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Explaining cross-country differences in participation rates and aggregate fluctuations☆

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Abstract

The empirical evidence shows that there exists a negative relationship between the ratio of employment to population and the standard deviation of (log of) GDP. In this paper, we build a Real Business Cycle model with an underground economy sector in order to quantitatively address this issue. The existence of an alternative to registered market activities for providing tradeable goods and services implies that population will be switching sectors in response to aggregate productivity shocks, amplifying the response of registered output. The level of participation in registered market activities will then be negatively related to fluctuations. This feature does not arise in a standard one sector model. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The empirical evidence shows that there exists a negative relationship between the ratio of employment to population and the standard deviation of (log of) GDP. In this paper, we build a real business cycle (RBC) model with an underground economy sector in order to quantitatively address this issue. The existence of an alternative to registered market activities for providing tradeable goods and services implies that population will be switching sectors in response to aggregate productivity shocks, amplifying the response of registered output. The level of participation in registered market activities will then be negatively related to fluctuations. This feature does not arise in a standard one-sector model.

Using the Summers–Heston data base we observe that countries with small ratios of employment over population (we will call this ratio the market participation rate) experience higher volatility of output. In particular, countries with participation rates as low as 25–30% might experience standard deviations of detrended output that are five times bigger than those experienced by countries with market participation rates above 50%.

We show that the opportunity cost of not participating in the market, i.e. the wage differential between market and non-registered activities, is the key element for understanding this pattern. If this opportunity cost is low, there will be smaller participation in the registered economy. Then, in response to aggregate technology shocks, there will be more movements in or out of the market sector, generating higher fluctuations. Notice that these differences arise even if the magnitude of shocks to total factor productivity is the same across countries.

In our paper the term "underground economy" refers to the production of goods and services that could otherwise be provided through registered market channels, but this activity is not registered in national income and product accounts. The tradeability of the output generated by this activities is the key feature distinguishing our approach from the household production literature (see Benhabib et al., 1991; McGrattan et al., 1997). Also, hours worked in the market are indivisible, while they are not in the underground economy. We use lotteries in order to handle this indivisibility (see Hansen, 1985, and Rogerson, 1987).

In Conesa et al. (2001) we also used the same theoretical framework in order to understand differences in fluctuations between the European and the US economies.

The paper is organized as follows. Section 2 presents the empirical evidence and motivation. Section 3 presents the model. Section 4 discusses the calibration and simulation procedures. Section 5 comments the simulations results. The conclusions close the paper.

2. Empirical evidence

We make international comparisons on yearly registered economic fluctuations using the Summers and Heston (1991) Penn World Table 5.5 (years 1960 to 1990). We construct a measure of capital stock using the perpetual inventory method. Fluctuations are measured by the standard deviation of the logarithm of GDP linearly detrended.

Notice that the way we treat the series is somewhat different from the standard procedure in the RBC literature. We use linearly detrended series. Also, the data base we use has annual instead of quarterly frequency and the series are not measured in market exchange rates but in purchasing power parity.

Fig. 1 displays average market participation rates versus standard deviations of registered output. We observe that the negative correlation between both variables is an empirical regularity in cross-country comparisons. The dashed line is the log-linear regression on the data.

In the data we observe that countries with participation rates as low as 25–30% might experience standard deviations of detrended output that are five times larger than those experienced by countries with market participation rates above 40%, which are mainly Western developed countries. The US economy has a participation rate of 45.4%, and standard deviation of registered output equal to 4.31%.

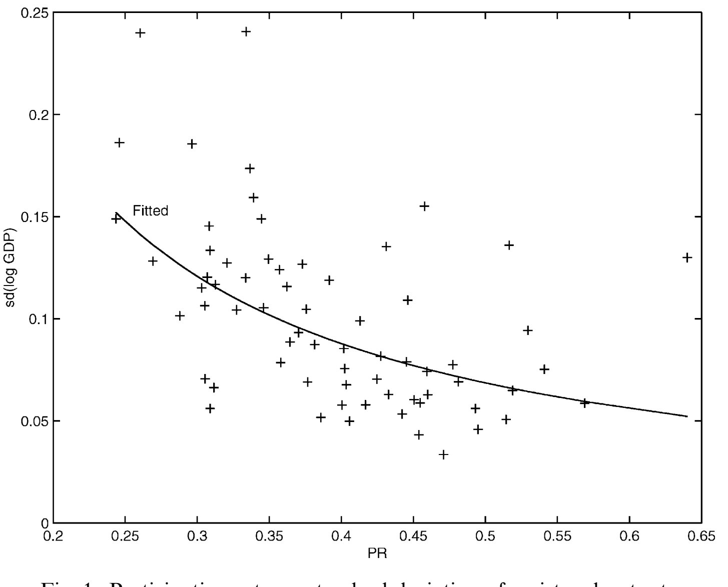


Fig. 1. Participation rate vs. standard deviation of registered output.

