taxpayers' income but also of the realization of this private cost. Therefore, other things equal, taxpayers facing a high (low) private cost will tend to conceal less (more) income from the tax authority in order to minimize the intensity of the audit conducted by the tax authority. The heterogeneity of that cost among taxpayers could arise from the different amount of time that taxpayers should devote to undergo a tax audit process. Thus, depending on the specific characteristics of each taxpayer's job (or even on the particular skill for efficiently filing tax documents), the opportunity cost in terms of foregone wage could vary across taxpayers. Furthermore, a tax inspection could trigger some psychological cost arising from the potential public exposure of the inspection process and from the typical distress associated with this kind of investigation. The exact value of the cost faced by an audited taxpayer is assumed to be unknown by the tax enforcement agency.

Another reason why tax reports could differ among individuals enjoying the same income stems from tax morale considerations. Thus, if a taxpayer dislikes cheating to the tax authority or views tax evasion as immoral, then he will tend to declare more income than another taxpayer with lower tax morale. Note that both the private cost of an inspection faced by the taxpayer and the private level of tax morale can be viewed as equivalent factors from the point of view of introducing noise in the tax report. Therefore, the report will not be the result of minimizing only the expected tax and penalty payment, since the aforementioned factors will be also crucial to determine the exact level of tax compliance of an individual.

Reinganum and Wilde (1988) considered instead a source of uncertainty faced by taxpayers, namely, that associated with the cost of conducting an audit by the tax agency. The exact realization of this cost is private information of the tax agency and it is unobservable by taxpayers. In the principal-agent model of Reinganum and Wilde the cut-off level of income triggering an inspection is a function of the audit cost faced by the agency. Therefore, taxpayers form non-degenerate beliefs about this cut-off income level from the distribution of the audit cost. These authors conclude that some degree of induced uncertainty about the audit cost of the agency improves compliance and, thus, increases the revenue collected by the agency. However, excessive uncertainty could decrease compliance.

The model we present in this paper considers sources of uncertainty similar to those appearing in the previous models. We will model the interaction between the tax enforcement agency and taxpayers using a game-theoretical approach where the agency does not commit to follow an inspection rule. Therefore, tax auditors will maximize the expected tax revenue and, to this end, they will devote an amount of effort to investigate taxpayers. This effort will be chosen after observing the amount of income reported by each taxpayer. This means that the equilibrium concept we use is that of sequential equilibrium, where the taxpayers move first and the auditors are the followers. The revenue accruing from the inspection is assumed to be proportional to the effort made by the auditor and to the amount of evaded taxes. As in Reinganum and Wilde (1988), the audit cost is private information of each auditor and, thus, taxpayers do not know the exact response of auditors after reading their income reports. However, we depart from Reinganum and Wilde (1988) by considering general audit strategies instead of cut-off ones and by not allowing commitment by the tax enforcement agency. Another even more important departure is that we consider a rational expectations model. This means that taxpayers' beliefs about the audit cost coincide with its true distribution, whereas in the paper of Reinganum and Wilde the true distribution was degenerate and thus the confusion suffered by taxpayers about the audit cost was incompatible with agents entertaining rational expectations. In our model, the distribution of the cost arises from the heterogenous quality of tax auditors, which is due to different natural auditing skills or non-homogeneous formal training. Moreover, in our model we assume a quadratic cost structure parametrized by the value of a coefficient parameter, which is private information of each auditor. We will show with the help of a couple of examples that the effects of increasing the variance of that parameter value are very sensitive to the specific distribution under consideration.

As we have said, we also allow for uncertainty concerning the private cost of suffering an inspection faced by taxpayers. This cost is known by each taxpayer but is unknown by the tax auditor. We will show that an increase in the variance of this idiosyncratic cost generates more revenue for the government. This result is a consequence of the fact that the inspection effort selected by auditors turns out to be more sensitive to the submitted report when the variance of the taxpayers' cost increases. In this case, auditors know that there will be a larger fraction of taxpayers displaying a very low cost and, thus, reporting a small amount of income. Therefore, auditors will put proportionally more effort on auditing taxpayers who submit low-income reports. This induces a bias in the reporting strategies, which results in turn in a larger amount of reported income.

Our paper analyzes also other three questions. First, we show that a larger variance of the income distribution reduces (not surprisingly) individual tax compliance, since auditors face more uncertainty about the relevant variable in the audit process. Second, we evaluate the effects of the different sources of uncertainty on taxpayers welfare. Our analysis shows that expected utility responds negatively to an increase in the variance of income and positively to an increase in the variance of the taxpayers' cost. The latter result is a consequence of the reduction in the expected cost of suffering an audit, since the audit intensity decreases when the variance of the taxpayers' cost rises. Finally, we analyze the progressive (or regressive) bias of the audit strategies followed by the tax auditors of our model. We show that the sign of this bias could be ambiguous since a tax inspection could now serve as an instrument to correct for the excessive tax contribution made by taxpayers facing a high cost of suffering an inspection. This ambiguity concerning the effective progressiveness of the tax system is in stark contrast to what is obtained in the standard model of tax compliance with strategic interaction between auditors and taxpayers, where the resulting effective tax system is always more regressive than the statutory one (see Reinganum and Wilde, 1986; Scotchmer, 1992).

The paper is organized as follows. Section 2 presents the model and derives the sequential equilibrium. Section 3 discusses some properties of the equilibrium. Section 4 contains the analysis of the potential progressive bias of the effective tax system. Section 5 discusses the implications of changes in the variance of the audit cost borne by the tax agency. Section 6 concludes the paper. All the proofs appear in the appendix.

2. The Model

Let us consider an economy with a continuum of taxpayers distributed on the interval [0, 1]. Assume that the income $\tilde{y}$ of each taxpayer is a normally distributed random variable with mean $\bar{y}$ and variance $V_y \geq 0$. The income of each taxpayer is independent of the others' income. Therefore, from the strong law of large numbers, the empirical average income is $\bar{y}$. The tax law establishes a statutory tax rate $\tau \in (0, 1)$ on income. Taxpayers also face a cost when they are inspected by the tax authority. We will assume that the total cost borne by a taxpayer is proportional to the audit effort e^* chosen by the tax authority. If ε^* is the cost of suffering an inspection per unit of effort exerted by inspectors, the total cost is $\varepsilon^* e^*$. Each taxpayer observes the realization ε^* of his cost parameter whereas this cost is unobservable by the tax authority. The value of the cost parameter of each taxpayer is assumed to be normally distributed and the cost of each taxpayer is independent of the others' cost. After observing the realizations of his income y and of his cost parameter ε^*, a taxpayer optimally decides the amount x of declared income.[2]

The tax enforcement agency has a pool of tax auditors and each income report is assigned randomly to one auditor. The auditor chooses the audit effort e^* applied to each taxpayer in order to maximize the expected net revenue (tax and penalty revenue, less audit cost) per taxpayer. Note that, due to the strong law of large numbers, this objective implies the maximization of the aggregate net revenue collected by the tax agency. The audit effort is contingent upon the report x observed by the tax auditor. We define the variable e as the product of the effort e^* and the penalty rate f on the amount of evaded taxes, $e = e^* f$.

Moreover, the resources that can be exacted by an audit are assumed to be proportional to the audit effort and to the amount of evaded taxes. Thus, the penalty revenue is

$$e^* f\tau(y - x) = e\tau(y - x). \tag{1}$$

Therefore, if the reported income x coincides with the true taxable income y of a taxpayer, then no new revenues will arise from an inspection. Moreover, no additional revenues are obtained by a tax auditor when either no effort is devoted to the inspection of potential tax evaders ($e^* = 0$) or no penalties are imposed on the amount of evaded taxes ($f = 0$).

For the rest of the paper we will take as given both the tax rate τ and the penalty rate f. Therefore, without loss of generality, we will use the variable e as the choice variable of tax auditors, since this variable is entirely determined by the endogenous audit effort e^* applied to a given taxpayer. Moreover, for the same reason, we can define the variable $\varepsilon = \frac{\varepsilon^*}{\tau f}$, which is proportional to the inspection cost ε^* faced by taxpayers. Thus, the total cost faced by a taxpayer suffering an inspection when the audit effort is e^* turns out to be

$$\varepsilon^* e^* = \varepsilon\tau f e^* = \varepsilon\tau e, \tag{2}$$

where the last inequality follows from the definition of the variable e. The random variable $\tilde{\varepsilon}$ is thus normally distributed and we assume that its mean is zero, while its variance is V_ε. The zero mean assumption is made without loss of generality in order to reduce the number of parameters of the model. As we will see, the random variable $\tilde{\varepsilon}$ can also be interpreted as the individual tax morale of taxpayers so that positive (negative) values of ε correspond to individuals who are willing to declare more (less) income due to considerations, like ethical values, social norms, or degree of satisfaction with the government. All these considerations are independent of the objective of tax payment minimization (see Alm, McClelland and Schulze, 1992; Erard and Feinstein, 1994b; Torgler, 2002, for this strand of the tax evasion literature). Note finally that the realizations of the income $\tilde{y}$ and of the cost $\tilde{\varepsilon}$ are private information of each taxpayer.

