Game Theory

International Doctorate in Economic Analysis (IDEA)
Universitat Autònoma de Barcelona (UAB). Jordi Massó

Problem Set #2

2.1.- Construct the normal forms and obtain their corresponding Nash equilibria in pure strategies of the games in extensive form of Figures 1, 2, and 3.

2.2.- Consider the game in extensive form of Figure 4.

(a) Find the set of Nash equilibria obtained by backwards induction.

(b) Obtain its normal form.

(c) Find all Nash equilibria (pure and mixed) of this normal form and verify that the equilibria obtained in (a) is a subset of the set of equilibria.

2.3.- Consider the game in extensive form with perfect information of Figure 5.

(a) How many strategies has each player?

(b) Write down all the strategies of the two players.

(c) Identify all Nash equilibria that can be obtained using backwards induction.

(d) Find a Nash equilibrium that can not be obtained by backwards induction.

2.4.- A committee with three members, $\{1, 2, 3\}$, has to choose a new member of a club among a set of four candidates, $\{a, b, c, d\}$. Each member of the committee has veto power which is used in a successive way, starting by member 1, and finishing with member 3. Each member of the committee has to veto one and only one of the candidates that have not been vetoed yet.

(a) Draw the extensive form of the game, writing in the terminal nodes the name of the elected candidate.

(b) How many strategies has each player? Do not try to write them out to count them (player 3 has more than 4,000).

(c) Specify two different sets of strict preferences for the members of the committee (on the set of candidates) and find the corresponding elected candidate associated to the Nash equilibrium obtained by backwards induction. If the members of the committee would not behave "strategically", would the elected candidate be different?, would the vetoed candidate be different?

2.5.- A game in extensive form with perfect information $\Gamma = (I, K, P, C, u)$ is called *perfectly competitive* if: (I) $I = \{1, 2\}$, and (II) for every $z, z' \in Z$, $u_1(z) \geq u_1(z')$ if and only if $u_2(z') \geq u_2(z)$. Show that if Γ is perfectly competitive the following statements are true:

(a) For every $s, s' \in S$, $h_1(s) \geq h_1(s')$ if and only if $h_2(s') \geq h_2(s)$.

(b) If $s, s' \in S^*$ then, $h_i(s) = h_i(s')$ for $i = 1, 2$.

(c) If $s, s' \in S^*$ then, $(s_1, s_2'), (s_1', s_2) \in S^*$.

2.6.- Let Γ_2 be an extensive-form game with imperfect information in which there are no chance moves, and assume that the game Γ_1 differs from Γ_2 only in that one of the information sets of player 1 in Γ_2 is split into two information sets in Γ_1. Show that all Nash equilibria in pure strategies of Γ_2 correspond to Nash equilibria of Γ_1. Show that the requirement that there be no chance moves is essential for this result.

2.7.- Consider the extensive-form game with imperfect information of Figure 6.

(a) For each player $i = 1, 2$ and for each mixed strategy $\sigma_i \in \Sigma_i$ obtain an equivalent behavioral strategy $\widehat{\sigma}_i \in \widehat{\Sigma}_i$.

(b) For each player $i = 1, 2$ and for each behavioral strategy $\widehat{\sigma}_i \in \widehat{\Sigma}_i$ obtain an equivalent mixed strategy $\sigma_i \in \Sigma_i$.

