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IMPERFECT COMPETITION IN A MULTI-SECURITY MARKET WITH RISK NEUTRALITY

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1. INTRODUCTION

THE CENTRAL PURPOSE OF THIS PAPER is to develop a model of insider trading (i.e., trading based on private information) in the context of an imperfectly competitive multi-security market with risk-neutral agents. Imperfect competition allows us to consider strategic behavior, and a multi-security market lets us study the effect of a correlated environment on equilibrium.

We employ the informational assumption that market makers can observe all order flows, and so portfolio diversification arises in this model for strategic reasons. Given correlated fundamentals, market makers can potentially learn about every security from each order flow. This causes even a risk neutral trader who does not face short-selling restrictions to refrain from determining the demand for each security independently. This contrasts with traditional multi-asset models, which focus on the incentive to reduce portfolio variance, or the effect of short-selling restrictions or budget constraints.

Under imperfect competition, correlation has two effects. One, ceteris paribus, it allows the uninformed to learn from additional variables since each order flow could potentially have information about all payoffs. On the other hand, it creates an incentive for informed traders to restrict what others can learn from public information. Thus, our analysis can be viewed as an application to the multi-security, heterogeneous-information model in Admati (1985) of the imperfectly competitive equilibrium concept which Kyle (1985) first applied to the single-security, homogeneous-information model of Grossman and Stiglitz (1980).

Our principal results include an explicit characterization of a linear equilibrium as a function of three general covariance matrices associated with payoffs, noise trading, and errors in private signals. Under general covariance structures, we show that there always exists an equilibrium in which the relationship between the vector of prices and the vector of order flows is governed by a symmetric positive definite matrix.

The plan of the paper is as follows. In Section 2, we introduce our model. We derive the equilibrium in Section 3, and Section 4 comments on the properties of the equilibrium. The proofs of the results are in the Appendix.

2. THE MODEL

We develop a K-trader, N-asset generalization of the single-trader, single-security model in Kyle (1985). It can be regarded as a model of a multi-good auction: the price is determined in the last stage of the game, after traders have made their quantity choices. This means that the informed traders select a quantity based on not an actual but an

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expected price, which captures the essence of a setting with market orders. Of course, in equilibrium the traders correctly anticipate the pricing rule, though not the actual price.

Our parametric assumptions are guided in part by Admati (1985) who studies a multi-asset market under perfect competition with a rich correlation structure.

We make the following assumptions:

A1: *There are N securities in the market, which will be indexed by n, $n = 1, \dots, N$, yielding a multivariate payoff vector, $\tilde{v} = (\tilde{v}_1, \tilde{v}_2, \dots, \tilde{v}_N)$, which is multivariate normally (MN) distributed with mean vector $\bar{v}$, and a nonsingular covariance matrix Σ_v.*

A2: *There are noise traders who generate a vector of random liquidity demands $\tilde{z}$ which is MN $(\bar{z}, \Sigma_z)$. The covariance matrix Σ_z is nonsingular. This is important in providing camouflage for informed trading.*

A3: *There are K informed traders indexed by k. Each insider k observes the realization of a vector of signals $\tilde{s}_k$.*[2] *The informational advantage of insider k with respect to outsiders is defined by the random vector $\tilde{\xi}_k = E(\tilde{v} | \tilde{s}_k) - \bar{v}$, which is assumed to be MN$(\underline{0}, \Sigma_\xi)$, where $\underline{0}$ denotes the zero vector and Σ_ξ is nonsingular, and $\tilde{\xi}_k$ is independent of the noise trade $\tilde{z}$, for all k. Moreover, we assume that the random vectors $\tilde{\xi}_k$ and $\tilde{\xi}_j$ have a joint normal distribution, and* $\operatorname{Cov}(\tilde{\xi}_k, \tilde{\xi}_j) = \Sigma_c$ *for every pair of traders with $k \neq j$, where Σ_c is also assumed to be symmetric positive definite.*[3] *The demand of insider k will be denoted by a random vector $\tilde{x}_k$ taking values in $\mathbb{R}^N$, which is a function of the random vector $\tilde{\xi}_k$.*

REMARK 2.1: Assumptions A1 and A3 readily imply that $\operatorname{Cov}(\tilde{v}, \tilde{\xi}_k) = \Sigma_\xi$ for all k, and $E(\tilde{\xi}_j | \tilde{\xi}_k) = \Sigma_c \Sigma_\xi^{-1} \tilde{\xi}_k$, for all $k \neq j$.