We assume that the total audit cost faced by the tax agency is quadratic in the effort devoted to auditing, $\frac{1}{2}c^* e^*$ with $c^* > 0$. This cost includes all the resources spent by the tax auditor in the process of inspection. Note that, by making $c = \frac{c^*}{f^2\tau} > 0$, the previous cost function becomes $\frac{1}{2}c\tau e^2$. The value of the cost parameter c^*, and thus of c, varies across auditors according to an exogenously given distribution. The value of that parameter could depend, for instance, on the natural skills and on the previous training of each tax auditor. The exact value of his cost parameter c is observable by each auditor but is not observable by taxpayers. Thus, from the taxpayers' viewpoint, the cost parameter is a random variable $\tilde{c}$ with a known distribution. The relevant realization of the random variable $\tilde{c}$ for a given taxpayer corresponds to the value c of the auditor assigned to him.

Finally, we assume that the random variables $\tilde{y}$, $\tilde{\varepsilon}$ and $\tilde{c}$ are mutually independent. The joint distribution of these random variables is common knowledge.

Let $e(x, c)$ be the audit effort of an auditor with a value c of his cost parameter who observes the amount x of reported income of a taxpayer. Tax auditors want to maximize the net revenue from each taxpayer they audit. Therefore, an auditor with an audit cost parameter equal to c chooses the audit effort e to be applied to a taxpayer declaring the

income level x according to the following audit strategy:

$$e(x, c) = \arg\max_{e} E\left[\tau x + e\tau(\tilde{y} - x) - \frac{1}{2}c\tau e^2 \,\middle|\, x\right], \tag{3}$$

where τx is the amount of taxes paid before the inspection has taken place, $e\tau(y - x)$ is the additional revenue that the auditor collects through the inspection (see (1)), and $\frac{1}{2}c\tau e^2$ is the cost borne by the agency. The first order condition of the auditor's problem is

$$E[\tilde{y} - x - ce \mid x] = 0.$$

The sufficient second order condition is simply $c > 0$, which is satisfied by assumption. Therefore, the audit effort is given by

$$e = \frac{1}{c}[E(\tilde{y} \mid x) - x]. \tag{4}$$

Since taxpayers do not observe the realization of the random variable $\tilde{c}$, they are uncertain about the effort that auditors will apply in their respective cases. Taxpayers are risk neutral and want to maximize the expected amount of their disposable income after the inspection has taken place. Note that $y - \tau x - e(x, c)\tau(y - x)$ is the income net of taxes and penalties. Moreover, recall that the cost of facing an inspection in terms of foregone income is given by expression (2). Therefore, the expected disposable income of a taxpayer with initial income y and individual cost ε will be

$$E[y - \tau\tilde{x} - e(x, \tilde{c})\tau(y - x) - \varepsilon\tau e(x, \tilde{c})]. \tag{5}$$

Taxpayers form rational expectations about the strategies followed by tax auditors. Since they observe their true income and their private cost of suffering an inspection, taxpayers follow the following report strategy:

$$x(y, \varepsilon) = \arg\max_{x} E\left[y - \tau x - e\left(x, \tilde{c}\right)\tau\left(y - x\right) - \varepsilon\tau e\left(x, \tilde{c}\right)\right]. \tag{6}$$

The first order condition of this problem is

$$-1 - E\left[\frac{\partial e(x, \tilde{c})}{\partial x}(y - x) - e(x, \tilde{c}) + \varepsilon\left(\frac{\partial e(x, \tilde{c})}{\partial x}\right)\right] = 0. \tag{7}$$

The second order condition is

$$-E\left[\frac{\partial^2 e(x, \tilde{c})}{\partial^2 x}(y - x) - 2\left(\frac{\partial e(x, \tilde{c})}{\partial x}\right) + \varepsilon\left(\frac{\partial^2 e(x, \tilde{c})}{\partial^2 x}\right)\right] < 0. \tag{8}$$

We see that, unlike the seminal papers of Allingham and Sandmo (1972) and Yitzhaki (1974), the taxpayer does not take as given the audit effort but takes into account the effect of his report on the effort that the tax auditor will devote to enforce the tax law. Note also that we consider the audit effort as the variable selected by auditors, whereas in previous models the choice variable used to be the probability of inspection.

An equilibrium of our model is thus a report strategy $x(y, \varepsilon)$ and an audit strategy $e(x, c)$ satisfying simultaneously (6) and (3). We will restrict our attention to linear report strategies, $x(y, \varepsilon) = \alpha + \beta y + \lambda\varepsilon$, and to audit strategies that are linear in the reported

income, $e(x, c) = \delta(c) + \gamma(c)x$. Note that for these linear audit strategies the sufficient second order condition (8) of the taxpayer's problem becomes simply $E[\gamma(\tilde{c})] < 0$. The next proposition provides the unique equilibrium belonging to this class:

Proposition 1. *Assume that $V_\varepsilon > V_y$. Then, there exists a unique equilibrium where both $x(\cdot, \cdot)$ and $e(\cdot, c)$ are linear. This equilibrium is given by*

$$x(y, \varepsilon) = \alpha + \beta y + \lambda\varepsilon,$$

where

$$\alpha = \frac{1}{2}\left\{\bar{y} - \frac{1}{E(1/\tilde{c})}\left[1 + \frac{V_y}{V_\varepsilon}\right]\right\}, \tag{9}$$

$$\beta = \frac{1}{2}, \tag{10}$$

$$\lambda = \frac{1}{2}; \tag{11}$$

and

$$e(x, c) = \delta(c) + \gamma(c)x,$$

where

$$\delta(c) = \frac{1}{c}\left\{\frac{1}{E(1/\tilde{c})}\left(\frac{V_y}{V_\varepsilon}\right) - \bar{y}\left(\frac{V_y - V_\varepsilon}{V_y + V_\varepsilon}\right)\right\}, \tag{12}$$

$$\gamma(c) = \frac{1}{c}\left(\frac{V_y - V_\varepsilon}{V_y + V_\varepsilon}\right). \tag{13}$$

Proof: See Appendix. □

It should also be pointed out that the linear equilibrium under consideration is in fact a sequential equilibrium with no additional restrictions, since the best response to the linear report strategy followed by a taxpayer consists on a linear audit strategy, and vice versa. It can be easily seen from our previous analysis and from the proof of Proposition (2.1) that the assumption of a quadratic cost function for the auditors, together with the linearity of the report strategies and the normality of the random variables $\tilde{y}$ and $\tilde{\varepsilon}$, yields linear optimal audit strategies.[3] Conversely, given the linear audit strategy followed by auditors, the assumption of risk neutrality for taxpayers yields optimal linear strategies for the tax reports.

For the rest of the paper we will maintain the assumption $V_\varepsilon > V_y$, which is necessary and sufficient for the second order condition (8) of the taxpayer problem. This condition requires in fact that the audit effort be decreasing in the amount of reported income. If the previous assumption were not imposed, the audit effort could be increasing in reported income and taxpayers would find optimal to report an infinite negative income level. In this respect, note that when $V_\varepsilon \leq V_y$ taxpayers introduce a quite small of noise in their reports and, thus, the reports they submit are very informative about their true income. In this case, since $\beta = 1/2$, the amount of evaded income $y - x(y, \varepsilon)$ rises with the true income

y and, hence, tax auditors maximize the penalty revenue by inspecting more intensively the taxpayers who submit high-income reports. On the contrary, if the variance of $\tilde{\varepsilon}$ is sufficiently high relative to that of income, as assumed in Proposition 2.1, the reports are not so informative and, hence, auditors attribute high-income reports to a high cost faced by the taxpayer. Moreover, since in this case the dispersion of income is small, the optimal audit strategy consists on inspecting more intensively the low-income reports, which are those having a higher probability of being submitted by taxpayers who underreport their true income because they face low cost when they are audited.[4]

It is important to remark that the particular definitions of the variables e, $\tilde{\varepsilon}$, and $\tilde{c}$ have been made in order to obtain relatively simple equilibrium strategies for taxpayers and auditors. In particular, note that neither the tax rate τ nor the penalty rate f explicitly appear in the equilibrium reporting and auditing strategies given in Proposition (2.1), since both τ and f are embedded on those transformed variables.

The functional form of the reporting strategy $x(y, \varepsilon)$ allows us to provide an alternative interpretation of the random variable $\tilde{\varepsilon}$. The realization of $\tilde{\varepsilon}$ can be viewed as the individual tax morale of the taxpayer under consideration. Thus, taxpayers for which $\varepsilon = 0$ are those for which the amount of income reported is exclusively driven by the objective of minimization of the expected amount of taxes and penalties to be paid (see (5)). A positive (negative) value of ε corresponds to taxpayers who are willing to pay more (less) taxes as a consequence of their attitude towards tax compliance based on tax morale considerations.