The differences in the amplitude of fluctuations might be a result of differences in the amplitude of technology shocks. We are not arguing that technological shocks do not differ among countries, but no insight is gained in order to explain the negative relationship between the level of market participation rates and aggregate fluctuations. Therefore, our question will be whether one can identify any feature, other than differences in technological shocks, that could account for cross country differences in aggregate registered fluctuations.

Our model is able to display this kind of pattern; namely, that the participation rate in registered activities will have a negative effect on the degree of fluctuation of investment and registered GDP when different economies face the same technological shocks.

3. The model

There exists a continuum of measure one of ex-ante identical agents which all of them are equally endowed with one unit of time. Their expected utility at period 0 takes the form

$$\mathrm{E}_0\left\{\sum_{t=0}^{\infty}\beta^t[U(c_t)+V(1-h_{mt}-h_t)]\right\},\tag{1}$$

where c_t is consumption in period t, $h_{mt}\in\{0,h^*\}$ is the indivisible time spent working in registered activities, $h_t\in[0,1]$ is time spent working in underground activities, and $\beta\in(0,1)$ is the discount factor. Functions U and V are assumed to be continuously differentiable, monotonically increasing and strictly concave. Also, we will assume that $h_t=0$ whenever $h_{mt}=h^*$, i.e. workers can only participate in one of the production sectors.

Therefore, individuals face two labor decisions in two stages: first, whether or not to participate in registered economic activities and, second, if they decide to engage in underground activities, how many hours to be dedicated to them. Individuals will take the first decision based on the difference between market wages in registered economic activities and the returns in the underground economy. Based on this opportunity cost, we endogenously get the participation rate in registered activities, which, together with the optimal amount of hours worked in underground activities and their productivity relative to working in the market, will allow us to obtain implicit estimates of the size of the underground economy.

The aggregate production function in registered activities is given by $e^{z_t}f(K_t,H_{mt})$, where the function $f(\cdot)$ is assumed to be constant returns to scale, monotonically increasing, strictly concave in both arguments and satisfies the Inada conditions; K_t is the amount of capital; H_{mt} is the total amount of labor in market activities; and z_t is a random shock that follows a stochastic first-order Markov process.

Each worker in the underground economy has an individual production function that is given by $g(h_t)$, where $g(\cdot)$ is assumed to be strictly concave and h_t is the number of hours worked in underground activities. Due to the labor-intensive specification of the production function for underground activities, we will assume that the technological shocks only affect the registered economy.

If we assume a perfectly insured market, we can interpret the decentralized competitive equilibrium as one in which agents purchase an infinite sequence of lotteries, see Hansen (1985) and Rogerson (1987). These lotteries are defined over consumption and hours worked in both activities. We define λ_t as the probability of working in underground activities at period t. Clearly, the probability of working in registered activities is $1-\lambda_t$.

The economy behaves as follows: at the beginning of every period t agents know the capital stock k_t and the realization of the stochastic shock z_t. Consumers, taking wages and rental rates for capital as given, will rent their capital stock to the firms in the registered economy, will decide how much to save and will choose a lottery, λ_t, that implies the probability of not participating in the registered economy, therefore consuming c_{bt} and working h_t hours with their individual production function. Otherwise, with probability $1-\lambda_t$, they work in the registered economy h^* hours, receive the competitive wage w_t and consume c_{mt}. The unique consumption good is homogeneous, regardless of whether it is produced in registered or in underground activities, and therefore it is sold at the same price in the market. Ex-post, according to the results of the lottery, the consumer will go to one of the two sectors. Firms in the registered economy will behave competitively. At the beginning of period t the shock is realized, and they hire capital and labor from the consumers taking prices as given. Free entry and exit in the registered economy will imply zero profits.

By definition $\lambda_t \in [0,1]$. Since there exists a continuum of agents of measure one, by the Law of Large Numbers, λ_t will be ex-post the fraction of workers in underground activities. The total number of hours worked in registered activities is then $H_{mt} = (1-\lambda_t)h^*$ and the aggregate production function of the underground economy is $\lambda_t g(h_t)$.

