

Pay-as-you-go Social Security and the Distribution of Altruistic Transfers

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This paper studies the impact of an unfunded social security system on the distribution of altruistic transfers in a framework where savings are due to both life cycle and random altruistic motivations. We show that the effect of social security on the distribution of these transfers depends crucially on the strength of the bequest motive in explaining savings behaviour. We measure this strength by the expected weight that individuals attach to the utility of future generations. On the one hand, if the bequest motive is strong, then an increase in the social security tax raises the bequests left by altruistic parents. On the other hand, when the importance of altruism in motivating savings is sufficiently low, the increase in the social security tax could result in a reduction of the bequests left by altruistic parents under some conditions on the attitude of individuals toward risk and on the relative returns associated with private saving and social security. Some implications concerning the transitional effects of introducing an unfunded social security scheme are also discussed.

1. INTRODUCTION

The effects of the social security system both on capital accumulation and on wealth distribution have received a great deal of attention among economists. Since most studies have considered life cycle economies with non-altruistic agents, the effects of social security on altruistic transfers have been mostly neglected. This seems an important omission since there is substantial evidence that intergenerational transfers are crucial for understanding capital accumulation and wealth distribution in the U.S. economy (see, for instance Kotlikoff and Summers, 1981).

This paper studies the impact of an unfunded social security system on the distribution of altruistic transfers in a framework where savings are due to both life cycle and random altruistic motivations. We show that the effect of social security on that distribution depends crucially on the importance of the bequest motive in explaining savings behaviour. We parametrize this importance by the expected weight that individuals attach to the utility of future generations. If the bequest motive is strong, then an increase in the social security tax raises the bequests left by altruistic parents. However, when the importance of altruism in motivating savings is sufficiently low, the increase in the social security tax could result in a reduction of the bequests left by altruistic parents under some conditions.

In this paper we develop a model where individuals save for both altruistic and life cycle motives that is simple enough to study distributional issues analytically. Individuals will live for two periods and they will work only during the first period. When individuals are young, they

save to finance their consumption during the second period of their life. Altruism is modelled as an uninsurable random shock on preferences as in Escolano (1992) and Dutta and Michel (1998) among others. However, in the latter paper individuals are assumed to live only for one period and all the savings accrue from the bequest motive. Of course, by not allowing the coexistence of at least two generations in the same period, such a framework is not suitable to analyse the effects of the *inter vivos* transfer implicit in the pay-as-you-go (PAYG) social security system. In our model individuals face an idiosyncratic shock that determines whether they love their children or not. The shock on altruism implies in fact that the intertemporal discount rate of utility is a random variable. Therefore, individuals will save to finance both their second period consumption and the transfer they will make to their children in case they turn out to be altruistic.

We characterize the distribution of altruistic transfers at the stationary equilibrium and study how this distribution changes with the social security tax. In this economy, the bequest motive is operative for altruistic agents depending on the values of both the interest and the discount rates. If the bequest motive is not operative at the stationary equilibrium, social security does not affect the long-run distribution of altruistic transfers since this distribution remains degenerate at zero. However, when the bequest motive is operative, social security affects the long-run distribution of transfers. Whether it increases or decreases the size of altruistic transfers depends both on the strength of the bequest motive as measured by the expected weight that individuals attach to the utility of future generations and on the return of social security relative to the return of capital.

We find that altruistic transfers increase with the size of pensions when the bequest motive is operative at the steady state and the probability of being altruistic is sufficiently high. Since an increase in pensions imposes higher mandatory transfers from the young to the old, altruistic parents find optimal to increase the size of the bequests they leave to their children. In the limit case of a probability of being altruistic equal to one, the increase in the voluntary transfers completely offsets the mandatory transfers of the PAYG social security system (see Barro, 1974). Therefore, when only a small subset of individuals are non-altruistic, a higher pension tax sparks off an increase in the wealth gap between children born in selfish families and children born in altruistic ones.

On the contrary, altruistic transfers can decrease with pension benefits when the expected weight that individuals attach to the indirect utility of their direct descendants is sufficiently low. More precisely, we prove that, if preferences exhibit decreasing absolute risk aversion and the return from saving is greater than the return of the social security system (which is given by the rate of population growth), such a decrease of transfers takes place and, thus, the wealth gap between “lucky” children born in altruistic families and “unlucky” children born in selfish ones becomes smaller. The intuition behind this result is one of insurance: individuals face wealth uncertainty because altruistic transfers are random. When the return from saving is greater than that of the social security, an increase in the social security tax reduces the present value of individuals’ lifetime income. Thus, with decreasing absolute risk aversion, in which case individuals dislike risk more the poorer they are, parents reduce the transfers they make so as to reduce the next period gap between consumption in the event of being altruistic and consumption in the event of being selfish. These lower transfers translate in fact into a reduction in the wealth uncertainty of their children, which in turn results in a lower wealth gap between lucky and unlucky children.

Another result of our paper refers to the transitional effects of social security. As we have said, the altruistic individuals of the economy could decide not to leave bequests in the steady state, that is, the bequest motive could be non-operative under some parametric restrictions (see Weil, 1987). In this case, the introduction of an unfunded social security system could force altruistic individuals to leave some bequests so as to absorb the initial impact of the mandatory transfers inherent in the social security system. Of course, non-altruistic individuals will remain

leaving zero bequests. Therefore, the introduction of social security could induce a transitional dynamics characterized by inequality in the distribution of wealth at birth. Such effect is just transitory since the bequests left by all the individuals will converge to zero in the long run.

Among the papers analysing the impact of fiscal policies on the distribution of bequests, we should mention the ones of Becker and Tomes (1979), Bevan and Stiglitz (1979) Atkinson (1980) and Davies (1986). These authors focus on intragenerational redistributive policies and their effects on wealth and income distribution. Our paper contributes to this literature with an analytical study on the effects of a PAYG social security system on the distribution of altruistic transfers.

The papers of Abel (1985) and Michel and Pestieau (1998) deserve special attention since they also look at the distribution of intergenerational transfers. Abel (1985) analyses the impact of social security in a framework where bequests are accidental and only arise because of lifetime uncertainty.¹ He finds that social security reduces accidental bequests, since it annuitizes the wealth of individuals. In his model the fraction of savings made compulsory by the social security is returned as a pension only if the corresponding individual survives. Therefore, thanks to this public provision of annuities, individuals will make less voluntary savings and thus accidental bequests will be smaller. Obviously, this results in a reduction of the intracohort variance of wealth. Michel and Pestieau (1998) consider instead a model where bequests are altruistically motivated and they obtain a non-degenerate distribution of intergenerational transfers by assuming that there are two types of dynasties: the altruistic and the selfish ones. In this case individuals know when they are young whether they are altruistic or not and, hence, they do not face any kind of individual risk. Obviously, an increase in the social security tax in this scenario would always increase the bequest left by all the altruistic agents in order to undo the transfer imposed by the social security system. This will also increase the wealth gap between the two types of dynasties, which is exactly the opposite of the result obtained by Abel. In our model we find that the distributional effects of social security could be ambiguous in the long run when the bequest motive is operative depending on the expected strength of that motive. This result differs from those obtained by Abel and Michel and Pestieau since, unlike Abel, we characterize the impact of PAYG social security on the distribution of intergenerational transfers when these are not accidental but altruistically motivated and, unlike Michel and Pestieau, the strength of the bequest motive is uncertain from the young individuals' viewpoint. Therefore, our model provides a unified framework where the aforementioned opposite effects could take place.

In a related paper Karni and Zilcha (1989) examine the effects of social security on income distribution. They emphasize the fact that, due to general equilibrium effects, social security decreases the return of labour relative to the return of capital. This effect leads to an increase in income inequality in their model because the only source of heterogeneity is an exogenous distribution of bequests (or initial capital). In contrast, our paper focuses on the effect of social security on the distribution of bequests, which is no longer viewed as exogenous.

We should point out that random altruism can be interpreted as a shock on the intertemporal discount rate of utility and that recently some authors have used models with stochastic discount factors.² In fact, random discounting has proven a useful device for generating realistic wealth heterogeneity in quantitative models.

An extensive empirical literature shows that *inter vivos* transfers and bequests play a significant role in real economies. For instance, Kotlikoff and Summers (1981) find that at least 80% of the U.S. capital stock is accumulated from *inter vivos* transfers and bequests, Gale and

1. Among other papers analysing the effects of social security under uncertain lifetimes we could mention the ones of Sheshinski and Weiss (1981) and Eckstein, Eichenbaum and Peled (1985).

2. See, for instance, Krusell and Smith (1998) or Karni and Zilcha (2000).

Scholz (1994) focus on the importance of *inter vivos* transfers in the U.S. economy, Laitner and Juster (1996) find that 50% of individuals save to leave an estate, Wolff (1999) shows that *inter vivos* transfers account for about one third of the lifetime accumulation of wealth in the U.S. for the period 1962–1992 and Mirer (1979) finds that retired individuals tend to increase their wealth over time. Furthermore, intergenerational altruism appears one of the most likely candidates for explaining such a substantial amount of intergenerational transfers. For instance, recent empirical work has found evidence that private transfers are increasing in the income of the donor and decreasing in the income of the recipient (see, for example Cox, 1987). Moreover, Leimer and Lesnoy (1982) find that private savings respond by very little to the existence of social security in the U.S. economy, which is consistent with the altruistic motive for savings. Of course, other explanations of the process leading to intergenerational transfers have been proposed like, for instance, strategic behaviour (Bernheim, Schleifer and Summers, 1985), joy-of-giving (Yaari, 1965) or the existence of an incomplete annuity market (Kotlikoff and Spivak, 1981). However, Bernheim (1991) argues that intergenerational transfers are not a consequence of incomplete annuity markets. In particular, he finds that social security annuity benefits decrease private annuities and increase life insurance holdings among elderly individuals, which is consistent with the existence of a bequest motive. However, the alternative theories aimed at explaining intergenerational transfers are not mutually exclusive and the available empirical evidence is not conclusive either. In particular, altruism-motivated transfers seem to play an important role for individuals enjoying high levels of income and wealth (see Hurd, 1987).

The paper is organized as follows. The next section describes the economic environment and the problem faced by individuals. Section 3 studies the dynamics of altruistic transfers and the operativeness of the bequest motive both in the short term and in the long run. Section 4 shows the existence and uniqueness of the invariant distribution of altruistic transfers and characterizes this distribution. Section 5 focuses on the effects of social security on the distribution of transfers when the bequest motive is not operative in a steady state, whereas Section 6 performs the comparative statics analysis for economies where the bequest motive is always operative for altruistic agents. Some extensions of our basic model are presented in Section 7. Finally, Section 8 concludes the paper. All the proofs appear in the Appendix.

2. THE MODEL

Let us consider an overlapping generations economy in discrete time with a continuum of individuals in each period. A generation of individuals with identical *ex ante* preferences is born in each period and individuals live for two periods. At the end of the first period of his life each agent has $N \geq 1$ children so that an individual is young when his parent is old.

When individuals become old, they realize if they love their children, that is, they know if they are altruistic or not. If an individual is altruistic, the indirect utility of each of his children appears as an argument in his utility function. That event occurs with probability $\pi \in (0, 1)$. We will assume that the total utility obtained by an altruistic old individual is the sum of the utility derived from his consumption and the discounted indirect utilities of their children. Old individuals are selfish with probability $1 - \pi$ and, then, they derive utility only from their own consumption. Therefore, in this large economy a mass π of individuals is altruistic whereas a mass $1 - \pi$ turns out to be selfish. There are no markets to buy insurance against the risk of becoming altruistic towards children.

We will assume that the economy under consideration exhibits constant interest rates and constant wages.³ Let $R > 0$ be the constant one-period gross rate of return on saving.

3. The assumption of constant rental prices for labour and capital could be the result of considering a small open economy with perfect capital mobility and no labour mobility. This means that the interest rate is constant and

Young individuals are endowed with a unit of labour time. They supply their labour endowment inelastically in exchange for the constant wage w . Old individuals are retired.