3. Properties of the Equilibrium

In this section we study some indicators of tax compliance in equilibrium. Note first that, using the equilibrium values of the coefficients α, β, λ, $\delta(c)$ and $\gamma(c)$, the equilibrium pair of strategies can be written as

$$x(y, \varepsilon) = \frac{1}{2}\left\{ y + \bar{y} + \varepsilon - \frac{1}{E(1/\tilde{c})}\left[1 + \frac{V_y}{V_\varepsilon}\right]\right\} \tag{14}$$

and

$$e(x, c) = \frac{1}{c}\left[\left(\frac{V_y - V_\varepsilon}{V_y + V_\varepsilon}\right)(x - \bar{y}) + \frac{1}{E(1/\tilde{c})}\left(\frac{V_y}{V_\varepsilon}\right)\right]. \tag{15}$$

We see that, on the one hand, the intended report $x(y, \varepsilon)$ is increasing in the true individual income y and, obviously, in the cost ε of being audited. Moreover, for a taxpayer with a given income level y, his report x increases with the variance V_ε of the taxpayers' cost, whereas it is decreasing in the variance V_y of income. Finally, the intended report is increasing in the expectation $E(1/\tilde{c})$. On the other hand, the inspection effort $e(x, c)$ applied to a taxpayer is decreasing in his income report x, as required by the second order condition (8), and decreasing in the auditors' cost parameter c. Moreover, for a given report x and a given realization of the value c of the auditors' cost parameter, the inspection effort e is

increasing in the variance V_y of income, decreasing in the variance V_ε of taxpayers' cost, and decreasing in the expectation $E(1/\tilde{c})$.

Let us discuss the previous properties of the equilibrium strategies. Consider a taxpayer with a given income level y. Clearly, as V_ε increases auditors know that the variance of the report will also increase. Therefore, the negative coefficient $\gamma(c)$ of the equilibrium audit strategy increases in absolute value when V_ε increases. This is so because the auditors would like to inspect more intensively low income reports in order to impose fines on taxpayers having low values of ε and, thus, evading more income. Since the audit effort becomes more decreasing in reported income, taxpayers know that low reports will be more intensively inspected, while high reports will not be exposed to so severe inspections. This new bias in the audit strategy induces in turn a change in the reporting strategies so that more income is declared in order to minimize the intensity of the audit.

The report x decreases with the income variance V_y, which is consistent with the fact that tax auditors are facing more uncertainty about the true income of taxpayers. Finally, if taxpayers believe that the expected audit cost is high (that amounts "ceteris paribus" to a low value of $E(1/\tilde{c})$), then they will expect a low audit effort by the tax auditors. Therefore, optimal reports must be increasing in $E(1/\tilde{c})$.

Concerning the audit effort for given values of x and c, we see that, as the variance V_y of income increases, tax auditors face more uncertainty about a variable that is private information of taxpayers. Since income is the relevant variable in the inspection process, more resources must be devoted to audit activities. The variance V_ε of the taxpayers' cost affects negatively the inspection effort. This is consistent with the fact that taxpayers raise the amount of income they report when V_ε increases and, hence, less effort should be devoted to audit taxpayers who underreport less income on average. Moreover, the audit effort is obviously decreasing in the cost parameter c and is also decreasing in $E(1/\tilde{c})$. Note that, if taxpayers expect a high value of the random variable $\tilde{c}$, then $E(1/\tilde{c})$ will tend to be low. In this case they will underreport more income, since they think that the auditors will not be very aggressive in their inspection strategy. The best response to this report strategy is to conduct an audit more aggressive than the one expected by taxpayers.

From (14) we can compute the expected reported income per capita in the economy,

$$E(\tilde{x}) = E[x(\tilde{y}, \tilde{\varepsilon})] = \bar{y} - \frac{1}{2E(1/\tilde{c})}\left[1 + \frac{V_y}{V_\varepsilon}\right].$$

Note that, as occurs with the report x, the expected reported income is increasing in both $E(1/\tilde{c})$ and V_ε, whereas is decreasing in V_y. Moreover, $\frac{\partial E(\tilde{x})}{\partial \bar{y}} = 1$, that is, an increase in the average income results in an equivalent increase in the average reported income.

We can compute now the expected audit effort to see how the different sources of uncertainty affect the audit strategy of the tax enforcement agency on average. To this end we compute the unconditional expectation of (15), which will give us the expected effort before observing the realization of the cost parameter $\tilde{c}$ of each auditor,

$$E(\tilde{e}) = E[e(x(\tilde{y}, \tilde{\varepsilon}), \tilde{c})] = \frac{1}{2}\left(1 + \frac{V_y}{V_\varepsilon}\right).$$

It is obvious that the expected audit effort $E(\tilde{e})$ is increasing in V_y, decreasing in V_ε and independent of both c and $E(1/\tilde{c})$. Clearly, as V_ε increases the reports become less reliable signals of the true income. Recall that high values of the variance of taxpayers' cost induce larger amounts of reported income. In this case tax auditors should reduce the average effort in order to lower the probability of applying to much effort in inspecting honest taxpayers. Again, more income uncertainty, parametrized by the variance V_y, requires more effort by the auditors. Finally, observe that, when computing the unconditional expectation, we are eliminating the asymmetry referred to the audit cost c between the agency and the taxpayer. When both the agency and the taxpayer face the same priors about the cost parameter c, the opposite effects of the distribution of $\tilde{c}$ on the reporting and inspection strategies cancel out on average.

We can now look at the expected revenue net of the audit cost raised by the tax enforcement agency and see also how this revenue is affected by the different sources of uncertainty. The random net revenue per taxpayer is

$$\tilde{R} = \tau x(\tilde{y}, \tilde{\varepsilon}) + \tau e(x(\tilde{y}, \tilde{\varepsilon}); \tilde{c})(\tilde{y} - x(\tilde{y}, \tilde{\varepsilon})) - \frac{1}{2}\tau \tilde{c}[e(x(\tilde{y}, \tilde{\varepsilon}), \tilde{c})]^2. \tag{16}$$

As we have already said, since there is a continuum of ex-ante identical taxpayers distributed uniformly on the interval [0, 1], the expected net resources extracted from a taxpayer coincide with the aggregate net revenue raised by the agency.

Corollary 1. *The expected net revenue $E(\tilde{R})$ is increasing in V_ε and decreasing in V_y. Moreover, $E(\tilde{R})$ is increasing in $E(1/\tilde{c})$.*

Proof: See Appendix. □

A larger value of the variance V_y of income means a larger disadvantage of tax auditors with respect to taxpayers and, hence, tax auditors end up putting to much effort on low income taxpayers, who are those paying less fines. When V_ε increases, tax report depart more from the true income and, as we argued before, low income reports will be more intensively audited. Moreover, the amount of reported income increases on average. Therefore, the agency will raise more revenues both from the penalties imposed on evaded taxes and from the taxes on the larger amount of voluntarily reported income. Finally, a low value of $E(1/\tilde{c})$ is typically associated with a large expected cost. Hence, the expected net revenue will be low since taxpayers anticipate that it is very costly for the auditors to conduct an audit.

Note that we have just looked at the unconditional expected net revenue raised by the tax agency. We could look now at the expected net revenue conditional to a given realization of the cost parameter, $E(\tilde{R} \mid \tilde{c} = c)$. The previous conditional expectation is the expected revenue raised by a tax auditor with a value of the cost parameter equal to c. In this case, the effects of changes in V_ε and V_y are the same as in the unconditional case. However, the effects of $E(1/\tilde{c})$ are generally ambiguous. To see this, consider the case where there is no income uncertainty, that is, $V_y = 0$ or, equivalently, $\tilde{y} = y$. This is in fact a situation very similar to that considered by Reinganum and Wilde (1988), where the agency knows the realization of its (homogeneous) audit cost parameter and taxpayers view this cost parameter as a random variable $\tilde{c}$ with a given distribution. More precisely, these authors consider a cut-off strategy where a taxpayer with a given income y is only inspected if his level of

underreporting is so large that the penalty revenue outweighs the audit cost faced by the tax enforcement agency. Finally, they assume a constant cost per inspection that each taxpayer views as if it were drawn from a uniform distribution. Coming back to our scenario, we can compute from (16) the following conditional expectation:

$$E(\tilde{R} \mid \tilde{c} = c, \tilde{y} = y) = \tau\left(y - \frac{1}{2E(1/\tilde{c})} + \frac{1}{8c[E(1/\tilde{c})]^2} + \frac{1}{8c}V_\varepsilon\right),$$

which is obviously decreasing in the value of the cost parameter c and increasing in the taxpayers' cost variance V_ε. However, it is immediate to obtain that

$$\frac{\partial E(\tilde{R} \mid \tilde{c} = c, \tilde{y} = y)}{\partial E(1/\tilde{c})} \gtrless 0 \text{ if and only if } E(1/\tilde{c}) \lessgtr \frac{1}{2c\tau}. \tag{17}$$

Therefore, if the tax authority can affect the distribution of its audit cost, then the expected revenue is maximized when $E(1/\tilde{c}) = \frac{1}{2c\tau}$. In the next section we will assume that the tax enforcement agency can affect the variance of the true distribution of $\tilde{c}$ and, then, we will make explicit the relation between $\mathrm{Var}(\tilde{c})$ and $E(1/\tilde{c})$ through a couple of examples.