REMARK 2.2: Admati (1985) assumes that each informed trader receives a signal about the payoff of each security which takes the form $\tilde{s}_k = \tilde{v} + \tilde{\varepsilon}_k$, with $\tilde{\varepsilon}_k \sim MN(\underline{0}, \Sigma_\varepsilon)$ for all k, and the random vectors $\tilde{\varepsilon}_k (k = 1, \dots, K)$ are mutually independent. The random vectors $\tilde{v}$, $\tilde{z}$, and $\tilde{\varepsilon}_k$ are also assumed to be mutually independent, for all k. In this case, the following equalities hold:

$$\tilde{\xi}_k = \Sigma_v (\Sigma_v + \Sigma_\varepsilon)^{-1} (\tilde{s}_k - \bar{v}),$$

$$\Sigma_\xi = \Sigma_v (\Sigma_v + \Sigma_\varepsilon)^{-1} \Sigma_v,$$

$$\Sigma_c = \Sigma_v (\Sigma_v + \Sigma_\varepsilon)^{-1} \Sigma_v (\Sigma_v + \Sigma_\varepsilon)^{-1} \Sigma_v,$$

and, therefore,

$$E\left(\tilde{\xi}_j | \tilde{\xi}_k\right) = \Sigma_v (\Sigma_v + \Sigma_\varepsilon)^{-1} \tilde{\xi}_k.$$

REMARK 2.3: We may also add a common noise term to Admati's structure. In this case, if $\tilde{s}_k = \tilde{v} + \tilde{u} + \tilde{\varepsilon}_k (k = 1, \dots, K)$, with $\tilde{u}$ independent of $\tilde{v}$ and $\tilde{\varepsilon}_k$, for all k, and

[2] Bhasin (1992) considers a similar N-asset, N-agent model in which each agent obtains information about one security, and with symmetry across securities, and some other restrictions.

[3] If the vector of signals $\tilde{s}_k$ is a random vector taking values in $\mathbb{R}^M$, which is assumed to be MN$(\bar{s}, \Sigma_s)$ with Cov $(\tilde{v}, \tilde{s}_k) = \Sigma_{vs} \in \mathbb{R}^{N \times M}$ for all k, and Cov $(\tilde{s}_k, \tilde{s}_j) = \Sigma_o$ for all pair of traders with $j \neq k$, it follows that $\tilde{\xi}_k = \Sigma_{vs} \Sigma_s^{-1} (\tilde{s}_k - \bar{s})$, $\Sigma_\xi = \Sigma_{vs} \Sigma_s^{-1} \Sigma_{vs}^T$, and $\Sigma_c = \Sigma_{vs} \Sigma_s^{-1} \Sigma_o \Sigma_s^{-1} \Sigma_{vs}^T$, where the superscript T denotes the transpose.

$\tilde{u} \sim MN(\underline{0}, \Sigma_u)$, the following equalities hold:

$$\tilde{\xi}_k = \Sigma_v(\Sigma_v + \Sigma_u + \Sigma_\varepsilon)^{-1}(\tilde{s}_k - \bar{v}),$$

$$\Sigma_\xi = \Sigma_v(\Sigma_v + \Sigma_u + \Sigma_\varepsilon)^{-1}\Sigma_v,$$

$$\Sigma_c = \Sigma_v(\Sigma_v + \Sigma_u + \Sigma_\varepsilon)^{-1}(\Sigma_v + \Sigma_u)(\Sigma_v + \Sigma_u\Sigma_\varepsilon)^{-1}\Sigma_v,$$

and, therefore, for all $j \neq k$,

$$E\left(\tilde{\xi}_j|\tilde{\xi}_k\right) = \Sigma_v(\Sigma_v + \Sigma_u + \Sigma_\varepsilon)^{-1}(\Sigma_v + \Sigma_u)\Sigma_v^{-1}\tilde{\xi}_k.$$

REMARK 2.4: If the insiders' information is common ($\tilde{s}_k = \tilde{s}$, for all k), it follows that $\Sigma_c = \Sigma_\xi$, and $E(\tilde{\xi}_j|\tilde{\xi}_k) = \tilde{\xi}_k$.

