Tax Evasion and Economic Growth*

Jordi Caballé† Judith Panadés‡

June 2000

Abstract

In this paper we analyze how the tax compliance policy affects the rate of economic growth. We consider a model of overlapping generations in which the paths of all the macroeconomic variables are endogenously determined and we perform the comparative statics analysis of changes in both the probability of inspection and the penalty fee imposed on tax evaders. We also show the non-optimality from the growth viewpoint of an inspection policy inducing truthful revelation of income for exogenously given levels of both the penalty and the tax rates. Finally, we show that "hanging evaders with probability zero" is the most growth enhancing policy among all the inspection policies inducing honest behavior by the taxpayers.

JEL classification code: H26, O41.

Keywords: Tax Evasion, Endogenous Growth, Fiscal Policy.

* Financial support from the Spanish Ministry of Education through grant PB96-1160-C02-02 and the Generalitat of Catalonia through grant SGR98-62 is gratefully acknowledged. We want to thank Inés Macho, Pau Olivella, and a referee of this journal for their valuable suggestions. Of course, they should not bear any responsibility for the remaining errors.

† Unitat de Fonaments de l’Anàlisi Econòmica and CODE. Universitat Autònoma de Barcelona.
‡ Unitat de Fonaments de l’Anàlisi Econòmica. Universitat Autònoma de Barcelona.

Correspondence address: Jordi Caballé. Universitat Autònoma de Barcelona. Departament d’Economia i d’Història Econòmica. Edifici B. 08193 Bellaterra (Barcelona). Spain.
1 Introduction

The aim of this paper is to analyze how the tax compliance policy affects the rate of economic growth. It is well known that proportional taxation matters for growth since taxes distort the accumulation of capital. It is usually found in standard growth models with infinite horizon that the rate at which either physical or human capital is accumulated increases with their private return (see, among many others, Lucas (1988), Lucas (1990), and Rebelo (1991)) and, hence, high tax rates on income are typically associated with low growth rates.

Moreover, in overlapping generation models displaying endogenous growth, individuals face a finite life span and the capital accumulation is a direct consequence of the saving of the young individuals who earn a wage in exchange of the labor they hire to the firms. In this kind of models young individuals must purchase all the capital installed in the economy during the next period. Therefore, an increase in the income tax rate reduces the disposable income of the workers and, thus, capital accumulation becomes also slower. Furthermore, since the after-tax interest rate decreases, savings will also fall provided the elasticity of intertemporal substitution is high enough to generate a saving function that is increasing in the interest rates (see Jones and Manuelli (1992)).

On the other hand, taxation is also affecting the process of economic growth since it generates resources to finance the supply of the productive inputs provided by the government (see, for instance, Barro (1990) and Turnovsky (1997)). Such inputs take usually the form of public goods, like roads or public education. Since firms are not charged by the use of these public goods, government spending plays the role of an externality for the productive sector. Such an externality ends up being a engine of endogenous growth since the resulting aggregate production function could display an uniformly high marginal productivity from private capital, and this makes perpetual capital accumulation possible (see Jones and Manuelli (1990)). Therefore, as pointed by Barro (1990), there is a tension between the role of taxation in disincentiving the accumulation of capital and the role of the public spending financed by these taxes in raising the return from private capital and, hence, the speed of accumulation.

Obviously, an effective tax system must be enforceable, that is, it must provide incentives to the taxpayers for tax compliance. Without these incentives nobody would pay taxes voluntarily in a competitive economy. Therefore, it seems pertinent to have a closer look to the instruments that allow the government to enforce the tax system. The two complementary instruments available to the tax collecting agency in order to enforce the tax legislation are the inspections and the fines imposed on tax evaders. We thus analyze the effects of changes in the parameters characterizing these two policy instruments on the rate of economic growth. To this end, we have to consider a dynamic general equilibrium model in which the paths of aggregate output, wages, interest rates, saving and consumption are all endogenously determined. We should mention at this point that the literature has paid little attention to the
Our general equilibrium approach forces us to an extreme stylization of the economy under study. Thus, we consider an overlapping generations model with production à la Diamond (1965) for which we parametrize both preferences and technologies. In such an economy young individuals obtain an income accruing from the labor services they supply to the firms. There is a proportional tax on declared labor income and collected taxes will finance productive inputs supplied by the government. Note that we have then all the elements necessary to reproduce the tension between government revenues and public spending since collected taxes will discourage savings whereas public spending will raise the marginal productivity of private capital.

We will assume that under-reporting of income is a risky and illegal activity. Agents are investigated with positive probability and, if a taxpayer is caught evading, she must pay a proportional penalty on the amount of evaded taxes, as in Yitzhaki (1974). The proceeds from penalties levied to the tax payers that are caught under-reporting are also used to finance public capital. Therefore, the enforcement policy has real effects since it generates funds to finance public capital formation through two channels: (i) it makes taxpayers to behave more honestly so that they end up paying more taxes and (ii) it generates additional resources accruing from the fines paid by audited evaders.

The combination of the penalty fee, the audit probability, and the tax rate determines not only the amount of declared income, but also the amount of labor income saved which will be used for consumption in the next period. Such a saving determines the private capital installed in the economy that, together with public capital, determines in turn the evolution of all the remaining macroeconomic variables. The economy will end up displaying a balanced growth path which is parametrized by its corresponding endogenous rate of long-term growth.

Our main findings include the comparative statics of the two tax compliance instruments on the aforementioned endogenous rate of long-term growth. Such a comparative statics is generally ambiguous and depends on the importance of publicly provided inputs in the production process. Such an ambiguity arises since the configurations of probability of audit and penalty for under-reporting that increase overall tax revenues will also increase growth through the public capital formation channel. However, since a greater overall tax revenue means less disposable income for individuals, there will be less saving, less investment, and lower rates of growth. Obviously, the net effect on growth will generally depend on the relative elasticity of output with respect to the two types of capital.

In our analysis the combination of the audit probability and the penalty fee determines whether individuals become partial evaders, total evaders (i.e., they...
do not even fill their income report), or honest taxpayers (i.e., they declare their total income). We characterize the effects on long-term growth of changing the tax compliance parameters in the previous three scenarios and, in some cases, we can make the result of the comparative statics exercises independent of the relative productivity of the two types of capital. For instance, when taxpayers behave honestly, an increase in the penalty rate has no consequences whereas an increase in the probability of inspection amounts to incur in a useless additional cost associated with the inspection effort. Such a waste of resources immediately translates into lower speed of accumulation in equilibrium.