We discuss next the comparative statics concerning taxpayers' total welfare under the assumption that the government revenue is not used to provide goods or services entering in the taxpayers' utility function. To introduce government spending in the taxpayers' utility function will give raise to an extra degree of freedom in our model. Since tax contributions decrease disposable income but they could increase the amount of government spending, the following results concerning welfare could be reversed depending on the importance of government spending in the preferences of taxpayers. Given the assumed linearity of preferences the ex-ante welfare is measured by the expected income $E(\tilde{n})$ of a taxpayer net of taxes, fines, and inspection costs before observing the realizations of his income y and of his private audit cost ε. Note that, because of the law of large numbers, the expected welfare of a given taxpayer is equal to the average welfare of the taxpayers of this economy. Recall that the random net income of a taxpayer is

$$\tilde{n} = \tilde{y} - \tau x(\tilde{y}, \tilde{\varepsilon}) - \tau e(x(\tilde{y}, \tilde{\varepsilon}), \tilde{c})(\tilde{y} - x(\tilde{y}, \tilde{\varepsilon})) - \tilde{\varepsilon}\tau e(x(\tilde{y}, \tilde{\varepsilon}), \tilde{c}). \tag{18}$$

The next proposition provides the comparative statics results concerning the taxpayers' expected welfare:

Corollary 2. (a) *The expected welfare $E(\tilde{n})$ of a taxpayer is increasing in V_ε and decreasing in V_y.*

(b) *The effects of changes in $E(1/\tilde{c})$ on $E(\tilde{n})$ are ambiguous.*

Proof: See Appendix. □

To understand part (a) of the previous corollary, we just have to remind that the expected audit effort is decreasing in the variance V_ε of taxpayers' cost and increasing in the variance V_y. The implications for the cost of suffering an inspection are thus immediate. In fact, in our model the effects on that cost dominate over the effects on the expected amount of reported income. In particular, we have already seen that, if the variance V_ε of taxpayers' cost increases, the expected amount of reported income increases. This first

effect tends to reduce the expected net income of taxpayers. However, the increase in V_ε directly reduces the expected value of the last term in (18), which collects the total expected cost borne by taxpayers. This last effect turns out to be the dominating one in our model and, thus, an increase in V_ε results in more expected welfare. Finally, the converse argument applies to explain the welfare effects of changes in the variance V_y of taxpayers' income.

Concerning part (b) of Corollary 2 note that a low expected value of the parameter $\tilde{c}$ (i.e., a large value of $E(1/\tilde{c})$) increases the expected effort of the audits and this has a direct negative effect on the expected income net of taxes and fines. However, a large value of $E(1/\tilde{c})$ increases the expected absolute value of the negative coefficient γ (see 13). This means that the audit strategy is more sensitive to the reports. Therefore, taxpayers facing high (low) values of the cost ε will be less (more) intensively inspected and this results in a smaller expected total cost of suffering an inspection, as can be seen from the last term of (18). Therefore, the previous two effects on taxpayers' net income go in opposite direction and the dominating effect will thus depend on the particular parameter values of the model.

4. The Bias of the Effective Tax System

Another question that can be analyzed in the present context is the degree of effective progressiveness exhibited by the tax system in equilibrium. It is a well established result in the literature that the effective tax rate displays less progressiveness than the statutory one when the relationship between auditors and taxpayers is strategic (see Reinganum and Wilde, 1986; Scotchmer, 1992). This is so because the agency will audit individuals reporting low income more intensively than individuals producing high income reports. Therefore, even if the optimal amount of reported income is increasing in true income, high-income individuals find more attractive to underreport a larger proportion of their income. This generates a regressive bias in the effective tax structure once we take into account the penalty payments.[5]

In order to analyze whether the effective tax structure of our model is progressive or regressive, we should compute the average expected tax rate faced by a taxpayer and see how this rate changes with the true income y. The expected payment to the government (including taxes and penalties) of a taxpayer having a level y of income is

$$g(y) = E[\tau x(y, \tilde{\varepsilon}) + e(x(y, \tilde{\varepsilon}), \tilde{c})\tau(y - x(y, \tilde{\varepsilon}))].$$

Note that in the previous expression we have to compute the expectation just with respect to the random variables $\tilde{\varepsilon}$ and $\tilde{c}$. The average expected tax rate is thus

$$\hat{\tau}(y) = \frac{g(y)}{y}.$$

Under effective proportionality $\hat{\tau}(y)$ should be independent of y, while under effective progressiveness (regressiveness) $\hat{\tau}(y)$ should be increasing (decreasing). The following corollary tells us that, unlike the previous literature, the function $\hat{\tau}(y)$ could be non-monotonic:

Corollary 3. *There exists an income level $\hat{y}$ such that the derivative of the average expected tax rate satisfies*

$$\hat{\tau}'(y) < 0 \quad \text{for all } y > \hat{y}.$$

Moreover, the function $\hat{\tau}(y)$ could be either

(a) *decreasing both on the interval* $(-\infty, 0)$ *and on the interval* $(0, \infty)$.
or
(b) *U-shaped on the interval* $(-\infty, 0)$ *and inverted U-shaped on the interval* $(0, \infty)$.

According to the first part of the corollary, the effective tax system is always locally regressive for sufficiently high income levels. Concerning the second part, the potential inverted U-shape of the average expected tax rate for positive income levels means that the effective tax system could display local regressiveness for sufficiently high levels of income, whereas it could display local progressiveness on a lower interval of positive income levels. In order to illustrate Corollary 3, Figure 1 displays the function $\hat{\tau}(y)$ for the following configuration of parameter values: $E(1/\tilde{c}) = 20/3$, $V_y = 1$, $V_\varepsilon = 10$, $\bar{y} = 3$ and $\tau = 0.2$. Figure 2 uses the same parameter values except that $V_\varepsilon = 4$. We see that the average expected tax rate can be monotonically decreasing (i.e., the tax system can be uniformly regressive), as in Figure 1, or inverted U-shaped on the interval $(0, \infty)$, as in Figure 2.

To understand the potential non-monotonic behavior of the average expected tax rate, we should bear in mind that individuals suffering an inspection might end up receiving a tax refund. This is so because they could have declared an amount of income larger than the true one due to the large cost they face in case of inspection. Note also that the existence of this cost makes taxpayers to declare a larger amount of income. Therefore, audits could detect this kind of excessive tax contribution. Since the audit effort is decreasing in the amount of reported income and reports are decreasing in true income, low-income individuals are more intensively inspected and, thus, they are more likely to get tax refunds. Note that

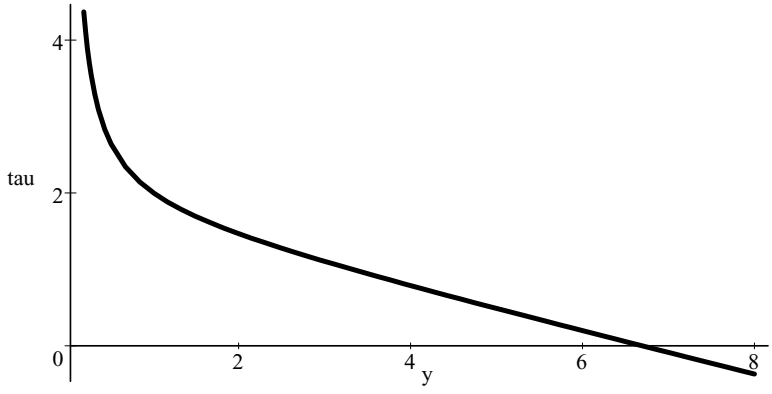

Figure 1. Average expected tax rate when $E(1/\tilde{c}) = 20/3$, $V_y = 1$, $V_\varepsilon = 10$, $\bar{y} = 3$ and $\tau = 0.2$.

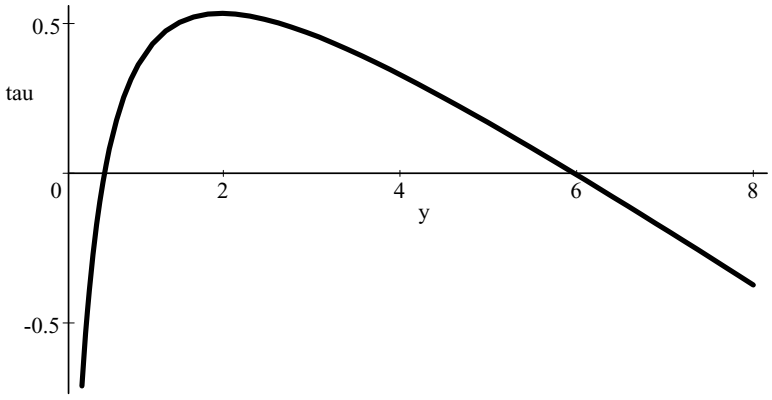

Figure 2. Average expected tax rate when $E(1/\tilde{c}) = 20/3$, $V_y = 1$, $V_\varepsilon = 4$, $\bar{y} = 3$ and $\tau = 0.2$.

this feature of the audit strategy induces a progressive bias in the tax system that could outweigh the aforementioned regressive bias present in strategic models of tax compliance. The potential non-monotonic behavior of $\hat{\tau}(y)$ just captures the trade-off between these two biases.