There is a government that administrates a balanced PAYG social security system. Young individuals contribute to the system by paying a constant lump-sum tax P with $0 \leq P < w$.⁴ Therefore, each old individual receives a pension benefit equal to NP .

The utility that an individual derives from his own consumption in each period is represented by a utility function u defined on $\mathbb{R}_{++}$ which is assumed to be bounded and twice continuously differentiable with $u' > 0$, $u'' < 0$ and to satisfy the Inada conditions $\lim_{c \rightarrow 0} u'(c) = \infty$ and $\lim_{c \rightarrow \infty} u'(c) = 0$.⁵

Young individuals only differ in the inheritance they receive from their respective parents. A young individual that has received the inheritance b from his parent solves the following stochastic dynamic programming problem when the pension is fixed at the level P :

$$V_y(b; P) = \max_{\{c_y, c_s, s\}} u(c_y) + \beta\{(1 - \pi)u(c_s) + \pi V_a(s; P)\}, \quad (2.1)$$

subject to

$$\begin{aligned} c_y &= w + b - P - s \geq 0, \\ c_s &= Rs + NP \geq 0, \end{aligned}$$

where c_y is the consumption of a young individual, s is the saving, c_s is the consumption of a selfish old individual, $V_y(b; P)$ is the value function of a young individual who has received an inheritance b when the level of pensions is P , and $V_a(s; P)$ is the value function of an altruistic old individual who has saved the amount s in his first period of life when the level of pensions is P . Note that the two weak inequalities in the constraints imply that $b \geq -w + P\left(\frac{R-N}{R}\right)$, where $-w + P\left(\frac{R-N}{R}\right) < 0$ since $P \in [0, w]$.⁶ The coefficient $\beta > 0$ is the time discount factor.

An altruistic old individual who saved the amount s when he was young solves the following programme:

$$V_a(s; P) = \max_{\{c_a, b'\}} u(c_a) + \delta N V_y(b'; P), \quad (2.2)$$

subject to

$$\begin{aligned} c_a &= Rs + NP - Nb' \geq 0, \\ b' &\geq 0, \end{aligned} \quad (2.3)$$

where c_a is the consumption of an altruistic old individual and b' is the bequest that he leaves to each of his children, respectively. The coefficient $\delta > 0$ is the discount factor on the indirect utility of each descendant. Concerning the inequality constraint (2.3), we should notice that, since

equal to its international level. Under a standard neoclassical production function and competitive input markets, the equilibrium capital-labour ratio turns out to be constant and, thus, the marginal productivity of labour (which is equal to the competitive real wage) is also constant.

4. When labour supply is inelastic, a lump-sum social security tax is equivalent to a flat rate tax on wages since no distortion is introduced in the labour market.

5. Instead of assuming that u is bounded we could assume specific functional forms that are not bounded but that are quite common in the literature. For instance, if we assume isoelastic preferences, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ with $\sigma > 0$, then all the results of this paper are still true when the condition $\beta\delta N\pi R^{1-\sigma} < 1$ for bounded value functions holds. Another commonly used utility function is the CARA $u(c) = -e^{-\gamma c}$ with $\gamma > 0$. All the main results of the paper apply for this function in spite of not satisfying the Inada condition at the origin.

6. Even if individuals can only leave non-negative bequests, we allow the received inheritance b to lie on the interval $\left(-w + P\left(\frac{R-N}{R}\right), \infty\right)$ in the individual optimization problem. Since $0 \in \left(-w + P\left(\frac{R-N}{R}\right), \infty\right)$, we are thus ensuring that the properties of the value function $V_y(\cdot; P)$ and of its corresponding policy functions, which will be presented in Lemmas 1 and 2, hold for all the potential equilibrium values of b .

altruism is unidirectional (it goes from parents to children) and there are neither institutions nor contracts to enforce liabilities on future generations, individuals cannot leave negative bequests to their children.

Plugging programme (2.1) on programme (2.2) and using the corresponding budget constraints, we obtain the following stochastic dynamic programming problem:

$$V_a(s; P) = \max_{\{b', s'\}} u(Rs + NP - Nb') + \delta Nu(w - P + b' - s') + \beta\delta N(1 - \pi)u(Rs' + NP) + \beta\delta N\pi V_a(s'; P) \quad (2.4)$$

subject to

$$\begin{aligned} Rs + NP &\geq Nb' \geq 0, \\ w - P + b' &\geq s' \geq -\frac{NP}{R}, \end{aligned}$$

where s' is the saving of each direct descendant of the agent under consideration.⁷ Note that the Inada conditions on the utility function imply that c_y , c_a and c_s are all strictly positive so that the solution of the programme (2.4) must satisfy $w - P + b' > s' > -\frac{NP}{R}$ and $Rs + NP > Nb'$. Of course, it remains the possibility of a corner solution for the altruistic transfer, namely, that $b' = 0$. Moreover, in order to have a well defined problem, we need that $s > -\frac{NP}{R}$ since neither c_a nor c_s could be positive otherwise. Finally, we assume that $\beta\delta N\pi < 1$ as $\beta\delta N\pi$ is in fact the discount factor of programme (2.4).

Let us define the policy functions corresponding to the previous programmes when the pension level is P . Concerning programme (2.1), we define the policy functions $c_y = \hat{c}_y(b; P)$, $c_s = \hat{c}_s(b; P)$, and $s = \hat{s}(b; P)$ for consumption of a young individual, consumption of a selfish old individual, and saving, respectively. For programme (2.2) we define $c_a = \hat{c}_a(s; P)$ and $b' = \hat{b}(s; P)$ as the policy functions of an altruistic old individual for consumption and bequest *per capita*, respectively. We also define the composite function $g(b; P) \equiv \hat{b}(\hat{s}(b; P); P)$, which is the transfer function giving the bequest *per capita* left by an altruistic old individual who had received the amount b from his parent when the pension remains fixed at the level P .

The following lemma establishes a basic result about the existence of a unique solution to the previous programmes. Its proof is omitted since it follows immediately from applying, for instance, Theorems 4.6–4.10 of Stokey and Lucas (1989) to programme (2.4). The existence and the properties of the value function $V_y(\cdot; P)$ follow directly from the existence and properties of the value function $V_a(\cdot; P)$ through programme (2.1).

Lemma 1. *There exists a unique value function $V_a(\cdot; P)$ associated with programme (2.4) which is continuously differentiable, strictly increasing, and strictly concave on $(-\frac{NP}{R}, \infty)$. The value function $V_y(\cdot; P)$ exists and is continuously differentiable, strictly increasing and strictly concave on $(-w + P(\frac{R-N}{R}), \infty)$. The programmes (2.1) and (2.2) have a unique solution, that is, the policy functions $\hat{c}_y(\cdot; P)$, $\hat{c}_s(\cdot; P)$, $\hat{c}_a(\cdot; P)$, $\hat{b}(\cdot; P)$ and $\hat{s}(\cdot; P)$ exist. The functions $\hat{c}_y(\cdot; P)$, $\hat{c}_s(\cdot; P)$, $\hat{s}(\cdot; P)$ and $g(\cdot; P)$ are all continuous on $(-w + P(\frac{R-N}{R}), \infty)$, whereas the functions $\hat{c}_a(\cdot; P)$ and $\hat{b}(\cdot; P)$ are continuous on $(-\frac{NP}{R}, \infty)$.*

Substituting the consumptions in the objective function of programme (2.1) and differentiating with respect to s , we obtain the following first-order condition:

$$u'(w + b - P - s) = \beta\{(1 - \pi)Ru'(Rs + NP) + \pi V'_a(s; P)\}, \quad (2.5)$$

7. We are implicitly assuming in problem (2.4) that parents choose the amount of saving s' of their children. This can be safely assumed since the sequentiality of the problem ensures that the saving of each son depends exclusively on the bequest he receives. Such a sequentiality prevents any kind of strategic behaviour between parents and children.

and the corresponding envelope condition is

$$V'_y(b; P) = u'(w + b - P - s). \quad (2.6)$$

Concerning programme (2.2), we substitute the consumption c_a into the objective function and differentiate with respect to b' to obtain the following first-order condition:

$$u'(Rs + NP - Nb') \geq \delta V'_y(b'; P), \quad \text{with } u'(Rs + NP - Nb') = \delta V'_y(b'; P) \text{ if } b' > 0, \quad (2.7)$$

and the corresponding envelope condition is

$$V'_a(s; P) = Ru'(Rs + NP - Nb'). \quad (2.8)$$

The following lemma provides additional properties of the policy functions:

Lemma 2. *The policy functions $\hat{c}_y(\cdot; P)$, $\hat{c}_s(\cdot; P)$, $\hat{c}_a(\cdot; P)$, $\hat{s}(\cdot; P)$ are all strictly increasing. The policy function $\hat{b}(\cdot; P)$ is non-decreasing and locally strictly increasing when $\hat{b}(s; P) > 0$. Moreover, the function $g(\cdot; P)$ is non-decreasing and locally strictly increasing when $g(b; P) > 0$.*

3. THE DYNAMICS OF ALTRUISTIC TRANSFERS WITHIN A DYNASTY

In this section we provide more properties of the function $g(\cdot; P)$ defining the bequest left by an altruistic individual as a function of the inheritance he has received from his parent under a stationary pension system. Note that $\beta\delta$ is the discount factor that a young individual would apply to the indirect utility of each of his sons if he were altruistic. This discount can be decomposed between the pure time discount β and the pure interpersonal discount δ . Several cases arise depending on both the discount factor $\beta\delta$ and the gross rate of return from saving R . The next proposition shows that, when the real interest rate $R - 1$ is lower than the discount rate $(1/\beta\delta) - 1$, the sequence of altruistic transfers is strictly decreasing even if all the members of a dynasty turn to be altruistic. Moreover, this sequence of transfers converges to zero.

Proposition 1. *Assume that $R\beta\delta \leq 1$. Then $g(b; P) < b$ for all $b > 0$, and $g(0; P) = 0$.*

Note that the previous proposition characterizes the shape of the mapping $b_{t+1} = g(b_t; P)$. Clearly, under the assumption of the proposition, the sequence of altruistic transfers within a family formed exclusively by altruistic individuals satisfies $b_{t+1} < b_t$ for all $b_t > 0$, and since $g(0; P) = 0$ it follows that $\lim_{t \rightarrow \infty} b_t = 0$.

The next proposition strengthens the previous one since it gives a sufficient condition for the altruistic transfers to become zero after a finite history of altruistic individuals within a dynasty. Such a sufficient condition is obtained by just making strict the weak inequality which was assumed in Proposition 1, that is, the interest rate should be strictly lower than the discount rate on utility.