Definition. A sequential competitive equilibrium for this economy is a sequence of allocations $\{c_{mt}, c_{bt}, k_t, \lambda_t, h_t, H_{mt}, K_t\}_{t=0}^{\infty}$ and prices $\{w_t, r_t\}_{t=0}^{\infty}$ such that:

(i) Consumers choose $\{c_{mt}, c_{bt}, k_t, \lambda_t, h_t\}$ to solve the problem

$$\max_{c_{mt}, c_{bt}, k_{t+1}, \lambda_t, h_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t [(1-\lambda_t)(U(c_{mt}) + V(1-h^*)) + \lambda_t (U(c_{bt}) + V(1-h_t))] \tag{2}$$

$$\text{s.t.} \quad (1-\lambda_t)c_{mt} + \lambda_t c_{bt} + k_{t+1} - (1-\delta)k_t \leq (1-\lambda_t)w_t + \lambda_t g(h_t) + r_t k_t \tag{3}$$

$\forall t$, given k_t, w_t, r_t and the initial capital stock k_0.

(ii) Firms choose $\{H_{mt}, K_t\}$ to solve

$$\max_{H_{mt},\, K_t} \; e^{z_t} f(K_t, H_{mt}) - w_t H_{mt} - r_t K_t \tag{4}$$

given z_t, w_t, r_t. Free entry and exit implies zero profits.

(iii) Market clearing: $k_t = K_t$, $(1-\lambda_t)h^* = H_{mt}$, $\forall t$.

(iv) Feasibility: $(1-\lambda_t)c_{mt} + \lambda_t c_{bt} + k_{t+1} - (1-\delta)k_t \leq e^{z_t} f(k_t, (1-\lambda_t)h^*) + \lambda_t g(h_t)$, $\forall t$.

It is straightforward to show that, given this framework, the Second Welfare Theorem applies. Therefore, we can specify the social planner's problem as one in which at every period t, $c_{mt}, c_{bt}, k_{t+1}, h_t$ (c_{mt} and c_{bt} denote consumption when working in registered and underground activities, respectively) and λ_t are chosen in order to solve the following maximization problem:

$$\max_{c_{mt},\, c_{bt},\, k_{t+1},\, \lambda_t, h_t} \; E_0 \sum_{t=0}^{\infty} \beta^t [(1-\lambda_t)(U(c_{mt}) + V(1-h^*)) + \lambda_t (U(c_{bt}) + V(1-h_t))] \tag{5}$$

$$\text{s.t.} \quad (1-\lambda_t)c_{mt} + \lambda_t c_{bt} + k_{t+1} - (1-\delta)k_t \leq e^{z_t} f(k_t, (1-\lambda_t)h^*) + \lambda_t g(h_t). \tag{6}$$

Standard first-order necessary conditions imply that at the optimum $c_{mt} = c_{bt}$. To see that, it suffices to show that first-order conditions imply equality of the marginal utility of consumption in all possible events. Thus, the problem can be rewritten as

$$\max_{c_t,\, k_{t+1},\, \lambda_t, h_t} \; E_0 \sum_{t=0}^{\infty} \beta^t [U(c_t) + (1-\lambda_t)V(1-h^*) + \lambda_t V(1-h_t)] \tag{7}$$

$$\text{s.t.} \quad c_t + k_{t+1} - (1-\delta)k_t \leq e^{z_t} f(k_t, (1-\lambda_t)h^*) + \lambda_t g(h_t). \tag{8}$$

The one-period objective function is continuous, strictly quasi-concave[1] in its three arguments (c_t, λ_t, h_t) and the constraint set is convex. It should be noticed

[1] Quasi-concavity of the one-period objective function, given the assumptions made on functions U and V, can be proven by the signs of the principal minors of the bordered Hessian, as shown in Arrow and Enthoven (1961).

