



**Course:** Probability and Statistics

**Faculty:** Jordi Caballé

**Term:** First Semester

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**Office Hours:** Upon request. Send an e-mail for an appointment (Office B3-146).

**Description:**

This course will cover the topics of probability theory, random variables, distributions, and stochastic processes. It will also cover the traditional topics of sampling, estimation, and hypothesis testing, which are needed for undertaking empirical research.

Slides and exercises can be downloaded from the professor's webpage

**Objective:**

The objective of this course is to provide the mathematical foundations and the tools of Probability and Statistics that will be used in courses dealing with Economics of Uncertainty and Econometrics.

**Outline:**

**1. Probability**

Combinatorics. Events and measurable spaces. Probability. Conditional probability. Theorem of total probability. Bayes' theorem.

## **2. Random Variables and Distributions**

Random variables. Probability distributions. Distribution function of a random variable. Discrete random variables and probability functions. Absolutely continuous random variables and densities. Random vectors and marginal distributions. Independent random variables. Generalized conditional probability and distribution.

## **3. Expectation**

Mathematical expectation. Moments. Chebyshev's inequality. The moment-generating function, the characteristic function, and the Laplace transform. Product moments and covariance. Mean and variance of linear combinations of random variables. Conditional expectation. The law of iterated expectations. Jensen's inequality.

## **4. Special Distributions**

The discrete uniform, Bernoulli, binomial, Pascal, geometric, and hypergeometric distributions. The multinomial and multivariate hypergeometric distribution. Integration by parts and by change of variable to polar coordinates. The uniform, gamma, exponential, chi-square, and beta distributions. The normal and the multivariate normal distributions. Multivariate normality and linear models.

## **5. Functions of Random Variables**

The distribution of a function of a random variable. The probability function of a function of a random variable. Convolutions. The density of a function of a random variable. The characteristic function, moment-generating function, and Laplace transform of a function of a random variable. Mixture distributions.

## **6. Stochastic Processes and Limiting Distributions**

Stochastic processes. Filtrations and martingales. Convergence in probability, in mean square, in distribution, and almost sure convergence. Convergence of distribution functions and of probability measures. Markov processes. The Poisson distribution as the limit of binomial distributions. The standard normal distribution as the limit of standardized binomial distributions. Laws of large numbers. The central limit theorem.

## **7. Sampling**

Random samples and statistics. The distribution of the sample mean. The distribution of the variance of a random sample from a normal population. The  $t$  distribution. The  $F$  distribution. Order statistics.

## **8. Estimation**

Point estimation. Efficiency of estimators. The sample mean and sample variance as unbiased estimators. The Cramér-Rao lower bound for unbiased estimators. Asymptotic properties of estimators: consistent estimators. Sufficient estimators and the Rao-Blackwell theorem. The method of moments. Maximum likelihood estimation. Bayesian estimation. Interval estimation.

## 9. Hypothesis Testing

Statistical hypotheses and their tests. The power function of a test. Uniformly most powerful tests and the Neyman-Pearson lemma. The monotone likelihood ratio property and the Karlin-Rubin theorem. Likelihood ratio, Wald, and score tests. Acceptance intervals. The  $p$ -value. Contingency tables. Goodness of fit.

### References:

- Ash, R.B. (1972), Real Analysis and Probability, Academic Press.
- Bierens, H.J. (2004), Introduction to the Mathematical and Statistical Foundations of Econometrics, Cambridge University Press.
- Billingsley, P. (1995), Probability and Measure, Wiley.
- Casella, G.H. and Berger, R.L. (2002), Statistical Inference, Duxbury/Thomson Learning.
- DeGroot, M.H. and Schervish, M.J. (2012), Probability and Statistics, Pearson.
- Hogg, R.V., McKean, J. and Craig A.T. (2012), Introduction to Mathematical Statistics, Pearson.
- Lindgren, B.V. (1993), Statistical Theory, Chapman and Hall/CRC.
- Mood, A.M., Graybill, F.A. and Boes, D.C. (1974), Introduction to the Theory of Statistics, McGraw Hill.
- Rice, J.A. (2007), Mathematical Statistics and Data Analysis, Cengage Learning.

### Grading:

Students must solve a series of problem sets. Problem sets will have a weight of 20% in the final grade. There will be a final exam, which will have a weight of 80% in the final grade.

**Note:** Some of this year's exercises have appeared on this course previously. Thus, it is possible, even likely, that you might be able to obtain solutions to these exercises that I have handed out earlier. However, I strongly recommend you not to look at these solutions when solving the exercises. By handing in your answers, you declare that the solutions are your own and that they are not based on solutions from previous years. If I catch you cheating, I will give you 0 points from the exercises. Even more significantly, you will suffer a reputation loss within IDEA (and academia in general) by presenting someone else's work as your own. Check out the definition of "plagiarism" and how it is viewed in academic circles if you do not immediately grasp what the consequences of cheating will be.