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ENDOGENOUS GROWTH, HUMAN CAPITAL, AND BEQUESTS IN A LIFE-CYCLE MODEL

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1. Introduction

THIS PAPER investigates the relationship between growth and efficiency in an overlapping generations economy with altruistic individuals in which human capital is a social factor of production.

Recent empirical work has made evident that both the stock of human capital and the investment in education play a determinant role in explaining the disparity across countries in macroeconomic variables such as productivity, income *per capita*, or rate of growth. For instance, the positive correlation between school enrollment rates (or initial human capital stocks) and rates of growth has been documented by several studies. (See Romer 1989; Kyriacou 1991; Barro 1991; Levine and Renelt 1992, among others.) This correlation can be justified by either assuming that the ability of a nation to adopt new technologies is a function of its human capital or appealing to transitional dynamics as in Barro and Sala-i-Martin (1992). The latter explanation relies on the fact that the speed of adjustment of an economy is an increasing function of the distance between the initial conditions and the steady state. Therefore, poor countries with a high initial endowment of human capital will grow very fast since their initial capital–labor ratio lies significantly below its steady state. On the other hand, and disregarding transitional dynamics, the importance of the stock of human capital as a productive input has been highlighted by the estimation of the neoclassical growth model made by Mankiw *et al.* (1992) and by the calibrations of King and Rebelo (1990).

Moreover, several authors have stressed the social nature and the scale effects associated with human capital (see, e.g. Azariadis and Drazen 1990; Lucas 1988; and Backus *et al.* 1992). As a consequence, an optimal policy should involve subsidies on education so as to internalize those external effects. It is also generally argued in the endogenous growth literature that the competitive rate of growth is lower than the efficient one when human capital exhibits those externalities and, therefore, a subsidy in education will raise the rate of growth (Arrau 1989). However, we will show that this presumption is not generically true in an economy populated by agents facing a finite life and who are altruistic toward their descendants. To this end, we consider another possibility for market failure, namely that capital imperfections prevent children from borrowing to finance their education. Thus, in our model parents decide how much to invest in their sons' human capital, and growth will be achieved by means of a technology for reproduction of labor which raises the income of each generation. This in turn will induce larger expenditures on children's education. We will show that the efficient rate of growth might be lower than the competitive one when the physical bequest motive is not operative at competitive equilibrium, i.e. when it involves the violation of the non-negativity constraint on physical bequests. In this case, the competitive level of physical bequests (zero) will be determined by a corner solution.

The discrepancy between the social and the private returns from investment in human capital means that the external effects in the accumulation of technical knowledge are not considered by parents when they decide how much to invest in their children's education. This implies that the competitive equilibrium is socially inefficient due to underinvestment in human capital. A second source of inefficiency that may appear in bequest-constrained economies comes from the typical overaccumulation problem. These two potential inefficiencies have effects on the rate of growth that are opposite in sign. If the physical motive is not operative, then an efficient solution could involve transfers from young to old agents, which cannot be implemented through a competitive equilibrium. Hence, both saving and the capital–labor ratio should decrease so as to achieve an optimal solution, and, as a by-product, shadow wages should also decrease. Therefore, since the return on human capital is proportional to the wages per efficiency unit, parents should invest less in their children's education, and this will lower the growth rate. However, if the external effects in human capital are efficiently internalized, then the investment in education increases since it becomes more profitable, and therefore, the rate of growth goes up. Thus, only when externalities in education are strong enough, does the latter positive effect on the growth rate offset the negative effect due to the non-negativity constraint on physical bequests.

In this framework we find conditions under which the physical bequest motive is operative, and we find a threshold level of altruism below which physical bequests cannot be positive. In the former case, the only source of inefficiency is that human capital is a social factor of production. This in turn means that we recover the standard relationship between inefficiency and low growth when the generations of the same dynasty are effectively linked by means of positive physical bequests.

We restrict the analysis to steady-state equilibria. Note that, even if all economies sharing the same fundamentals display common growth and interest rates at a steady state, the long-run levels of both income and human capital *per capita* will not be generically the same. As the literature on conditional convergence makes clear, the convergence in levels is achieved only when the initial stocks of human capital are the same for all countries. (See Mankiw *et al.* 1992; Barro and Sala-i-Martin 1992; Caballé and Santos 1993.)

We also discuss the implications of different fiscal policies for the interest rate, the capital–labor ratio and the growth rate when the bequest motive is operative and when it is not. Obviously, and as our previous discussion indicates, the same set of fiscal instruments may have opposite effects depending on whether the lump-sum physical transfers within the dynasties are strictly positive or not. In particular, a subsidy on education may decrease the rate of growth in a bequest-constrained economy.

Overlapping generations models in which agents are altruistic toward their descendants have also been studied by Drazen (1978) and Weil (1987) to illustrate cases in which the Ricardian proposition about the neutrality of government debt and lump-sum taxes, proposed by Barro (1974), does not hold. Government deficits are not neutral when the physical bequest motive is not operative since agents cannot offset, by means of private transfers within their dynasty, the effects of lump-sum transfers made by the government. The non-neutrality of lump-sum taxation also holds in our model when the economy is bequest-constrained. Moreover, our formulation will allow us to investigate the effects of government deficits on growth.

The paper is structured as follows. Section 2 presents the basic assumptions of the model and some preliminary results. The competitive, balanced path of growth is characterized in Section 3. Section 4 explores necessary and sufficient conditions under which the physical bequest motive is operative in a balanced equilibrium. We compare the competitive equilibrium with the efficient one in Section 5. Section 6 studies the effects of diffential fiscal policies. Section 7 concludes.

2. Physical bequests and education

The model is a variant of the overlapping generations (OLG) model with production introduced by Diamond (1965) and Samuelson (1968). The economy consists of many agents who live for three periods. Every individual has offspring in the middle period of life so that a new generation is born in each period, Agents make economic decisions during the last two periods of their lives only, and they receive an endowment of human capital from their parents in the first period of life.[1] Parents are altruistic toward their children and may give them two kinds of transfers: bequests and education. There is a single commodity in this economy which can be either consumed or invested, and the investment can be either in physical or in human capital.

Agents supply inelastically one unit of labor in the middle period of their life, and they are retired in the last period. Each generation is indexed by the period in which its members work. A worker distributes his earnings from labor and inheritance received from his parent among own consumption, investment in his children's human capital (which will make them more efficient when they work), and saving. Therefore, the budget constraint of a worker in period t is

$$\hat{w} + b_t = c_t + ne_t + s_t \tag{1}$$

where $\hat{w}_t$ denotes labor income, b_t is the bequest per descendant left by

[1] The first period of life is a dummy period in which individuals only acquire the human capital necessary for being productive when they become workers.

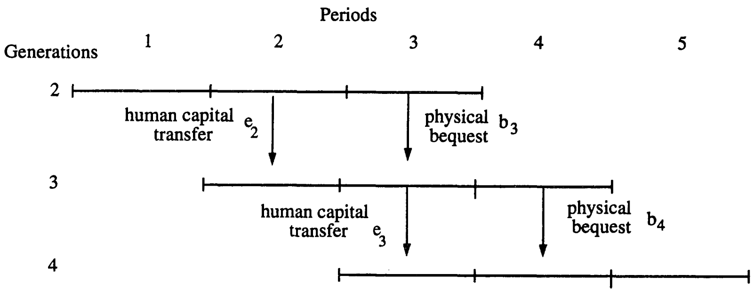

FIG. 1. Transfers within a family

generation (working in period) $t-1$, c_t is the consumption in period t (worker consumption), n is the exogenous number of children per parent (i.e. n is the gross rate of population growth), e_t is the investment in each of his children's human capital, and s_t is the saved income. When an agent is old, he receives a return from his saving. This return is distributed between consumption and physical bequest to his heirs. Thus, the budget constraint of an old agent is

$$R_{t+1}s_t = x_{t+1} + nb_{t+1} \tag{2}$$

where R_{t+1} is the gross rate of return on savings, and x_{t+1} is the consumption in period $t+1$ (old consumption).

We can combine (1) and (2) to get the life-cycle budget constraint

$$(\hat{w}_t + b_t - ne_t - c_t)R_{t+1} = x_{t+1} + nb_{t+1} \tag{3}$$

We also have the following non-negativity constraint

$$\mathrm{b}_t \geqslant 0 \tag{4}$$

This constraint is institutional since parents cannot force their children to give them gifts when they (the parents) are old as a payment for their investment in education.