5. The Effects of the Variance of the Auditors' Cost

The comparative statics exercises of the previous section have been performed in terms of the expectation $E(1/\tilde{c})$. In this section we analyze how this expectation could be affected by the moments of the primitive distribution of $\tilde{c}$. In order to motivate this exercise, assume that the tax enforcement agency has a given budget to provide some training to its inspectors. Let us assume that the amount of resources available per auditor is equal to $\hat{b}$. There is a stochastic training technology relating the value of the cost parameter c of a tax auditor with the amount b invested in his training,

$$\tilde{c} = h(b, \tilde{\xi}),$$

where h is strictly decreasing in b and $\tilde{\xi}$ is a random variable independent of the amount b. As we already know, if the tax authority wants to maximize its aggregate revenue, then it has to maximize the expected revenue per taxpayer. According to Corollary 1, it is obvious that the agency should try to reach the largest possible value for $E(1/\tilde{c})$.

The following natural question arising in this context is whether the tax agency should give identical training to all the auditors, or should allow for some non-homogeneous training that would give rise in turn to some dispersion in the idiosyncratic values of the audit cost parameter. We are thus implicitly assuming that the tax enforcement agency can control, at some extent, some statistical properties of the random variable $\tilde{\xi}$ at zero cost. To answer the previous question we analyze how the value of $E(1/\tilde{c})$ is affected by the variance of the distribution of $\tilde{c}$ in two particular cases, namely, when the random variable $\tilde{c}$ is uniformly

distributed and when it is log-normal. The choice of these two distributions allows us to be consistent with the second order condition of the tax auditor problem requiring that the value c of his cost parameter be strictly positive.

Assume first that $\tilde{c}$ has a uniform density. In particular, let

$$h(b, \tilde{\xi}) = \hat{h}(b) + \tilde{\xi},$$

where $\tilde{\xi}$ has a uniform density with zero mean and $\hat{h}(b)$ is a positive valued and strictly decreasing mapping. Therefore, the mean of $\tilde{c}$ is

$$E(\tilde{c}) = \hat{h}(b) \tag{19}$$

and the variance is

$$\text{Var}(\tilde{c}) = \text{Var}(\tilde{\xi}). \tag{20}$$

The density of $\tilde{c}$ can be thus written as,

$$f(c) = \begin{cases} \dfrac{1}{2\eta} & \text{for } c \in (\bar{c} - \eta, \bar{c} + \eta) \\ 0 & \text{otherwise} \end{cases}$$

with $\eta > 0$ and $\bar{c} - \eta > 0$, so that $\tilde{c}$ takes always on positive values. Therefore, it holds that $E(\tilde{c}) = \bar{c}$ and $\text{Var}(\tilde{c}) = \eta^2/3$. It is then clear that $\text{Var}(\tilde{c})$ is a strictly increasing function of η. Then,

$$E(1/\tilde{c}) = \int_{\bar{c}-\eta}^{\bar{c}+\eta} \left(\frac{1}{2\eta}\right)\left(\frac{1}{c}\right) dc = \frac{\ln(\bar{c} + \eta) - \ln(\bar{c} - \eta)}{2\eta}.$$

After some simplifications we obtain the following derivatives:

$$\frac{\partial E(1/\tilde{c})}{\partial \bar{c}} = -\frac{1}{\bar{c}^2 - \eta^2} < 0 \tag{21}$$

and

$$\frac{\partial E(1/\tilde{c})}{\partial \eta} = -\frac{1}{\bar{c}^2 - \eta^2} - \frac{\ln(\bar{c} + \eta) - \ln(\bar{c} - \eta)}{2\eta^2} < 0.$$

Hence, we have that

$$\frac{\partial E(1/\tilde{c})}{\partial\ \text{Var}(\tilde{c})} < 0. \tag{22}$$

Since the audit cost of an auditor is strictly decreasing in the amount of resources devoted to his training, the derivative (21) implies that the expected value of $\tilde{c}$ should be minimized and, thus, the agency should select $b = \hat{b}$, which implies that $E(\tilde{c}) = \hat{h}(\hat{b})$, as follows from (19). This means that the agency should exhaust all the resources for training. Moreover, according to (22), if the randomization device of the training technology generates a uniform distribution of the cost parameter $\tilde{c}$, a tax enforcement agency aiming at the maximization of its net revenue should try to minimize the variance of $\tilde{c}$. Obviously, this is achieved by minimizing the variance of the random variable $\tilde{\xi}$ (see (20)).

Assume now that the cost parameter $\tilde{c}$ is log-normally distributed. More precisely, assume that

$$h(b, \tilde{\xi}) = \hat{h}(b)\tilde{\xi}, \tag{23}$$

where $\tilde{\xi}$ is log-normal with $E(\tilde{\xi}) = 1$ and $\hat{h}(b)$ has the same properties as before. Therefore, the mean of the random variable $\tilde{c}$ is

$$E(\tilde{c}) = \hat{h}(b), \tag{24}$$

and its variance is

$$\text{Var}\,(\tilde{c}) = [\hat{h}(b)]^2 \text{Var}(\tilde{\xi}). \tag{25}$$

Let $E[\ln(\tilde{\xi})] = \mu$ and $\text{Var}[\ln(\tilde{\xi})] = \sigma^2$. Therefore, the mean of $\tilde{\xi}$ is

$$E(\tilde{\xi}) = \exp\left(\mu + \frac{\sigma^2}{2}\right) = 1,$$

and, hence, $\mu = -\sigma^2/2$. Moreover,

$$\text{Var}(\tilde{\xi}) = [E(\tilde{\xi})]^2[\exp(\sigma^2) - 1] = \exp(\sigma^2) - 1. \tag{26}$$

Since $\tilde{c}$ is log-normal, the random variable $\ln(\tilde{c})$ is normally distributed. Therefore, from (23), we have that

$$E[\ln(\tilde{c})] = \ln(\hat{h}(b)) + \mu = \ln(\hat{h}(b)) - \frac{\sigma^2}{2}$$

and

$$\text{Var}[\ln(\tilde{c})] = \sigma^2.$$

Similarly, the random variable $1/\tilde{c}$ is log-normal as $\ln(1/\tilde{c})$ is normal. Since $\ln(1/\tilde{c}) = -\ln(\tilde{c})$, we get

$$E[\ln(1/\tilde{c})] = -\ln(\hat{h}(b)) + \frac{\sigma^2}{2} \tag{27}$$

and

$$\text{Var}[\ln(1/\tilde{c})] = \sigma^2. \tag{28}$$

Therefore, using (27) and (28), we can obtain the mean of the random variable $1/\tilde{c}$,

$$E(1/\tilde{c}) = \exp\left[E[\ln(1/\tilde{c})] + \frac{\text{Var}[\ln(1/\tilde{c})]}{2}\right] = \exp[-\ln(\hat{h}(b)) + \sigma^2]. \tag{29}$$

A revenue-maximizing tax enforcement agency should select the largest feasible value of $E(1/\tilde{c})$ (see Corollary 1), and it is obvious from (29) that this is achieved by choosing simultaneously the lowest feasible value for $\ln(\hat{h}(b))$ and the largest feasible value for σ^2. The minimization of $\ln(\hat{h}(b))$ is accomplished again by selecting $b = \hat{b}$ and, hence, $E(\tilde{c}) = \hat{h}(\hat{b})$, as follows from (24). Having picked optimally the value of $E(\tilde{c})$, note from

(26) that the maximization of the variance σ^2 means that the variance of $\tilde{\xi}$ has to reach its largest feasible value. Moreover, the previous policy implies that, for a given value of resources per auditor $\hat{b}$, the variance of $\tilde{c}$ must be set as large as possible by the tax enforcement agency (see (25)).

We see that the effect of the variance of the cost parameter $\tilde{c}$ on the expectation $E(1/\tilde{c})$ under a log-normal distribution is the opposite to that obtained under a uniform distribution. Thus, if the results contained in Corollaries 1 and 2, and in expression (17) were written in terms of the variance of $\tilde{c}$, the corresponding comparative statics exercises would be extremely dependent on the specific distribution of $\tilde{c}$ under consideration.

6. Conclusion

In the context of a model of strategic interaction between tax auditors and taxpayers, we have analyzed the effects of different sources of uncertainty on tax compliance. Besides the typical uncertainty faced by tax auditors associated with the income of taxpayers, we add two additional sources of uncertainty. The first one refers to individual cost borne by taxpayers suffering a tax inspection. This cost is unobservable by tax auditors and results in tax reports not fully informative about the true income of taxpayers. The noise introduced in the reports could also arise from the idiosyncratic tax morale of each taxpayer. The second source of uncertainty refers to the fact that the cost of conducting an audit is private information of the tax auditors and the inspection strategy is thus viewed as random by the taxpayers. Our main results can be summarized as follows:

- Larger variance of the distribution of taxpayers' cost results in more average income reported, less average audit effort, more net revenue for the government, and more expected welfare for the taxpayers.
- Larger variance of the income distribution results in less average income reported, more average audit effort, less net revenue for the government, and less expected welfare for the taxpayers.
- Larger average audit cost borne by the tax agency typically results in less average income reported and less net revenue for the government.
- The relation between the average expected tax rate and the true income could be non-monotonic. Therefore, the tax system could be locally progressive on some range of income levels and locally regressive on another range.