Proposition 2. *Assume that $R\beta\delta < 1$. Then there exists a threshold level of altruistic transfers $\underline{b}(P) > 0$ such that $g(b; P) = 0$ if and only if $b \leq \underline{b}(P)$.*

Obviously, the shape of the mapping $b_{t+1} = g(b_t; P)$ implied by the previous proposition leads to a sequence of altruistic transfers that becomes zero after some period. This is so because there exists a period T^* for which $g(b_{T^*}; P) \leq \underline{b}(P)$ and, hence, $b_t = 0$ for all $t > T^*$. Figure 1 describes the dynamics of altruistic transfers within a dynasty formed exclusively by altruistic individuals under the assumption made in Proposition 2. However, as follows from Proposition 1,

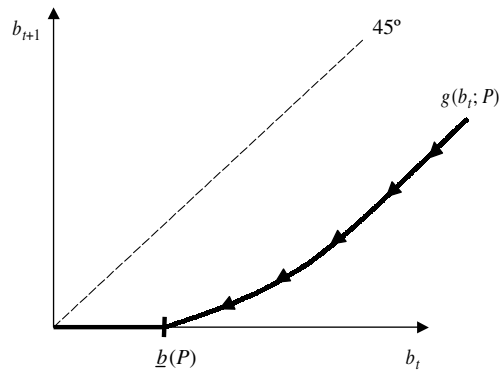


FIGURE 1

The dynamics of altruistic transfers when $R\beta\delta < 1$. Transfers converge to zero in finite time

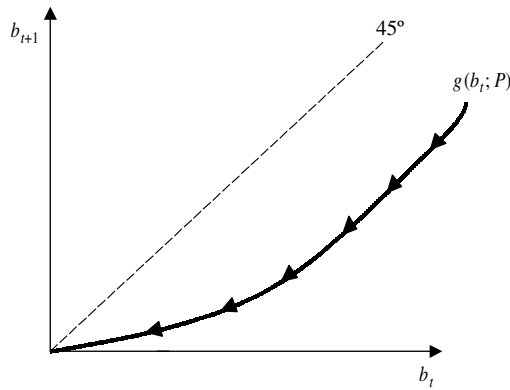


FIGURE 2

If $R\beta\delta = 1$ altruistic transfers converge to zero but not necessarily in finite time

we can only guarantee the asymptotic convergence of transfers to zero when $R\beta\delta = 1$. In this case the zero value for altruistic transfers is not necessarily reached in finite time (see Figure 2).

The next proposition shows that the bequest motive is always operative when both the interest rate and the discount factor are high enough.

Proposition 3. *If $R\beta\delta > 1$, then $g(b; P) > 0$ for all $b \geq 0$.*

Figure 3 depicts the dynamics of altruistic transfers when $R\beta\delta > 1$. Notice that the previous proposition leaves open the possibility that the sequence of transfers generated by the difference equation $b_{t+1} = g(b_t; P)$ be either bounded or unbounded. We will come back briefly to this issue in Section 4.

The properties of the transfer function $g(\cdot; P)$ we have just discussed will be extensively used in the next section in order to explore the existence and the properties of the stationary distribution of altruistic transfers in this economy.

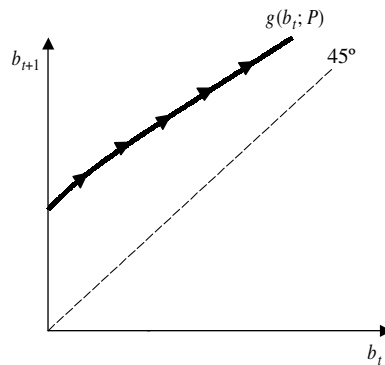


FIGURE 3

Altruistic transfers are always strictly positive when $R\beta\delta > 1$

4. THE DISTRIBUTION OF ALTRUISTIC TRANSFERS

The distribution of altruistic transfers across individuals in each period is a probability measure defined on the measurable space $(\mathbb{R}_+, \mathcal{B})$ where $\mathcal{B}$ is the σ -algebra of Borel sets of $\mathbb{R}_+$. In this section we will show that, given a constant level P of the pension, the probability measure of transfers converges to a unique invariant (or stationary) probability measure $\bar{\mu}(\cdot; P)$ on $(\mathbb{R}_+, \mathcal{B})$.

In this large economy a proportion $1 - \pi$ of individuals receives a zero transfer from their parents while a proportion π of individuals receives a transfer governed by the function $g(\cdot; P)$. Clearly, if an individual has an altruistic parent who received the inheritance b_t , then he will receive an inheritance equal to $g(b_t; P)$. Hence, the law of motion of altruistic transfers within a dynasty is the following:

$$b_{t+1} = \begin{cases} 0 & \text{with probability } 1 - \pi, \\ g(b_t; P) & \text{with probability } \pi. \end{cases}$$

Therefore, the distribution of altruistic transfers $\mu_t(\cdot; P)$ evolves along time as dictated by the following functional equation:

$$\mu_{t+1}(B; P) = (1 - \pi)\mathcal{I}_B(0) + \pi \int_{g^{-1}(B; P)} \mu_t(db; P), \quad (4.1)$$

where $\mathcal{I}_B$ is the indicator function of the Borel set B and

$$g^{-1}(B; P) = \{b \geq 0 \text{ such that } g(b; P) \in B\}.$$

Proposition 4. *There exists a unique probability measure $\bar{\mu}(\cdot; P)$ on the measurable space $(\mathbb{R}_+, \mathcal{B})$ such that, for every initial distribution of altruistic transfers $\mu_0(\cdot; P)$, the sequence of distributions defined by equation (4.1) satisfies*

$$\lim_{t \rightarrow \infty} |\mu_t(B; P) - \bar{\mu}(B; P)| = 0, \quad \text{for all } B \in \mathcal{B},$$

and the convergence is uniform for all sets in $\mathcal{B}$.

The next two propositions characterize the invariant distribution $\bar{\mu}(\cdot; P)$ of altruistic transfers for different levels of interest and discount rates. Note that such an invariant distribution satisfies

$$\bar{\mu}(B; P) = (1 - \pi)\mathcal{I}_B(0) + \pi \int_{g^{-1}(B; P)} \bar{\mu}(db; P), \quad (4.2)$$

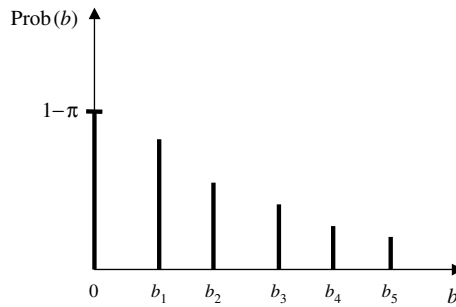


FIGURE 4

The distribution of altruistic transfers when $R\beta\delta > 1$

as follows from (4.1). We will show that the stationary distribution of altruistic transfers is degenerate and has unitary mass at zero when the interest rate is lower than the discount rate. Conversely, if the interest rate is higher than the intertemporal discount rate, the invariant distribution of altruistic transfers is non-degenerate.

Proposition 5. *If $R\beta\delta \leq 1$, then the invariant distribution of altruistic transfers is degenerate at zero, $\bar{\mu}(\{0\}; P) = 1$. Moreover, if $R\beta\delta < 1$ then the convergence to this degenerate distribution is achieved in finite time, that is, there exists a $T^* > 0$ such that, for all $t > T^*$,*

$$\mu_t(B; P) = \bar{\mu}(B; P), \quad \text{for all } B \in \mathcal{B}.$$

Proposition 6. *If $R\beta\delta > 1$, then the invariant distribution of altruistic transfers is non-degenerate and is given by the probability measure satisfying $\bar{\mu}(\{b_i\}; P) = (1 - \pi)\pi^i$, for $i = 0, 1, \dots$, where $b_{i+1} = g(b_i; P)$ and $b_0 = 0$.*

Figure 4 provides a picture of the probability function associated with the measure $\bar{\mu}$ when $R\beta\delta > 1$.

Clearly, the average altruistic transfer under the stationary distribution is $(1 - \pi) \sum_{i=0}^{\infty} \pi^i b_i$, where $b_{i+1} = g(b_i; P)$, and $b_0 = 0$. The support of the stationary distribution $\bar{\mu}(\cdot; P)$ could be either bounded or unbounded depending on the parameter constellation. In fact, when $R\beta\delta > 1$ and the stationary distribution is thus non-degenerate, it can be proved that the support of $\bar{\mu}(\cdot; P)$ is bounded for $R \leq N$, while it is unbounded for $R\beta\delta\pi \geq 1$ (see Caballé and Fuster, 2000). Note that these sufficient conditions for having either bounded or unbounded support are independent of the pension tax.

It should be noticed that the timing of events assumed in our model plays a crucial role in order to get a non-degenerate distribution of altruistic transfers when $R\beta\delta > 1$. In our OLG model with individuals living for two periods, if young individuals received the inheritance after they had made their consumption and saving decisions, no heterogeneity on the amount of bequests will arise. To see why in this scenario the distribution of bequests would have all its mass concentrated in a single point, simply note that old individuals would observe the realization of the shock on the altruism factor after their consumption had taken place. In this case, old individuals will save for precautionary reasons as they face the risk of being altruistic towards their children. If they turn out to be altruistic, they will use that precautionary saving to make a voluntary transfer to their direct descendants, while if they turn out to be selfish, they will leave exactly the same amount as accidental bequest, since they will have already consumed. Note that in order to get a non-degenerate distribution of bequests we need individuals observing the

realization of the altruism factor towards their children before consumption in the last period of their life occur. Our model contains thus the minimal set of ingredients with the previous feature.

5. TRANSITIONAL EFFECTS OF SOCIAL SECURITY WHEN THE BEQUEST MOTIVE IS INOPERATIVE IN THE LONG RUN

In this section we analyse how an unanticipated permanent introduction of a PAYG social security system affects individual decisions and the corresponding distribution of altruistic transfers when the bequest motive is not operative in the long run, that is, when $R\beta\delta \leq 1$. Note that, when the introduction of the social security scheme takes place, the saving of an old agent is already fixed and is equal to $\hat{s}(0; 0)$, whereas the bequest he leaves if he becomes altruistic will be $\hat{b}(\hat{s}(0; 0); P)$, which is not necessarily equal to either $g(0; 0) \equiv \hat{b}(\hat{s}(0; 0); 0)$ or $g(0; P) \equiv \hat{b}(\hat{s}(0; P); P)$. Moreover, young individuals at the moment of the policy change will select their saving according to the function $\hat{s}(\cdot; P)$. We will see that, when interest rates are low, there is a range of pension levels for which the unanticipated introduction of the social security system does not even affect the distribution of altruistic transfers in the short run, since it remains degenerate at zero. However, for higher values of the interest rate, there is a threshold level of pensions above which the introduction of the social security induces a non-degenerate distribution of altruistic transfers in the short run. Such a distribution will converge in the long-run to the degenerate one in accordance with Propositions 4 and 5.

Before stating the precise result, we need to establish the following lemma characterizing the optimal decisions of a young individual when the bequest motive is not operative:

Lemma 3. *Let $P_1 > P_2$ and assume that $g(b; P_1) = g(b; P_2) = 0$, then*

$$\hat{s}(b; P_1) < \hat{s}(b; P_2) \quad \text{and} \quad \hat{c}_y(b; P_1) \geq \hat{c}_y(b; P_2) \quad \text{if } R \leq N.$$

Proposition 7. *Let $R\beta\delta < 1$ and assume that the initial distribution of altruistic transfers is the degenerate, invariant one (given in Proposition 5).*

- (a) *If $R \leq N$, then there exists a pension tax $\underline{P} > 0$ such that the distribution of altruistic transfers remains degenerate at zero after the unanticipated introduction of a PAYG social security system with a constant pension tax $P \in (0, \min\{\underline{P}, w\})$.*
- (b) *If $R \geq N$, then there exists a pension tax $\bar{P} \in (0, w)$ such that*
 - (b.i) *positive altruistic transfers appear when the PAYG social security system is unanticipatedly introduced with a constant pension tax $P \in (\bar{P}, w)$. In this case, the distribution of altruistic transfers becomes non-degenerate when social security is introduced and it converges back to the degenerate distribution;*
 - (b.ii) *the distribution of altruistic transfers remains degenerate at zero after the unanticipated introduction of a PAYG social security system with a constant pension tax $P \in (0, \bar{P})$.*

The previous proposition tells us that, in order to obtain a transitory effect on the distribution of altruistic transfers, we need an introduction of the PAYG social security with a pension tax P sufficiently large. Note that, if the interest rate is higher than the implicit rate of return of the social security system, then the present value of lifetime income of the descendants will decrease after the introduction of social security. Clearly, $w > w - P + \frac{NP}{R}$ if and only if $R > N$. In this situation, altruistic parents will react by returning part of the pension they receive to their

descendants. However, the sons of non-altruistic parents will not enjoy such a compensation and this would give rise to a non-degenerate distribution of altruistic transfers. This means that the introduction of the PAYG system would be the source of some inequality of initial wealth in the short run if and only if the pension is introduced at a level larger than $\bar{P}$. Such an effect is indeed transitory when $R\beta\delta < 1$, since altruistic transfers tend to zero in the long run even within dynasties displaying an infinite sequence of altruistic individuals.⁸ Needless to say, the non-degenerate distribution of transfers in the short run is associated with a corresponding non-degenerate distribution of both consumption and saving through their respective policy functions.