We also provide two other results also found in partial equilibrium analyses of the tax evasion problem. The first one refers to the non-optimality from the growth viewpoint of an inspection policy inducing truthful revelation of income for exogenously given levels of both the penalty and the tax rates. This result follows since, if there is truthful revelation, then a slight reduction in the costly inspection effort reduces negligibly the amount of collected taxes whereas the resources liberated by the tax collection agency can be devoted to the provision of growth enhancing public services. The second result refers to the growth maximizing combination of the two instruments. Such a combination depends on how productive is public capital relative to private capital. In particular, if inducing complete tax compliance is optimal, then a policy with an arbitrarily high penalty fee and a low inspection probability allows the implementation of a growth rate arbitrarily close to that of an economy without tax evasion.

The paper is organized as follows. Section 2 presents the taxpayer optimization problem. Section 3 determines the equilibrium of the economy from the interaction among consumers, firms and the government. Section 4 analyzes the implications of the tax enforcement policy for economic growth. Section 5 concludes the paper.

2 The Tax Evasion Problem

Let us consider an overlapping generations (OLG) economy populated by a continuum of identical individuals living for two periods. A new generation is born in each period and there is no population growth. Generations are indexed by the period in which they are born. Individuals own a unit of labor when they are young (the first period of their lives) and this unit of labor is supplied inelastically to the firms in exchange of a wage. Labor income is subjected to a proportional tax and the tax rate is \( \tau \in (0, 1) \). An individual of generation \( t \) declares a level \( x_t \) of labor income during the first period of life. Therefore, the amount of taxes paid voluntarily will be \( \tau x_t \). Since tax evasion is possible, \( x_t \) might be less than the real wage \( w_t \). With probability \( p \in (0, 1) \) individuals are subjected to investigation by the tax authority and, if such an investigation takes place, the tax collecting agency detects the true labor income earned by the taxpayer. In such a case, the taxpayer will have to pay a proportional penalty rate \( \pi > 1 \) on the amount of evaded taxes \( \tau (w_t - x_t) \). Note that, even if there is no uncertainty in our model, the tax authority must audit an individual to indisputably certify that she is an evader and to impose her the corresponding penalty.
Our specification of the tax evasion problem is thus the same as in Yitzhaki (1974) since the penalty is imposed on evaded taxes while Allingham and Sandmo (1972) assume instead that the penalty is on undeclared income. Note however that, if the tax rate $\tau$ is exogenously given, all our analysis can be adapted to the setup of Allingham and Sandmo by replacing the penalty rate $\pi$ by $\pi / \tau$, where $\pi$ would be the penalty rate on unreported income.

We now introduce two doses of realism in the tax system in order to prevent counterfactual behavior by the taxpayers. First, if an individual has declared more than her true labor income, and this individual is audited, then the excess tax contribution is just returned. In other words, the penalty rate applying to ”negative” tax evasion is 1. Under such an assumption, no individual will declare more than her true wage because excess tax contribution is in fact a risky investment having a negative risk premium.\(^2\)

Second, the tax legislation does not feature a ”loss offset”. This means that the tax rate applying to negative income is zero. Hence, the tax code establishes that only agents declaring positive income must fill the tax form and pay the corresponding taxes on declared income. Note that individuals not filling the tax form are in fact implicitly declaring that they have earned a labor income equal to zero.

The sequence of events is the following. First, young individuals work and receive their wages. Then, they fill the report where they voluntarily declare the labor income they have earned and they pay the corresponding taxes. Consumption in the first period of life takes place. Let $s_t$ denote the income disposable after an individual has consumed and paid the taxes on declared income. Then, the potential inspection occurs with probability $p$. Obviously, the effective saving of an agent which has not been audited is $s_t$ while the saving of an audited agent will be $s_t - \pi \tau (w_t - x_t)$. The gross rate of return on the amount effectively saved is $R_{t+1}$. Capital income will be consumed when individuals are old (i.e., in the second period of life). An old individual does not have any other source of income and thus her consumption will be $R_{t+1} (s_t - \pi \tau (w_t - x_t))$ if she has been audited, or $R_{t+1} s_t$ if she has not. Since the inspection occurs after consumption has taken place, taxing the income of old agents is not enforceable and, therefore, capital income is tax exempt.

The following table summarizes the sequence of events within each period of life:

\(^2\)Recall that risk averse agents take risky positions if and only if the associated risk premium is strictly positive (see Arrow (1970)).
First period of life

<table>
<thead>
<tr>
<th>Individuals work.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wages are paid.</td>
</tr>
<tr>
<td>Individuals declare their labor income and pay the corresponding taxes.</td>
</tr>
<tr>
<td>Young consumption takes place.</td>
</tr>
<tr>
<td>Tax inspection occurs with probability ( p ) and the corresponding penalty is paid.</td>
</tr>
</tbody>
</table>

Second period of life

<table>
<thead>
<tr>
<th>Return on saving is paid.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Old consumption takes place.</td>
</tr>
</tbody>
</table>

The preferences of an agent of generation \( t \) are represented by the time-additive Von Neumann-Morgenstern utility function

\[
u(C_1^t) + \delta E \left( u(\tilde{C}_{t+1}^2) \right),
\]

where \( C_1^t \) denotes consumption in the individual’s first period of life (young consumption) and \( \tilde{C}_{t+1}^2 \) is the random consumption in the second period of life (old consumption). The random variable \( \tilde{C}_{t+1}^2 \) takes two values, \( C_{t+1}^{2A} \) and \( C_{t+1}^{2N} \), which correspond to old consumption if the individual has been audited, and old consumption if she has not been audited, respectively. The parameter \( \delta > 0 \) is the discount factor.