Figure 1 gives a graphical representation of the transfers within a family. Three generations of a family and five time periods are represented in that figure.

The utility of an individual belonging to generation t, V_t, is represented by

$$V_t = U(c_t, x_{t+1}) + \beta V_{t+1} \tag{5}$$

where V_{t+1} is the utility of each of his heirs.[2] The function $U(\cdot, \cdot)$ is assumed to be twice continuously differentiable and homothetic. These homothetic preferences will allow a steady state in spite of labor-augmenting technological growth. Let $U_c(c_t, x_{t+1})$ and $U_x(c_t, x_{t+1})$ be the derivatives of $U(c_t, x_{t+1})$

[2] This is not a model of 'joy of giving' as the one in Yaari (1965) or Abel (1986a). In those models the utility was a function of the size of the bequest left to one's heir. Our model follows Barro's (1974) formulation in which parents derive utility from their heirs' utilities.

with respect to its first and second arguments respectively. We assume that

$$U_c(c_t, x_{t+1}) > 0, \quad U_x(c_t, x_{t+1}) > 0, \quad \lim_{c_t \to 0} U_c(c_t, \bar{x}) = \lim_{x_{t+1} \to 0} U_x(\bar{c}, x_{t+1}) = \infty$$

for $\bar{c} > 0$ and $\bar{x} > 0$, and that $U(\cdot, \cdot)$ is strictly concave. The parameter $\beta > 0$ is the altruism factor which may be a function of the number of children per parent.

In order to concentrate our analysis on the steady state in which the extensive variables grow geometrically and the intensive ones are constant, we have to assume that all reproducible goods are produced by means of linearly homogeneous production functions. In particular, the total output Y_t is produced according to the twice continuously differentiable, increasing, strictly concave and linearly homogeneous production function, $Y_t = F(K_t, L_t)$, where K_t is the amount of capital and L_t is the labor measured in efficiency units.Capital fully depreciates after one period. Expressing the production function in terms of efficency units of labor, we can write $y_t = f(k_t)$ whenever $L_t > 0$, where y_t and k_t denote output and capital per unit of efficiency labor services, respectively. We assume that $f'(\cdot) > 0$, $f''(\cdot) < 0$, for $k > 0$, $f(0) = 0$, and the Inada conditions

$$\lim_{k \to 0} f'(k) = \infty \quad \text{and} \quad \lim_{k \to \infty} f'(k) = 0$$

The unit of labor supplied by agent i in period t is converted into efficiency units of labor L_t^i by the following relationship

$$L_t^i = \psi(e_{t-1}^i, \bar{e}_{t-1}) \tag{6}$$

where $\psi_t(\cdot, \cdot)$ is a function that depends on the investment in human capital e_{t-1}^i made by agent i's parent in period $t-1$, and on the average level $\bar{e}_{t-1}$ of education (or skill) in the economy. This extra term embodies the same kind of externality as in Arrow (1962), Romer (1986), and Lucas (1988), which is a consequence of the social nature of production. Since our economy is large, no parent takes into account this external effect when deciding how much to invest in his children's human capital. We also assume that $\psi(\cdot, \cdot)$ is linearly homogeneous and

$$\psi_1(e_t^i, \bar{e}_t) \equiv \frac{\delta \psi(e_t^i, \bar{e}_t)}{\partial e_t} > 0, \quad \psi_2(e_t^i, \bar{e}_t) \equiv \frac{\partial \psi(e_t^i, \bar{e}_t)}{\partial \bar{e}_t} \; 0, \quad \psi_{11}(e_t^i, \bar{e}_t) \equiv \frac{\partial^2 \psi(e_t^i, \bar{e}_t)}{\partial e_t^2} \leqslant 0$$

and

$$\psi(0, \cdot) = 0$$

Since all agents are assumed to be equal, we have $e_t^i = e_t$ for all i. The aggregate number of efficiency units supplied in period t is $L_t = \psi(e_{t-1}, \bar{e}_{t-1})N_t$, where N_t is the number of workers in period t, each supplying a single unit of physical labor.

Firms are able to observe the level of skill of each worker, and are competitive. Therefore, factors are paid their marginal products

$$R_t = R(k_t) \equiv f'(k_t) \tag{7}$$

and

$$w_t = w(k_t) \equiv f(k_t) - f'(k_t)k_t \tag{8}$$

where w_t is the wage per efficiency unit; therefore the wage per worker is

$$\hat{w}_t = \psi(e_{t-1}, \bar{e}_{t-1})w_t \tag{9}$$

We are saying implicitly that savings are converted into productive capital by banks or financial intermediaries that lend capital to firms. Banks are competitive and make zero profits. Therefore, the interest on bank accounts or savings is equal to the competitive rate of return on capital. Since skill of workers is uniformly distributed across firms, the return on capital is a function of the average capital-efficient labor ratio k_t. Therefore, the wage per efficiency unit also depends on k_t exclusively. No parent has enough weight in this large economy to change k_t and affect the competitive price of an efficiency unit of labor. However, each parent is able to control the efficiency units of labor that his children are going to supply when they become workers given the average level $\bar{e}_{t-1}$ of education.

The maximization of (5) with respect to $\{c_t, x_{t-1}, e_t, b_{t+1}\}$ subject to (3), (4), and (9), is equivalent to solving the following dynamic programming problem

$$\begin{aligned} V_t^*(e_{t-1}, b_t) = \max_{\{c_t, e_t, b_{t+1}\}} & \left[U(c_t, (\psi(e_{t-1}, \bar{e}_{t-1})w_t + b_t - ne_t - c_t)R_{t+1} - nb_{t+1})\right. \\ & \left. + \beta V_{t+1}^*(e_t, b_{t+1})\right] \end{aligned} \tag{C}$$

such that $b_{t+1} \geqslant 0$, and $\bar{e}_t$, w_t and R_{t+1} are exogenous parameters, for all t.

Using the envelope theorem to evaluate

$$\frac{\partial V_{t+1}^*}{\partial e_t} \quad \text{and} \quad \frac{\partial V_{t+1}^*}{\partial b_{t+1}}$$

the following first order conditions, corresponding to the derivatives with respect to c_t, e_t and b_{t+1} respectively, are obtained

$$U_c(c_t, x_{t+1}) = U_x(c_t, x_{t+1})R_{t+1} \tag{10}$$

$$nU_c(c_t, x_{t+1}) = \beta U_c(c_{t+1}, x_{t+2})\psi_1(e_t, \bar{e}_t)w_{t+1} \tag{11}$$

$$nU_x(c_t, x_{t+1}) \geqslant \beta U_c(c_{t+1}, x_{t+2}) \quad \text{with equality if } b_{t+1} > 0 \tag{12}$$

Equation (10) gives us the optimal allocation of consumption for an agent over his lifetime. One unit of consumption saved in period t increases consumption in $t + 1$ by R_{t+1} units. At a maximum, the untility lost in period t has to be equal to the utility gained in period $t + 1$.

Equation (11) gives the optimal investment in human capital. An agent of generation t reduces his consumption as a worker until this loss equates the increment in the discounted utility *per capita* of his children. The utility of each child is raised because a higher level of human capital implies a higher wage. Given the assumptions on $U(\cdot, \cdot)$, $f(\cdot)$, and $\psi(\cdot, \cdot)$ at the origin, e_t is always strictly positive, and eq. (11) holds with equality, as in Drazen (1978) and Nerlove *et al.* (1988).[3]

Finally, eq. (12) characterizes the optimal physical bequest. When an agent belonging to generation t reduces his consumption in one unit when old, he raises the utility of each of his heirs by

$$\frac{U_c(c_{t+1}, x_{t+2})}{n}$$

When the bequest motive is operative ($b_{t+1} > 0$), the loss of parents must be equal to the discounted utility gain of their heirs due to higher bequest. If the non-negativity constraint is strictly binding, then the parents would like to receive gifts from their children. But, in our model with one-sided altruism, gifts are ruled out, and therefore, there is an excess of consumption by generation $t + 1$ in the middle period of life. This means that inequality (12) becomes strict.