We should mention that the tax evasion literature has considered additional sources of randomness in the relation between taxpayers and auditors. The fact that tax codes are complex, vague, and ambiguous has been recognized by several studies (see the abundant references in Section 9.1 of Andreoni, Erard and Feinstein, 1998). This aspect of tax codes makes difficult for the taxpayers to apply the law even if they want to do so (see Rubinstein, 1979). Scotchmer and Slemrod (1989) consider a model were the ambiguity of tax laws gives raise to an audit policy yielding random outcomes depending on the interpretation of the law made by the auditors. Scotchmer (1989) and Jung (1991) consider instead models where tax complexity makes taxpayers uncertain about their true taxable income. Pestieau, Possen and Slutsky (1998) analyze the welfare implications of explicit randomization in tax laws. An

even more direct source of mistakes committed by taxpayers arises from the design of the income report form that, in many circumstances, induces taxpayer confusion. For instance, if the sources of income are diverse and, thus, the report has to contain multiple components (as in Rhoades, 1999), then the final report could easily contain some imprecisions. Finally, Broadway and Sato (2000) consider also the possibility of unintentional administrative errors committed by tax auditors. The previous theoretical papers and the experiments conducted by Alm, Jackson and McKee (1992) tend to conclude that randomness in reports or in audits induces more tax compliance.

In our paper we obtain a similar result concerning the positive association between the expected amount of reported income and the variance of the distribution of the cost faced by audited taxpayers. However, in our paper no ambiguity in the law is present and no errors are committed by taxpayers. The noise appearing in an individual tax report is observable by each taxpayer but is unknown by the tax enforcement agency. This is so because this noise arises either from the private cost of suffering an inspection or from idiosyncratic moral sentiments towards tax compliance. A somewhat surprising implication of our model is that, even if taxpayers declare more income when the variance of the taxpayers' cost rises, their expected welfare increases thanks to the induced reduction in the total cost they face when they are audited.

Appendix A

Proof of Proposition 1: The tax auditor observes the reported income x and the value c of the cost parameter and chooses the audit effort e in order to solve (3). Therefore, the optimal audit effort is given by (4). The auditor conjectures that taxpayers follow linear report strategies, i.e.,

$$x(\tilde{y}, \tilde{\varepsilon}) = \alpha + \beta\tilde{y} + \lambda\tilde{\varepsilon}.$$

Note that observing a realization of the random variable $\tilde{x}$ is informationally equivalent to observing a realization of the random variable

$$\frac{\tilde{x} - \alpha}{\beta} = \tilde{y} + \frac{\lambda\tilde{\varepsilon}}{\beta},$$

which has mean equal to $\bar{y}$ and variance equal to $V_y + (\lambda^2 V_\varepsilon/\beta^2)$. Therefore,

$$E(\tilde{y} \mid \tilde{x}) = E\left(\tilde{y} \,\middle|\, \frac{\tilde{x} - \alpha}{\beta}\right) = E\left(\tilde{y} \mid \tilde{y} + \frac{\lambda\tilde{\varepsilon}}{\beta}\right).$$

Since $\tilde{y}$ and $\tilde{\varepsilon}$ are mutually independent, we can apply the projection theorem for normally distributed random variables to get

$$E(\tilde{y} \mid \tilde{x}) = \bar{y} + \frac{V_y}{V_y + (\lambda^2 V_\varepsilon/\beta^2)}\left(\frac{\tilde{x} - \alpha}{\beta} - \bar{y}\right). \tag{A.1}$$

Plugging (A.1) in (4) and collecting terms we obtain

$$e = \frac{1}{c}\left\{\left[1 - \frac{V_y}{V_y + (\lambda^2 V_\varepsilon/\beta^2)}\right]\bar{y} - \left[\frac{V_y}{V_y + (\lambda^2 V_\varepsilon/\beta^2)}\right]\frac{\alpha}{\beta}\right\} + \frac{\tau}{c}\left\{\frac{V_y}{[V_y + (\lambda^2 V_\varepsilon/\beta^2)]\beta} - 1\right\}\tilde{x}.$$

The previous expression confirms that the audit strategy is linear in the observed report x. Therefore, letting

$$e(\tilde{x}, \tilde{c}) = \delta(\tilde{c}) + \gamma(\tilde{c})\tilde{x}$$

and equating coefficients, we get

$$\delta(\tilde{c}) = \frac{1}{\tilde{c}}\left\{\left[1 - \frac{V_y}{V_y + (\lambda^2 V_\varepsilon/\beta^2)}\right]\bar{y} - \left[\frac{V_y}{V_y + (\lambda^2 V_\varepsilon/\beta^2)}\right]\frac{\alpha}{\beta}\right\}, \tag{A.2}$$

and

$$\gamma(c) = \frac{1}{c}\left\{\frac{V_y}{[V_y + (\lambda^2 V_\varepsilon/\beta^2)]\beta} - 1\right\}. \tag{A.3}$$

A taxpayer observes his true income y and conjectures that the tax auditor will follow an audit strategy that is linear in x, $e(x, c) = \delta(c) + \gamma(c)x$. Therefore, the objective of the taxpayer is to maximize

$$E\{y - \tau x - [\delta(\tilde{c}) + \gamma(\tilde{c})x)]\tau(y - x) - \varepsilon\tau[\delta(\tilde{c}) + \gamma(\tilde{c})x)]\}.$$

The optimal intended report x must satisfy the following first order condition (see (7)):

$$-1 - E[\gamma(\tilde{c})(y - x) - \delta(\tilde{c}) - \gamma(\tilde{c})x + \gamma(\tilde{c})\varepsilon] = 0.$$

We can solve for x in the previous equation,

$$x = \frac{1}{2}\left[y + \varepsilon + \frac{(1 - E[\delta(\tilde{c})])}{E[\gamma(\tilde{c})]}\right]. \tag{A.4}$$

The second order condition (8) becomes simply $E[\gamma(\tilde{c})] < 0$. Note that (A.4) confirms that the report strategies used by taxpayers are linear, i.e., $x(y) = \alpha + \beta y + \lambda\varepsilon$. Therefore, equating coefficients we obtain,

$$\alpha = \frac{1}{2}\left(\frac{1 - E[\delta(\tilde{c})]}{E[\gamma(\tilde{c})]}\right), \tag{A.5}$$

$$\beta = \frac{1}{2}, \tag{A.6}$$

and

$$\lambda = \frac{1}{2}. \tag{A.7}$$

Thus, we obtain the equilibrium values of β and λ given in (10) and (11). We must compute now the expected values of the coefficients $\delta(\tilde{c})$ and $\gamma(\tilde{c})$. To this end, we compute the expectation of (A.2) and (A.3) to obtain

$$E[\delta(\tilde{c})] = E(1/\tilde{c})\left\{\left[1 - \frac{V_y}{V_y + (\lambda^2 V_\varepsilon/\beta^2)}\right]\bar{y} - \left[\frac{V_y}{V_y + (\lambda^2 V_\varepsilon/\beta^2)}\right]\frac{\alpha}{\beta}\right\} \tag{A.8}$$

and

$$E[\gamma(\tilde{c})] = E(1/\tilde{c})\left\{\frac{V_y}{[V_y + (\lambda V_\varepsilon/\beta^2)]\beta} - 1\right\}. \tag{A.9}$$

Using (A.6) and (A.7), we can find the values of α, $E[\delta(\tilde{c})]$, and $E[\gamma(\tilde{c})]$ solving the system of equations (A.5), (A.8) and (A.9). After some tedious algebra we obtain the values of α given in (9) and

$$E[\delta(\tilde{c})] = \frac{V_y}{V_\varepsilon} - E(1/\tilde{c})\bar{y}\left(\frac{V_y - V_\varepsilon}{V_y + V_\varepsilon}\right),$$
$$E[\gamma(\tilde{c})] = E(1/\tilde{c})\left(\frac{V_y - V_\varepsilon}{V_y + V_\varepsilon}\right).$$

Note that the second order condition $E[\gamma(\tilde{c})] < 0$ is satisfied since both $\tilde{c} > 0$ and $V_\varepsilon > V_y$ hold by assumption.

We can now find the coefficients δ and γ defining the audit strategy. To this end we only have to plug the values of α, β and λ we have just obtained into (A.2) and (A.3). Some additional algebra yields the values of $\delta(c)$ and $\gamma(c)$ given in (12) and (13). □

Proof of Corollary 1: The expected net revenue raised by a tax auditor before observing the realization of the cost $\tilde{c}$ and the report $\tilde{x}$ is

$$\begin{aligned} E(\tilde{R}) &= E\left\{\tau x(\tilde{y}, \tilde{\varepsilon}) + e(x(\tilde{y}, \tilde{\varepsilon}), \tilde{c})\tau(\tilde{y} - x(\tilde{y}, \tilde{\varepsilon})) - \frac{1}{2}c[e(x(\tilde{y}, \tilde{\varepsilon}), \tilde{c})]^2\right\} \\ &= E\left\{\tau(\alpha + \beta\tilde{y} + \lambda\tilde{\varepsilon}) + [\delta(c) + \gamma(c)(\alpha + \beta\tilde{y} + \lambda\tilde{\varepsilon})]\tau(\tilde{y} - \alpha - \beta\tilde{y} - \lambda\tilde{\varepsilon})\right. \\ &\quad \left. - \frac{1}{2}c[\delta(c) + \gamma(c)(\alpha + \beta\tilde{y} + \lambda\tilde{\varepsilon})]^2\right\}. \end{aligned}$$