Therefore, an individual of generation \( t \) chooses both the declared income \( x_t \in [0, w_t] \) and the intended saving \( s_t \) in order to solve the following program:

\[
\text{Max} \ \{ u(C_1^t) + (1 - p) \delta u(C_{t+1}^{2N}) + p \delta u(C_{t+1}^{2A}) \},
\]

subject to

\[
C_1^t = w_t - \tau x_t - s_t,
\]

\[
C_{t+1}^{2N} = R_{t+1} s_t, \text{ and}
\]

\[
C_{t+1}^{2A} = R_{t+1} (s_t - \pi \tau (w_t - x_t)).
\]

For tractability we will assume that the expected utility representation \( u \) is logarithmic, i.e., \( u(C) = \ln C \). The analysis can be generalized to an isoelastic utility, \( u(C) = \frac{C^{1-\sigma}}{1-\sigma} \) with \( \sigma > 0 \). However, this generalization will yield a saving function that will not be independent of the interest rate and this will substantially complicate the analysis. In fact, the relationship between saving and the interest rate is an unsolved empirical question and to build a model abstracting from such a relationship is thus a reasonable, defensive position.\(^{3}\) Clearly, our results will be

---

\(^{3}\)It should be noticed that the Von Neumann-Morgenstern utility function (1) is time-additive, homothetic, and exhibits a saving function that is independent of the interest rate if and only if \( u \) is logarithmic.
qualitatively similar if we assume instead isoelastic utilities having a parameter \( \sigma \) sufficiently close to 1. As we will see, a policy leading to greater enforcement (due to an increase either in \( p \) or in \( \pi \)) will modify the long-term interest rate and, thus, if saving were not independent of the interest rate, we will have a new channel through which the tax compliance policy could influence the rate of capital accumulation.

**Lemma 1** The solution to the individual’s optimization program (2) is given by

\[
x_t = X w_t, \quad \text{and} \quad s_t = S w_t,
\]

where

\[
X = \begin{cases} 
0 & \text{if } p \pi \leq \frac{(\tau + \delta)p}{\tau(1 + \delta) + \delta(1 - \tau)p}, \\
\frac{(1 - p)\tau(1 + \delta p \pi) - (1 - p \pi)(p \delta + \tau)}{p \tau(\pi - 1)(1 + \delta)} & \text{if } \frac{(\tau + \delta)p}{\tau(1 + \delta) + \delta(1 - \tau)p} < p \pi < 1, \\
1 & \text{if } p \pi \geq 1,
\end{cases}
\]

and

\[
S = \begin{cases} 
S_1 & \text{if } p \pi \leq \frac{(\tau + \delta)p}{\tau(1 + \delta) + \delta(1 - \tau)p}, \\
\frac{\pi \delta(1 - p)(1 - \tau)}{(\pi - 1)(1 + \delta)} & \text{if } \frac{(\tau + \delta)p}{\tau(1 + \delta) + \delta(1 - \tau)p} < p \pi < 1, \\
\frac{\delta(1 - \tau)}{(1 + \delta)} & \text{if } p \pi \geq 1,
\end{cases}
\]

with

\[
S_1 = \frac{(\delta + (1 + \delta(1 - p)) \pi \tau) + \sqrt{((\delta + (1 + \delta(1 - p)) \pi \tau)^2 - 4 \delta \pi \tau (1 - p)(1 + \delta)}}}{2(1 + \delta)}.
\]

**Proof.** See the appendix.

Notice that intended saving before tax inspection \( s_t \) and reported income \( x_t \) are both linear in actual wages. From (3) we see that the condition for obtaining an interior solution for declared income, \( x_t \in (0, w_t) \), can be rewritten as

\[
\frac{(\tau + \delta)}{\tau(1 + \delta) + \delta(1 - \tau)p} < p \pi < \frac{1}{p}.
\]

It should be pointed out that the inequalities in (6) are satisfied by a plausible parameter configuration like

\[
\tau = 0.25, \quad \pi = 3, \quad p = 0.05, \quad \delta = 0.425.
\]
Figure 1 shows the regions of the parameters $p$ and $\pi$ for which we obtain either interior or corner solutions for the declared income $x_t$. In the interior of region B, the optimal solution satisfies $x_t \in (0, w_t)$ whereas $x_t = w_t$ in region A and $x_t = 0$ in region C. The function of $p$ defining the frontier between regions B and C is the decreasing and convex hyperbole given by the first expression in (6). Note that such an expression becomes equal to \( \frac{(\tau + \delta)}{\tau(1 + \delta)} \) whenever $p = 0$. The frontier between regions A and B is clearly another hyperbole given by the locus satisfying $p\pi = 1$.

Therefore, for a given value of $p \in (0, 1)$, it is clear that the set of values of the penalty rate $\pi$ for which $x_t \in (0, w_t)$ constitutes an open interval $(\pi, \pi)$ with

$$\pi = \frac{(\tau + \delta)}{\tau(1 + \delta) + \delta(1 - \tau)p} \in \left( \frac{1}{p} \right)$$

and $\pi = \frac{1}{p}$. Moreover, for a given value of the penalty rate $\pi > 1$, the set of values of the audit probability $p$ for which $x_t \in (0, w_t)$ constitutes also an open interval $(p, \bar{p})$ with $\bar{p} = \frac{1}{\pi}$. The infimum $\underline{p}$ of this interval is equal to zero when the mild condition

$$\pi \geq \frac{(\tau + \delta)}{\tau(1 + \delta)}$$

is imposed whereas $\underline{p} \in (0, \frac{1}{\pi})$ when the weak inequality in (8) does not hold.

The following partial derivatives concerning the behavior of both the propensity to declare $X$ and the propensity to save $S$ for an interior solution are obtained from (3) and (4):

$$\frac{\partial X}{\partial p} = \frac{\pi \delta(1 - \tau)}{\tau(\pi - 1)(1 + \delta)} > 0,$$

$$\frac{\partial X}{\partial \pi} = \frac{\delta(1 - p)(1 - \tau)}{\tau(\pi - 1)^2(1 + \delta)} > 0,$$

$$\frac{\partial S}{\partial p} = \frac{-\delta \pi(1 - \tau)}{(\pi - 1)(1 + \delta)} < 0,$$

$$\frac{\partial S}{\partial \pi} = \frac{-\delta(1 - p)(1 - \tau)}{(\pi - 1)^2(1 + \delta)} < 0.$$

As expected, reported income is increasing in both the probability of investigation and the penalty rate $\pi$. Since individuals increase the income reported with $p$ and $\pi$, this immediately translates into a decrease of intended saving.