6. LONG-RUN EFFECTS OF SOCIAL SECURITY WHEN THE BEQUEST MOTIVE IS ALWAYS OPERATIVE

In this section we will explore how a permanent marginal change in the social security tax affects both the bequest that individuals leave to their descendants and the long-run distribution of intergenerational transfers when the bequest motive is always operative for altruistic agents. This situation occurs when $R\beta\delta > 1$ and, thus, the stationary distribution of altruistic transfers is non-degenerate in this case.

If the bequest motive were non-random and always operative within a dynasty, then individuals would adjust their transfers in order to completely offset the change in the pension level. In fact, individuals would only care about the net intergenerational transfer $b - P$ from parents to each descendant. This means that when P increases, the transfer b should increase by the same amount. However, if the altruism is random, individuals could react by decreasing their saving when the pension level increases. In fact this is the typical reaction of non-altruistic agents, as the social security system shifts the individuals' income from the first period of life to the second one. As we will prove in this section, when the probability of being altruistic is low, then such a reduction in the amount of savings could encompass a reduction of the bequests left by the few individuals that turn out to be altruistic.

The next proposition holds for economies populated basically by altruistic individuals:

Proposition 8. *Let $R\beta\delta > 1$ and $P_1 > P_2$. There exists a $\hat{\pi} > 0$ such that $g(b; P_1) > g(b; P_2)$ for all $\pi \in (\hat{\pi}, 1)$. Moreover, $\lim_{\pi \rightarrow 1} \frac{\partial g(b; P)}{\partial P} = 1$.*

Note that the implication of the previous proposition is that the stationary average size of altruistic transfers of this economy increases with the pension level when the probability π of being altruistic is high enough and the altruistic agents leave positive bequests ($R\beta\delta > 1$). Moreover, if we divide the population of the economy into two groups of individuals: the ones that receive inheritances and the ones that do not, then all the values of the discrete support of the stationary distribution of altruistic transfers for the former group will suffer an increase when the pension is raised. Obviously, the inheritances for the latter group remain unaltered at zero. This means that the expected wealth gap at birth between descendants of non-altruistic parents and descendants of altruistic ones increases with the pension tax in this scenario. Figure 5 describes the effect of raising the pension tax on the probability function associated with the invariant distribution $\bar{\mu}$ when $g(b; P)$ increases with P . We see in that figure that all the strictly positive points in the support of the invariant distribution of altruistic transfers shift to the right. A final obvious implication of the previous proposition and Figure 5 is that the distribution of transfers

8. In a general equilibrium framework, an increase in the pension level may have a permanent effect on the distribution of bequests. Indeed, a numerical example in Section 7 shows that an increase in the pension level could induce the interest rate to become higher than $1/\beta\delta$, and this results in a non-degenerate distribution of transfers in the long run.

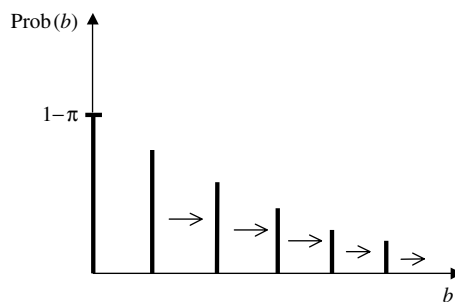


FIGURE 5

The effect of a rise in the social security tax on the distribution of altruistic transfers when $g(b_t; P)$ increases with P

associated with a lower pension is dominated by the distribution with a higher pension in the sense of first-order stochastic dominance. This is so because the probability of having a transfer lower than a given value $\bar{b}$ falls as the pension P increases, for all $\bar{b} > 0$ (see Hadar and Russell, 1969).

It is possible however to find examples for which the increase in the pension tax results in a reduction of the expected difference in wealth at birth between the two population groups we have just defined. Proposition 8 tells us that in order to find such examples we will need an operative bequest motive and, simultaneously, a low value of π . Moreover, if the individuals' preferences exhibit decreasing (increasing) absolute risk aversion (see Pratt (1964) and Arrow (1970)), we will also need to impose that the gross return R from savings be higher (lower) than the gross rate N of population growth. When individuals are assumed to be non-altruistic with a very high probability, their decisions concerning their profile of consumption are mostly driven by the present value of their lifetime income and they abstract from bequest considerations. If $R > N$, the increase in the pension P translates into a decrease in the present value of the lifetime income ($w + b - P + \frac{NP}{R}$) of all agents. This in turn makes agents more (less) risk averse under decreasing (increasing) absolute risk aversion. Therefore, the few altruistic parents of this economy would react by decreasing (increasing) the bequest left to their sons. This is so because the increase (decrease) in risk aversion induces agents to decrease (increase) the difference between the old consumption corresponding to the event of being altruistic and the one corresponding to the event of being selfish. Of course, the opposite argument applies when $R < N$. The propositions of this section will formalize the previous discussion by making explicit the interaction between the wealth effect and the behaviour of the risk attitude.

The next proposition applies to an economy populated basically by non-altruistic agents, where every altruistic old agent attaches to the indirect utility of his children a weight that is at least as large as the one attached to his own utility, i.e. δ is assumed to be larger than or equal to one.

Proposition 9. *Let $\delta \geq 1$, $R\beta > 1$ and $P > 0$. There exists a $\pi^* > 0$ such that $\frac{\partial g(b; P)}{\partial P} < 0$ for all $\pi \in (0, \pi^*)$ whenever any of the following two conditions is satisfied:*

- (i) $R > N$ and the index of absolute risk aversion of u , $\mathcal{A} = -u''/u'$, is strictly decreasing.
- (ii) $R < N$ and the index of absolute risk aversion of u is strictly increasing.

Concerning the distribution of altruistic transfers (and, thus, of consumptions and savings), we see that, in the scenario depicted by the assumptions of Proposition 9, an increase in the

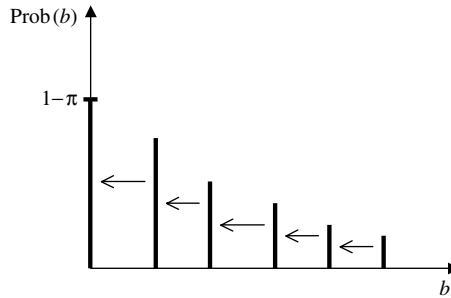


FIGURE 6

The effect of a rise in the social security tax on the distribution of altruistic transfers when $g(b_t; P)$ decreases with P

pension P results in a reduction of the bequests left by all the altruistic agents so that the expected wealth gap at birth between children of altruistic parents and children of egoist ones decreases with the pension level. Note that this is in sharp contrast to the situation where the probability of being altruistic was sufficiently high, since in that scenario the increase in the pension tax widened the gap of initial wealth between these two population groups. Figure 6 depicts the effect of an increase in the pension tax on the probability function associated with the invariant distribution altruistic transfers when $g(b; P)$ is decreasing in P . We see in that figure that all the strictly positive points in the support of the invariant distribution of altruistic transfers shift to the left. Moreover, under the assumptions of Proposition 9, it is clear from Figure 6 that the probability of having a transfer lower than a given value $\bar{b}$ rises as the pension P increases, for all $\bar{b} > 0$. Clearly, this means that the distribution of intergenerational transfers with a lower social security tax dominates the one with a higher tax in the sense of first degree stochastic dominance.

Even if the probability π of being altruistic is very low, an increase in the pension could trigger an increase in the bequests left by altruistic agents. To get such a result we just need to make the assumptions opposite to the ones made in Proposition 9 and apply exactly the same reasoning. The proof of the following proposition is obvious from that of Proposition 9 and is thus omitted.

Proposition 10. *Let $\delta \geq 1$, $R\beta > 1$ and $P > 0$. There exists a $\pi^* > 0$ such that $\frac{\partial g(b; P)}{\partial P} > 0$ for all $\pi \in (0, \pi^*)$ whenever any of the following two conditions is satisfied:*

- (i) $R > N$ and the index of absolute risk aversion of u is strictly increasing.
- (ii) $R < N$ and the index of absolute risk aversion of u is strictly decreasing.

The conclusion of the previous two propositions can be also reached if we assume instead small values of the inter-personal discount factor δ . In this case we should ensure that the bequest motive is operative, i.e. $R\beta\delta > 1$. The following propositions are the counterparts of Propositions 9 and 10:

Proposition 11. *Let $P > 0$ and $R\beta\delta > 1$. There exists a strictly positive pair (π^*, δ^*) with $\delta^* > 1/R\beta$ such that $\frac{\partial g(b; P)}{\partial P} < 0$ for all pairs $(\pi, \delta) \in (0, \pi^*) \times (1/R\beta, \delta^*)$ whenever any of the following two conditions is satisfied:*

- (i) $R > N$ and the index of absolute risk aversion of u is strictly decreasing.

(ii) $R < N$ and the index of absolute risk aversion of u is strictly increasing.

Proposition 12. *Let $P > 0$ and $R\beta\delta > 1$. There exists a strictly positive pair (π^*, δ^*) with $\delta^* > 1/R\beta$ such that $\frac{\partial g(b; P)}{\partial P} > 0$ for all pairs $(\pi, \delta) \in (0, \pi^*) \times (1/R\beta, \delta^*)$ whenever any of the following two conditions is satisfied:*

- (i) $R > N$ and the index of absolute risk aversion of u is strictly increasing.
- (ii) $R < N$ and the index of absolute risk aversion of u is strictly decreasing.

The proof of the last proposition is omitted since it follows from just making the assumptions opposite to the ones of Proposition 11.

Let us discuss the testable implications of our analysis. Under the empirically reasonable assumptions of decreasing absolute risk aversion and $R > N$, part (i) of Propositions 9 and 11 predicts that, if the fraction of altruistic individuals is low, the distribution of bequests suffers a deterioration in terms of first-order stochastic dominance. However, according to Proposition 8, if the fraction of altruistic individuals is large enough, then the distribution of bequests improves in terms of first-order stochastic dominance. Thus, the prevailing effect will be determined by the value of the parameter π giving the probability of being altruistic. The estimation of the parameter π is indeed a complex task. However, an upper bound on the value of π could be obtained by the fraction of individuals in the economy who effectively give positive bequests, whereas a lower bound would be given by one minus the fraction of single-headed, retired households who have purchased an annuity preventing the young members of the household from receiving inheritances.

Let us introduce now some welfare considerations. On the one hand, it is obvious that the welfare of the generation that is old at the time of a pension increase is always improved by such a policy change. On the other hand, if we were interested in the effects on welfare at the steady state, we should restrict our attention to the expected utility of the newborns under the stationary distribution of altruistic transfers. Note that, when the probability of being altruistic is very low, the *ex ante* welfare of individuals depends basically on the statistical properties of the present value of their lifetime income. In this case, the effects of social security on the expected utility of newborns are generally ambiguous as usually happens in the standard overlapping generations model. However, in the scenario depicted by part (i) of Propositions 9 and 11, we can say that the *ex ante* welfare of a newborn is reduced since the present value of non-inherited lifetime income $(w - P + \frac{NP}{R})$ decreases and the distribution of transfers becomes less desirable as a consequence of the shift in terms of first-order stochastic dominance. The opposite holds in the scenario of part (ii) of Propositions 10 and 12 and, thus, the *ex ante* welfare of a newborn increases in such a case. Concerning the remaining cases considered in the previous four propositions, the results are ambiguous since the present value of lifetime income and the shift on the distribution of altruistic transfers have opposite effects on welfare. It should be noted that the inequality $R > (<)N$ is the standard condition for dynamic efficiency (inefficiency) of a non-stochastic OLG economy at the steady state. If the economy is dynamically inefficient (efficient), then an increase in the tax of the unfunded social security system improves (worsens) the welfare of every representative newborn at the stationary equilibrium. Moreover, in the stochastic environment considered in Propositions 9–12, the impact on welfare of the shift in the bequest distribution should be modest, since the probability of being altruistic is quite low. Therefore, when π is low, we should expect that the effect of changing the social security tax on welfare will depend almost exclusively on the relationship between the return on capital and the rate of population growth, as it happens in the non-stochastic version of the model.