When $b_{t+1} > 0$, it is easy to combine (10)–(12) to obtain

$$R_{t+1} = \psi_1(e_t, \bar{e}_t)w_{t+1} \tag{13}$$

This means that parents invest in the human capital of each child until the marginal productivity of human capital equates the return on saving. This is an arbitrage condition between human and nonhuman capital as means of transferring wealth intertemporally. This arbitrage condition is also found in Nerlove *et al.* (1988). However, if $b_{t+1} = 0$ and $nU_x(c_t, x_{t+1}) > \beta U_c(c_{t+1}, x_{t+2})$, that is, if the non-negativity constraint is strictly binding, then $R_{t+1} < \psi_1(e_t, \bar{e}_t)w_{t+1}$ and this suggests underinvestment in human capital for a given $\bar{e}_t$. Parents would like to invest more in the education of their children if they could obtain part of the return on this investment by means of gifts. But, as we have already pointed out, b_t cannot be negative since there is no institutional mechanism to enforce such a liability with future generations. Therefore, parents cut back their expenditures on education.

3. Balanced competitive equilibrium

To maintain the tractability of the analysis we are going to assume, as in Lucas (1988), Abel (1986a), or Laitner (1988), that the utility function $U(c_t, x_{x+1})$ is not only homothetic, but also additively separable (i.e. it belongs to the Bergson class). Therefore, we can write

$$U(c_t, x_{t+1}) = u(c_t) + \delta u(x_{t+1})$$

[3] Note that the investment in human capital includes not only education but also food, medical services, and other factors that increase the 'quality' of future workers.

and, from Katzner (1970, Theorem 2.4-4), $u(\cdot)$ admits the following representation

$$u(z) = \begin{cases} \dfrac{z^{1-\gamma}}{1-\gamma} & \text{for} \quad \gamma \neq 1 \\ \ln z & \text{for} \quad \gamma = 1 \end{cases}$$

where $\gamma > 0$ is the inverse of the elasticity of intertemporal substitution in consumption.

We also assume $\delta \in (0, 1]$. This condition on the intertemporal discounting factor, combined with (22) below, will be sufficient for bounded dynastic utility at a balanced equilibrium.

We can write the savings function for an agent of generation t as

$$s_t = s(Y_{w,t}, Y_{o,t+1}, R_{t+1}) = \psi(e_{t-1}, \bar{e}_{t-1})w_t + b_t - ne_t - c_t$$

where $Y_{w,t} \equiv \psi(e_{t-1}, \bar{e}_{t-1})w_t + b_t - ne_t$ is the endowment of a worker, $Y_{o,t+1} \equiv -nb_{t+1}$ is the non-positive endowment of an old agent, and c_t, e_t and b_{t+1} are solutions to problem (C). It can be shown that this specific utility function yields the following savings function

$$s(Y_w, Y_o, R) = (1 - [1 + \delta^{1/\gamma}R^{(1-\gamma)/\gamma}]^{-1})Y_w - \frac{1}{R}(1 + \delta^{1/\gamma}R^{(1-\gamma)/\gamma})^{-1}Y_o \quad (14)$$

Note that saving is strictly increasing in the interest rate if and only if $\gamma < 1$ and provided both Y_w and Y_o are positive. In this case consumption when a worker and when old are strict gross substitutes.

The next step is to define an equilibrium path for this economy. First, we should write the market clearing condition that states that the amount saved by generation t is equal to the physical capital available as productive input in period $t + 1$

$$K_{t+1} = N_t s(\psi(e_{t-1}, \bar{e}_{t-1})w_t + b_t - ne_t, -nb_{t+1}, R_{t+1})$$

Divide both sides by N_t and use (6) to obtain

$$n\psi(e_t, \bar{e}_t)k_{t+1} = s(\psi(e_{t-1}, \bar{e}_{t-1})w_t + b_t - ne_t, -nb_{t+1}, R_{t+1}) \quad (15)$$

Definition 1 A competitive equilibrium path for the economy described above is a sequence $\{k_{t+1}, c_t, x_{t+1}, e_t, b_{t+1}\}_{t=0}^{\infty}$ such that : (i) c_t, x_{t+1}, e_t, and b_{t+1} maximize (5) subject to (3), (4), and (9) with w_t, R_{t+1}, and $\bar{e}_t$ taken as exogenous for $t = 1, 2, \ldots$; (ii) factors are paid their marginal product according to (7) and (8); (iii) the market clears (eq. (15) holds); (iv) $k_1 = k_1^v > 0$, $e_0 = e_0^v > 0$ and $c_0 = c_0^v > 0$ are the exogenous initial conditions; and (v) $e_t = \bar{e}_t$ for $t = 0, 1, 2, \ldots$.

Since we are going to confine attention to balanced equilibrium paths, we introduce the following definition:

Definition 2 A balanced equilibrium path is a competitive equilibrium path in which k_t is constant and c_t, x_t, e_t, and b_t are constant or grow exponentially at the same gross rate g which is endogenously determined in equilibrium.

Dividing both sides of (15) by $\psi(e_{t-1}, \bar{e}_{t-1})$, which is the technological level available in period t, and using the homotheticity of the utility function that makes the savings function linearly homogeneous in the income profile, and the equilibrium condition $e_t = \bar{e}_t$, for all t, we obtain

$$\frac{n\psi(\bar{e}_t, \bar{e}_t)k_{t+1}}{\psi(\bar{e}_{t-1}, \bar{e}_{t-1})} = s\left(w_t + \frac{b_t}{\psi(\bar{e}_{t-1}, \bar{e}_{t-1})} - \frac{n\bar{e}_t}{\psi(\bar{e}_{t-1}, \bar{e}_{t-1})}, \frac{-nb_{t+1}}{\psi(\bar{e}_{t-1}, \bar{e}_{t-1})}, R_{t+1}\right) \tag{16}$$

Define $\bar{\psi} = \psi(1, 1)$. Using the linear homogeneity of $\psi(\cdot, \cdot)$, we can write (16) as

$$ng_{t+1}k_{t+1} = s\left(w_t + \frac{b_t}{\bar{\psi}\bar{e}_{t-1}} - \frac{n\bar{e}_t}{\bar{\psi}\bar{e}_{t-1}}, \frac{-nb_{t+1}}{\bar{\psi}\bar{e}_{t-1}}, R_{t+1}\right) \tag{17}$$

where

$$g_{t+1} \equiv \frac{\psi(\bar{e}_t, \bar{e}_t)}{\psi(\bar{e}_{t-1}, \bar{e}_{t-1})} = \frac{\bar{e}_t}{\bar{e}_{t-1}}$$

is the gross rate of growth of the technological level from t to $t + 1$.

Since along a balanced path, and under labor-augmenting technological progress, k_t is constant and all extensive variables (c_t, x_t, b_t, e_t) grow at the constant rate of technological growth, $g_t = g$, we can write the steady-state market clearing condition

$$ngk = s\left(w(k) + \hat{b} - \frac{ng}{\bar{\psi}}, -ng\hat{b}, R(k)\right) \tag{18}$$

where

$$\hat{b} = \frac{b_t}{\bar{\psi}\bar{e}_{t-1}}$$

for all t, i.e. $\hat{b}$ is the constant bequest per child and per efficiency unit of labor services in each period. Note that, along a balanced equilibrium path, $\hat{b}_t > 0$ if and only if $b_t > 0$ for all t.

Note that, since in equilibrium $e_t = \bar{e}_t$ and $\psi(\cdot, \cdot)$ is linearly homogeneous, $\psi_1(e_t, \bar{e}_t)$ is constant regardless of the value of e_t. Let us denote this derivative as ψ_1. Therefore, the balanced path equilibrium is compactly defined as a non-negative triplet $(g_c, k_c, \hat{b}_c)$ that solves the following system of equations in g, k, and $\hat{b}$

$$ngk = s\left(f(k) - kf'(k) + \hat{b} - \frac{ng}{\bar{\psi}}, -ng\hat{b}, f'(k)\right) \tag{19}$$

$$ng^{\gamma} = \beta\psi_1(f(k) - kf'(k)) \tag{20}$$

$$ng^{\gamma} \geqslant \beta f'(k) \quad \text{(with equality if } \hat{b} > 0) \tag{21}$$

where: (19) comes from (7), (8), and (18); (20) comes from (11), and the fact that

$$\frac{U_c(c_t, x_{t+1})}{U_c(c_{t+1}, x_{t+2})} = g^{\gamma}$$

at a balanced path; and (21) comes similarly from combining (10) and (12).