The effects of marginal changes in the policy parameters on the propensity to save $S$ when $p\pi < \frac{(\tau + \delta)p}{\tau(1 + \delta) + \delta(1 - \tau)p}$ can be obtained directly from implicitly differentiating the first order condition of problem (2). Since $x_t = 0$ in such a parameter region, the first order condition with respect to $s_t$ is

$$-u'(w_t - s_t) + (1 - p)R_{t+1}u'(R_{t+1}s_t) + pR_{t+1}u'(R_{t+1}(s_t - \pi tw_t)) = 0.$$  

(9)
Implicitly differentiating (9) we get

\[
\frac{\partial S_t}{\partial \pi} = \frac{p(R_{t+1})^2 \tau w_t u''(R_{t+1} (s_t - \pi \tau w_t))}{u''(w_t - s_t) + (1 - p)(R_{t+1})^2 u''(R_{t+1}s_t) + p(R_{t+1})^2 u''(R_{t+1} (s_t - \pi \tau w_t))} > 0,
\]

and

\[
\frac{\partial S_t}{\partial p} = \frac{R_{t+1} [u'(R_{t+1}s_t) + u'(R_{t+1} (s_t - \pi \tau w_t))]}{u''(w_t - s_t) + (1 - p)(R_{t+1})^2 u''(R_{t+1}s_t) + p(R_{t+1})^2 u''(R_{t+1} (s_t - \pi \tau w_t))} > 0,
\]

where the latter inequality comes from the fact that \( u'(R_{t+1} (s_t - \pi \tau w_t)) > u'(R_{t+1}s_t) \). Therefore, we can conclude that \( \frac{\partial S}{\partial \pi} > 0 \) and \( \frac{\partial S}{\partial p} > 0 \) in the interior of region C of Figure 1.

On the other hand, it is obvious from (4) that marginal changes in the tax compliance policy have no effects on the propensity to save \( S \) when \( p\pi > 1 \), i.e., when truthful revelation of income is already achieved.

3 Equilibrium

There are competitive firms in the economy that produce a single good according to the following Cobb-Douglas gross production function:

\[
Y_t = BK_t^\alpha \hat{L}_t^{1-\alpha}, \quad \text{with } B > 0, \alpha \in (0, 1),
\]

where \( Y_t \) is the gross output, \( K_t \) is the private capital used by each firm, and \( \hat{L}_t \) denotes the efficiency units of labor hired by each firm. Note that \( K_t \) might be interpreted as a composite capital embodying both physical and human capital. Efficiency units of labor are proportional to both the number \( L_t \) of physical units of labor and the level \( g_t \) of capital supplied by the government per worker, i.e.,

\[
\hat{L}_t = DL_t g_t, \quad \text{with } D > 0.
\]

Therefore, we are assuming that public capital increases proportionally the productivity of each worker as in Barro (1990). The services provided by public capital are assumed completely rival for the users so that is the amount of public capital per capita and not the total amount that enters in the production function. Moreover, we assume that there are neither user charges nor additional congestion effects associated with public services. Examples of such public services include public education, transportation systems, maintenance of law and order, etc.\(^4\) Public capital is thus a productive externality from the firms viewpoint. Hence, the production function (10) can be written as

\[
Y_t = AK_t^\alpha L_t^{1-\alpha} g_t^{1-\alpha},
\]

where \( A = BD^{1-\alpha} \). We assume that both private and public capital fully depreciate after one period.

---

\(^4\)Barro and Sala-i-Martín (1992) and Turnovsky (1997) discuss the growth implications of alternative assumptions on the nature of publicly provided services.
Taking \( g_t \) as given, the optimal demands for private capital and workers by firms must satisfy the first order conditions for profit maximization

\[
w_t = A(1 - \alpha)K_t^\alpha L_t^{-\alpha} g_t^{1-\alpha}, \quad (11)
\]

and

\[
R_t = A\alpha K_t^{\alpha-1} L_t^{-\alpha} g_t^{1-\alpha}. \quad (12)
\]

Given the constant returns to scale assumption, competitive firms will make zero profits and its number remains thus indeterminate. We normalize the number of firms to one per worker. Hence, equilibrium in the labor market implies that \( L_t = 1 \) for all \( t \). Therefore, (11) and (12) become in equilibrium

\[
w_t = A(1 - \alpha)K_t^\alpha g_t^{1-\alpha}, \quad (13)
\]

and

\[
R_t = A\alpha K_t^{\alpha-1} g_t^{1-\alpha}. \quad (14)
\]

On the other hand, equilibrium in the capital market implies that effective saving must be equal to the private capital installed in the next period,

\[
K_{t+1} = (1 - p)s_t + p(s_t - \pi \tau (w_t - x_t)). \quad (15)
\]

Since in this large economy a fraction \( p \) of individuals is subjected to tax investigation, the first term on the RHS of (15) is the effective saving of the non-audited population whereas the second term is the effective saving of the audited population. Substituting \( s_t \) and \( x_t \) by their optimal values given in Lemma 1, (15) becomes

\[
K_{t+1} = Mw_t, \quad (16)
\]

where

\[
M = S - p\pi \tau (1 - X). \quad (17)
\]

Note that \( M > 0 \) since effective saving after inspection is strictly positive. Using (3) and (4) to substitute for \( X \), and \( S \), we get the following explicit expression for \( M \):

\[
M = \begin{cases} 
S_1 - p\pi \tau & \text{if } p\pi \leq \frac{(\tau + \delta) p}{\tau(1 + \delta) + \delta(1 - \tau)p}, \\
\frac{\pi \delta (1 - \tau)(1 - 2p + p^2 \pi)}{(\pi - 1)(1 + \delta)} & \text{if } \frac{(\tau + \delta) p}{\tau(1 + \delta) + \delta(1 - \tau)p} < p\pi < 1, \\
\frac{\delta (1 - \tau)}{1 + \delta} & \text{if } p\pi \geq 1,
\end{cases}
\]

where \( S_1 \) is given in (5).

The government finances the stock of public capital by means of both the proportional taxes on declared income and the penalty fees collected from the audited taxpayers in the previous period. We assume that the government faces a proportional
inspection cost $c$ per unit of audited income. Therefore, the budget constraint of the government is

$$g_{t+1} = (1-p)\tau x_t + p(\tau x_t + \pi \tau (w_t - x_t)) - cpw_t,$$

(19)

where the first term on the RHS of (19) are the taxes paid by the non-audited taxpayers, the second term are the taxes plus the penalty fees paid by audited taxpayers, and the last term is the cost associated with tax inspection. Substituting the equilibrium value of $x_t$ given in Lemma 1, we get

$$g_{t+1} = Gw_t,$$

(20)

where

$$G = (1-p\pi)\tau X + p\pi \tau - cp.$$  

(21)

We can use (3) to get an explicit expression for $G$ in terms of the exogenous parameters,

$$G = \begin{cases} 
  p(\pi \tau - c) & \text{if } p\pi \leq \frac{(\tau + \delta)p}{\tau(1+\delta)+(1-\tau)p}, \\
  \frac{(1-p\pi)(\tau(\pi-1)+\delta\tau(1-p)-\delta(1-p\pi))}{(\pi-1)(1+\delta)} + p\pi \tau - cp & \text{if } \frac{(\tau + \delta)p}{\tau(1+\delta)+(1-\tau)p} < p\pi < 1, \\
  \tau - cp & \text{if } p\pi \geq 1.
\end{cases}$$