REMARK 2.5: If insiders are perfectly informed ($\tilde{s}_k = \tilde{v}$), it follows that $\tilde{\xi}_k = \tilde{v} - \bar{v}$, $\Sigma_\xi = \Sigma_c = \Sigma_v$, and, obviously, $E(\tilde{\xi}_j|\tilde{\xi}_k) = \tilde{v} - \bar{v}$.

A4: *The price vector $\tilde{p}$ is determined by the following rule*:

$$(1) \qquad \tilde{p} \equiv p(\tilde{\omega}) = E(\tilde{v}|\tilde{\omega}), \qquad \textit{almost surely},$$

where $\tilde{\omega} \equiv \sum_{k=1}^K \tilde{x}_k + \tilde{z}$ is the vector of order flows. Thus, the pricing rule is such that, conditional on any set of public signals (order flows), market makers can expect to make zero profits in each market.

Typically, that zero-expected profit condition used in the literature (see Kyle (1985) and Admati and Pfleiderer (1988)) has been justified by invoking Bertrand-type competition among risk-neutral market makers. It is important to realize that for this story to be exactly true a rather restrictive game among market makers must be specified. For instance, each market maker must be able to observe the aggregate order flow, not just the portion of the aggregate that is directed towards him, as would be the case in telephone-based exchanges such as the London Stock Exchange. As Dennert (1993) shows, in such exchanges market makers may obtain positive expected profits in equilibrium.

Among well-known examples of financial markets, this description would apply closely only to the call auction that characterizes the opening trade in some exchanges such as the NYSE and Tokyo Stock Exchange. While most exchanges have a variety of detailed trading protocols (see, e.g., the Floor Officials' Manual of the NYSE) which are intended to ensure that all registered floor officials behave "competitively," whether a zero expected profit condition serves as an adequate reduced-form description is quite unclear. It is possible to show, however, that the zero-profit condition is consistent with a Walrasian market-clearing framework, as in Vives (1992), in which there is a risk neutral competitive market making sector submitting limit orders (demand schedules) while informed and liquidity traders submit market orders (quantities). In this setup, the demand of the market making sector is bounded only if prices equate the expected payoffs conditional on all public information.

3. EQUILIBRIUM

Profits of the informed trader k are given by $\tilde{\pi}_k = (\tilde{v} - \tilde{p})^T\tilde{x}_k$, where the vector of prices $\tilde{p}$ depends on the pricing rule, $\tilde{p} = p(\tilde{\omega})$, with $\tilde{\omega} = \sum_{k=1}^K \tilde{x}_k + \tilde{z}$. Since the strategies of informed traders are functions from the realizations of the random variables $\tilde{\xi}_k$ to quantities traded, $\tilde{x}_k = x_k(\tilde{\xi}_k)$, $k = 1, \ldots, K$, we may write

$$\tilde{\pi}_k = \left(\tilde{v} - \tilde{p}\left(x_1(\tilde{\xi}_1) + \cdots + x_k(\tilde{\xi}_k) + \cdots + x_K(\tilde{\xi}_K) + \tilde{z}\right)\right)^T x_k(\tilde{\xi}_k). \tag{2}$$

An equilibrium is a vector of $K+1$ functions $(x_1(\cdot), \ldots, x_K(\cdot), p(\cdot))$ such that the following conditions hold:

(a) Profit maximization:

$$\begin{aligned} &E\left[\left(\tilde{v} - \tilde{p}\left(x_1(\tilde{\xi}_1) + \cdots + x_k(\tilde{\xi}_k) + \cdots + x_K(\tilde{\xi}_K) + \tilde{z}\right)\right)^T x_k(\tilde{\xi}_k)\right] \\ &\quad \geqslant E\left[\left(\tilde{v} - \tilde{p}\left(x_1(\tilde{\xi}_1) + \cdots + x_k'(\tilde{\xi}_k) + \cdots + x_K(\tilde{\xi}_K) + \tilde{z}\right)\right)^T x_k'(\tilde{\xi}_k)\right], \end{aligned} \tag{3}$$

for any alternative trading strategy $x_k'(\cdot)$, and for $k = 1, \ldots, K$.