Finally, in order to guarantee that the dynastic utility is bounded above and the steady-state equilibrium satisfies the transversality conditions, we also assume

$$\beta g_c^{1-\gamma} < 1 \tag{22}$$

As we have already pointed out, the intercohort utility discount factor β may be a function of the number of children per parent. We can rewrite $\beta = n\delta\beta'$ as in Weil (1987) to indicate that each parent cares equally about his n children. Thus, β' is the pure interpersonal discount factor. Note from (20) and (21) that, when the economy is not bequest-constrained, the equilibrium rate of growth is increasing in β'. This is the same relationship between thriftiness and growth found in Lucas (1988).

Note also that the equilibrium rate of growth in a unconstrained economy does not depend on the rate of population growth under the linear altruism. However, this result does not hold if we assume concave altruism, as in Abel (1986b) or Barro and Becker (1989). Let us rewrite $\beta = n^{\varepsilon}\delta\beta'$, where $0 < \varepsilon < 1$. In this case the equilibrium rate of growth is decreasing in n, as the empirical evidence suggests.

4. Operative and inoperative physical bequest motive

Our next goal will be to find conditions under which parents leave positive physical bequests to their heirs. The propositions in this section state the necessary and sufficient conditions under which the physical bequest motive is operative.

Let us define $\bar{\beta}$ as the level of altruism such that the non-negativity constraint is just binding in equilibrium, i.e. $\hat{b} = 0$ and (21) holds with equality. This means that

$$g = \left(\frac{\beta R(k)}{n}\right)^{1/\gamma} \tag{23}$$

Also, from combining (20) and (21), we can define k^* as the unique positive solution to

$$f'(k) = \psi_1[f(k) - kf'(k)] \tag{24}$$

Given the assumptions on $f(k)$, there exists a unique solution to eq. (24).[4] Then,

[4] The strict concavity of $f(\cdot)$ implies that $f(k) - kf'(k)$ is strictly increasing in k and

$$\lim_{k \to 0} [f(k) - kf'(k)] = 0$$

These properties combined with the assumptions on $f'(k)$ are sufficient to prove the existence of a unique positive solution to eq. (24).

plugging (23) into (19), using (14), and making $\hat{b}$ equal to zero, we obtain

$$n\left(\frac{\beta R(k^*)}{n}\right)^{1/\gamma} k^* = [1 - H(k^*)]\left[w(k^*) - \frac{n\left(\frac{\beta R(k^*)}{n}\right)^{1/\gamma}}{\bar{\psi}} \right] \tag{25}$$

where

$$H(k) \equiv [1 + \delta^{1/\gamma} R(k)^{(1-\gamma)/\gamma}]^{-1}$$

Solving for β, we obtain the following critical level of atruism

$$\bar{\beta} = \frac{n^{1-\gamma}}{R(k^*)}\left[\frac{w(k^*)}{\frac{k^*}{1 - H(k^*)} + \frac{1}{\bar{\psi}}}\right]^{\gamma} \tag{26}$$

Proposition 1 If $\beta \leqslant \bar{\beta}$, then $\hat{b}_c = 0$ in a balanced equilibrium path.

Proof (By contradiction). Suppose $\bar{b}_c > 0$. Then, the equilibrium triplet $(g_c, k_c, \bar{b}_c)$ must satisfy

$$ng_c k_c = s\left(w(k_c) + \hat{b} - \frac{ng_c}{\bar{\psi}}, -ng_c\hat{b}_c, R(k_c)\right)$$

and (21) holds with equality. Therefore, (20) and (21) imply that $k_c = k^*$, regardless of the value of β. But, by the definition of k^*

$$ng^*k^* = s\left(w(k^*) - \frac{ng^*}{\bar{\psi}}, 0, R(k^*)\right)$$

where

$$g^* = \left(\frac{\bar{\beta} R(k^*)}{n}\right)^{1/\gamma}$$

On the other hand, since $\beta \leqslant \bar{\beta}$, the equilibrium rate of growth g_c associated with β is lower than or equal to g^* (from either (20) or (21)). Therefore, $g_c \leqslant g^*$ and $k_c = k^*$ imply

$$ng_c k^* \leqslant ng^*k^* = s\left(w(k^*) - \frac{ng^*}{\bar{\psi}}, 0, R(k^*)\right)$$
$$< s\left(w(k^*) + \hat{b}_c - \frac{ng_c}{\bar{\psi}}, -ng_c\hat{b}_c, R(k^*)\right)$$

The last inequality follows since savings are strictly increasing in worker income and strictly decreasing in last period income. Therefore, (g_c, k^*) cannot characterize a balanced equilibrium path with positive physical bequests. □

Before proceeding with the next proposition, we need to rewrite the steady-state market clearing condition (19) in a more convenient way. First, use (14), and divide both sides of (19) by ng, to obtain

$$k = [1 - H(k)]\left[\frac{w(k)}{ng} + \frac{\hat{b}}{ng} - \frac{1}{\bar{\psi}}\right] + \frac{H(k)}{R(k)}\hat{b} \tag{27}$$

Using (20), (27) becomes

$$k = [1 - H(k)]\left[\frac{w(k)^{(\gamma-1)/\gamma}}{n\left(\dfrac{\beta\psi_1}{n}\right)^{1/\gamma}} + \frac{\hat{b}}{n\left(\dfrac{\beta\psi_1 w(k)}{n}\right)^{1/\gamma}} - \frac{1}{\bar{\psi}}\right]$$

$$+ \frac{H(k)}{R(k)}\hat{b} \equiv \hat{s}(k, \beta, \hat{b}) \tag{28}$$

where the expression $\hat{s}(k, \beta, \hat{b})$, defined in the last identity, gives the saving per efficiency unit of labor in the next period as a function of the capital–labor ratio, the altruism factor, and the bequest per efficiency unit. Note that, by definition, k^* satisfies $k^* = \hat{s}(k^*, \bar{\beta}, 0)$.

We introduce also the following assumption

Assumption A $k > \hat{s}(k, \bar{\beta}, 0)$ for $k > k^*$

Note that this assumption is weaker than the one found in Abel (1987) and Weil (1987). Since $k^c \geqslant k^*$ in a bequest-contrained equilibrium, assumption (A) readily implies that k^* is the unique steady-state of the capital–labor ratio when the altruism factor is $\bar{\beta}$.

Lemma 1 gives a sufficient condition for (A), and Lemma 2 characterizes the behavior of the function $\hat{s}(k, \bar{\beta}, 0)$ when the sufficient condition of Lemma 1 is not satisfied.

Lemma 1 If $\gamma \leqslant 1$, then (A) holds, and k^* is the unique positive solution to equation

$$k = \hat{s}(k, \hat{\beta}, 0) \tag{29}$$

Proof Note that, from (28)

$$\hat{s}(k, \bar{\beta}, 0) = [1 - H(k)]\left[\frac{w(k)^{(\gamma-1)/\gamma}}{n\left(\dfrac{\bar{\beta}\psi_1}{n}\right)^{1/\gamma}} - \frac{1}{\bar{\psi}}\right] \tag{30}$$

For $\gamma = 1$, $\hat{s}(k, \bar{\beta}, 0)$ is equal to k^* for all k. For $\gamma < 1$, is is easy to check that the function $\hat{s}(k, \bar{\beta}, 0)$ satisfies

$$\lim_{k \to 0} \hat{s}(k, \bar{\beta}, 0) = \infty$$

and is strictly decreasing in k when it is positive-valued since both

$$[1 - H(k)] \equiv [1 - [1 + \delta^{1/\gamma} R(k)^{(1-\gamma)/\gamma}]^{-1}] \quad \text{and} \quad w(k)^{(\gamma-1)/\gamma}$$

are strictly decreasing in k. The implications in the statement of the lemma then follow immediately. □

Lemma 2 If $\gamma > 1$ then k^* is not generically the unique positive solution to eq. (29).

Proof For $\gamma > 1$, we see from (30) that there exists an $\eta > 0$ such that, for all $k \in (0, \eta)$, $\hat{s}(k, \bar{\beta}, 0) < 0$, Moreover, it can be checked that

$$\lim_{k \to \infty} [1 - H(k)] = 1 \quad \text{and} \quad \lim_{k \to \infty} \left[\frac{1}{k} \frac{w(k)^{(\gamma - 1)/\gamma}}{n\left(\frac{\bar{\beta}\psi_1}{n}\right)^{1/\gamma}} - \frac{1}{\bar{\psi}} \right] = 0$$

This implies that

$$\lim_{k \to \infty} \left\{ \frac{\hat{s}(k, \bar{\beta}, 0)}{k} \right\} = 0$$

and, therefore, the function $\hat{s}(k, \bar{\beta}, 0)$ lies below the 45°-line for k sufficiently high. This, together with the fact that $k^* = \hat{s}(k^*, \bar{\beta}, 0)$, implies that the function $\hat{s}(k, \bar{\beta}, 0)$ crosses generically the 45°-line at least twice. □

Even if consumption when a worker and when old are not gross substitutes (i.e. when $\gamma > 1$), assumption (A) might hold provided γ is close enough to one. For instance, if the production function is Cobb-Douglas ($f(k) = Ak^{\alpha}$) with $A = 1$, $\alpha = 0.9$, $\delta = 1$, and $n = 1$, $\psi_1 = 0.04$ and $\bar{\psi} = 0.05$, we have $k^* = 15$. If $\gamma = 1.2$, then eq. (29) has only one other solution at $k = 0.58$, and (A) holds there. However, if $\gamma = 3$, that equation has a further solution at $k = 22.4$ and, therefore, (A) does not hold.