(22)

We will assume that $\tau > c$ since this assumption, together with the fact that $X \in [0,1]$, ensures that $G$ is strictly positive. That is, if the unitary cost of inspection $c$ is lower than the tax rate, the tax system always generates resources for positive public spending. Plugging (13) into (20) we obtain

$$g_{t+1} = GA(1-\alpha)K_t^{\alpha}g_t^{1-\alpha},$$

which can be rewritten as

$$\frac{g_{t+1}}{g_t} = GA(1-\alpha)\left(\frac{K_t}{g_t}\right)^\alpha.$$  

(23)

Furthermore, divide (16) by (20) and get

$$\frac{K_t}{g_t} = \frac{M}{G},$$

(24)

so that (23) becomes

$$\Gamma \equiv \frac{g_{t+1}}{g_t} = GA(1-\alpha)\left(\frac{M}{G}\right)^\alpha = A(1-\alpha)M^\alpha G^{1-\alpha}.$$  

(25)

Therefore, the gross rate of growth $\Gamma$ of public spending is constant for all $t$ along an equilibrium path. Hence, combining (13) and (24) we get

$$w_t = A(1-\alpha)\left(\frac{M}{G}\right)^\alpha g_t,$$

(26)
and, thus, wages also grow at the same gross rate $\Gamma$. Since the reported income $x_t$ and the intended savings $s_t$ are proportional to wages, and the same occurs with the several consumptions, as dictated by the constraints of problem (2), all these variables also grow at the rate $\Gamma$. Finally, from (14) and (24), the equilibrium interest rate is constant and equal to

$$R_t = A\alpha \left( \frac{G}{M} \right)^{1-\alpha}. \quad (27)$$

Note that this economy is always in a balanced growth path and thus displays no transition. This should not be surprising since the constant returns to scale assumption, together with the fact that public spending is proportional to installed capital (see (24)), implies that the model becomes of the $Ak$ type. Recall that the infinite horizon versions of the $Ak$ models of Barro (1990) and Rebelo (1991) did not display transition either.

4 Growth Effects of the Tax Compliance Policy

The effects of changes in the tax enforcement parameters on the gross rate $\Gamma$ of economic growth are exclusively determined by the induced changes in $M$ and $G$ as it can be seen from (25). The following partial derivatives for interior solutions can be obtained from (18) and (22) after some tedious algebra:

$$\frac{\partial M}{\partial \pi} = -\frac{\delta(1-p\pi)(1-\tau)(p(\pi-2)+1)}{(\pi-1)^2(1+\delta)} < 0, \quad (28)$$

$$\frac{\partial M}{\partial p} = -\frac{2\delta \pi(1-\tau)(1-p\pi)}{(\pi-1)(1+\delta)} < 0, \quad (29)$$

$$\frac{\partial G}{\partial \pi} = \frac{\delta(1-p\pi)(1-\tau)(p(\pi-2)+1)}{(\pi-1)^2(1+\delta)} > 0, \quad (30)$$

$$\frac{\partial G}{\partial p} = \frac{2\delta \pi(1-\tau)(1-p\pi) - c(1+\delta)(\pi-1)}{(\pi-1)(1+\delta)}. \quad (31)$$

The sign of the last partial derivative is ambiguous. However, $\frac{\partial G}{\partial p} > 0$ if and only if

$$2\delta \pi(1-\tau)(1-p\pi) > c(1+\delta)(\pi-1). \quad (32)$$

This condition is satisfied whenever the unitary cost of inspection is sufficiently low for given values of $p$ and $\pi$. For instance, the parameter configuration in (7) exhibits a positive derivative of $G$ with respect to $p$ if and only if $c < 0.553$. This is a mild restriction indeed since a reasonable calibration of the model would place the value of $c$ around 0.03.\(^5\) Under condition (32), the qualitative effects of $p$ and $\pi$ on $M$ are

\(^5\)Let us mention incidentally that, if the parameter values are set according to (7), $c = 0.03$, $\alpha = 0.34$, and $A = 11$, the resulting values of the gross rate of growth and of the gross interest rate per period would be $\Gamma = 1.49$ and $R = 2.53$, respectively. If we view a period in our model as having a length of 25 years, the corresponding net interest rate per year, $r \equiv (R)^{1/25} - 1$, would be 3.77\%, and the yearly net rate of growth, $\gamma \equiv (\Gamma)^{1/25} - 1$, would be 1.61\%.
the opposite of those on $G$. Therefore, in such a case, the private to public capital ratio $\frac{\partial G}{\partial \pi}$ is decreasing in both the probability of inspection $p$ and the penalty rate $\pi$, as follows from (24). Moreover, from (27), we see that the equilibrium interest rate is then increasing in both parameters of the tax compliance policy.

At the interior of the parameter region for which $X = 0$, the effects of changes in $p$ and $\pi$ on $M$ are ambiguous since the derivatives $\frac{\partial M}{\partial p}$ and $\frac{\partial M}{\partial \pi}$ might be either positive or negative depending on a quite complicated and non-intuitive relation involving $p$, $\pi$, $\delta$, and $\tau$. Inspection of (22) in such a region reveals clearly that $G$ is locally increasing in both the penalty rate $\pi$ and the audit probability $p$.

Finally, if $p\pi > 1$, which means that there is complete tax compliance, then $\frac{\partial M}{\partial \pi} = \frac{\partial M}{\partial p} = \frac{\partial G}{\partial p} = 0$, and $\frac{\partial G}{\partial \pi} < 0$. Obviously, penalties have no effect since no taxpayer is evading in this scenario. However, to raise the costly inspection effort results in less resources available for productive government spending.

An interesting, unambiguous comparative statics result refers to the effects of the parameter $c$ on the rate of growth. Since $G$ is always decreasing in the inspection cost $c$ (see (21)), and this cost does not affect $M$, improvements in the inspection technology directly translate into higher growth rates.