that the choice of (λ_t, h_t) is a static problem embodied in a dynamic framework. This is clear from the first-order conditions with respect to those variables, Eqs. (10) and (11). Therefore, the Kuhn–Tucker–Lagrange conditions are necessary and sufficient for a maximum, according to the results of Arrow and Enthoven (1961).[2] In addition, strict quasi-concavity implies unicity of the solution. Since first-order conditions imply necessarily strictly positive amounts of (c_t, λ_t, h_t), the optimality conditions will be

$$U'(c_t) = E_t\{U'(c_{t+1})\beta[1 + e^{z_{t+1}} f_1(k_{t+1}, (1-\lambda_{t+1})h^*) - \delta]\}, \tag{9}$$

$$V(1-h_t) - V(1-h^*) = U'(c_t)[e^{z_t} f_2(k_t, (1-\lambda_t)h^*)h^* - g(h_t)], \tag{10}$$

$$V'(1-h_t) = U'(c_t)g'(h_t). \tag{11}$$

Condition (9) is the standard Euler condition. Condition (10) is the optimality condition for participation in registered activities. This decision is made based on the opportunity cost of not participating in market activities (the expression in brackets), that will be closely related to our definition of the wage premium. Finally, condition (11) is the optimality condition for the number of hours worked in underground activities.

4. Quantitative analysis and calibration procedure

4.1. Functional forms and steady-state computation

We choose the following functional forms:

$$U(c_t) = \log c_t, \tag{12}$$

$$V(1-h_{mt}-h_t) = A\log(1-h_{mt}-h_t), \tag{13}$$

$$f(k_t, (1-\lambda_t)h^*) = k_t^{\theta}((1-\lambda_t)h^*)^{1-\theta}, \tag{14}$$

$$g(h_t) = Bh_t^{\gamma}, \tag{15}$$

where A and B are positive constants, $\theta \in (0,1)$ and $\gamma \in (0,1)$.

The technological shock is assumed to be of the form $z_t = \rho z_{t-1} + \varepsilon_t$, where ε_t is iid with normal distribution $(0, \sigma_\varepsilon^2)$. The optimality conditions, given the

[2] See Theorem 1 in Arrow and Enthoven (1961).

functional forms assumed, yield in the deterministic steady-state the following equations:

$$1 = \beta[1 + \theta k^{\theta-1}[(1-\lambda)h^*]^{1-\theta} - \delta], \tag{16}$$

$$A\log(1-h) - A\log(1-h^*) = \frac{1}{c}\{(1-\theta)k^{\theta}[(1-\lambda)h^*]^{-\theta}h^* - Bh^{\gamma}\}, \tag{17}$$

$$\frac{A}{1-h} = \frac{1}{c}B\gamma h^{\gamma-1}. \tag{18}$$

4.2. Parametrization and calibration

According to Ghez and Becker (1975) and Juster and Stafford (1991) households allocate one-third of their discretionary time to market activities. Therefore, the natural choice of h^* is 1/3, since we are restricting ourselves to underground economic activities versus registered activities.

The model has been calibrated to the US economy. To compute the model, we follow Kydland and Prescott (1982) and approximate the objective function using a second-order Taylor series expansion around its deterministic steady state. That yields a linear quadratic approximation of the problem, implying linear decision rules.

The capital share is chosen to be $\theta = 0.36$, which is standard in the RBC literature. The underground activity parameter γ has been chosen assuming that the production function for self-employed workers in the US is of the same form (i.e. labor intensive with decreasing returns). Therefore, from the NIPA we have output and number of self-employed workers, so that we can estimate γ by OLS (roughly 0.65). Notice that this value implies that the individual production function in underground activities is roughly the same as the registered economy Cobb–Douglas production function with a fixed amount of capital. The constant A is chosen so that the steady-state value of λ matches the US average registered participation rate, for the period considered.

In order to calibrate the constant B, we proceed as follows. We assume that the technology in the underground economy is such that workers can make the minimum wage by working full time in underground activities. We follow Mankiw et al. (1992) in assuming that workers with no skill qualification would make the minimum wage. Then, the implicit assumption is that underground activities do not require any skill. We consider the wage premium (defined as the ratio of market wages to minimum wage) as a policy objective institutionally given. The justification could be that governments, through several policy instruments (minimum wage, tax policy, transfers and other welfare policy instruments), choose a certain wage premium as a policy objective, that would be considered as exogenously given in our model. Therefore, B is chosen so that in steady state and for full-time employment, the wage premium matches the ratio between average wage and minimum wage for the US. Dolado et al. (1996) derived an estimate of 2.5 for the year 1993. We will use that figure as a benchmark for simulation purposes. Thus, we introduce a constraint in the steady state computation to pin down the parameter B: $w/B(1/3)^{\gamma} = 2.5$, where w denotes the steady-state wage of an individual working full time (i.e. 1/3) in the market. Therefore, the calibration of the parameter B and the computation of the steady state are done simultaneously.