Proposition 2 Assume that (A) holds. If $\beta > \bar{\beta}$, then $\hat{b}_c > 0$ in a balanced equilibrium path.

Proof (By contradiction). Assume that $\hat{b}_c = 0$ and $\beta > \bar{\beta}$. It follows from (20) and (21) that $k_c \geqslant k^*$ and $g_c \geqslant g^*$. The steady-state market clearing condition (28) becomes

$$k_c = [1 - H(k_c)] \left[\frac{w(k_c)^{(\gamma - 1)/\gamma}}{n\left(\frac{\beta\psi_1}{n}\right)^{1/\gamma}} - \frac{1}{\bar{\psi}} \right] \equiv \hat{s}(k_c, \beta, 0) \tag{31}$$

and by definition of $\bar{\beta}$

$$k^* = [1 - H(k^*)] \left[\frac{w(k^*)^{(\gamma - 1)/\gamma}}{n\left(\frac{\bar{\beta}\psi_1}{n}\right)^{1/\gamma}} - \frac{1}{\bar{\psi}} \right] \equiv \hat{s}(k^*, \bar{\beta}, 0)$$

if $\beta > \bar{\beta}$, then $\hat{s}(k, \beta, 0) < \hat{s}(k, \bar{\beta}, 0)$ for all k. Therefore, from assumption (A), k_c must be smaller than k^* in order to satisfy the equilibrium condition with zero bequest (31), and this yields a contradiction. □

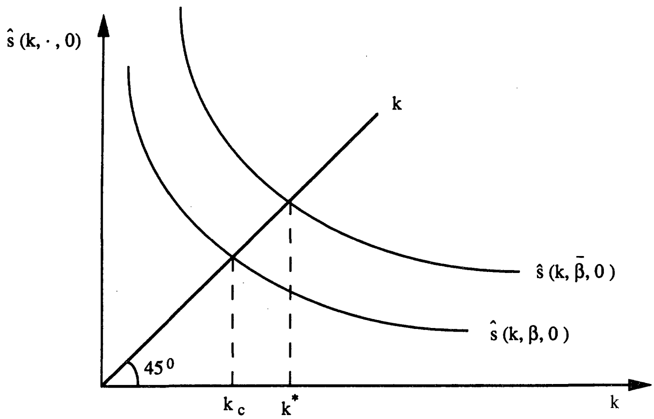

FIG. 2. Proposition $2(\beta > \bar{\beta})$

Figure 2 gives a graphic illustration of Proposition 2. Note that $\hat{s}(k, \beta, \hat{b})$ may have a positive derivative with respect to k at k^*, and satisfy (A) simultaneously.

When $\beta > \bar{\beta}$, the economy has no transitional dynamics since the capital–labor ratio jumps to the level k^* after the first period, as dictated by the arbitrage condition (13).[5] However, the economy behaves as in the standard OLG model with production when $\beta < \bar{\beta}$ and therefore multiple steady states (some of them unstable) with zero bequests may exist. As usual, we will concentrate on the comparative statics of locally stable steady-states.

Under assumption (A), we have shown that parents leave positive physical bequests if and only if the altruism factor β is greater than the critical level $\bar{\beta}$. Therefore, an interesting question that arises is: do externalities in the accumulation of human capital make the non-negativity constraint on bequests more likely to be binding in a steady-state equilibrium? The parameter ψ_1 measures the degree of externalities of human capital. A lower value of this parameter means a higher level of externalities, and the maximum value of ψ_1 is $\bar{\psi}$, which corresponds to an economy without externalities.[6] We cannot provide a general result about the relationship between ψ_1 and $\bar{\beta}$ except for the following example with logarithmic utility ($\gamma = 1$) and Cobb-Douglas technology.

Example If $\gamma = 1$, the condition (22) for bounded dynastic utility becomes simply $\beta < 1$. By virtue of Lemma 1, (A) holds in this sample. Let us assume the production function $f(k_t) = Ak_t^{\alpha}$. For this case, the solution to equation

[5] This lack of transition is also found for the same reasons in the Ak models by Barro (1990) and Rebelo (1991).

[6] To prove that $\bar{\psi} \geqslant \psi_1$, observe that in equilibrium, and since $\psi(\cdot, \cdot)$ is linearly homogeneous, $\psi(e_t, \bar{e}_t) = \bar{\psi} e_t = \psi_1 e_t + \psi_2 \bar{e}_t \geqslant \psi_1 e_t$, and the result follows.

(24) is

$$k^* = \frac{\alpha}{(1-\alpha)\psi_1} \tag{32}$$

and the following relationships hold

$$R(k^*) = A\alpha^\alpha(1-\alpha)^{1-\alpha}\psi_1^{1-\alpha} \tag{33}$$

$$w(k^*) = \frac{A\alpha^\alpha(1-\alpha)^{1-\alpha}}{\psi_1^\alpha} \tag{34}$$

$$1 - H(k) = \frac{\delta}{1+\delta} \tag{35}$$

Therefore, plugging (32)–(35) into (26), the critical level of altruism becomes

$$\bar{\beta} = \frac{1}{\dfrac{\alpha/(1-\alpha)}{\delta/(1+\delta)} + \dfrac{\psi_1}{\bar{\psi}}}$$

which is clearly decreasing in ψ_1. Hence, when the accumulation of human capital exhibits large externalities (ψ_1 is low), the economy is more likely to be bequest-constrained ($\bar{\beta}$ is high). Note also that the physical bequest motive is always inorperative when

$$\psi_1 < \bar{\psi}\left(1 - \frac{\alpha/(1-\alpha)}{\delta/(1+\delta)}\right)$$

since $\bar{\beta}$ is greater than 1 in this case, whereas β must be less than 1 so as to satisfy the transversality condition at infinity.

The impossibility of finding a definite sign for the relationship between ψ_1 and $\bar{\beta}$ when $\gamma \neq 1$ is illustrated by means of the numerical example in Table 1.

A relation between $\bar{\beta}$ and the gross rate of population growth can also be obtained from (26). Assuming linear altruism, we can write $\bar{\beta} = n\delta\bar{\beta}'$, and (26) becomes

$$\bar{\beta}' = \frac{n^{-\gamma}}{R(k^*)}\left[\frac{w(k^*)}{\dfrac{k^*}{1-H(k^*)} + \dfrac{1}{\bar{\psi}}}\right]^\gamma$$

which is clearly decreasing in n. This means that higher rates of population growth make the non-negativity constraint on bequests less likely to be binding. Under concave altruism ($\bar{\beta} = n^\varepsilon\delta\bar{\beta}'$, $\varepsilon \in (1, 1)$), the critical intercohort discount factor $\bar{\beta}'$ is decreasing in n if and only if ε is greater than $1-\gamma$.

5. The planned solution

From the point of view of a social planner who takes into account all external effects in the accumulation of human capital, the competitive equilibrium path is inefficient because no parent takes into account these externalities when

TABLE 1
The relationship between ψ_1 and $\bar{\beta}$

$\gamma = 2, \delta = 0.8, n = 1$ $A = 1, \alpha = 0.5, \bar{\psi} = 4$		$\gamma = 0.2, \delta = 0.2, n = 2$ $A = 1, \alpha = 0.8, \bar{\psi} = 0.5$	
ψ_1	$\bar{\beta}$	ψ_1	$\bar{\beta}$
0.5	0.1104	0.25	0.3789
1.5	0.1210	0.30	0.3523
2.5	0.1152	0.35	0.3312
3.5	0.1070	0.40	0.3607

deciding how much to invest in his children's education. Let us assume that a time-consistent social planner in period zero is able to select the optimal sequence $\{c_t, x_{t+1}, K_{t+1}, e_t\}_{t=0}^{\infty}$, and has the same preferences and the same degree of altruism towards future generations as the individual agents.