(b) Semi-strong market efficiency: The pricing rule $p(\cdot)$ satisfies

$$p(\tilde{\omega}) = E(\tilde{v} \mid \tilde{\omega}), \qquad \text{almost surely.} \tag{4}$$

We now provide an explicit characterization of a linear equilibrium.

Proposition 3.1: *There always exists an equilibrium defined as follows:*
The price function is

$$\begin{aligned} &p(\tilde{\omega}) = \bar{v} + \frac{\sqrt{K}}{2} A(\tilde{\omega} - \bar{z}), \qquad \text{where} \\ &A = \Sigma_z^{-1/2} M^{1/2} \Sigma_z^{-1/2}, \end{aligned} \tag{5}$$

where $M^{1/2}$ and $\Sigma_z^{1/2}$ are the unique symmetric positive definite square roots of $M = \Sigma_z^{1/2} G \Sigma_z^{1/2}$ and Σ_z respectively, and G is the symmetric positive definite matrix defined as

$$\begin{aligned} G = &\left\{ \left[\Sigma_\xi^{-1} + \frac{(K-1)}{2} \Sigma_\xi^{-1} \Sigma_c \Sigma_\xi^{-1}\right]^{-1} \right. \\ &\left. - \left[\frac{2}{K-1} \Sigma_c^{-1} + 2\Sigma_\xi^{-1} + \frac{(K-1)}{2} \Sigma_\xi^{-1} \Sigma_c \Sigma_\xi^{-1}\right]^{-1} \right\}. \end{aligned} \tag{6}$$

The demand strategy for each insider is $x_k(\tilde{\xi}_k) = B\tilde{\xi}_k$, where

$$B = \frac{1}{\sqrt{K}} A^{-1} \left[I + \frac{(K-1)}{2} \Sigma_c \Sigma_\xi^{-1}\right]^{-1}. \tag{7}$$

Moreover, this is the unique equilibrium for which A is symmetric.

Remark 3.1: The unique symmetric positive definite square root of the symmetric positive definite matrix M is given by $M^{1/2} = E\Lambda^{1/2}E^T$, where $\Lambda^{1/2}$ is a diagonal matrix with the positive square roots of the (positive) eigenvalues of M along the diagonal, and the corresponding orthogonal eigenvectors are the rows of the matrix E (see Bellman (1970, pp. 93–94)).

Remark 3.2: It is straightforward to prove (e.g. by adapting the backward-reaction mapping technique proposed in Novshek (1984) to our setting with private information) that, given a linear pricing rule, the equilibrium of the game among insiders is unique, and it involves the K traders adopting symmetric linear strategies.

REMARK 3.3: With a common information structure (see Remark 2.4), the equilibrium matrices satisfy

$$G = \frac{4}{(K+1)^2}\Sigma_\xi \quad \text{and} \quad B = \frac{2}{(K+1)\sqrt{K}}A^{-1}.$$

If private information is perfect (see Remark 2.5), Σ_ξ simplifies to Σ_v. Finally, when there is a single insider ($K=1$), the equilibrium matrices satisfy $G = \Sigma_\xi$ and $B = A^{-1}$, and we have proved elsewhere (Caballé and Krishnan (1990)) that there is a unique linear equilibrium in this case, so that symmetry of A obtains as a derived result.

In the general case with many insiders, the existence of an equilibrium in which A is not symmetric remains an open question. However, even with many insiders, the next proposition shows that we can still assert uniqueness of the linear equilibrium subject to a restriction on Σ_z.

PROPOSITION 3.2: *Assume $\Sigma_z = \sigma_z^2 I$, i.e., the noise trading has identical variance and is uncorrelated across markets; then the equilibrium of Proposition 3.1. is the unique linear equilibrium. In this case, $A = (1/\sigma_z)G^{1/2}$.*

4. GENERAL PROPERTIES OF EQUILIBRIUM

Note that in our framework investors are aware of the impact of their trades on prices, and so react cautiously to their private information. Hence, prices are not fully revealing even with risk neutrality (as in Kyle (1985)). In contrast, in a Gaussian setting with risk aversion, Hellwig (1980) and Admati (1985) consider the problem of how market prices aggregate information across traders in a large competitive market. Given our risk-neutral framework, to compare their results with ours, we must consider their results in the limit as the risk aversion parameter vanishes. It is easily seen from their pricing formulae that prices tend to fully reveal all private information as the risk aversion parameter tends to zero when the market is perfectly competitive.