TABLE 1

Effects of social security on the distribution of altruistic transfers and on welfare for $\hat{\beta} = 0.96$, $\hat{\delta} = 1$ and $\pi = 0.2$. We consider time periods of 30 years. The rates with a hat are average compound rates ($\hat{x} = x^{1/30}$)

$\mathcal{A}$	$\hat{R} - 1$ (%)	$\hat{N} - 1$ (%)	Average bequest	P	Gini coefficient	Welfare
Case I: Isoelastic utility with coefficient of relative risk aversion $\sigma = 4$ and $R > N$						
Decreasing	5	1.2	0.023	0	0.819	-3.05
Decreasing	5	1.2	0.022	0.064	0.818	-3.78
Case II: Quadratic utility: $u(c) = c - 0.25c^2$ and $R < N$						
Increasing	5	5.9	0.076	0	0.804	0.63
Increasing	5	5.9	0.075	0.064	0.804	0.65
Case III: Quadratic utility: $u(c) = c - 0.25c^2$ and $R > N$						
Increasing	5	1.2	0.21	0	0.81	0.66
Increasing	5	1.2	0.22	0.064	0.81	0.63
Case IV: Isoelastic utility with coefficient of relative risk aversion $\sigma = 4$ and $R < N$						
Decreasing	5	5.9	0.00805	0	0.805	-3.11
Decreasing	5	5.9	0.00824	0.064	0.806	-2.90

The numerical examples of Table 1 illustrate the main results of this section and refer to the scenarios depicted by Propositions 9 and 10. The probability of being altruistic is set relatively low ($\pi = 0.2$) and we assume that the total utility of an altruistic old individual is equal to the sum of the utility derived from his consumption and the indirect utility of their children (*i.e.* $\delta = 1$). Under isoelastic preferences, which obviously display decreasing absolute risk aversion, we observe in Case I that the average bequest decreases when the social security system is introduced for $R > N$, as dictated by part (i) of Proposition 9. Looking at the Gini coefficient of the stationary bequest distribution, we see that inequality decreases slightly. Furthermore, if we measure welfare by the *ex ante* expected lifetime utility of an agent under the stationary distribution of altruistic transfers, we see that welfare decreases, which agrees with our previous discussion about the normative implications of social security. For Case IV with $R < N$ and quadratic utility (and thus with increasing absolute risk aversion) the opposite effects concerning the distribution of bequest hold, which agrees with part (ii) of Proposition 10. Moreover, in this case *ex ante* welfare goes up as the present value of lifetime increases and the new distribution of bequests dominates the initial one in terms of first-order stochastic dominance. Cases II and III give rise to changes in the distribution of altruistic transfers that agree with part (ii) of Proposition 9 and part (i) of Proposition 10, respectively. In these cases, the impact on welfare of the introduction of social security is quite small and is completely driven by the relative values of R and N . Finally, it can be checked for Cases I and II that the optimal bequest left by altruistic individuals decreases for all initial wealth levels, while the opposite holds for Cases III and IV.

7. EXTENSIONS: WAGE HETEROGENEITY AND GENERAL EQUILIBRIUM

7.1. Wage heterogeneity

We could easily extend the model of our paper to investigate the impact of social security on the distribution of altruistic transfers for economies with different degrees of wage inequality and with intergenerational correlation of wages. To this end, let us assume that the logarithm of

TABLE 2

Impact of social security under different wage structures. Isoelastic utility with coefficient of relative risk aversion $\sigma = 4$ and $R > N$ ($\hat{R} - 1 = 5\%$, $\hat{N} - 1 = 1.2\%$), $\hat{\beta} = 0.96$, $\hat{\delta} = 1$ and $\pi = 0.2$. We consider time periods of 30 years. The rates with a hat are average compound rates ($\hat{x} = x^{1/30}$)

Wage distribution				
Wages	Average wage	Gini coefficient	ρ	
Deterministic	0.641	0.0	1.0	
Empirical correlation	0.641	0.31	0.4	
Zero correlation	0.641	0.31	0.0	
Social security effects				
Wages	$P = 0$		$P = 0.064$	
	Avg. bequest	Welfare	Avg. bequest	Welfare
Deterministic	0.0235	-3.05	0.0220	-3.78
Empirical correlation	0.1134	-22.77	0.1168	-85.98
Zero correlation	0.1427	-21.74	0.1468	-82.53

wages is governed by the following first-order Markov process:

$$\log w_t = \rho \log w_{t-1} + \varepsilon_t,$$

where $\rho \in [0, 1)$, ε_t is i.i.d. with $\varepsilon_t \sim N(\bar{\varepsilon}, \sigma_\varepsilon^2)$, and the subindex t denotes the generation in a particular dynasty. We choose values for the parameters ρ , $\bar{\varepsilon}$, and σ_ε and then, we approximate the above autoregressive process by a five-state first-order Markov chain following Tauchen (1986).

In economies where wages are stochastic altruistic transfers play the role of imperfect insurance against wage risk. The bequest left by an altruistic individual depends on the inheritance he receives, the wage w_t he earns, and the wage w_{t+1} earned by his children. Therefore, for each realization (w_t, w_{t+1}) of the wage pair, there is an optimal transfer given by $b_{t+1} = g(w_t + b_t, w_{t+1}; P)$. The function $g(w_t + b_t, w_{t+1}; P)$ is decreasing with respect to the children's wage w_{t+1} and, as in Section 3, increasing with respect to first period income $w_t + b_t$. Now, the distribution of bequests will emerge from two independent sources or risk, namely, the random intergenerational discount and the random wage processes.

Table 2 describes the economies that we compare in this example. These economies differ with respect to the intergenerational correlation of wages and the degree of wage inequality. We concentrate the analysis on the case where individual utility is isoelastic and $R > N$. The first row of Table 2 describes the benchmark economy where there is no heterogeneity ($\sigma_\varepsilon^2 = 0$) and $\bar{\varepsilon}$ is set equal to zero. This case coincides with Case I in Table 1. The other rows correspond to economies where wages are stochastic. In order to keep comparability between the different economies, the average wage and the Gini coefficient of the distribution of wages do not differ across stochastic wage economies. Moreover, the average wage in the stochastic economies equals the wage in the benchmark economy. To this end, we choose $\bar{\varepsilon}$ so that the average wage is always equal to 0.64. Finally, for the economies with stochastic wages, we choose a value for σ_ε^2 generating a value of the Gini coefficient of wages which is close to that of the Gini coefficient of lifetime earnings that Knowles (1999) reports for the U.S. economy.

The second and third rows describe economies where wages are stochastic and the intergenerational correlation coefficient ρ is set equal to 0.4, as Solon (1992) and Zimmerman (1992) report for the U.S. economy, and equal to zero, respectively. Note that in the economies with stochastic wages the statement of part (i) of Proposition 9 does not apply since the average

TABLE 3

General equilibrium effects with isoelastic utility having coefficient of relative risk aversion $\sigma = 2$ and $\hat{\beta} = 0.96$, $\hat{N} - 1 = 1.2\%$, $\pi = 0.2$, $y = k^\alpha$, $\alpha = 0.36$ and $P = \tau w$. We consider time periods of 30 years. The rates with a hat are average compound rates ($\hat{x} = x^{1/30}$)

$R\beta\delta$	k	$\hat{R} - 1$ (%)	Average bequest	τ	Gini coefficient	Welfare
Case I: Isoelastic utility and $\hat{\delta} = 0.989$						
<1	1	4.8	0	0	0	-9.96
>1	0.58	5.9	0.0053	10%	0.82	-12.13
Case II: Isoelastic utility and $\hat{\delta} = 1.0$						
>1	1	4.6	0.0052	0	0.82	-9.92
>1	0.61	5.7	0.0154	10%	0.82	-11.85

bequest increases with the pension. In order to understand this result, notice that bequests play a new role in this environment, since now altruistic transfers also provide insurance to altruistic parents against the wage risk of their descendants. That is, old altruistic individuals can use bequests to compensate a low after-tax wage of their children. As $R > N$ and individual preferences display decreasing risk aversion, the introduction of the PAYG system enhances the precautionary role of altruistic transfers as individuals face a lower lifetime income. This introduces a new effect that could end up outweighing the negative effect on bequests prevailing under deterministic wages. We can also see that the lower is the correlation of wages, the higher is the average bequest, since the more important is the insurance role of altruistic transfers.

Finally, note that welfare decreases with social security for all wage configurations. Since the return of capital is higher than the return of social security, the present value of lifetime income decreases with social security and this has a dominating negative impact on welfare in all the examples under consideration.

7.2. General equilibrium effects

Our analysis has been conducted for an economy where both the interest rate and the wage were fixed. The introduction of general equilibrium effects inducing changes in the rental prices for labour and capital could alter our previous results substantially. From Ricardian equivalence, the effects on wages and interest rates of PAYG social security should be modest when individuals are most likely altruistic, *i.e.* when π takes values near to one. However, when the vast majority of agents are selfish, then the rental prices of inputs could be affected in a non-negligible way. As it is well known from the standard OLG model, the introduction of an unfunded social security system crowds out capital so that the equilibrium interest rate increases. Table 3 illustrates two types of effects triggered exclusively by the endogenous determination of rental prices for a closed economy with a Cobb–Douglas aggregate production function. The capital installed in each period will be equal to the aggregate saving of the previous period and prices are competitive. Pensions will be now financed by means of a flat rate tax on wages. In Case I we have $R\beta\delta < 1$ for the economy without social security. The introduction of PAYG social security crowds out capital and increases the equilibrium interest rate. In fact, $R\beta\delta > 1$ for the economy where the social security tax is 10%. Consequently, the bequest motive becomes operative and the introduction of social security is a source of permanent wealth heterogeneity, which is in stark contrast to what is claimed in part (ii) of Proposition 7. Notice also that the *ex ante* welfare of a

newborn is lower in the economy with social security since the return of social security is lower than that of capital and because there is a significant decrease in wages due to the 40% crowding-out of capital induced by social security. In these economies, individuals save mostly for retirement because the probability of being altruist is very low ($\pi = 0.2$). Thus, social security crowds out a large amount of savings. In Case II the bequest motive is operative before and after the introduction of social security. Notice also that $R > N$ in both economies. Bequests increase with social security in spite of the fact that the assumptions of part (i) of Proposition 9 hold. The reason lying behind this result is that social security raises the interest rate and this makes old individuals enjoy a larger capital income. This in turn, raises the bequest left by the agents that turn out to be altruistic. Finally, welfare is lower in the economy with social security as it happens in Case I.

8. CONCLUSIONS

In this paper we have characterized the distribution of altruistic transfers in an economy where a PAYG social security system is present. We have analysed both short-run and long-run effects on that distribution triggered by changes in the pension tax. We have shown that the effects of social security on the distribution of these intergenerational transfers depends crucially on the importance of the bequest motive. On the one hand, if individuals are most likely altruists, then the introduction of social security increases the size of altruistic transfers. On the other hand, when individuals are most likely non-altruistic the introduction of social security could reduce these transfers under some conditions on the attitude of individuals towards risk and the relative returns associated both with private saving and with social security.