Note that the planner does not discriminate between e_t and $\bar{e}_t$. Hence, we can write the human capital technology as perceived by the planner in the following way

$$\psi(e_t, \bar{e}_t) = \bar{\psi} e_t \tag{36}$$

Therefore, the maximization problem faced by the planner is

$$\max_{\{c_t, x_{t+1}, K_t, e_t\}} V_0 = \sum_{t=0}^{\infty} \beta^t U(c_t, x_{t+1}) \tag{P}$$

subject to

$$K_{t+1} = F(K_t, \bar{\psi} e_{t-1} N_t) - N_t c_t - N_{t-1} x_t - N_{t+1} e_t \tag{37}$$

with the exogenous initial conditions $c_0 = c_0^v > 0$, $K_1 = K_1^v > 0$ and $e_0 = e_0^v > 0$.

Equation (37) is the aggregate budget constraint that states that the total production in period t has to be distributed between capital accumulation and consumption of old individuals, workers and children (investment in human capital). The constraint can be written in intensive terms as

$$n\bar{\psi} e_t k_{t+1} = \bar{\psi} e_{t-1} f(k_t) - c_t - \frac{x_t}{n} - n e_t \tag{38}$$

Solving for x_t in (38), substituting into the objective function, and performing the derivatives with respect to c_t, e_t, and k_t, respectively, we obtain the following first order conditions for the planner's optimization problem

$$nU_x(c_{t-1}, x_t) = \beta U_c(c_t, x_{t+1}) \tag{39}$$

$$nU_c(c_t, x_{t+1}) = \beta U_c(c_{t+1}, x_{t+2}) f'(k_{t+1}) \tag{40}$$

$$nU_c(c_t, x_{t+1}) = \beta U_c(c_{t+1}, x_{t+2}) \bar{\psi} [f(k_{t+1}) - k_{t+1} f'(k_{t+1})] \tag{41}$$

Equation (39) tallies with eq. (12) in the individual maximization problem when the non-negativity constraint on physical bequests is not strictly binding. The equation gives the intergenerational optimal allocation. Combining (39) and (40), we get

$$U_c(c_t, x_{t+1}) = U_x(c_t, x_{t+1})f'(k_{t+1}) \tag{42}$$

which tallies in equilibrium with eq. (10) in the individual maximization problem. Finally, (41) is similar to eq. (11).

We need also the constraint (38) to fully characterize the solution of the planner's problem. It can be easily seen that this constraint is also embedded in the individual agents' budget constraints, the market clearing condition, and the competitive payments to factors.

Hence, the two unique differences between the competitive solution and the efficient one are: (i) there is a discrepancy between (11) and (41) due to the divergence between the social return ($\bar{\psi}$) and the private return (ψ_1) from investment in human capital; and (ii) the first order condition (12) always holds with equality at the efficient equilibrium because the planner is not facing non-negativity constraints, and she may redistribute resources from young to old agents.

Using our specific functional form for the utility function, and combining (39)–(41), the values of k and g in a balanced efficient path of growth are described as the solution of the following two equations

$$ng^{\gamma} = \beta\bar{\psi}[f(k) - kf'(k)] \tag{43}$$

$$ng^{\gamma} = \beta f'(k) \tag{44}$$

Let us denote the solutions to the above system as k_e and g_e.[7] Assume that the competitive economy is not bequest-constrained, and that k_c and g_c are the solutions to eqs (20) and (21) when $\hat{b} > 0$. Then, since $\bar{\psi} > \psi_1$, It is obvious that $k_e < k_c$ and $g_e > g_c$. Therefore, the economy under *laissez faire* in which the bequest motive is operative grows at a lower rate than the planned economy does. Moreover, the planned economy exhibits a lower capital intensity and higher shadow interest rates.

If the economy is bequest-constrained ($\hat{b} = 0$), then it is obvious that the capital–labor ratio in the planned economy is lower than in the *laissez faire*, bequest-constrained economy. As in Weil (1987) the inoperativeness of the physical bequest motive is linked to the problem of overaccumulation of capital. The comparison between growth rates in the two regimes is ambiguous when externalities are present since, on the one hand, $\bar{\psi} > \psi_1$, but, on the other hand, the planned solution involves a smaller capital–labor ratio (see (20) and (43)). Figure 3 shows graphically why this ambiguity may arise, depending on whether

[7] The solutions of these two equations m[illegible] the condition for bounded utility. This condition is like (22), with g_e replacing g_c. Moreover, from (40) and (41), we have the arbitrage condition $f'(k_{t+1}) = \bar{\psi}[f(k_{t+1}) - k_{t+1}f'(k_{t+1})]$ for all $t \geqslant 0$. Therefore, the efficient path displays no transition, and $k_{t+1} = k_e$ for all $t \geqslant 0$.

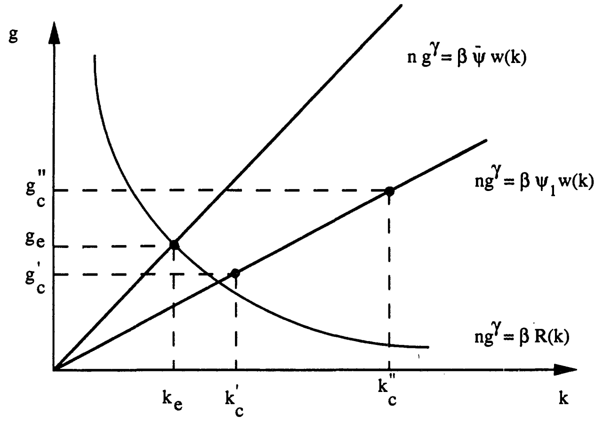

FIG. 3. Efficient and competitive growth rates

the bequest-constrained competitive equilibrium is k_c' or k_c''. However, if external effects are absent ($\bar{\psi} = \psi_1$), the efficient rate of growth is always lower than the competitive one in a bequest-constrained economy. Since wages per efficiency unit are decreasing in the capital–labor ratio, the optimal return from investment in education decreases and thus the economy grows slowly at a balanced efficient path. Therefore, the optimal rate of growth is higher than the competitive one in a bequest-constrained economy only when the externalities in human capital are sufficiently large.

The ambiguity in the comparison of growth rates when the non-negativity constraint on bequest is binding can be also illustrated with our previous logarithmic, Cobb-Douglas example.

Example (Continued) If the nonnegativity constraint is strictly binding, the altruistic factor β must satisfy

$$\beta < \frac{1}{\dfrac{\alpha/(1-\alpha)}{\delta/(1+\delta)} + \dfrac{\psi_1}{\bar{\psi}}} = \bar{\beta} \tag{45}$$

The competitive steady-state values of k and g are

$$k_c = \frac{\delta}{1+\delta}\left[\frac{1}{\beta\psi_1} - \frac{1}{\bar{\psi}}\right], \qquad g_c = \frac{\beta\psi_1 A(1-\alpha)}{n}\left[\frac{\delta}{1+\delta}\left(\frac{1}{\beta\psi_1} - \frac{1}{\bar{\psi}}\right)\right]^{\alpha} \tag{46}$$

and the efficient steady-state values are

$$k_e = \frac{\alpha}{(1-\alpha)\bar{\psi}}, \qquad g_e = \frac{\beta\bar{\psi} A(1-\alpha)}{n}\left[\frac{\alpha}{(1-\alpha)\bar{\psi}}\right]^{\alpha} \tag{47}$$

Using (45), it is easy to check that $k_e < k_c$. Combining (46) and (47), we see

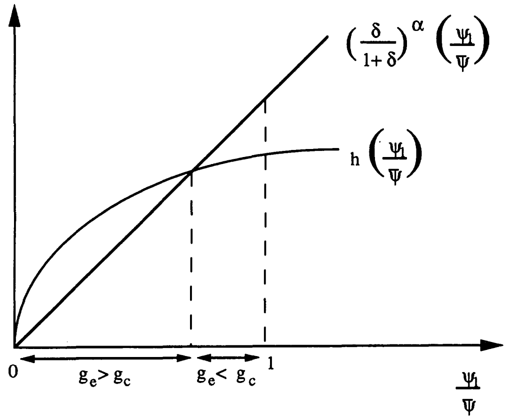

FIG. 4. Logarithmic Cobb-Douglas example

that $g_c > (<) g_e$ if and only if

$$\left(\frac{\delta}{1+\delta}\right)^{\alpha}\left(\frac{\psi_1}{\bar{\psi}}\right) > (<) \left[\frac{\dfrac{\alpha}{(1-\alpha)}}{\dfrac{1}{\beta(\psi_1/\bar{\psi})} - 1}\right]^{\alpha} \equiv h\left(\frac{\psi_1}{\bar{\psi}}\right)$$

where the function $h(\cdot)$ is positive, and has the following properties:

$$h(0) = 0, \qquad \lim_{x \to 0} h'(x) = \infty \quad \text{and} \quad h(1) < \left(\frac{\delta}{1+\delta}\right)^{\alpha}$$ [8]

Therefore, the comparison between g_c and g_e is clearly ambiguous (see Fig. 4), except when ψ_1 is close enough either to zero or to $\bar{\psi}$.