The following corollary also shows that the informational content of the equilibrium price vector is independent of the variance of noise trading, and this generalizes the Kyle (1985) result that more noise leads to more aggressive trading, so that the informativeness of order flows is independent of the level of noise.

COROLLARY 4.1: *The informativeness of prices, I_p, measured by the reduction in the prior covariance matrix of the return vector, after conditioning on the vector of prices, $\Sigma_v - Var(\tilde{v}|\tilde{p})$, is given by the positive definite matrix $K[2\Sigma_\xi^{-1} + (K-1)\Sigma_\xi^{-1}\Sigma_c\Sigma_\xi^{-1}]^{-1}$, which is independent of Σ_z.*

REMARK 3.4: With common information (see Remark 2.4),

$$I_p = \frac{K}{K+1}\Sigma_\xi.$$

If we added risk aversion to our setting, there would be yet another reason for traders to be cautious, and we would expect the relationship between prices and private information to be more complex. In this respect we should point out that the independence of the informativeness of prices from the noise matrix only holds under risk neutrality, as was noted by Kyle (1985) and made evident by Subrahmanyam (1991).

We also record the following result about profits:

COROLLARY 4.2: *The informed trader's ex-ante expected profits are given by*

$$\left(1/2\sqrt{K}\right) \text{ trace } (A\Sigma_z).$$

It is clear from the expected profits formula and (5) that, if $\Sigma_z = \mu\hat{\Sigma}_z$, where μ is a positive scalar and $\hat{\Sigma}_z$ is a symmetric positive definite matrix, then the insiders' expected profits are increasing in the noise level μ. Note that an increase in the variance of noise trading enhances the camouflage opportunities for the insiders.

It should also be noticed that the equilibrium of Proposition 3.1 exhibits a matrix $A_1 \equiv (\sqrt{K}/2)A$ which is symmetric and positive definite. This implies that the price of each asset is increasing in its own orders, and that each asset is comparatively more sensitive to its own orders than to the orders for any other asset.

While positive definiteness follows simply from nonmanipulability (see the second-order condition of the informed traders' portfolio selection problem), the generic existence of an equilibrium in which the matrix A is symmetric is more puzzling. This tells us that regardless of the extent of asymmetry across assets, the nth price responds to the qth order flow exactly as the qth price responds to the nth order flow, for any n and q. While this ultimately reflects the balance between various complex interactive effects, it will help build intuition to consider a heuristic explanation of how strategic behavior helps achieve this balance.

Assume that asset n is characterized by a very high level of liquidity noise; this makes the trader more aggressive in trading asset n, relative to some other asset q which has less noise, since there is more camouflage. This makes the informativeness of order flows the same for both assets: order flow n is as useful in predicting payoff q as order flow q in predicting payoff n. So priors are modified in the same way for every asset. The matrix of response coefficients are like a ratio of prior-to-posterior precisions, since, by virtue of the market efficiency requirement, equilibrium prices equal expected payoffs, i.e., prices are like regression functions. Since the prior covariance matrix is symmetric, given the same degree of improvement in precision from observing order flows, this symmetry is preserved in the pricing rule. This is an argument to show that A_1 is symmetric even when liquidity noise varies across assets. A similar argument can be constructed to account for differences in payoff variances and error variances.

The symmetry (and positive definiteness) property of the matrix governing the equilibrium relationship between order flows and prices is a testable proposition, given the recent availability of transaction data, which permits us to construct measures of order flows. Evidence presented in Caballé, Krishnan, and Patel (1991), based on an empirical specification whose systematic component is specified as in our theory, and with error structure allowing for general heteroskedasticity and serial correlation, suggests that symmetric positive definiteness cannot be rejected, though diagonal structure given positive definiteness can be rejected.

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APPENDIX

Before proving the results, we state the following two definitions:

DEFINITION $A.1$: The $N \times N$ matrix D is *positive definite* if, for any vector $x \in \mathbb{R}^N$ different from $\underline{0}$, $x^T D x > 0$.