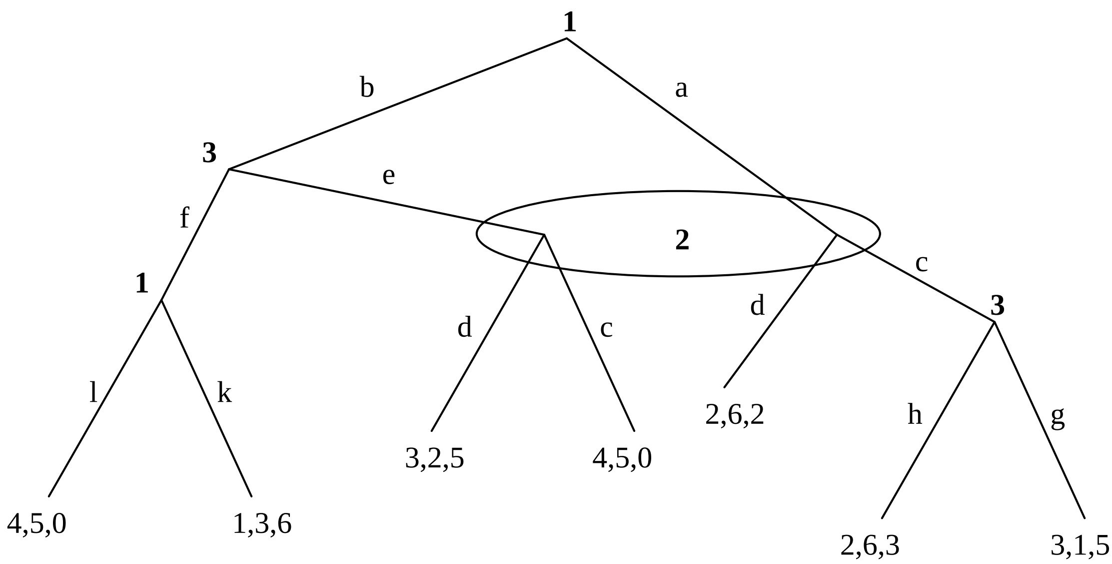

Figure 1

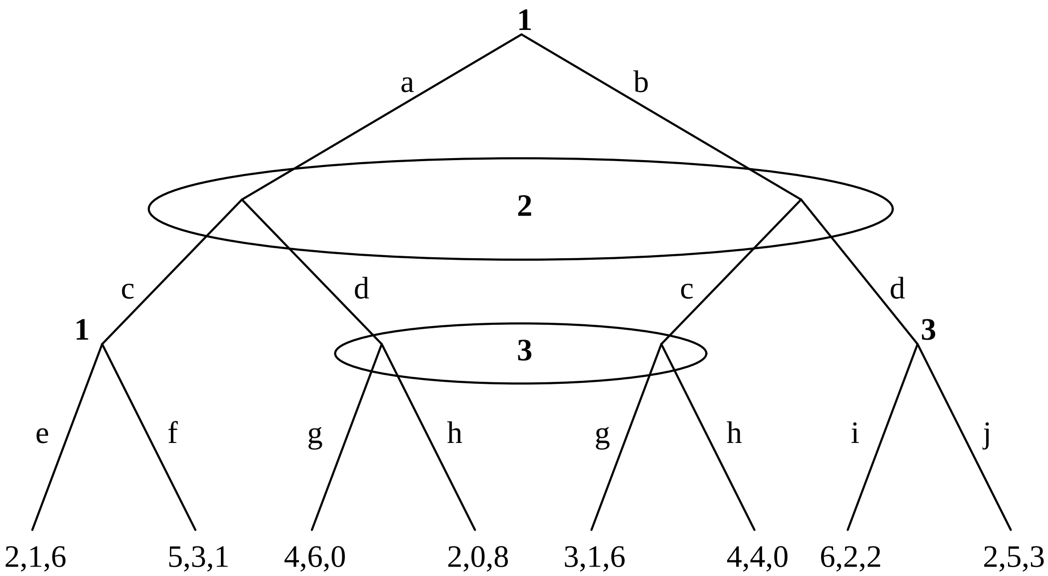

Figure 2

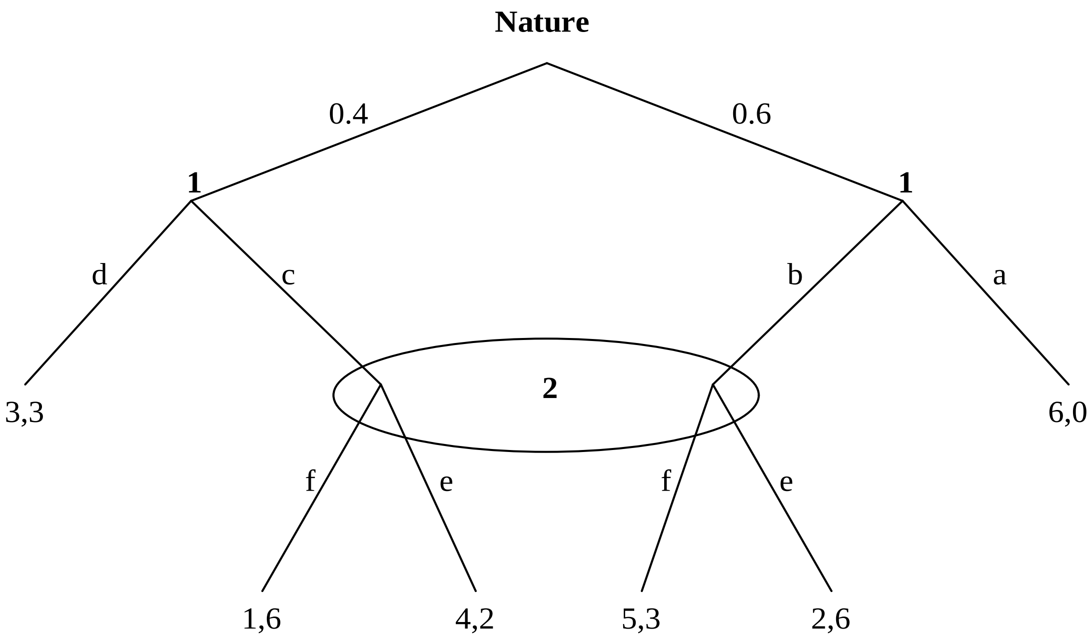

Figure 3

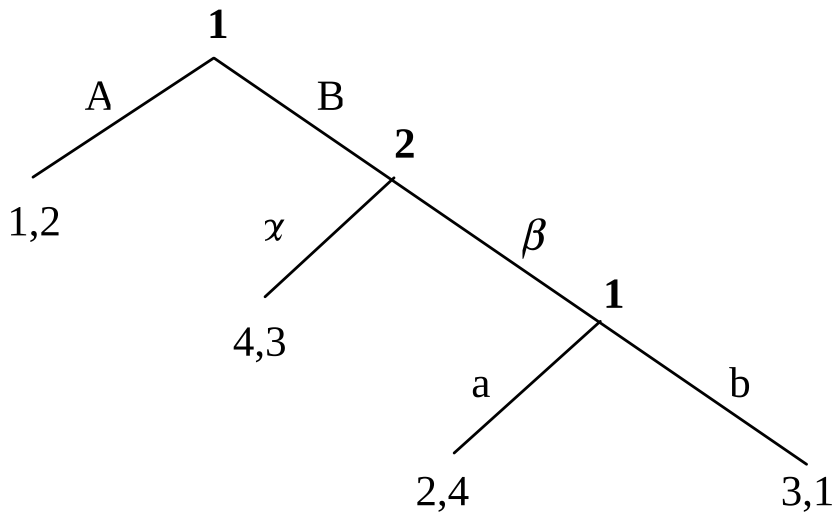

Figure 4