6. The effects of fiscal policy

It is easy to analyze the effects of different fiscal policies on the steady-state equilibrium. We will study the effects of marginal changes in taxes on both the competitive rate of growth and the capital-labor ratio (and, as a by-product,

[8] To prove

$$h(1) < \left(\frac{\delta}{1+\delta}\right)^{\alpha}$$

note that condition (45) can be rewritten as

$$\frac{\delta}{1+\delta} > \frac{\dfrac{\alpha}{1-\alpha}}{\dfrac{1}{\beta} - 1} = [h(1)]^{1/\alpha}$$

on the interest rates and wages per efficiency unit of labor). We should distinguish in our analysis the case in which the non-negativity constraint on physical bequests is binding and the one in which it is not. Finally, we will assume for simplicity that the government runs a balanced budget and the revenue from proportional taxes is spent entirely on public goods that either do not enter agents' utility function or enter in an additive way.

When the economy is unconstrained, our analysis resembles the one of Abel aand Blanchard (1983). We assume that the non-negativity constraint on bequest is not binding either before or after the implementation of the change in policy.

When $\hat{b} > 0$, the equilibrium values of k and g are exclusively determined by eqs (20) and (21), with (21) holding with equality. Since these marginal relationships are not altered, lump-sum taxes and transfers do not affect the equilibrium values of k_c and g_c. Since all agents of the same dynasty are inked by means of intergenerational transfers, marginal changes in lump-sum taxes will only modify the amount of physical bequest left from parents to their descendants.

Let us proceed to study the effects of proportional taxation. We will consider the following fiscal policies: a tax on inheritances, a tax on interest income, a tax on labor (or wages), and a subsidy on education or child maintenance. The proportional tax (subsidy) rates will be t_b, t_c, t_w, and s_e, respectively.

With the previous set of fiscal instruments, and assuming that the economy is not bequest-constrained, eqs (10)–(12) become

$$U_c(c_t, x_{t+1}) = (1 - t_c)U_x(c_t, x_{t+1})R_{t+1} \tag{48}$$

$$(1-s_e)nU_c(c_t, x_{t+1}) = (1 - t_w)\beta U_c(c_{t+1}, x_{t+2})\psi_1(e_t, \bar{e}_t)w_{t+1} \tag{49}$$

$$nU_x(c_t, x_{t+1}) = (1 - t_b)\beta U_c(c_{t+1}, x_{t+2}) \tag{50}$$

From the previous equations, we obtain the following ones that determine the steady-state values k_c and g_c

$$(1 - s_e)ng^\gamma = (1 - t_w)\beta\psi_1 w(k) \tag{51}$$

$$ng^\gamma = (1 - t_b)(1 - t_c)\beta R(k) \tag{52}$$

which are equivalent to (20) and (21).

The effects of marginal changes in tax rates are summarized in the following proposition:

Proposition 3 Assume that the bequest motive is operative both before and after the change in fiscal policy. Then, the capital–labor ratio is decreasing in t_b, t_c, ands_e, and increasing in t_w. The equilibrium rate of growth is decreasing in t_b, t_c, and t_w, and increasing in s_e.

Proof Obvious from (51) and (52). □

The equivalence between taxes on interest income and on bequests is clear since these fiscal instruments reduce the return on savings, and agents save for both old consumption and bequest reasons. Therefore, when either capital income or inheritances are taxed, savings and bequests must decrease. This implies a lower capital–labor ratio and, from (51), a lower rate of growth since the return on eduction has decreased.

Both a reduction in the subsidy rate on education and an increase in the tax rate on labor reduce the return from investment in human capital and thus decrease the equilibrium rate of growth.

The effects on the capital–labor ratio and on the interest rate of these two pairs of fiscal instruments are opposite in direction. Both taxes on interest income and on inheritances increase the equilibrium interest rate, while the opposite is true for the policies that either increase the tax rate on labor or reduce the subsidy rate on education.

Finally, note that all previous effects on g and k also hold if the government remits the proceeds from taxation in a lump-sum fashion to the private sector when the physical bequest motive is operative both before and after the change in fiscal policy.

The previous set of policies, with the exception of the tax on bequests, can also be analyzed when the economy is bequest-constrained in equilibrium ($\hat{b}_c = 0$). We concentrate our analysis to equilibrium values of k such that satisfy the following assumption

Assumption B There exists a real number $\zeta > 0$ such that $k < \hat{s}(k_c, \beta, 0)$ for $k \in (k_c - \zeta, k_c)$, and $k > \hat{s}(k_c, \beta, 0)$ for $k \in (k_c, k_c + \zeta)$, where k_c is the capital–labor ratio in a steady-state equilibrium for a bequest-constrained economy with a level of altruism equal to β ($\leqslant \bar{\beta}$).

Assumption (B) is a local version of a similar assumption made by Weil (1987) and Abel (1987), which turns out to be sufficient for the local stability of the steady state in their models. Note that, by definition, $k_c = \hat{s}(k_c, \beta, 0)$. Assumption (B) is automatically satisfied when consumption as a worker and when old are gross substitutes ($\gamma \leqslant 1$), and k_c is also the unique solution to the equation $k = \hat{s}(k, \beta, 0)$ (see the proof of Lemma 1). If $\gamma > 1$, then there is an even number of balanced equilibria (or none), as can be seen from the proof of Lemma 2, and therefore, we restrict attention to marginal changes of fiscal policy around the balanced equilibrium for which (B) holds.

Moreover, the dynamic behavior of the capital–labor ratio in a bequest-constrained economy is governed by the following relationship (which comes from (14), (17), and (20))

$$k_{t+1} = [1 - H(k_{t+1})]\left[\frac{w(k_t)^{(\gamma-1)/\gamma}}{h\left(\frac{\beta\psi_1}{n}\right)^{1/\gamma}} - \frac{1}{\bar{\psi}}\right] \tag{53}$$

When $\gamma < 1$, implicit differentiation of (53) shows that the capital labor–ratio displays cycles around the steady-state, and that assumption (B) is necessary for the stability of this steady state. However, if $\gamma > 1$, then assumption (B) can be shown to be equivalent to the assumption that the steady state is locally stable and that the convergence to k_c is non-oscillatory. Obviously, there is no transition when $\gamma = 1$.

For bequest-constrained economies, the equations equivalent to (20) and (28) are (51) and the following

$$k = [1 - [1 + \delta^{1/\gamma}[(1 - t_c)R(k)]^{(1-\gamma)/\gamma}]^{-1}]\left[\frac{\{(1 - t_w)w(k)]^{(\gamma-1)/1}}{n\left[\dfrac{\beta\psi_1}{(1 - s_e)n}\right]^{1/\gamma}} - \frac{1}{\bar{\psi}}\right] \quad (54)$$

The next proposition, referring to the marginal introduction of proportional taxes, follows easily from (51) and (54).

Proposition 4 Assume that (B) holds and that the economy is bequest-constrained in equilibrium both before and after the change in fiscal policy.

(i) If $\gamma < 1$, then the capital-labor ratio is increasing in t_w and decreasing in t_c, the equilibrium rate of growth is decreasing in t_c, and the effect of a change in t_w on the growth rate is ambiguous.
(ii) If $\gamma > 1$, then both the capital–labor ratio and the rate of growth are decreasing in t_w and increasing in t_c.
(iii) The capital–labor ratio is always decreasing in s_e. The effect of a change in s_e on the growth rate is ambiguous.