DEFINITION $A.2$: The $N \times N$ matrix D is *symmetric positive definite* if both symmetric and positive definite.

PROOF OF PROPOSITION 3.1: Let us check that the pricing rule and the demand strategies given in the statement of the theorem constitute an equilibrium. First, note that the equilibrium takes the following linear functional form:

$$p(\tilde{\omega}) = A_0 + A_1 \tilde{\omega}, \quad \text{and} \quad x(\tilde{\xi}) = B_0 + B_1 \tilde{\xi}, \quad \text{for} \quad k = 1, \dots, K.$$

The quantities $\tilde{x}_k$ demanded by each agent k $(k = 1, \dots, K)$ must maximize

$$E\left[\left(\tilde{v} - A_0 - A_1\left(\sum_{j \neq k}\left(B_0 + B_1 \tilde{\xi}_j\right) + \tilde{x}_k + \tilde{z}\right)\right)^T \tilde{x}_k \middle| \tilde{\xi}_k\right].$$

The first order condition is

$$\text{(A.1)} \qquad E\left(\tilde{v} | \tilde{\xi}_k\right) - A_0 - A_1 \bar{z} - (K-1) A_1 B_0 - A_1 B_1 \left[\sum_{j \neq k} E\left(\tilde{\xi} | \tilde{\xi}_k\right)\right] = 2 A_1 \tilde{x}_k,$$

where we have used the symmetry of A_1. The second-order condition is satisfied since A_1 is positive definite. Recall that, for all $j \neq k$, $E(\tilde{\xi}_j | \tilde{\xi}_k) = \Sigma_c \Sigma_\xi^{-1} \tilde{\xi}_k$, and, by definition, $E(\tilde{v} | \tilde{\xi}_k) = \tilde{\xi}_k + \bar{v}$. Plugging these expectations in (A.1), using the conjecture that $x_k(\tilde{\xi}_k) = B_0 + B_1 \tilde{\xi}_k$, and equating coefficients, we obtain

$$\text{(A2)} \qquad (K+1) A_1 B_0 = \bar{v} - A_0 - A_1 \bar{z}$$

and

$$\text{(A.3)} \qquad 2 A_1 B_1 = I - (K-1) A_1 B_1 \Sigma_c \Sigma_\xi^{-1}.$$

The vector of order flows $\tilde{\omega} = K B_0 + B_1 \Sigma_{k=1}^K \tilde{\xi}_k + \tilde{z}$ is multivariate normally (MN) distributed as $\tilde{\omega} \sim \text{MN}(K B_0 + \bar{z},\ K B_1 \Sigma_\xi B_1^T + K(K-1) B_1 \Sigma_c B_1^T + \Sigma_z)$, and Cov $(\tilde{v}, \tilde{\omega}) = K \Sigma_\xi B_1^T$. Therefore,

$$E(\tilde{v} | \tilde{\omega}) = \bar{v} + K \Sigma_\xi B_1^T \left[K B_1 \Sigma_\xi B_1^T + K(K-1) B_1 \Sigma_c B_1^T + \Sigma_z\right]^{-1} \cdot \left(\tilde{\omega} - K B_0 - \bar{z}\right).$$

Since $p(\tilde{\omega}) = E(\tilde{v} | \tilde{\omega})$, and making the conjecture that $p(\tilde{\omega}) = A_0 + A_1 \tilde{\omega}$, we can equate coefficients to obtain

$$\text{(A.4)} \qquad A_0 = \bar{v} - K A_1 B_0 - A_1 \bar{z},$$

and, using the invertibility of B_1, we also get

$$\text{(A.5)} \qquad A_1 = \left[B_1 + (K-1) B_1 \Sigma_c \Sigma_\xi^{-1} + \frac{\Sigma_z \left(B_1^T\right)^{-1} \Sigma_\xi^{-1}}{K}\right]^{-1}.$$

The solutions A_0, A_1, B_0, and B_1 given in the statement of Theorem 3.1 must solve the system formed by the equations (A.2), (A.3), (A.4), and (A.5). Solving for A_1 in (A.3), we obtain

$$\text{(A.6)} \qquad A_1 = \left[2 B_1 + (K-1) B_1 \Sigma_c \Sigma_\xi^{-1}\right]^{-1}.$$