On the other hand, the tax compliance policy might have ambiguous effects on the rate of growth depending on the technological parameter $\alpha$, as it can be seen by logarithmically differentiating (25),

\[
\frac{\partial (\ln \Gamma)}{\partial p} = \left( \frac{\alpha}{M} \right) \frac{\partial M}{\partial p} + \left( 1 - \frac{\alpha}{G} \right) \frac{\partial G}{\partial p},
\]

\[
(33)
\]

\[
\frac{\partial (\ln \Gamma)}{\partial \pi} = \left( \frac{\alpha}{M} \right) \frac{\partial M}{\partial \pi} + \left( 1 - \frac{\alpha}{G} \right) \frac{\partial G}{\partial \pi}.
\]

\[
(34)
\]

We have already mentioned that, if we had assumed preferences leading to a saving function that were not independent of the interest rate, a modification in the policy parameters $p$ and $\pi$ would also affect the amount of saving in equilibrium as a result of the induced change in the marginal productivity of private capital (see (27)).\(^6\) This will certainly add a new source of growth effects to the tax enforcement policy. In particular, if the condition (32) holds and the optimal reported income is interior, $X \in (0,1)$, then to raise any of the parameters $p$ and $\pi$ of the tax enforcement policy will induce higher interest rates (see (27), (28), (29), (30), and (31)). Therefore, if savings were increasing in the interest rate, greater enforcement would reinforce the positive growth effects of public capital since it would directly stimulate a faster accumulation of private capital. This would actually somewhat neutralize the negative effects due to the fall in the after-tax income of taxpayers.

As an immediate consequence of the previous discussion and from inspection of (18), (22), (28), (30) and (34), we can state the following proposition referred to marginal changes in the inspection cost $c$, and the penalty rate $\pi$ on evaded taxes:

---

\(^6\) Under isoelastic preferences, $u(C) = \frac{C^{1-\sigma}}{1-\sigma}$ with $\sigma > 0$, the propensity to save out of the present value of lifetime income is increasing (decreasing) in the interest rate whenever $\sigma < (>)1$. 

---
Proposition 2  

(a) The rate of growth $\Gamma$ is decreasing in the unitary inspection cost $c$.

(b) The rate of growth $\Gamma$ is not affected by marginal changes in the penalty rate $\pi$ when $p\pi > 1$.

(c) Consider a tax compliance policy pair $(p, \pi)$ such that there is under-reporting of income, i.e., $p\pi < 1$. If $\alpha$ is sufficiently close to zero, then the rate of growth $\Gamma$ is locally increasing in the penalty rate $\pi$.

(d) Consider a tax compliance policy pair $(p, \pi)$ such that $X \in (0, 1)$. If $\alpha$ is sufficiently close to one, then the rate of growth $\Gamma$ is locally decreasing in the penalty rate $\pi$.

Clearly, the parameter $\alpha$ measures the importance of private capital in the production process. If $\alpha$ is close to one, then the contribution of government spending to aggregate output is small so that, at an interior solution, a decrease in the penalty rate will reduce the resources devoted to government spending while it will increase private capital accumulation ($M$ will increase). The latter effect will dominate the reduction in public resources due to the decrease in $G$. The converse argument applies when $\alpha$ is close to zero.

We should point out that the nature of the two instruments available to the tax authority is quite different. Usually, the tax legislation establishes the levels of both the tax rate $\tau$ and the penalty rate $\pi$ on evaded taxes whereas the probability $p$ of inspection depends on the effort made by the tax collection agency. Such an effort is not verifiable and, therefore, an specific value of $p$ cannot be enforced by law. Moreover, the probability $p$ of inspection can be almost instantaneously adjusted by the tax authority whereas the modification of the penalty rate should undergo a rather lengthy parliamentary process. Therefore, let us assume now that the penalty rate is fixed at a finite level, and consider a tax authority trying to maximize the rate of economic growth for a given tax rate. The following proposition establishes the non-desirability from the growth viewpoint of auditing policies inducing taxpayers to be honest.

Proposition 3  

For every given finite value of $\pi$, the rate $\Gamma$ of economic growth is never maximized by selecting an audit probability $p$ which induces taxpayers to declare their true labor income.

Proof. See the appendix.

The intuition behind the previous proposition is quite obvious. First, note that condition (32) does not hold when the expected penalty rate $p\pi$ takes a value around 1 and, according to (31), $G$ is locally decreasing in $p$ in such a case. Then, given a finite penalty rate and an initial position of complete tax compliance, the tax collection agency may increase $G$ (the government revenues) by reducing, say, the number of tax inspectors whereas the induced reduction in $M$ (the private capital accumulation) is almost zero around $p = \frac{1}{\pi}$ (see (29)).

As an illustration of Proposition 3, the two panels of Figure 2 show examples of growth rates as functions of the probability $p$ on the domain $(0, 1)$. In the case
considered in Panel 2A, we obtain a kind of Laffer curve, and growth is maximized when the audit probability is $p = 0.0775 \in (p, \bar{p})$. However, Panel 2B shows a situation in which public spending is so unproductive ($\alpha = 0.9$) that the growth rate is strictly decreasing in $p$ on the whole domain. Note that the two plotted functions are strictly decreasing in $\left(\frac{1}{\pi} - \varepsilon, 1\right)$ for some $\varepsilon > 0$.

[Insert Figure 2 about here]

The argument of Proposition 3 would not apply if we fixed the probability of inspection and modified instead the penalty rate. In such a case, the growth rate might be monotonically increasing, monotonically decreasing or non-monotonic in the penalty rate $\pi$, as the different panels of Figure 3 show. In particular, for the case analyzed in Panel 3A, which corresponds to an economy with highly productive public capital ($\alpha = 0.2$), to induce complete tax compliance through high penalties is a growth maximizing policy. However, recall that it is impossible to obtain an uniformly increasing function when the variable $p$ is in the horizontal axis, as follows from Proposition 3. The Panels 3B and 3C correspond to situations where the growth maximizing penalty rates are $\pi$ and 1, respectively, so that declared income equals zero in both cases when growth is maximized. Panel 3D corresponds to a situation in which maximum growth is achieved at an interior solution of the taxpayer optimization problem since the growth maximizing penalty rate is $\pi = 2.1558 \in (\pi, \bar{\pi})$. Finally, notice that the rate of growth remains always unaffected by changes in the penalty rate whenever $\pi \geq \frac{1}{p}$ since then truthful revelation of income is already achieved and no additional revenues accrue from tax inspections.

[Insert Figure 3 about here]

To conclude this discussion about the effects of changes in a single parameter of the tax enforcement policy, we state a couple of additional comparative statics results concerning the audit probability $p$. Both results are provided for the sake of completeness and they follow directly from performing straightforward derivatives.

**Proposition 4** (a) The rate of growth $\Gamma$ is locally decreasing in the audit probability $p$ when $p\pi > 1$.

(b) Consider a tax compliance policy pair $(p, \pi)$ such that $X = 0$. If $\alpha$ is sufficiently close to zero, then the rate of growth $\Gamma$ is locally increasing in the audit probability $p$.