Using the equilibrium values of α, β, λ, $\gamma(c)$ and $\delta(c)$ obtained in Proposition 1, and after some cumbersome algebra, we obtain

$$\begin{aligned} E(\tilde{R}) = \frac{\tau}{8V_\varepsilon^2(V_y + V_\varepsilon)E(1/\tilde{c})}&\big[V_y^3 - V_y^2 V_\varepsilon - 5V_y V_\varepsilon^2 - 3V_\varepsilon^3 \\ &+ 8\bar{y}V_y V_\varepsilon^2 E(1/\tilde{c}) + 8\bar{y}V_\varepsilon^3 E(1/\tilde{c}) - 2V_y V_\varepsilon^3[E(1/\tilde{c})]^2 \\ &+ V_\varepsilon^4[E(1/\tilde{c})]^2 + V_y^2 V_\varepsilon^2[E(1/\tilde{c})]^2\big]. \end{aligned}$$

We can compute now the following derivative:

$$\frac{\partial E(\tilde{R})}{\partial V_\varepsilon} = \frac{\tau}{2V_\varepsilon^3(V_y+V_\varepsilon)^2E(1/\tilde{c})}\big[2V_y^2V_\varepsilon^2 + 2V_yV_\varepsilon^3 - V_y^4 - 2V_y^3V_\varepsilon - 3V_y^2V_\varepsilon^3[E(1/\tilde{c})]^2 + 2V_yV_\varepsilon^4[E(1/\tilde{c})]^2 + 2V_\varepsilon^5[E(1/\tilde{c})]^2\big].$$

It can be shown that the previous derivative becomes equal to zero only when $V_\varepsilon = V_y$, whereas it is positive whenever $0 < V_y < V_\varepsilon$, which holds by assumption.

Concerning the effects of V_y, we compute

$$\frac{\partial E(\tilde{R})}{\partial V_y} = \frac{\tau}{8V_\varepsilon^2(V_y+V_\varepsilon)^2E(1/\tilde{c})}\big[2V_y^3 + 2V_y^2V_\varepsilon + V_y^2V_\varepsilon^2[E(1/\tilde{c})]^2 - 2V_yV_\varepsilon^2 - 3V_\varepsilon^4[E(1/\tilde{c})]^2 - 2V_\varepsilon^3 + 2V_yV_\varepsilon^3[E(1/\tilde{c})]^2\big]. \tag{A.10}$$

The previous derivative becomes equal to zero whenever

$$V_y = V_\varepsilon, \tag{A.11}$$

$$V_y = V_\varepsilon\left[-1 - \frac{V_\varepsilon[E(1/\tilde{c})]^2}{4} + \tau E(1/\tilde{c})\sqrt{V_\varepsilon\left(\frac{V_\varepsilon[E(1/\tilde{c})]^2}{16} - 1\right)}\right], \tag{A.12}$$

or

$$V_y = V_\varepsilon\left[-1 - V_\varepsilon[E(1/\tilde{c})]^2 - E(1/\tilde{c})\sqrt{V_\varepsilon\left(\frac{V_\varepsilon[E(1/\tilde{c})]^2}{16} - 1\right)}\right]. \tag{A.13}$$

The roots (A.12) and (A.13) are imaginary when $V_\varepsilon \in (0, 16/E(1/\tilde{c}))$. In this case, the single real root is the one given by (A.11). If $V_\varepsilon \geq 16/E(1/\tilde{c})$, then the roots (A.12) and (A.13) are real. The root (A.13) is obviously negative. Concerning the root (A.12), it can be easily checked that it is also negative when $V_\varepsilon \geq 16/E(1/\tilde{c})$. Therefore, (A.10) does not change its sign in all the parameter region satisfying $0 < V_y < V_\varepsilon$. Since the last chain of inequalities holds by assumption, we only have to check numerically that (A.10) is negative in that region.

Finally, we can compute the following derivative with respect to $E(1/\tilde{c})$:

$$\frac{\partial E(\tilde{R})}{\partial E(1/\tilde{c})} = \frac{\tau^2}{8V_\varepsilon^2(V_y+V_\varepsilon)[E(1/\tilde{c})]^2}\big[-V_y^3 + V_y^2V_\varepsilon^2[E(1/\tilde{c})]^2 + V_y^2V_\varepsilon - 2\tau^2V_yV_\varepsilon^3[E(1/\tilde{c})]^2 + 5V_yV_\varepsilon^2 + V_\varepsilon^4[E(1/\tilde{c})]^2 + 192V_y^3\big].$$

The previous derivative is always positive, as it can be shown by checking that it has only two imaginary roots for $E(1/\tilde{c})$ and, hence, it never changes its sign for all positive real values of $E(1/\tilde{c})$. □

Proof of Corollary 2: (a) The expected disposable income net of audit costs of a taxpayer is

$$\begin{aligned} E(\tilde{n}) &= E[\tilde{y} - \tau x(\tilde{y},\tilde{\varepsilon}) - e(x(\tilde{y},\tilde{\varepsilon}),\tilde{c})\tau(\tilde{y} - x(\tilde{y},\tilde{\varepsilon})) - \tilde{\varepsilon}\tau e(x(\tilde{y},\tilde{\varepsilon}),\tilde{c})] \\ &= E\{\tilde{y} - \tau(\alpha+\beta\tilde{y}+\lambda\tilde{\varepsilon}) - [\delta(\tilde{c}) + \gamma(\tilde{c})(\alpha+\beta\tilde{y}+\lambda\tilde{\varepsilon})]\tau(\tilde{y} - \alpha - \beta\tilde{y} - \lambda\tilde{\varepsilon}) \\ &\quad - \tilde{\varepsilon}\tau[\delta(\tilde{c}) + \gamma(\tilde{c})(\alpha+\beta\tilde{y}+\lambda\tilde{\varepsilon})]\}. \end{aligned}$$

Using the equilibrium values of α, β, λ, $\delta(c)$ and $\gamma(c)$ given in (9)–(13) and simplifying, we obtain

$$E(\tilde{n}) = \frac{1}{4E(1/\tilde{c})V_\varepsilon^2(V_y+V_\varepsilon)}\big[4\bar{y}V_yV_\varepsilon^2[E(1/\tilde{c})] + 4\bar{y}V_\varepsilon^3[E(1/\tilde{c})] - 4\tau\bar{y}V_yV_\varepsilon^2[E(1/\tilde{c})] - 4\tau\bar{y}V_\varepsilon^3[E(1/\tilde{c})] - \tau V_y^2V_\varepsilon + \tau V_yV_\varepsilon^2 + \tau V_\varepsilon^3 - \tau V_y^3 + \tau V_yV_\varepsilon^3[E(1/\tilde{c})]^2 + \tau V_\varepsilon^4[E(1/\tilde{c})]^2\big].$$

We can compute then the derivative of $E(\tilde{n})$ with respect to V_ε,

$$\frac{\partial E(\tilde{n})}{\partial V_\varepsilon} = \frac{\tau}{4V_\varepsilon^3(V_y+V_\varepsilon)^2E(1/\tilde{c})}\big[4V_y^3V_\varepsilon + V_\varepsilon^5[E(1/\tilde{c})]^2 + 2V_y^2V_\varepsilon^2 + 2V_y^4 + 2V_yV_\varepsilon^4[E(1/\tilde{c})]^2 - V_y^2V_\varepsilon^3[E(1/\tilde{c})]^2\big].$$

It can be shown that the previous derivative is strictly positive whenever $V_\varepsilon > (\sqrt{2}-1)V_y$, which holds by assumption.

Similarly, for the effects of V_y on $E(\tilde{n})$ we can compute

$$\frac{\partial E(\tilde{n})}{\partial V_y} = -\frac{\tau}{2V_\varepsilon^2(V_y+V_\varepsilon)^2E(1/\tilde{c})}\big[2V_y^2V_\varepsilon + V_yV_\varepsilon^2 + V_y^3 + V_\varepsilon^4[E(1/\tilde{c})]^2\big] < 0.$$

(b) We can compute the following derivative:

$$\frac{\partial E(\tilde{n})}{\partial E(1/\tilde{c})} = \frac{\tau}{4V_\varepsilon^2(V_y+V_\varepsilon)[E(1/\tilde{c})]^2}\big[V_y^2V_\varepsilon - V_yV_\varepsilon^2 + V_y^3 - V_\varepsilon^3 - V_yV_\varepsilon^3[E(1/\tilde{c})]^2 + V_\varepsilon^4[E(1/\tilde{c})]^2\big].$$

Let

$$\theta = \frac{V_y+V_\varepsilon}{(V_\varepsilon)^{3/2}},$$

Then, it can be easily verified that

$$\frac{\partial E(\tilde{n})}{\partial E(1/\tilde{c})} \lesseqgtr 0 \quad \text{for all } E(1/\tilde{c}) \lesseqgtr \theta.$$

□

Proof of Corollary 3: The average expected tax rate is

$$\hat{\tau}(y) = \frac{g(y)}{y} = \frac{E[\tau x(y,\tilde{\varepsilon}) + e(x(y,\tilde{\varepsilon}),\tilde{c})\tau(y-x(y,\tilde{\varepsilon}))]}{y} = \frac{E\{\tau(\alpha+\beta y+\lambda\tilde{\varepsilon}) + [\delta(\tilde{c})+\gamma(\tilde{c})(\alpha+\beta y+\lambda\tilde{\varepsilon})]\tau(y-\alpha-\beta y-\lambda\tilde{\varepsilon})\}}{y}.$$