Combining (A.5) and (A.6), we get

$$B_1 = \frac{\Sigma_z (B_1^T)^{-1} \Sigma_\xi^{-1}}{K}, \tag{A.7}$$

and, plugging (A.7) into (A.5), we obtain

$$A_1 = \left[\frac{2\Sigma_z (B_1^T)^{-1} \Sigma_\xi^{-1}}{K} + (K-1) B_1 \Sigma_c \Sigma_\xi^{-1} \right]^{-1}. \tag{A.8}$$

Using the invertibility of A_1, (A.6) implies that

$$B_1 = A_1^{-1} \left[2I + (K-1) \Sigma_c \Sigma_\xi^{-1} \right]^{-1}, \quad \text{and} \quad (B_1^T)^{-1} = A_1 \left[2I + (K-1) \Sigma_\xi^{-1} \Sigma_c \right]. \tag{A.9}$$

Replacing (B_1) and $(B_1^T)^{-1}$ in (A.8), we obtain

$$A_1^{-1} = \frac{2\Sigma_z A_1 \left[2\Sigma_\xi^{-1} + (K-1) \Sigma_\xi^{-1} \Sigma_c \Sigma_\xi^{-1} \right]}{K} + (K-1) A_1^{-1} \left[2\Sigma_\xi \Sigma_c^{-1} + (K-1) I \right]^{-1},$$

which implies that

$$A_1^{-1} \left[I - (K-1) \left[2\Sigma_\xi \Sigma_c^{-1} + (K-1) I \right]^{-1} \right] \left[2\Sigma_\xi^{-1} + (K-1) \Sigma_\xi^{-1} \Sigma_c \Sigma_\xi^{-1} \right] = \frac{2}{K} \Sigma_z A_1,$$

which, after some algebra, simplifies to

$$\frac{K}{4} A_1^{-1} G = \Sigma_z A_1, \tag{A.10}$$

where G is defined by expression (6) in the text. G is symmetric positive definite by virtue of the Rayleigh's principle, and A_1 must be a symmetric positive definite solution to (A.10). To solve for A_1, write (A.10) as

$$\frac{K}{4} \Sigma_z^{1/2} G \Sigma_z^{1/2} = \Sigma_z^{1/2} A_1 \Sigma_z^{1/2} \Sigma_z^{1/2} A_1 \Sigma_z^{1/2}, \tag{A.11}$$

where $\Sigma_z^{1/2}$ is the unique symmetric positive definite square root of Σ_z. Since the LHS of (A.11) is symmetric positive definite, $\Sigma_z^{1/2} A_1 \Sigma_z^{1/2}$ is its unique symmetric positive definite square root. Therefore,

$$A_1 = \frac{\sqrt{K}}{2} \Sigma_z^{-1/2} M^{1/2} \Sigma_z^{-1/2} \equiv \frac{\sqrt{K}}{2} A,$$

where $M \equiv \Sigma_z^{1/2} G \Sigma_z^{1/2}$. Obviously, this is the unique symmetric matrix which solves (A.11). Finally, (A.9) gives us $B_1 = B$, where B is given in the statement of the proposition. From (A.2) and (A.4), we immediately obtain $A_0 = \bar{v} - A_1 \bar{z}$ and $B_0 = \underline{0}$. Verifying that both A_1 and B_1 are invertible completes the proof. *Q.E.D.*

To prove Proposition 3.2. we first state the following lemma:

LEMMA: *If the $N \times N$ matrix D is positive definite and D^2 is symmetric, then D is symmetric positive definite.*

PROOF: Let us define $S = \frac{1}{2}(D + D^T)$ and $T = \frac{1}{2}(D - D^T)$. The matrix S is symmetric positive definite, whereas T is skew-symmetric, i.e., $T^T = -T$. Furthermore, $D = S + T$, and $D^2 = (S+T)^2 = S^2 + T^2 + ST + TS$. The matrix S^2 is obviously symmetric and, since $(T^2)^T = (T^T)^2 = (-T)^2 = T^2$, the matrix T^2 is also symmetric. Given that D^2 is symmetric, $ST + TS$ must be also symmetric. Therefore, $(ST + TS)^T = T^T S^T + S^T T^T = -(TS + ST)$. Since $ST + TS$ is symmetric and equal to its negative, we conclude that $ST + TS = \hat{0}$, where $\hat{0}$ denotes the null matrix.