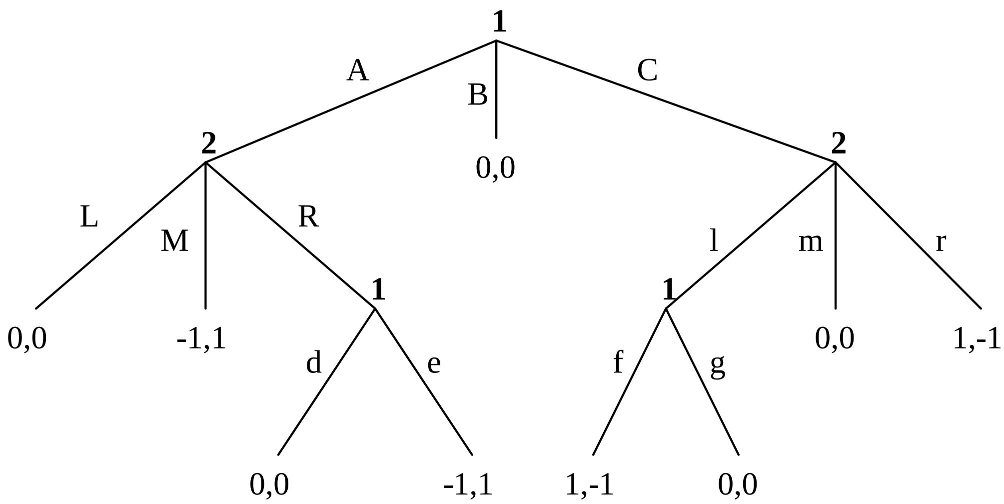

Figure 5

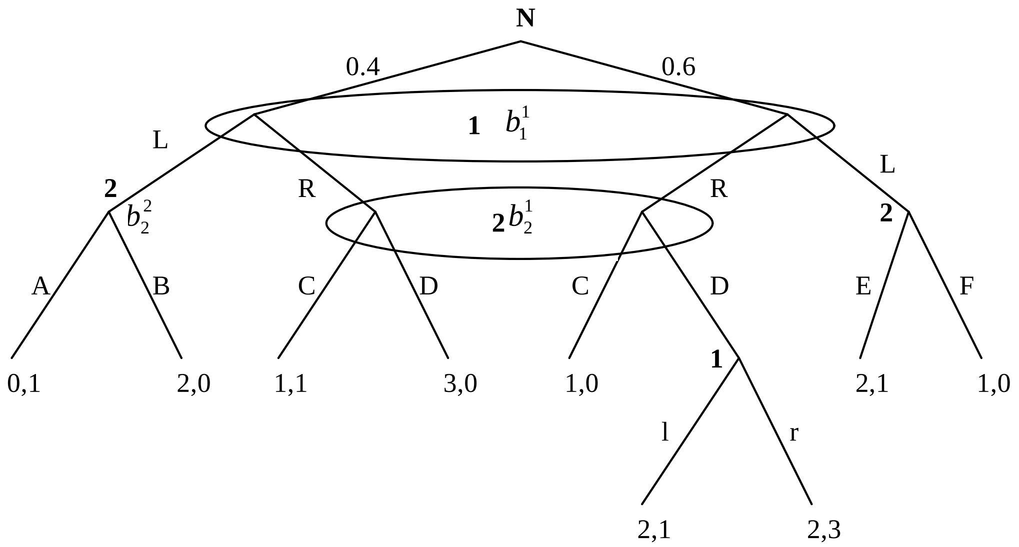
N
0.4
0.6
1 b_1^1
L
R
R
L
2 b_2^2
2 b_2^1
2
A
B
C
D
C
D
E
F
1
0,1
2,0
1,1
3,0
1,0
2,1
1,0
l
r
2,1
2,3

Figure 6