Using the equilibrium values of the parameters characterizing the audit and report strategies given in (9)–(13) and computing the expectation with respect to $\tilde{c}$ and $\tilde{\varepsilon}$, we get an expression of the following type:

$$\hat{\tau}(y) = \frac{my^2+ny+q}{sy},$$

where the coefficients m, n, q and s depend on the parameters of the model. In particular,

$$m = V_\varepsilon^2(V_y - V_\varepsilon)[E(1/\tilde{c})]^2$$

and

$$s = 4yV_\varepsilon^2(V_y + V_\varepsilon)E(1/\tilde{c}).$$

Note that $m < 0$ as $V_y < V_\varepsilon$, whereas $s > 0$. Therefore,

$$\lim_{y\to\infty} \hat{\tau}(y) = -\infty,$$
$$\lim_{y\to-\infty} \hat{\tau}(y) = \infty,$$

and the function $\hat{\tau}(y)$ is discontinuous at $y = 0$. Moreover, the equation $\hat{\tau}'(y) = 0$ has two conjugate solutions,

$$\frac{\pm\sqrt{\Delta}}{V_\varepsilon E(1/\tilde{c})},$$

with

$$\Delta = V_y^2 - 2\bar{y}V_yV_\varepsilon E(1/\tilde{c}) + 2V_yV_\varepsilon - 2\bar{y}V_\varepsilon^2E(1/\tilde{c}) - V_\varepsilon^3[E(1/\tilde{c})]^2 + V_\varepsilon^2 + \bar{y}^2V_\varepsilon^2[E(1/\tilde{c})]^2.$$

These two solutions are both real with opposite sign when the term Δ is positive. Otherwise, the two solutions are imaginary. Therefore, on the one hand, when Δ is negative, the function $\hat{\tau}(y)$ is decreasing on the interval $(-\infty, 0)$ and is also decreasing on the interval $(0, \infty)$. On the other hand, if Δ is positive then the function $\hat{\tau}(y)$ is U-shaped on the interval $(-\infty, 0)$ and inverted U-shaped on the interval $(0, \infty)$. Note that in both cases there exists an income level $\hat{y}$ such that $\hat{\tau}'(y) < 0$, for all $y > \hat{y}$. Finally, note that Δ can be positive or negative depending on the parameter values. For instance, let $E(1/\tilde{c}) = 20/3$, $V_y = 1$, $\bar{y} = 3$ and $\tau = 0.2$. In this case, if $V_\varepsilon = 4$, then $\Delta = 2780.6$. However, if $V_\varepsilon = 10$, then $\Delta = -8723.4$. □

Acknowledgments

This paper has benefited from comments made by participants in the Encuentro de Economia Pública (Santa Cruz de Tenerife) and by two anonymous referees of this Journal. Of course, they should not bear any responsibility for the remaining errors. Financial support from the Institute of Fiscal Studies (Spanish Ministry of Finance), Spanish Ministry of Science and Technology through grants SEC2003-306 and HI2001-0039, and from the Generalitat of Catalonia through the Barcelona Economics program of CREA and grant SGR2001-00162 is gratefully acknowledged.

Notes

1. The previous basic models have been enriched in several directions. For instance, Border and Sobel (1987) allow for general objective functions for the tax agency; Mookherjee and P'ng (1989) study the implications of having risk averse agents; Sánchez and Sobel (1983) analyze the conditions under which cut-off policies are optimal from the expected revenue viewpoint; and Erard and Feinstein (1994a) introduce a fraction of honest taxpayers that always produce truthful reports. Alm, Bahl and Murray (1993) provide strong empirical support for the game-theoretical approach of the tax evasion phenomenon versus the alternative models based on just random audit policies.
2. We suppress the tilde to denote the realization of a random variable.
3. Note that the assumption of normality of $\tilde{y}$ and $\tilde{\varepsilon}$ gives rise to the linearity of the conditional expectation $E(\tilde{y} \mid x)$ with respect to the report x (see (A.1)). The linearity of this conditional expectation is crucial for obtaining linear strategies in equilibrium.
4. Note that, if we assume empirically plausible parameter values for the tax and the penalty rates, like $\tau = 0.2$ and $f = 2$, we obtain that $V_\varepsilon = \text{Var}(\tilde{\varepsilon}^*)/0.16$ as $\varepsilon = \frac{\varepsilon^*}{\tau f}$. Recalling that the variable $\tilde{\varepsilon}^*$ is the primitive cost faced by taxpayers per unit of audit effort, our assumption $V_\varepsilon > V_y$ becomes $\text{Var}(\tilde{\varepsilon}^*) > 0.16 V_y$.
5. Scotchmer (1987) and Galmarini (1997) analyze the size of the regressive bias under cut-off audit policies when taxpayers are sorted into income classes and when taxpayers differ in terms of their risk aversion, respectively. These two modifications imply a reduction in the size of the regressive bias.

References

Allingham, M. G. and A. Sandmo. (1972). "Income Tax Evasion: A Theoretical Analysis," *Journal of Public Economics* 1, 323–338.

Alm, J., R. Bahl and M. N. Murray. (1993). "Audit Selection and Income Tax Underreporting in the Tax Compliance Game," *Journal of Development Economics* 42, 1–33.

Alm, J., B. Jackson and M. McKee. (1992). "Institutional Uncertainty and Taxpayer Compliance," *American Economic Review* 82, 1018–1026.

Alm, J., G. H. McClelland and W. D. Schulze. (1992). "Why Do People Pay Taxes," *Journal of Public Economics* 48, 21–38.

Andreoni, J., B. Erard and J. Feinstein. (1998). "Tax Compliance," *Journal of Economic Literature* 36, 818–860.

Border, K. and J. Sobel. (1987). "Samurai Accountant: A Theory of Auditing and Plunder," *Review of Economic Studies* 54, 525–540.

Broadway, R. and M. Sato. (2000). "The Optimality of Punishing Only the Innocent: The Case of Tax Evasion," *International Tax and Public Finance* 7, 641–664.

Erard, B. and J. Feinstein. (1994a). "Honesty and Evasion in the Tax Compliance Game," *RAND Journal of Economics* 25, 1–19.

Erard, B. and J. Feinstein. (1994b). "The Role of Moral Sentiments and Audit Perceptions in Tax Compliance," *Public Finance/Finances Publiques* 48, 21–48.

Galmarini, U. (1997). "On the Size of the Regressive Bias in Tax Enforcement," *Economic Notes by Banca Monte dei Paschi di Siena SpA* 26, 75–102.

Jung, W. (1991). "Tax Reporting Game under Uncertain Tax Laws and Asymmetric Information," *Economics Letters* 37, 323–329.

Mookherjee, D. and I. P'ng. (1989). "Optimal Auditing, Insurance, and Redistribution," *Quartely Journal of Economics* 104, 399–415.

Pestieau, P., U. M. Possen and S. M. Slutsky. (1998). "The Value of Explicit Randomization in the Tax Code," *Journal of Public Economics* 67, 87–103.

Reinganum, J. and L. Wilde. (1985). "Income Tax Compliance in a Principal-Agent Framework," *Journal of Public Economics* 26, 1–18.

Reinganum, J. and L. Wilde. (1986). "Equilibrium Verification and Reporting Policies in a Model of Tax Compliance," *International Economic Review* 27, 739–760.

Reinganum, J. and L. Wilde. (1988). "A Note on Enforcement Uncertainty and Taxpayer Compliance," *Quarterly Journal of Economics* 103, 793–798.

Rhoades, S. C. (1999). "The Impact of Multiple Component Reporting on Tax Compliance and Audit Strategies," *The Accounting Review* 74, 63–85.

Rubinstein, A. (1979). "An Optimal Conviction Policy for Offenses that May Have Been Committed by Accident," *Applied Game Theory* 406–413.

Sánchez, I. and J. Sobel. (1993). "Hierarchical Design and Enforcement of Income Tax Policies," *Journal of Public Economics* 50, 345–369.

Scotchmer, S. (1987). "Audit Classes and Tax Enforcement Policy," *American Economic Review* 77, 229–233.

Scotchmer, S. (1989). "Who Profits from Taxpayer Confusion," *Economics Letters* 29, 49–55.

Scotchmer, S. (1992). "The Regressive Bias in Tax Enforcement." In P. Pestieau (ed.), *Public Finance in a World of Transition, Proceedings of the 47th Congress of the IIPF*. St. Petersburg. 1991. Supplement to *Public Finance/Finances Publiques* 47, 366–371.

Scotchmer, S. and J. Slemrod. (1989). "Randomness in Tax Enforcement," *Journal of Public Economics* 38, 17–32.

Torgler, B. (2002). "Speaking to Theorists and Searching for Facts: Tax Morale and Tax Compliance in Experiments," *Journal of Economic Surveys* 5, 658–683.

Yitzhaki, S. (1974). "A Note on Income Tax Evasion: A Theoretical Analysis," *Journal of Public Economics* 3, 201–202.