In contrast to our results, if the distribution of intergenerational transfers were generated by uncertain lifetimes, then an increase in the pension tax would result unambiguously in a smaller wealth gap between the individuals who have received a positive inheritance and the ones that have not. This is so because social security acts as a public annuity which reduces the size of accidental bequests (see Abel, 1985). A non-degenerate distribution of intergenerational transfers could also arise when there are two types of dynasties, the altruistic and the non-altruistic ones (as in Michel and Pestieau, 1998). Since individuals know in this case which kind of dynasty they belong to, an increase in the social security tax will always increase the bequest left by altruistic agents and this will also increase the wealth gap between any two agents belonging to different types of dynasties.

A model which appears to be between ours and that of Michel and Pestieau (1998) would be one where individuals know at birth whether they are altruistic or not. However, they do not know if they will have altruistic children. One implication of this different timing of events is that old individuals will observe whether their children are altruistic. This contrasts to our model where old agents do not know the altruism factor of their children. If an altruistic old agent observes that his offspring is altruistic, then the amount of bequest he leaves will increase with the pension tax in order to undo the transfer of the PAYG system. However, if an altruistic old agent knows that his offspring is selfish, he will try to offset the change in the present value of the lifetime income of his children due to a pension tax modification. Note that the direction of the change in the present value of the lifetime income is ultimately determined by the relative values of the rate of population growth and the rate of return on saving, as it occurs in the model of this paper. Therefore when $R > (<)N$ altruistically motivated bequests will increase (decrease) with the amount of the pension.

We have focused on the analysis of an unfunded social security system since, as we have seen, it is a system for which a quite rich plethora of results arises depending on the parametric assumptions of the model. If we had considered instead a fully funded social security system, the marginal changes in the social security tax would translate immediately into a change by the

same amount in the voluntary savings. Therefore, the bequest left by an individual would remain unchanged since this bequest only depends on his effective saving regardless of whether it is compulsory or voluntary.

Finally, we point out that it could be interesting to characterize the distribution of intergenerational transfers and the corresponding impact of social security if we introduced uncertainty on alternative sources of intended transfers. Among the different approaches, we mention the model of “joy-of-giving” in which parents derive direct utility from the size of the bequest they leave (Yaari, 1965) or the model of “strategic altruism” in which intergenerational transfers arise as payments for services provided by children (Bernheim *et al.*, 1985). We leave this for future research.

APPENDIX

Proof of Lemma 2. (a) $\hat{s}(\cdot; P)$ is strictly increasing. Let us proceed by contradiction. Let $b_1 > b_2$ and assume that $s_1 \leq s_2$, where $s_1 = \hat{s}(b_1; P)$ and $s_2 = \hat{s}(b_2; P)$, then

$$\beta\{(1-\pi)Ru'(Rs_1 + NP) + \pi V'_a(s_1; P)\} \geq \beta\{(1-\pi)Ru'(Rs_2 + NP) + \pi V'_a(s_2; P)\},$$

as follows from the concavity of both u and $V_a(\cdot; P)$. From (2.5) and the previous inequality, we obtain

$$u'(w + b_1 - P - s_1) \geq u'(w + b_2 - P - s_2).$$

We get a contradiction by noting that $b_1 - s_1 > b_2 - s_2$, which is incompatible with the concavity of u .

(b) $\hat{c}_s(\cdot; P)$ is strictly increasing. Obvious from part (a) and the fact that $\hat{c}_s(b; P) = R\hat{s}(b; P) + NP$.

(c) $\hat{c}_y(\cdot; P)$ is strictly increasing. From the envelope condition (2.6), $V'_y(b; P) = u'(c_y(b; P))$, and the concavity of both u and $V_y(\cdot; P)$, the result immediately follows.

(d) $\hat{c}_a(\cdot; P)$ is strictly increasing. From the envelope condition (2.8), $V'_a(s; P) = u'(c_a(s; P))$, and the concavity of both u and $V_a(\cdot; P)$, the result immediately follows.

(e) $\hat{b}(\cdot; P)$ is non-decreasing and strictly increasing when $\hat{b}(s; P) > 0$. Assume that s_1 is an amount of saving for which $\hat{b}(s_1; P) = 0$. Then, for every $s_2 > s_1$, we have $\hat{b}(s_2; P) \geq 0$ because of the non-negativity constraint on altruistic transfers. Therefore, $\hat{b}(s_2; P) \geq \hat{b}(s_1; P)$. Assume now that s_1 is such that $\hat{b}(s_1; P) > 0$. Then, condition (2.7) holds with equality and the concavity of both u and $V_y(\cdot; P)$ yields $\hat{b}(s_2; P) > \hat{b}(s_1; P)$ whenever $s_2 > s_1$.

(f) $g(\cdot; P)$ is non-decreasing and locally strictly increasing when $g(b; P) > 0$. Obvious from (a) and (e). $\parallel$

Proof of Proposition 1. Let $b > 0$ be such that $b' = g(b; P) > 0$. Then, we have

$$u'(\hat{c}_y(b; P)) = R\beta\{(1-\pi)u'(\hat{c}_s(b; P)) + \pi u'(\hat{c}_a(\hat{s}(b; P); P))\} <$$

$$R\beta u'(c_a(\hat{s}(b; P); P)) = R\beta\delta V'_y(g(b; P); P) = R\beta\delta u'(\hat{c}_y(g(b; P); P)) \leq u'(\hat{c}_y(g(b; P); P)),$$

where the first equality comes from substituting the envelope condition (2.8) into the first-order condition (2.5), the strict inequality comes from the fact that $\hat{c}_a(\hat{s}(b; P); P) < \hat{c}_s(b; P)$ when $b' > 0$, the second equality is just the first-order condition (2.7) when $b' > 0$, whereas the third equality is the envelope condition (2.6). Finally, the weak inequality comes from the assumption that $R\beta\delta \leq 1$. Since $\hat{c}_y(\cdot; P)$ is strictly increasing and u is strictly concave, we get $g(b; P) < b$.

If $g(b; P) = 0$ for $b > 0$, then it trivially follows that $g(b; P) < b$.

Finally, let $b = 0$. Since $g(b; P) < b$ for all $b > 0$, the continuity of $g(\cdot; P)$ on $\left(-w + P\left(\frac{R-N}{R}\right), \infty\right)$ implies that $\lim_{b \rightarrow 0} g(b; P) = g(0; P) \leq 0$. Hence, the non-negativity constraint on transfers allows us to conclude that $g(0; P) = 0$. $\parallel$

Proof of Proposition 2. Notice that

$$V'_y(0; P) = u'(c_y(0; P)) = \beta\{(1-\pi)Ru'(R\hat{s}(0; P) + NP) + \pi V'_a(\hat{s}(0; P); P)\}, \quad (\text{A.1})$$

where the first equality is the envelope condition (2.6) while the second is the first-order condition (2.5). From Proposition 1, we know that $g(0; P) = 0$ so that the envelope condition (2.8) becomes

$$V'_a(\hat{s}(0; P); P) = Ru'(R\hat{s}(0; P) + NP). \quad (\text{A.2})$$

Combining (A.1) and (A.2), we get

$$V'_y(0; P) = R\beta u'(R\hat{s}(0; P) + NP),$$

and using the fact that $R\beta\delta < 1$, we obtain

$$\delta V'_y(0; P) = R\beta\delta u'(R\hat{s}(0; P) + NP) < u'(R\hat{s}(0; P) + NP).$$

We can thus define $\underline{b}(P)$ implicitly by

$$\delta V'_y(0; P) = u'(R\hat{s}(\underline{b}(P); P) + NP). \quad (\text{A.3})$$

Note that $\underline{b}(P) > 0$ since $\hat{s}(\cdot; P)$ is strictly increasing and u is strictly concave. Therefore, the following weak inequality holds for all $b \in [0, \underline{b}(P)]$:

$$\delta V'_y(0; P) \leq u'(R\hat{s}(b; P) + NP). \quad (\text{A.4})$$

We can show next that $g(b; P) = 0$ for $b \in [0, \underline{b}(P)]$. We proceed by contradiction and assume instead that $g(b; P) > 0$. Since both $V_y(\cdot; P)$ and u are strictly concave, we have that

$$V'_y(0; P) > V'_y(g(b; P); P)$$

and

$$u'(R\hat{s}(b; P) + NP - Ng(b; P)) > u'(R\hat{s}(b; P) + NP).$$

These two inequalities, together with (A.4), imply that

$$\delta V'_y(g(b; P); P) < u'(R\hat{s}(b; P) + NP - Ng(b; P)),$$

which according to the first-order condition (2.7) implies that $g(b; P) = 0$, and this is the desired contradiction.

To prove that $g(b; P) > 0$ for $b > \underline{b}(P)$ assume instead that $g(b; P) = 0$ to get a contradiction. Such a contradiction is easily obtained since $g(b; P) = 0$ implies that

$$V'_a(\hat{s}(0; P); P) \leq Ru'(R\hat{s}(b; P) + NP),$$

as dictated by the first-order condition (2.7). Moreover, $b > \underline{b}(P)$ implies that

$$V'_a(\hat{s}(0; P); P) > Ru'(R\hat{s}(b; P) + NP), \quad (\text{A.5})$$

because of the definition of $\underline{b}(P)$ in (A.2) and the monotonicity of $\hat{s}(\cdot; P)$. We obtain thus the desired contradiction. $\parallel$

Proof of Proposition 3. We proceed by contradiction and we assume thus that $g(b; P) = 0$. The first-order condition (2.5) and the envelope condition (2.8) imply that

$$u'(\hat{c}_y(b; P)) = R\beta\{(1 - \pi)u'(R\hat{s}(b; P) + NP) + \pi u'(R\hat{s}(b; P) + NP)\} > \frac{u'(R\hat{s}(b; P) + NP)}{\delta}, \quad (\text{A.6})$$

where the inequality comes from the assumption that $R\beta\delta > 1$. Moreover, when $g(b; P) = 0$ we have

$$u'(R\hat{s}(b; P) + NP) \geq \delta V'_y(0; P) = \delta u'(\hat{c}_y(0; P)), \quad (\text{A.7})$$

where the weak inequality is the corresponding first-order condition (2.7) and the equality is the envelope condition (2.6). Combining (A.6) with (A.7), we get

$$u'(\hat{c}_y(b; P)) > u'(\hat{c}_y(0; P)).$$

From the concavity of u and the fact that $\hat{c}_y(\cdot; P)$ is increasing, it follows that $b < 0$, which is the desired contradiction. $\parallel$

Proof of Proposition 4. Let $Q(b, B; P)$ be the transition function of the Markov process of transfers when the pension is P . This transition function gives the probability that an individual receiving an inheritance equal to b leaves a bequest lying in the Borel set B . Therefore,

$$Q(b, B; P) = (1 - \pi)\mathcal{I}_B(0) + \pi\mathcal{I}_B(g(b; P)). \quad (\text{A.8})$$

Let B^c be the complementary of the Borel set B in $\mathbb{R}_+$. It is obvious from (A.8) that for every $B \in \mathcal{B}$, $Q(b, B; P) \geq 1 - \pi$ if $0 \in B$, for all $b \in B$. Moreover, $Q(b, B^c; P) \geq 1 - \pi$ if $0 \in B^c$, for all $b \in B$. Note that this means that Condition M in Section 11.4 of Stokey and Lucas (1989) holds and, therefore, from their Theorems 11.12 and 11.6 we get the desired uniform convergence result. $\parallel$

Proof of Proposition 5. We will show that the distribution $\bar{\mu}(\{0\}; P) = 1$ satisfies the functional equation (4.2), that is,

$$\bar{\mu}(\{0\}; P) = (1 - \pi)\mathcal{I}_{\{0\}}(0) + \pi \int_{g^{-1}(\{0\}; P)} \bar{\mu}(db; P) = 1 - \pi + \pi \int_{g^{-1}(\{0\}; P)} \bar{\mu}(db; P).$$

