Game Theory

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Problem Set #0

0.1.- Suppose that $\succeq$ is rational and satisfies the independence axiom. Prove that for any two outcomes $x \succeq y$ and any $1 > \alpha > \beta > 0$

$$\alpha\delta_x + (1-\alpha)\delta_y \succeq \beta\delta_x + (1-\beta)\delta_y,$$

where δ_x is the degenerate lottery with sure outcome x.

0.2.- Take $X = \{x_1, x_2\}$ and give an example of a preference on Δ^2 satisfying continuity but not satisfying independence.

0.3.- Take $X = \{x_1, x_2, x_3\}$ and give an example of a preference on Δ^3 satisfying independence but not satisfying continuity.

0.4.- Suppose an agent who evaluates each lottery by the number of prizes that can be realized with positive probability, such that $p \succeq q$ if $|supp(p)| \leq |supp(q)|$. Check whether the axioms of independence and continuity are satisfied.

0.5.- Determine whether these utility criteria satisfy expected utility:

1. Judge by highest probability: $v(p) = \max_{x \in X} p(x)$.
2. Suppose a subset of outcomes $G \subseteq X$, interpreted as good outcomes, the criterion judges by the probability of good outcomes: $v(p) = \sum_{x \in G} p(x)$.
3. Judge by lowest possible utility: $v(p) = \min_{x \in X}\{u(x) | p(x) > 0\}$.
4. Judge by most likely prize: $v(p) = \arg\max_{x \in X} p(x)$.

0.6.- Suppose an agent whose preferences satisfy the independence axiom.

(a) Consider four lotteries $p, q, r, s \in \Delta^3$ over prizes in $X = \{x, y, z\}$ where

- $p = (0.2, 0.3, 0.5)$,
- $q = (0.25, 0.35, 0.4)$,
- $r = (0.8, 0, 0.2)$,
- $s = (0.9, 0.1, 0)$.

What does $p \succeq q$ imply for r vs s?

(b) For the same lotteries, suppose that sure prizes can be ranked such that $z \succeq y \succeq x$. Show that $p \succeq_{FSD} q$ (*i.e.*, p first-order stochastically dominates q) and that the independence axiom implies $p \succeq q$.

0.7.- "*It is a difference of opinion that makes horse races.*" [Mark Twain]

(a) Show that a rational agent with a smooth Bernoulli utility function u who is offered favorable (better than fair) odds will accept a sufficiently small bet.

(b) The two residents of Fatland, Al and Betty, have smooth, strictly concave Bernoulli utility functions u_A and u_B, respectively. In Fatland, there will, or there will not, be a solar eclipse on March 1. Whether or not this eclipse occurs affects no person's utility directly. Show that Al and Betty will not bet with each other if they agree about the probability of its occurring. Prove also that Al and Betty will bet with each other if and only if their probabilities are either $p_A < 0.5 < q_B$ or $q_B < 0.5 < p_A$.

0.8.- Show that if preferences $\succeq$ over simple lotteries with three outcomes are representable by a Bernoulli utility function which has the expected utility form, then indifference curves are parallel straight lines.

0.9.- A soccer fan has a Bernoulli utility function $u(w) = \ln w$ and has a subjective probability p that his team will win the next game and a probability $1 - p$ that it will not. He decides to bet x Euros for his team, so if it wins, the fan wins x Euros and if it loses, he loses x Euros (the fan decides the size of x). We know that the fan's initial income is w_0 Euros. How would you determine the subjective probability p observing the size of the bet x?

0.10.- The risk averse residents of a flood plain have 50% chance of suffering a loss of L Euros and a 50% chance of suffering no loss. The government offers these people flood insurance at actuarially fair rates. People who do not buy flood insurance and who suffer damages are compensated by the government disaster relief program for half the monetary damages they suffer. How much insurance will the flood plain residents buy?

0.11.- An agent with an automobile faces the risk of a minor accident with probability p_1 and a major accident with probability p_2, but cannot have both. The resultant losses are L_1 and L_2 Euros respectively where $L_1 < L_2$. Assume that the agent has a Bernoulli utility function defined on wealth, and is risk averse. He must choose between a deducible policy (where the first D Euros of loss is not reimbursed) and a coinsurance policy (where the insured must pay a fraction α of any loss, $0 < \alpha < 1$). Assume that D and α are such that the expected value of the loss is equal for both policies and is equal to the premium, r, for each, *i.e.*,

$$r = p_1(L_1 - D) + p_2(L_2 - D) = p_1(1 - \alpha)L_1 + p_2(1 - \alpha)L_2.$$

Show that the agent will always purchase the deductible policy.

0.12.- Compute the Arrow-Prat measure of absolute and relative risk aversion for the Bernoulli utility function

$$u(x) = 3x^{1/3}$$

at $x = 5$.

0.13.- Consider a Bernoulli utility function $u(x) = 2\sqrt{x}$ and a fair coin flip. If heads show up she gets 71, if tails show up she gets 15.

(a) Determine the risk premium (the maximum amount of money one is willing to pay in order to avoid a pure risk) associated to this gamble at wealth level 10.

(b) Calculate the degrees of absolute and relative risk aversion at wealth levels w. Would the risk premium change if wealth decreased to 1?

0.14.- Show that preferences of an agent with strictly decreasing relative risk aversion exhibit strictly decreasing absolute risk aversion, but the converse is not necessarily true.

0.15.- Let u_1 be a Bernoulli utility function on wealth levels x and $g : \mathbb{R} \to \mathbb{R}$ a strictly increasing and strictly concave function. Define a new Bernoulli utility function u_2 by $u_2(x) = g(u_1(x))$ for all $x \in X = \mathbb{R}$. Show that u_2 exhibits more risk aversion than u_1 in the sense of the coefficient of absolute risk aversion.

0.16.- Let u be a Bernoulli utility function of an agent. Show that the following two statements are equivalent.

1. u is concave.

2. $\pi(x, \varepsilon, u) \geq 0$ for all $x \in X$ and $\varepsilon > 0$.