Let $z_1, \dots, z_N$ be a basis of eigenvectors of S, and $\lambda_1, \dots, \lambda_n$ be the eigenvalues of S. Then $Sz_n = \lambda_n z_n$, $n = 1, \dots, N$. Since $ST + TS = \hat{0}$, $STz_n + TSz_n = STz_n + \lambda_n Tz_n = (S + \lambda_n I)Tz_n = \underline{0}$. The matrix $S + \lambda_n I$ is symmetric positive definite and, therefore, $Tz_n = \underline{0}$, $n = 1, \dots, N$, that is, the rows of T are orthogonal to the basis of $\mathbb{R}^N$. This implies that $T = \hat{0}$ and, therefore, $D = S$. The symmetry and positive definiteness of D is thus established. *Q.E.D.*

Proof of Proposition 3.2: First, note that the matrix A_1 must be positive definite so as to satisfy the second-order condition of the investors' stochastic maximization problem.

If A_1 is not restricted to be symmetric, (A.3) becomes

$$\left(A_1 + A_1^T\right)B_1 = I - (K-1)A_1 B_1 \Sigma_c \Sigma_\xi^{-1},$$

which can be rewritten as

$$A_1^T B_1 = I - A_1 B_1 - (K-1)A_1 B_1 \Sigma_c \Sigma_\xi^{-1}. \tag{A.12}$$

On the other hand, (A.5) can be rewritten as

$$\frac{A_1 \Sigma_z \left(B_1^T\right)^{-1} \Sigma_\xi^{-1}}{K} = I - A_1 B_1 - (K-1)A_1 B_1 \Sigma_c \Sigma_\xi^{-1}. \tag{A.13}$$

Combining (A.12) and (A.13) we get

$$A_1^T B_1 = \frac{A_1 \Sigma_z \left(B_1^T\right)^{-1} \Sigma_\xi^{-1}}{K},$$

which is equivalent to

$$K B_1 \Sigma_\xi B_1^T = \left(A_1^T\right)^{-1} A_1 \Sigma_z. \tag{A.14}$$

Since $\Sigma_z = \sigma_z^2 I$ and the LHS of (A.14) is a symmetric matrix, it follows that $(A_1^T)^{-1}A_1$ is a symmetric matrix. That is, $(A_1^T)^{-1}A_1 = A_1^T A_1^{-1}$, which implies in turn that $(A_1)^2 = ((A_1)^2)^T$. This proves the symmetry of $(A_1)^2$. Finally, since A_1 is positive definite, we can use the previous lemma so as to conclude that A_1 must be symmetric positive definite. Hence, the linear equilibrium is unique (see the proof of Proposition 3.1. and Remark 3.2.). It is straightforward to see that, in this case, $A = (1/\sigma_z)G^{1/2}$. *Q.E.D.*

Proof of Corollary 4.1: We just have to compute Var $(\tilde{v}\,|\,\tilde{\omega})$, which is equal to Var $(\tilde{v}\,|\,\tilde{p})$. We use the properties of the joint distribution of $(\tilde{v}, \tilde{\omega})$, given in the previous proof, to compute that conditional variance, and use the fact that in equilibrium $\Sigma_z = K B_1 \Sigma_\xi B_1^T$ (see (A.7)) to obtain

$$\operatorname{Var}(\tilde{v}\,|\,\tilde{\omega}) = \Sigma_v - K\left[2\Sigma_\xi^{-1} + (K-1)\Sigma_\xi^{-1}\Sigma_c\Sigma_\xi^{-1}\right]^{-1}.$$

The result then follows. *Q.E.D.*

Proof of Corollary 4.2: Obvious, since in our zero-sum game the expected profits for the insiders are equal to the total expected cost of trading for the noise traders, which is equal to $E((\tilde{z} - \bar{z})^T A_1 \tilde{z})$. The expectation is easily computed using Graybill (1983, p. 341). After dividing by the number of insiders K, and, since $A_1 \equiv (\sqrt{K}/2)A$, the result follows. *Q.E.D.*

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