Finally, σ_{ε} is chosen to match the US aggregate registered fluctuations (standard deviation of detrended log of GDP of 0.0431), so that $\sigma_{\varepsilon} = 0.014$. Also, $\rho = 0.93$ is taken from the yearly Solow residual for the US economy.

The model has been simulated for 31 periods and reproduces the key features of the business cycle facts. However, our model displays a degree of fluctuation of investment much higher than in previous studies. Hansen (1985) showed that indivisibilities increase fluctuations of investment by 35%, but his model still fell short in accounting for the high standard deviation registered in investment (the fluctuation of investment in his model was 2/3 of that of the US economy). In our model, investment fluctuates 12%, compared to 9% in the US data. As it was to be expected, registered output fluctuations are higher than those of total production and consumption. The results are reported in the second column of Table 1.

The exercise performed implies that any person belonging to the working age population but not employed in the registered economy, will work an average of 0.0126 h in underground activities (compared to the one-third corresponding to workers in registered activities). Finally, we indirectly get an estimate of the size of the underground economy (measured by underground activities output over registered output) of roughly 3.6%. A survey of estimates based on different methodologies for European countries and the US can be found in Marrelli (1987). The estimate for the US is of 4.5% of GDP.

5. The quantitative experiment

The next exercise is to determine the extent to which exogenous changes in the wage premium can account for the wide range of cross-country differences in

Table 1
US data and computation results

	US data	WP = 2.5	WP = 1.3
Participation rate	0.4539	0.4539	0.2116
St. dv. registered output	0.0431	0.0431	0.1105
St. dv. investment	0.0904	0.1200	0.4401
St. dv. total output		0.0395	0.0408
Hours in underground econ.		0.0126	0.0903
Size undergr. sector (% of registered)		3.57	78.47

aggregate fluctuations. In that sense, our exercise will analyze how economies that only differ in their market wage premium will react to the same technological shocks.

Therefore, we keep the same parameters (except for the constant B) and technological shocks used to calibrate the US economy. A smaller wage premium will induce a lower participation rate since the opportunity cost of not participating in registered activities decreases. Then, with more population out of the registered economy, the effects of technological shocks are amplified. In response to every shock in the registered economy, more population will be switching between the two sectors. As a result, economies only differing in their wage premiums would perform differently when subject to the same technological shocks.

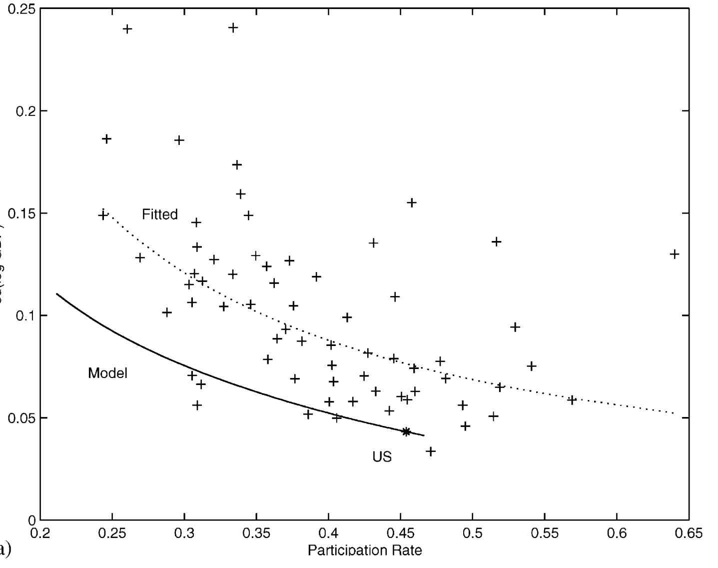
Which value should the market wage premium take to match the lowest participation rates found in the world sample given the same technological shocks.

Keeping the same parameters as before and recomputing for a wage premium of 1.3 (market average wage is 30% above what could be made working full time in underground activities) we obtain participation rates as low as 21% with standard deviations of registered output of 11%. Overall, this economy would display an underground economy sector of 78.5% of GDP.[3] Those are the numbers displayed in the fourth column of Table 1.