The ambiguity of the effect on g_c when a subsidy on education is introduced can be explained in a similar fashion as in the discussion of Section 5, where we compared the efficient and competitive rates of growth. When a subsidy on education is introduced, the profitability of investment in human capital increases, and the private return from the investment approaches its social return. This allows the capital–labor ratio to decrease and, therefore, wages go down. If the reduction in wages outweighs the increase in the subsidy, the economy will grow more slowly so as to satisfy the optimality condition (51).

When the economy is bequest-constrained, lump-sum taxes are also an effective fiscal instrument. We restrict attention again to economies that satisfy assumption (B), and that are bequest-constrained in equilibrium both before and after the change in fiscal policy.

First, consider the marginal introduction of an unfunded social security system. This system collects funds from workers and distributes them equally among old agents. Let T_t be the social security tax paid by workers and S_t be the transfer received by old agents in period t. In order to have balanced paths, we also assume that taxes increase at the same rate as the average level of technological progress $\psi(\bar{e}_t, \bar{e}_t)$. Let us also define

$$\hat{T} = \frac{T_t}{\psi(\bar{e}_{t-1}, \bar{e}_{t-1})} \quad \text{and} \quad \hat{S} = \frac{S_t}{\psi(\bar{e}_{t-1}, \bar{e}_{t-1})}$$

Note that $\hat{T}$ is directly proportional to the total taxes per output. Since the social security system runs a balanced budget, we have that $n\hat{T} = \hat{S}$. After the introduction of the social security system, the steady-state market clearing condition (28) becomes

$$k = [1 - H(k)]\left[\frac{w(k)^{(\gamma-1)/\gamma}}{n\left(\frac{\beta\psi_1}{n}\right)^{1/\gamma}} - \frac{\hat{T}}{n\left(\frac{\beta\psi_1 w(k)}{n}\right)^{1/\gamma}} - \frac{1}{\bar{\psi}}\right] - \frac{H(k)}{R(k)}\hat{T} \equiv \hat{s}(k, \beta, -\hat{T})$$

Since the derivative of $\hat{s}(k, \beta, -\hat{T})$ with respect to $\hat{T}$ is negative, assumption (B) readily implies that the new equilibrium value of k must be lower after the marginal introduction of an unfunded social security system. We also see from (20) that the introduction of this social security system reduces the stationary rate of growth of the economy.

Another effective policy for a bequest-constrained economy consists of a change in lump-sum taxes, accompanied by the issue of government debt. As in the previous case, we assume that the amount of both government bonds and taxes are proportional to the level of technological progress. Denoting government debt, taxes on workers, and taxes on old agents per efficiency unit in period t as $\hat{d}_t$, $\hat{\tau}_{1t}$, and $\hat{\tau}_{2t}$ respectively, we can write the following government budget constraint

$$ng_{t+1}\hat{d}_{t+1} = R_t\hat{d}_t - \hat{\tau}_{1t} - \frac{1}{n}\hat{\tau}_{2t}$$

Assuming that $\hat{d}_t$, $\hat{\tau}_{1t}$, and $\hat{\tau}_{2t}$ are constant, we obtain the steady-state government budget constraint

$$\hat{\tau}_1 = (R - ng)\hat{d} - \frac{1}{n}\hat{\tau}_2 \tag{55}$$

Using (55), the steady-state market clearing condition can be written as

$$F(k, \hat{d}, \hat{\tau}_2) \equiv s\left[w(k) - (R(k) - ng(k))\hat{d} + \frac{1}{n}\hat{\tau}_2, -g(k)\hat{\tau}_2, R(k)\right] - ng(k)(k + \hat{d}) = 0$$

where the function

$$g(k) = \left(\frac{\beta f'(k)}{n}\right)^{1/\gamma}$$

comes from (20) and gives the growth rate as a function of the capital-labor ratio.

We now analyze the effects of a marginal introduction of public debt, accompanied by an introduction of lump-sum taxes (or subsidies) per efficiency unit paid by workers, keeping the lump-sum taxes paid by old agents at the zero level (uncompensated public debt).

One can prove that

$$\frac{\partial F(k, \hat{d}, \hat{\tau}_2)}{\partial \hat{d}} = -ng(k)\left(1 - \frac{\partial s}{\partial Y_1}\right) - R(k)\frac{\partial s}{\partial Y_1} < 0$$

where the last inequality follows since both worker and old consumption are normal goods, i.e.

$$0 < \frac{\partial s}{\partial Y_1} < 1$$

Notice that $F(k, \hat{d}, \hat{\tau}_2)$ at $\hat{d} = \hat{\tau}_2 = 0$ can be rewritten as $ng(k)[s(k, \beta, 0) - k]$. Note also that Assumption (B) implies that the derivative of $\hat{s}(k, \beta, 0)$ with respect to k is less than one at the equilibrium. Combining the previous facts, we obtain

$$\frac{\partial F(k, \hat{d}, \hat{\tau}_2)}{\partial k} < 0 \quad \text{at} \quad \hat{d} = \hat{\tau}_2 = 0$$

Therefore, implicit differentiation yields

$$\frac{\partial k}{\partial \hat{d}} < 0$$

This, combined with (2), implies that the introduction of public deficits are associated with lower values of the long run rate of growth. It can be shown similarly that the same effects hold when the taxes per efficiency unit paid by workers are kept constant at the zero level, and the introduction of public debt is accompanied by an introduction of lump-sum taxes (or subsidies) paid by old agents.

Summing up our discussion, we can state the following:

Proposition 5 Assume that (B) holds and that the economy is bequest-constrained in equilibrium both before and after the change in fiscal policy. The introduction of either an unfunded social security system or uncompensated public debt lowers both the capital–labor ratio and the rate of growth.

To end this section we discuss the design of optimal fiscal policies. Recall that the optimal capital–labor ratio, k_e, and the optimal rate of growth, g_e, are given by the solutions to the system (43)–(44). Moreover, from comparing the first order conditions for an efficient path, (39), (41), and (42), with those of the competitive solution, (48)–(50), we just need to select tax rates satisfying

$$t_b = t_c = 0 \quad \text{and} \quad \frac{(1 - t_w)}{(1 - s_e)} = \frac{\hat{\psi}}{\psi_1}$$

The cost of this optimal policy will be financed through lump-sum taxes on workers. Furthermore, we must solve for $\hat{b}$ in eq. (19), when $k = k_e$ and $g = g_e$. Let $\hat{b}_e$ be the unique solution, which can be interpreted as the optimal bequest per child and efficiency unit in each period. If $\hat{b}_e > 0$, it is unnecessary to introduce additional lump-sum taxation since the members of each dynasty will be linked endogenously by a stationary bequest equal to $\hat{b}_e$. However, if $\hat{b}_e < 0$, old agents must receive a transfer at an optimal path.[9] Therefore, it is easy to see from (19) and (55), that in this case the optimal lump-sum taxes per efficiency unit must satisfy $\hat{\tau}_1 = -\hat{b}_e > 0$ and $\hat{\tau}_2 = n\hat{b}_e < 0$. Obviously, the government runs a balanced budget with this policy mix.

7. Conclusion

This paper has developed a life-cycle model for endogenous growth with human capital as a social factor of production. We have compared the *laissez-faire* equilibrium with the efficient path in which the externalities are taken into account by a social planner. The most striking result of this comparison is that an efficient equilibrium displays a higher rate of growth in a bequest-constrained economy only if external effects are sufficiently large. In this case, we recover the argument in favor of high rates of growth from a welfare viewpoint.

We have also shown that the same fiscal policies may have very different effects when they are applied to bequest-contrained economies and to economies with an operative physical bequest motive. In particular, subsidies on education may have negative effects on the equilibrium rate of growth when the economy is bequest-constrained.

We believe that a promising avenue of future research would be the simulation of a large-scale model (as in Auerbach and Kotlikoff 1987; Laitner 1990; Arrau 1989) within the framework provided by our model. By replacing the assumption on the life length from three periods to, say, 75, we will obtain a more realistic setup that will enhance the income effects associated with permanent policy changes. This simulation will also allow us to analyze quantitatively the transition path from one balanced equilibrium to another.

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[9] Agents face a higher return from education when optimal taxes on labor income and subsidies on education are introduced. Even if the economy was not bequest-constrained before this non-marginal change in policy, it might become it after the new taxes are implemented, and vice-versa, as Table 1 suggests. However, for the logarithmic Cobb-Douglas example considered before, if the economy was unconstrained before the change in policy, then it remains unconstrained after the change, since the threshold level of altruism for operative physical bequest motive decreases when external effects are internalized.

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