It should be pointed that the comparative statics analysis of changes in $p$ is even less clear than that of $\pi$. This is so because the sign of the relation between $p$ and $G$ when $X \in (0, 1)$ is ambiguous depending on whether condition (32) holds or not.

Our previous discussion is dramatically modified if the tax authority can control simultaneously both instruments of the tax enforcement policy. Observe that agents report their true wages when $p\pi \geq 1$. Then, the rate of growth under such a policy is given by

$$\Gamma = A(1 - \alpha) \left(\frac{\delta(1 - \tau)}{1 + \delta}\right)^{\alpha} (\tau - cp)^{1-\alpha},$$

Panel 3C is obtained under an unrealistically high audit probability $p = 0.9$. 

14
as follows from (18), (22) and (25). Such a growth rate is strictly decreasing in $p$ and it is clear that a growth rate arbitrarily close to

$$\Gamma^* = A(1 - \alpha) \left( \frac{\delta(1 - \tau)}{1 + \delta} \right)^{\alpha} \tau^{1 - \alpha}$$  \hspace{1cm} (35)

can be implemented by means of a complete tax compliance policy displaying a probability $p$ of inspection arbitrarily low and a penalty rate $\pi$ arbitrarily high with $p\pi \geq 1$ (see Figure 1). Such a policy consisting on “hanging evaders with probability zero” has received attention in the theoretical literature on tax evasion when the government seeks to maximize its revenues (see, among many others, Kolm (1973)).\(^8\)

It is easy to check algebraically from (18) and (22) that, if $X \in (0, 1)$ then $M > \frac{\delta(1 - \tau)}{1 + \delta}$ and $G < \tau$. Hence, the desirability from the growth viewpoint of complete tax compliance will depend on whether the technological parameter $\alpha$ is high or low, as it can bee seen from comparing (25) with (35). In particular, to induce honest behavior by the taxpayers is desirable whenever public capital is very productive, i.e., when $\alpha$ is sufficiently close to zero. The following proposition summarizes more precisely the results:

**Proposition 5** (a) Consider the set of tax compliance policies inducing true reports of labor income, that is, policies satisfying $p\pi \geq 1$. Then, the supremum of the set of rates of growth associated with such policies is $\Gamma^*$. Moreover, for all $\varepsilon > 0$, there exists a policy pair $(p(\varepsilon), \pi(\varepsilon))$, with $p(\varepsilon) \cdot \pi(\varepsilon) = 1$, such that the rates of growth associated with the policies $(p(\varepsilon), \pi)$, with $\pi \geq \pi(\varepsilon)$, are equal to $\Gamma^* - \varepsilon$. Furthermore, the function $p(\varepsilon)$ is strictly increasing while $\pi(\varepsilon)$ is strictly decreasing, and $\lim_{\varepsilon \to 0} p(\varepsilon) = 0$ while $\lim_{\varepsilon \to 0} \pi(\varepsilon) = \infty$.

(b) If $\alpha$ is sufficiently close to zero, then there exists a policy pair $(p, \pi)$ inducing complete tax compliance that displays faster economic growth than any other policy inducing tax evasion. Conversely, if $\alpha$ is sufficiently close to one, then there exists a policy pair $(p, \pi)$ inducing tax evasion that displays faster economic growth than any other policy inducing complete tax compliance.

Let us point out that Proposition 3 does not contradict the first sentence in part (b) of Proposition 5. In the former we were keeping fixed the penalty rate at a finite level whereas in the latter both $p$ and $\pi$ were moving simultaneously in opposite directions with $\pi$ tending to infinity and $p$ tending to zero. Obviously, the growth rate given in (35) is never achieved by a complete compliance policy but it is just arbitrarily approximated.

## 5 Summary and Final Remarks

We have developed a simple OLG model to analyze the implications for economic growth of different tax compliance policies. A crucial assumption of our model is

\(^8\)Moreover, Friedland, Maital and Rutenberg (1978) have documented the effectiveness of such an extreme policy in their experimental work.
that both private and public capital are needed for production. We have shown that the effects of greater enforcement depend on the relative productivity of these two types of capital. Even if greater enforcement leads to a reduction of saving since individuals will enjoy less disposable income, the overall effect might be growth enhancing. This is so because enforcement generates resources that are used to finance public capital formation. Public capital becomes a source of endogenous growth because allows private capital to keep its marginal productivity at a high level and, thus, it stimulates savings. We also show that the symmetry between the two instruments of tax enforcement that we have considered is far from complete. In this respect, we have seen that the policy of “hanging evaders with probability zero”, that consists on imposing very high penalties to evaders with a very low probability of inspection, is the one that allows the government to better approximate the highest growth rate among all the policies inducing honest behavior. Moreover, for a given level of penalties, long-term growth is never maximized for a probability of inspection inducing truthful revelation of income since the marginal cost of such in inspection effort is always greater than the marginal revenue generated by such a policy.

It should be noticed that in this paper we have conducted just a positive analysis of the growth effects of changes in the tax enforcement policy. The normative analysis in an OLG model like ours will depend on the objective function of the social planner and, in particular, on the weights assigned to each generation in his objective function. It is indeed very easy to construct examples illustrating how the preferences of the social planner might conflict with the objective of maximizing economic growth.9

Due to the extreme simplicity of the model we have just considered, many extensions are possible. We just mention four of them.

The first one refers to the explicit recognition of involuntary mistakes in the process of filling the tax form (see Rubinstein (1979)). In this case, the penalty fee on detected tax evaders should be set at a moderate level since both the inefficiency and the inequality generated by severe penalties applied infrequently could be politically unbearable. The analysis could give rise then to an endogenous penalty rate, and it will thus provide further support for the non-optimality of complete tax compliance in the spirit of our Proposition 3. The relevance of such a proposition relies indeed on the fact that legislators do not set very severe penalties on tax evaders since they are perhaps aware that many taxpayers commit unverifiable mistakes by accident when they fill their tax forms. Therefore, fines cannot tend to infinity, which conforms with the assumption of Proposition 3. However, this extension would require agents working for more than one period since the repeated interaction between taxpayers and the tax collecting agency would be now a key element of the model.

The second extension would be to consider inspection policies for which the probability of an audit depends on the income declared as in Reinganum and Wilde (1985). These authors show that net fiscal revenues could increase by appropriately designing a policy belonging to that class. The growth implications of those inspection policies remain thus unexplored.

