

Matching

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Program

1 Introduction

- 1.1.- Matching Markets: Examples
- 1.2.- Questions to be Answered
- 1.3.- Plan of the Course
- 1.4.- History

2 The Marriage Model: Stability

- 2.1.- Agents and Preferences
- 2.2.- Matchings
- 2.3.- Optimal Stable Matchings
- 2.4.- Two-sided Matching versus the Roommate Problem and Many-sided
- 2.5.- The Deferred Acceptance Algorithm (DAA)
- 2.6.- A Sequential Description of the DAA
- 2.7.- The Core

3 The Marriage Model: Structure of the Set of Stable Matchings

- 3.1.- Example and Preliminary Remarks
- 3.2.- Optimal Matchings
- 3.3.- Opposition of Interests
- 3.4.- Decomposition Lemma and the “Rural Hospital” Theorem
- 3.5.- Preliminaries on Lattices
- 3.6.- Lattice Theorem
- 3.7.- An Algorithm to Compute all Stable Matchings

- 3.8.- The Blocking Lemma
- 3.9.- The Weak Pareto Optimality Theorem
- 3.10.- Paths to Stability: Knuth's Example
- 3.11.- The Roth and Vande Vate Theorem: The RVV Sequential Algorithm
- 3.12.- Another Sequential Algorithm to Find Stable Matchings
- 3.13.- Vacancy Chains
- 3.14.- On the Number of Stable Matchings
- 3.15.- Indifferences
- 3.16.- Axiomatic Characterizations of the Core
- 3.17.- Von Neumann-Morgenstern Stable Sets of Matchings
- 3.18.- A Bargaining Set

4 The Marriage Model: Strategic Incentives

- 4.1.- Stable mechanisms
- 4.2.- On the Impossibility of Strategy-proof Stable Mechanisms
- 4.3.- Partial Strategy-proofness of the DAAs
- 4.4.- Nash Equilibria of the DAAs
- 4.5.- Stability of the Outcomes of some Nash Equilibria of the DAAs
- 4.6.- Partial Group Strategy-proofness of the DAAs

5 The College Admissions Problem

- 5.1.- Many-to-one Matching Models
- 5.2.- Agents, Quotas and Rankings
- 5.3.- Matchings
- 5.4.- Stable Matchings
- 5.5.- Responsive Extensions
- 5.6.- The Deferred Acceptance Algorithms (DAAs)
- 5.7.- A Related Marriage Market
- 5.8.- Results on Stability and Incentives
- 5.9.- Giving Advice
- 5.10.- Equilibria under Incomplete Information
- 5.11.- Random Stable Mechanisms

6 Many-to-one Matching Models

- 6.1.- Agents, Preferences, and Matchings
- 6.2.- Pair-wise Stable Matchings
- 6.3.- Substitutable Preferences: Results
- 6.4.- The Deferred Acceptance Algorithms (DAAs)
- 6.5.- Quota q -separable Preferences: The Core and Results
- 6.6.- Substitutable Preferences and the Law of Aggregate Demand
- 6.7.- Core Matchings as Fixed Points
- 6.8.- Preferences over Colleagues
- 6.9.- Many-to-many Matchings: Pair-wise, Set-wise, and Core Stability
- 6.10.- An Algorithm to Compute the Set of Set-wise Stable Matchings

7 Matching Models with Money

- 7.1.- The Assignment Game
- 7.2.- Competitive Equilibria: Definition and Existence
- 7.3.- Structure of the Set of Competitive Equilibria
- 7.4.- Generalized Assignment Games

8 Other Topics

- 8.1.- Decentralized Matching Markets
- 8.2.- The House Allocation Problem
- 8.3.- The Assignment of a Set of Indivisible Objects (without Money)
- 8.4.- The Envy-free Assignment of a Set of Indivisible Objects (with Money)
- 8.5.- The School Choice Problem
- 8.6.- Kidney Exchange
- 8.7.- Matching with Contracts

Text Books

Alvin E. Roth and Marilda A. Oliveira Sotomayor. *Two-sided Matching: A Study in game-theoretic Modelling and Analysis*. Econometric Society Monograph No. 18 Cambridge University Press, 1990.

Dan Gusfield and Robert W. Irving. *The Stable Marriage Problem: Structure and Algorithms*. The MIT Press, Cambridge, 1989.