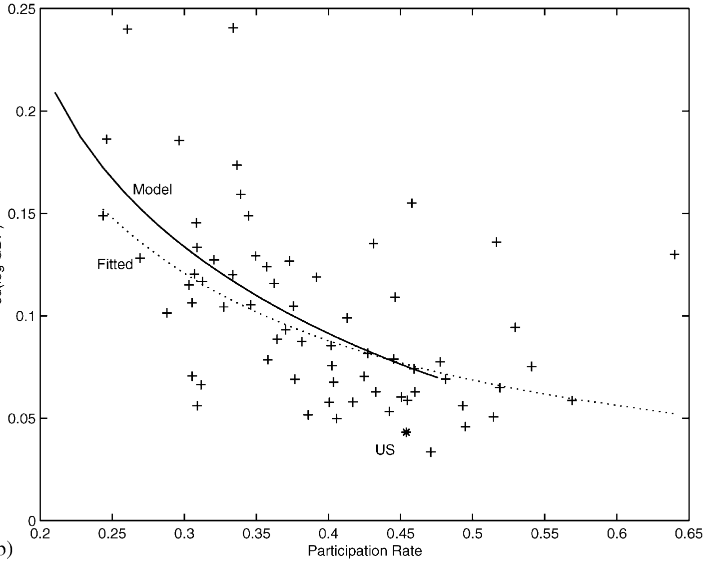
Fig. 2a shows what the model predictions are in terms of the relationship between the participation rate and the standard deviation of registered output as compared to the evidence found in the data. We use the model calibrated to the US economy and we redo the simulations for a grid of wage premia to get participation rates and fluctuations of the model economy when subject to the same technological shocks. Obviously, by construction, the model prediction goes through the US data point, which is in the lowest range of the sample. However, if we increase the variance of the stochastic component of TFP, then we obtain standard deviations of registered output in the range of those fitted to the data (see Fig. 2b). Notice then that the performance of the model is substantially closer to the data.

Finally, Fig. 3 shows the model predictions in terms of the relationship between participation rates and the standard deviation of investment. It is important to note that we observe in the data those economies with very small participation rates displaying standard deviations of investment well in the range of 40% as predicted by the model.

[3] It is important to note that registered GDP in this economy is half of what we obtained in the benchmark economy, even though total factor productivity is the same.



(a)



(b)

Fig. 2. (a) Model performance compared to data. Calibrated to US. (b) Model performance compared to data. Calibrated to fitted value.

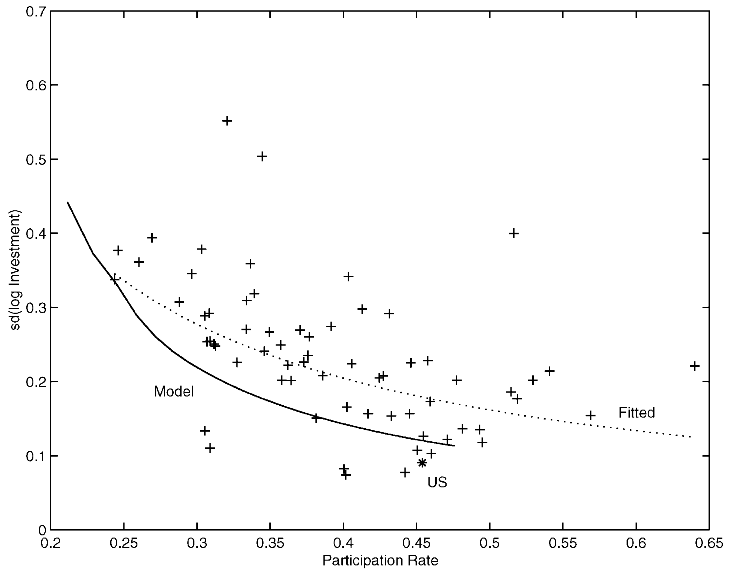


Fig. 3. Model performance compared to data, PR vs. SD (investment).

6. Conclusion

In this paper, we argue that the existence of underground economic activities is a plausible explanation for cross-country differences in aggregate fluctuations. The empirical evidence shows that there exists a negative correlation between the market participation rate and the standard deviation of registered output. Thus, we show how economies that are otherwise equal will respond differently to the same technological shocks. The reason will be that the participation rates and the size of their underground economies are different.

We construct a RBC model in which individuals will participate or not in registered economic activities according to an exogenously specified wage premium, i.e. the ratio of market wages to the minimum wage. Those individuals that do not participate in the registered economy can develop production activities through an individual production function.

The simulations performed show that our model can account for the high range of aggregate fluctuations, both in registered output and investment, found across countries. Small wage premia induce a small participation in registered activities, large size of the underground economy, and fluctuations of registered output and investment close to those found in the data.

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