Third, in our model growth is achieved by means of the accumulation of both private and public capital. However, there are other ways in which sustained growth

9 Some examples are available under request to the authors.
can be achieved, like for instance through human capital accumulation (see, among many others, Caballé and Santos (1993)). An advantage of the models displaying an explicit mechanism of human capital formation that raised the efficiency units of labor is that labor and public capital could be more properly distinguished. Note that in our model the exponents for labor and public capital are the same. Moreover, even if we reinterpreted private capital as a composite input embodying both physical and human capital, these two kinds of capital would be considered as perfect substitutes (see Rebelo (1991)). Therefore, in both cases we are making quite restrictive assumptions indeed. The analysis of changes in the tax compliance policy on such richer models of human capital accumulation could also provide insights on both the short-run and the long-run effects. This is so because those models typically display some transitional dynamics while such a dynamics is absent in the model considered in this paper.

Finally, since in our model inspection occurs in every period after consumption has taken place, the income of old agents cannot be audited and, hence, capital income is not taxed. A more general OLG model, like one with more periods of life (as in Auerbach and Kotlikoff (1987)) or the one of perpetual youth of Blanchard (1985), would allow us to consider taxation on capital income as well. In this context, to fight against evasion will have direct effects on the interest rate that will affect in turn both savings and the rate of capital accumulation.
APPENDIX

Proof of Lemma 1. To obtain the solution for the individual’s problem (2) we must first assume that the solution is interior, that is, \( x_t \in (0, w_t) \). In this case, the first order conditions with respect to \( x_t \) and \( s_t \) yield, after some tedious algebra, the following solution:

\[
x_t = \frac{(1 - p)\tau (1 + \delta p\pi) - (1 - p\pi)(p\delta + \tau)}{p\tau (\pi - 1)(1 + \delta)} w_t, \tag{A1}
\]

and

\[
s_t = \frac{\pi \delta (1 - p)(1 - \tau)}{(\pi - 1)(1 + \delta)} w_t.
\]

It can be checked from (A1) that such a conjectured solution satisfies in fact \( x_t > 0 \) if and only if

\[
p\pi > \frac{(\tau + \delta)p}{\tau(1 + \delta) + \delta(1 - \tau)p}. \tag{A2}
\]

On the other hand, it can be seen from manipulating (A1) that a necessary and sufficient condition for \( x_t < w_t \) is

\[
p\pi < 1. \tag{A3}
\]

Since \( x_t \in [0, w_t] \) as a consequence of the aforementioned tax code, we have that \( x_t = 0 \) when (A1) is non-positive, which means that the agent will not fill the tax form in such a circumstance. Hence, to obtain the solution for problem (2) when condition (A2) does not hold, we impose \( x_t = 0 \) and solve the maximization problem for \( s_t \). The corresponding optimal propensity to save is then given in (5). Furthermore, \( x_t = w_t \) when \( p\pi \geq 1 \) so that, in such a case, we solve the maximization problem (2) for \( s_t \) after imposing \( x_t = w_t \) in its constraints.

Proof of Proposition 3. Observe that the gross growth rate \( \Gamma \) is a continuous function of the inspection probability \( p \) for all \( p \in (0, 1) \). Hence, we only have to prove that there exist a number \( \varepsilon \in (0, \frac{1}{\pi} - p) \) such that the rate of growth \( \Gamma \) is strictly decreasing in \( p \) on the interval \( \left( \frac{1}{\pi} - \varepsilon, 1 \right) \). We will first prove that the derivative \( \frac{d\Gamma}{dp} \) is strictly negative for \( p \in \left( \frac{1}{\pi}, 1 \right) \). Notice that \( X = 1 \) when \( p \in \left( \frac{1}{\pi}, 1 \right) \). Thus, from (18) and (22), it holds that \( M = \frac{\delta (1 - \tau)}{1 + \delta}, G = \tau - cp, \) and

\[
\Gamma = A(1 - \alpha) \left( \frac{\delta (1 - \tau)}{1 + \delta} \right)^\alpha (\tau - cp)^{1-\alpha}, \tag{A4}
\]

as follows from evaluating (25) in such a parameter region. Clearly, (A4) is strictly decreasing in \( p \). Next, since \( \Gamma \) has continuous derivatives with respect to \( p \) on \( (p, \frac{1}{\pi}) \), we must compute the left derivative of \( \Gamma \) with respect to \( p \) at \( \frac{1}{\pi} \). In order to compute \( \lim_{p \to (1/\pi)} \frac{d\Gamma}{dp} \) we only have to evaluate the derivative of (25) at an interior solution and take the limit as \( p \) tends to \( \frac{1}{\pi} \). From (29) and (31) we obtain

\[
\lim_{p \to (1/\pi)} \frac{\partial M}{\partial p} = 0 \text{ and}
\]
\[ \lim_{p \to (1/\pi)^-} \frac{\partial G}{\partial p} = -c < 0. \] Therefore, from (33), we get \[ \lim_{p \to (1/\pi)^-} \frac{\partial (\ln \Gamma)}{\partial p} < 0. \] We have thus proved that for some \( \varepsilon > 0 \) the rate of growth is strictly decreasing in the interval \( \left( \frac{1}{\pi} - \varepsilon, 1 \right) \) and, hence, a policy inducing complete tax compliance cannot be growth maximizing. ■
References


FIGURE 1: Interior and boundary solutions for the taxpayer problem.
FIGURE 2: The relation between $\Gamma$ and $p$.

$\delta = 0.425$, $\pi = 3$, $\tau = 0.25$, $c = 0.03$, $A = 11$ ($p = 0$, $\bar{p} = 0.33$)

Panel 2A: $\alpha = 0.6$

Panel 2B: $\alpha = 0.8$
**FIGURE 3:** The relation between $\Gamma$ and $\pi$.

\[\delta = 0.425, \; \tau = 0.25, \; c = 0.03, \; A = 11\]

Panel 3A: $\alpha = 0.2, \; p = 0.05 \; (\pi = 1.8136 \text{ and } \bar{\pi} = 20)$

Panel 3B: $\alpha = 0.95, \; p = 0.05 \; (\pi = 1.8136 \text{ and } \bar{\pi} = 20)$
Panel 3C: $\alpha = 0.9$, $p = 0.9$ ($\pi = 1.0496$ and $\bar{\pi} = 1.11$)

Panel 3D: $\alpha = 0.8$, $p = 0.05$ ($\pi = 1.8136$ and $\bar{\pi} = 20$)