Observe that Proposition 1 applies and $\{0\} \in g^{-1}(\{0\}; P)$ since $R\beta\delta \leq 1$. Therefore,

$$\pi \int_{g^{-1}(\{0\}; P)} \bar{\mu}(db; P) = \pi.$$

The last part of the proposition follows immediately from Propositions 1 and 2. $\parallel$

Proof of Proposition 6. We will show that the distribution in the statement satisfies (4.2). If $b_i > 0$, we have from Proposition 3 that $b_{i+1} = g(b_i; P) > 0$. Then, since $g(\cdot; P)$ is strictly increasing as is established by Lemma 2, $g^{-1}(\{b_{i+1}\}; P) = b_i$ and equation (4.2) becomes

$$\bar{\mu}(\{b_{i+1}\}; P) = (1 - \pi) \mathcal{I}_{\{b_{i+1}\}}(0) + \pi \int_{b_i} \bar{\mu}(db; P) = 0 + \pi \bar{\mu}(\{b_i\}; P) = (1 - \pi) \pi^{i+1},$$

since $\bar{\mu}(\{b_i\}; P) = (1 - \pi) \pi^i$. Moreover, if $b_i = 0$, then

$$\bar{\mu}(\{0\}; P) = (1 - \pi) \mathcal{I}_{\{0\}}(0) + \pi \int_{g^{-1}(\{0\}; P)} \bar{\mu}(db; P) = 1 - \pi,$$

since Proposition 3 implies that the set $g^{-1}(\{0\}; P)$ is empty and, thus, has zero measure. $\parallel$

Proof of Lemma 3. Combine the first-order condition (2.5) and the envelope condition (2.8) evaluated at $b' = 0$ to get

$$u'(w + b - P - s) = R\beta u'(Rs + NP).$$

Implicitly differentiating the previous equation, we obtain

$$\frac{ds}{dP} = - \frac{u''(w + b - P - s) + RN\beta u''(Rs + NP)}{u''(w + b - P - s) + R^2\beta u''(Rs + NP)} < 0. \quad (\text{A.9})$$

It is straightforward to check that $\left| \frac{ds}{dP} \right| \geq 1$ if $R \leq N$. Since $c_y = w + b - P - s$, it follows that $\frac{dc_y}{dP} \geq 0$ if $R \leq N$. $\parallel$

Proof of Proposition 7. (a) Recall that the stationary distribution of altruistic transfers is degenerate at zero when $R < 1/\beta\delta$ for all $P \in [0, w)$ as dictated by Proposition 5. Let

$$\underline{P} = \frac{R}{N} [\hat{s}(\underline{b}(0); 0) - \hat{s}(0; 0)],$$

where $\underline{b}(0)$ is defined in equation (A.3). Note that the saving of the agents who were old at the moment of the introduction of social security is given by $\hat{s}(0; 0)$. Clearly, $\underline{P} > 0$, since the function $\hat{s}(\cdot; 0)$ is strictly increasing and $\underline{b}(0) > 0$ (see the proof of Proposition 2). If $P \in (0, \min\{\underline{P}, w\})$, we get

$$u'(R\hat{s}(0; 0) + NP) > u'(R\hat{s}(0; 0) + N\underline{P}) = u'(R\hat{s}(\underline{b}(0); 0)) = \delta V'_y(0; 0), \quad (\text{A.10})$$

where the inequality is a consequence of the strict concavity of u , the first equality follows from the definition of $\underline{P}$, and the last equality follows from the definition of $\underline{b}(0)$ given in (A.3).

From Lemma 3, if $R \leq N$ and $g(0; 0) = g(0; P) = 0$, the first period consumption is non-decreasing in the pension level, that is, $\hat{c}_y(0; P) \geq \hat{c}_y(0; 0)$. Therefore, from the concavity of u and the envelope condition (2.6), we get

$$\delta V'_y(0; 0) \geq \delta V'_y(0; P). \quad (\text{A.11})$$

Therefore,

$$u'(R\hat{s}(0; 0) + NP) > \delta V'_y(0; P) \geq \delta V'_y(\hat{b}(\hat{s}(0; 0); P); P),$$

where the strict inequality follows from combining (A.10) with (A.11) and the weak inequality is a consequence of the concavity of $V'_y(\cdot; P)$ together with the non-negativity constraint on bequests. The first-order condition (2.7) at the period where the PAYG system is introduced implies that, if

$$u'(R\hat{s}(0; 0) + NP) > \delta V'_y(\hat{b}(\hat{s}(0; 0); P); P),$$

then $\hat{b}(\hat{s}(0; 0); P) = 0$ for all $P \in (0, \min\{\underline{P}, w\})$. Therefore, the bequest motive remains inoperative and, thus, the degenerate initial distribution of altruistic transfers is not affected by the introduction of this social security scheme.

(b.i) From combining (A.3) and (2.6) when $P = 0$, we have that

$$u'(R\hat{s}(\underline{b}(0); 0)) = \delta u'(\hat{c}_y(0; 0)).$$

Therefore, since $\underline{b}(0)$ is strictly positive and $\hat{s}(\cdot; 0)$ is strictly increasing, the following condition holds before introducing the social security scheme:

$$u'(R\hat{s}(0; 0)) > \delta u'(\hat{c}_y(0; 0)). \quad (\text{A.12})$$

Define the threshold pension $\bar{P}$ as the one that solves the following equation:

$$u'(R\hat{s}(0; 0) + N\bar{P}) = \delta u'(\hat{c}_y(0; \bar{P})). \quad (\text{A.13})$$

From (A.12) and (A.13) it is clear that the threshold pension $\bar{P}$ is strictly positive. This is so because u is strictly concave and $\hat{c}_y(0; \bar{P})$ is non-increasing in P for $R \geq N$ whenever $g(0; \bar{P}) = 0$, as follows from Lemma 3. Moreover, $\bar{P} < w$ as a consequence of the Inada conditions at the origin.

Let us consider the threshold pension tax $\bar{P}$ defined in (A.13). Then,

$$u'(R\hat{s}(0; 0) + NP) < \delta u'(\hat{c}_y(0; P)), \quad (\text{A.14})$$

for all $P \in (\bar{P}, w)$, since u is concave and $\hat{c}_y(0; \bar{P}) \geq \hat{c}_y(0; P)$ under the assumptions imposed for this case. Clearly, (A.14) is incompatible with conditions (2.6) and (2.7). Therefore, the bequest $b' = \hat{b}(\hat{s}(0; 0); P)$ left by altruistic parents in the period where the pension is introduced should be positive. In this case, the same conditions (2.6) and (2.7) imply that the bequest b' satisfies

$$u'(R\hat{s}(0; 0) - Nb' + NP) = \delta u'(\hat{c}_y(b'; P)).$$

Note that $b' > 0$, since u is strictly concave and $\hat{c}_y(\cdot; P)$ is strictly monotonically increasing. The bequest motive becomes thus operative immediately after the introduction of a pension P at a level larger than $\bar{P}$.

Finally, as follows from Propositions 4 and 5, the distribution of altruistic transfers after the introduction of the pension converges to the degenerate one since $\lim_{i \rightarrow \infty} b_i = 0$ when $b_{i+1} = g(b_i; P)$ and $b_0 \geq 0$.

(b.ii) For this case, we get

$$u'(R\hat{s}(0; 0) + NP) > \delta u'(\hat{c}_y(0; P)) = \delta V'_y(0; P) \geq \delta V'_y(\hat{b}(\hat{s}(0; 0); P); P),$$

for all $P \in (0, \bar{P})$, where the strict inequality follows from (A.13), the strict concavity of u , and the fact that now $\hat{c}_y(0; \bar{P}) \leq \hat{c}_y(0; P)$; the equality follows from the envelope condition (2.6); and the weak inequality is a consequence of the concavity of $V'_y(\cdot; P)$ and the non-negativity constraint on bequests. Therefore, $\hat{b}(\hat{s}(0; 0); P) = 0$ as dictated by the first-order condition (2.7) evaluated at the period where the PAYG system is introduced. This means that the bequest motive remains inoperative after the introduction of this social security scheme and, thus, the degenerate initial distribution of altruistic transfers is not affected by the introduction of a pension lower than $\bar{P}$. $\parallel$

Proof of Proposition 8. For $\pi = 1$ the result follows directly from Barro (1974) since the altruistic agents completely offset the effects of social security by means of adjustments in the amount of altruistic transfers within the dynasty when the bequests motive is always operative, i.e. when $R\beta\delta > 1$. In fact, from (2.6) and (2.7), we have that

$$u'(R\hat{s}(b; P_1) - Ng(b; P_1) + NP_1) = \delta u'(w - P_1 + g(b; P_1) - \hat{s}(g(b; P_1); P_1)),$$

when the pensions were at level P_1 and the bequest motive is operative, whereas when pensions are set at level P_2 we have

$$u'(R\hat{s}(b; P_2) - Ng(b; P_2) + NP_2) = \delta u'(w - P_2 + g(b; P_2) - \hat{s}(g(b; P_2); P_2)).$$

Therefore, these two equations are compatible when $\hat{s}(b; P_1) = \hat{s}(b; P_2)$, $g(b; P_1) = g(b; P_2) + (P_1 - P_2)$ and $\hat{s}(g(b; P_1); P_1) = \hat{s}(g(b; P_2); P_2)$. By continuity, if π is close to one, then altruistic transfers increase when social security is introduced. $\parallel$

Proof of Proposition 9. First, observe that from (A.9) we get

$$\lim_{\pi \rightarrow 0} \frac{ds}{dP} = -\frac{1 + R\beta NQ}{1 + R^2\beta Q}, \quad (\text{A.15})$$

where $Q \equiv \frac{u''(\hat{c}_s(b; P))}{u''(\hat{c}_y(b; P))}$, and $\hat{c}_s(b; P) = Rs + NP$ and $\hat{c}_y(b; p) = w + b - P - s$ satisfy

$$u'(\hat{c}_y(b; P)) = R\beta u'(\hat{c}_s(b; P)), \quad (\text{A.16})$$

which is the limit of the first-order condition (2.5) when π tends to zero. Note that the Inada conditions on the utility function u prevent the limit of $V'_a(s; P)$ from tending to infinity. This is so because $V'_a(s; P)$ has to be equal to $R\delta u'(\hat{c}_y(b'; P))$, as follows from combining (2.6), (2.7) and (2.8).

Combine now conditions (2.5) and (2.8) corresponding to the maximization problem of a son of the individual under consideration to get

$$u'(w + b' - P - s') = R\beta\{(1 - \pi)u'(Rs' + NP) + \pi u'(Rs' + NP - Nb(s'; P))\}. \quad (\text{A.17})$$

Implicit differentiation of (A.17) yields the following limit:

$$\lim_{\pi \rightarrow 0} \frac{db'}{dP} = 1 + R\beta N Q' + (1 + R^2 \beta Q') \left(\lim_{\pi \rightarrow 0} \frac{ds'}{dP} \right), \quad (\text{A.18})$$

where $Q' \equiv \frac{u''(\hat{c}_s(b'; P))}{u''(\hat{c}_y(b'; P))}$, and $\hat{c}_s(b'; P) = Rs' + NP$ and $\hat{c}_y(b'; p) = w + b' - P - s'$ satisfy

$$u'(\hat{c}_y(b'; P)) = R\beta u'(\hat{c}_s(b'; P)). \quad (\text{A.19})$$

Note also that we can rewrite (A.19) as

$$R\beta Q' = \frac{\mathcal{A}(\hat{c}_s(b'; P))}{\mathcal{A}(\hat{c}_y(b'; P))}, \quad (\text{A.20})$$

where $\mathcal{A}(c) = \frac{-u''(c)}{u'(c)}$ denotes the index of absolute risk aversion evaluated at c , and $\hat{c}_s(b'; P)$ and $\hat{c}_y(b'; p)$ satisfy condition (A.19). Analogously, we have that

$$R\beta Q = \frac{\mathcal{A}(\hat{c}_s(b; P))}{\mathcal{A}(\hat{c}_y(b; P))}. \quad (\text{A.21})$$

Combine conditions (2.7) and (2.6) to get

$$u'(Rs + NP - Nb') = \delta u'(w + b' - P - s'). \quad (\text{A.22})$$

Implicit differentiation of the previous equation yields

$$\lim_{\pi \rightarrow 0} \frac{ds'}{dP} = \left(1 + \frac{NZ}{\delta}\right) \left(\lim_{\pi \rightarrow 0} \frac{db'}{dP}\right) - \left(1 + \frac{NZ}{\delta}\right) - \frac{RZ}{\delta} \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP}\right), \quad (\text{A.23})$$

where $Z \equiv \frac{u''(\hat{c}_a(b; P))}{u''(\hat{c}_y(b'; P))}$ and $\hat{c}_y(b'; P) = w + b' - P - s'$ and $\hat{c}_a(b; P) = Rs + NP - Nb'$. Combining (A.18) and (A.23), we get

$$\lim_{\pi \rightarrow 0} \frac{db'}{dP} = \frac{R^2 \beta Q' - R\beta N Q' + \left(\frac{Z}{\delta}\right) (1 + R^2 \beta Q') [N + R \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP}\right)]}{\frac{NZ}{\delta} + R^2 \beta Q' \left(1 + \frac{NZ}{\delta}\right)}. \quad (\text{A.24})$$

The sign of the above expression is the same as that of its numerator.

Note also that we can rewrite (A.22) as

$$\frac{Z}{\delta} = \frac{\mathcal{A}(\hat{c}_a(b; P))}{\mathcal{A}(\hat{c}_y(b'; P))}. \quad (\text{A.25})$$

We will proceed now with the proof of part (i). Assume thus that $R > N$ and that the index of absolute risk aversion is decreasing. A decreasing index of absolute risk aversion implies that $\mathcal{A}(\hat{c}_s(b; P)) < \mathcal{A}(\hat{c}_y(b; P))$ because $\hat{c}_s(b; P) > \hat{c}_y(b; P)$ as follows from (A.16) and the fact that $R\beta > 1$. Hence, equation (A.21) implies that $R\beta Q < 1$. Similarly, from (A.19) and (A.20), it holds that $R\beta Q' < 1$. Moreover, since $\delta \geq 1$, we have that $\hat{c}_a(b; P) \leq \hat{c}_y(b'; P)$ and, given a decreasing index of absolute risk aversion, $\mathcal{A}(\hat{c}_a(b; P)) \geq \mathcal{A}(\hat{c}_y(b'; P))$. Hence, from equation (A.25) we get that $Z/\delta \geq 1$.

From equation (A.17) we get the following expression:

$$N + R \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP} \right) = -\frac{1}{R\beta Q} \left[1 + \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP} \right) \right]. \quad (\text{A.26})$$

Note that $N + R \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP} \right) < 0$ since $\lim_{\pi \rightarrow 0} \frac{ds}{dP} > -1$ when $R > N$ (see equation (A.17)). Moreover,

$$\frac{1}{R\beta Q} \left[1 + \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP} \right) \right] > 1 + \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP} \right), \quad (\text{A.27})$$

since $R\beta Q < 1$. Therefore, from (A.26) and (A.27), it follows that

$$N + R \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP} \right) < - \left[1 + \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP} \right) \right]. \quad (\text{A.28})$$

If we apply inequality (A.28) to the numerator of $\lim_{\pi \rightarrow 0} \frac{db'}{dP}$, which was obtained in (A.24), we get

$$\begin{aligned}
 R^2 \beta Q' - R \beta N Q' + \left(\frac{Z}{\delta} \right) (1 + R^2 \beta Q') \left[N + R \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP} \right) \right] \\
 &= R \beta Q' (R - N) + \left(\frac{Z}{\delta} \right) \left[N + R \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP} \right) \right] + \left(\frac{Z}{\delta} \right) R^2 \beta Q' \left[N + R \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP} \right) \right] \\
 &< R \beta Q' (R - N) + \left(\frac{Z}{\delta} \right) \left[N + R \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP} \right) \right] - \left(\frac{Z}{\delta} \right) R^2 \beta Q' \left[1 + \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP} \right) \right] \\
 &= -R \beta Q' \left[N + R \left(\frac{Z}{\delta} - 1 \right) + \left(\frac{Z}{\delta} \right) R \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP} \right) \right] + \left(\frac{Z}{\delta} \right) \left[N + R \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP} \right) \right] \\
 &= -R \beta Q' \left(\frac{Z}{\delta} - 1 \right) (R - N) + \left(\frac{Z}{\delta} \right) (1 - R \beta Q') \left[N + R \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP} \right) \right] < 0, \tag{A.29}
 \end{aligned}$$

where the last inequality follows from $R \beta Q' < 1$ and $N + R \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP} \right) < 0$ and $Z/\delta \geq 1$ and $R > N$.

To prove part (ii) we just have to follow the same arguments as in part (i) and note that in this case the relevant inequalities are $R \beta Q > 1$ and $\lim_{\pi \rightarrow 0} \frac{ds}{dP} < -1$ so that inequalities (A.27) and (A.28) still hold. However, in this case we have that

$$N + R \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP} \right) > 0, \tag{A.30}$$

as can be seen from equation (A.26) and the fact that now $\lim_{\pi \rightarrow 0} \frac{ds}{dP} < -1$. Finally, combining inequality (A.30) with $R \beta Q' > 1$ and $Z/\delta \leq 1$ and $R < N$, we obtain that inequality (A.29) also holds in this case. $\parallel$

Proof of Proposition 11. Replicating the first steps of the proof of Proposition 9, we also obtain expressions (A.17), (A.24) and (A.25). Recall that the sign of expression (A.24) is the same as that of its numerator, since its denominator is positive.

We will proceed now with the proof of part (i). Assume thus that $R > N$ and that the index of absolute risk aversion is decreasing. First we show that $R \beta Q' < Z/\delta$ and $R \beta Q < Z/\delta$ because these properties will be useful later on. Since $R \beta \delta > 1$, the sequence of altruistic bequests is non-decreasing. Therefore, $b' \geq b$ and $\hat{c}_s(b'; P) \geq \hat{c}_s(b; P)$ since second period consumption is increasing with respect to bequest. Moreover, $\hat{c}_s(b; P) > \hat{c}_a(b; P)$ because $b' > 0$. Thus, $\hat{c}_s(b'; P) > \hat{c}_a(b; P)$. A decreasing index of absolute risk aversion implies that $\mathcal{A}(\hat{c}_s(b'; P)) < \mathcal{A}(\hat{c}_a(b; P))$. Hence, the last inequality implies that

$$R \beta Q' = \frac{\mathcal{A}(\hat{c}_s(b'; P))}{\mathcal{A}(\hat{c}_y(b'; P))} < \frac{\mathcal{A}(\hat{c}_a(b; P))}{\mathcal{A}(\hat{c}_y(b'; P))} = \frac{Z}{\delta}.$$

Moreover, since first period consumption increases with the received inheritance, we have that $\hat{c}_y(b; P) \leq \hat{c}_y(b'; P)$ and $\mathcal{A}(\hat{c}_y(b; P)) \geq \mathcal{A}(\hat{c}_y(b'; P))$. Thus,

$$R \beta Q = \frac{\mathcal{A}(\hat{c}_s(b; P))}{\mathcal{A}(\hat{c}_y(b; P))} < \frac{\mathcal{A}(\hat{c}_a(b; P))}{\mathcal{A}(\hat{c}_y(b'; P))} = \frac{Z}{\delta},$$

since $\mathcal{A}(\hat{c}_s(b; P)) < \mathcal{A}(\hat{c}_a(b; P))$ when $b' > 0$.

From equation (A.17) we get

$$N + R \left(\lim_{\pi \rightarrow 0} \frac{ds}{dP} \right) = \frac{N - R}{1 + R^2 \beta Q}. \tag{A.31}$$

Substituting (A.31) into the numerator of (A.24), we get

$$\begin{aligned}
 R^2 \beta Q' - R \beta N Q' + \left(\frac{Z}{\delta} \right) (1 + R^2 \beta Q') \left(\frac{N - R}{1 + R^2 \beta Q} \right) &= (R - N) \left[R \beta Q' - \left(\frac{Z}{\delta} \right) \left(\frac{1 + R^2 \beta Q'}{1 + R^2 \beta Q} \right) \right] \\
 &= \frac{R - N}{1 + R^2 \beta Q} \left[R \beta Q' (1 + R^2 \beta Q) - \left(\frac{Z}{\delta} \right) (1 + R^2 \beta Q') \right].
 \end{aligned}$$

When δ tends to $1/R\beta$, intergenerational transfers tend to zero at the steady state and, hence,

$$\lim_{\delta \rightarrow 1/R\beta} \left(\lim_{\pi \rightarrow 0} \frac{db'}{dP} \right) = \frac{\frac{R-N}{1+R^2\beta Q} \left[R \beta Q' (1 + R^2 \beta Q) - \left(\frac{Z}{\delta} \right) (1 + R^2 \beta Q') \right]}{\frac{NZ}{\delta} + R^2 \beta Q' \left(1 + \frac{NZ}{\delta} \right)} \bigg|_{b'=b=0} = 0 \tag{A.32}$$

because $R \beta Q = R \beta Q' = Z/\delta$ when $b' = b = 0$.

Now, we are going to show that the numerator of (A.32) becomes negative when δ increases marginally around $1/R\beta$. We proceed by differentiating the expression between brackets with respect to δ and we evaluate the resulting derivative at $\delta = 1/R\beta$. Such a derivative is

$$\begin{aligned} & \left[(1 + R^2\beta Q) \frac{d(R\beta Q')}{d\delta} + (R^2\beta Q') \frac{d(R\beta Q)}{d\delta} - (1 + R^2\beta Q') \frac{d(Z/\delta)}{d\delta} - R \left(\frac{Z}{\delta} \right) \frac{d(R\beta Q')}{d\delta} \right] \Big|_{\delta=1/R\beta} \\ &= \left[\frac{d(R\beta Q')}{d\delta} - \frac{d(Z/\delta)}{d\delta} \right] \Big|_{\delta=1/R\beta} + R^2\beta Q \left[\frac{d(R\beta Q)}{d\delta} - \frac{d(Z/\delta)}{d\delta} \right] \Big|_{\delta=1/R\beta} < 0, \end{aligned} \quad (\text{A.33})$$

where the equality follows since $R\beta Q = R\beta Q' = Z/\delta$ when $\delta = 1/R\beta$. The inequality in (A.33) holds since $\frac{d(R\beta Q)}{d\delta} \Big|_{\delta=1/R\beta} < \frac{d(Z/\delta)}{d\delta} \Big|_{\delta=1/R\beta}$ and $\frac{d(R\beta Q')}{d\delta} \Big|_{\delta=1/R\beta} < \frac{d(Z/\delta)}{d\delta} \Big|_{\delta=1/R\beta}$ as $Z/\delta > R\beta Q$ and $Z/\delta > R\beta Q'$ for $\delta > 1/R\beta$.

To prove part (ii) we just have to follow the same arguments as in part (i) and notice that in this case the relevant inequalities are $R\beta Q > Z/\delta$ and $R\beta Q' > Z/\delta$ for $\delta > 1/R\beta$. Therefore,

$$\left[\frac{d(R\beta Q')}{d\delta} - \frac{d(Z/\delta)}{d\delta} \right] \Big|_{\delta=1/R\beta} + R^2\beta Q \left[\frac{d(R\beta Q)}{d\delta} - \frac{d(Z/\delta)}{d\delta} \right] \Big|_{\delta=1/R\beta} > 0,$$

and the numerator of expression (A.32) becomes thus negative, since $R < N$ in this case. $\parallel$

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