

ASPIRATIONS, HABIT FORMATION, AND BEQUEST MOTIVE*

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We consider an economy where individuals use their standard of living based on past consumption in order to evaluate the utility arising from current consumption. We analyse how this process of preference formation affects the bequest motive. We show that habits (based on one's own past consumption) reduce the willingness of individuals to leave bequests, while aspirations (based on the standard of living of parents) make the existence of positive bequests easier. The long-run effects of both habits and aspirations on capital stock and on the amount of bequests depend on whether or not the bequest motive is operative.

There is a longstanding debate in the literature about the relative importance of bequests and life-cycle saving in the mechanics of capital accumulation, wealth inequality, and public policy. More specifically, as part of this discussion, theoretical and empirical works have studied whether intergenerational transfers motivated by parental altruism might play a crucial role in accounting for the observed saving patterns. In this article we aim to contribute to this debate by analysing how the presence of preferences displaying habits and aspirations affects the altruistic bequest motive from parents to children. By habits we mean that the utility associated with a given amount of current consumption depends on the past experience of consumption of the individual under consideration. Meanwhile, by aspirations we mean that the utility of individuals depends on the consumption experience of their predecessors. In both cases, past consumption is used as a reference with which current consumption is compared, thus implying that preferences are time non-separable.

The extensive empirical literature shows that bequests may play a significant role in explaining some empirical saving patterns. For instance, Kotlikoff and Summers (1981) find that at least 80% of US capital stock is accumulated through intergenerational transfers, and Laitner and Juster (1996) find that 50% of individuals save in order to leave an estate. However, several alternative motives leading to intergenerational transfers have been proposed by different authors. These motives include strategic behaviour, joy-of-giving, existence of incomplete annuity markets and pure intergenerational altruism. Although there is still some controversy over the reasons why individuals make intergenerational transfers, some empirical research has found evidence that intergenerational altruism is one of the most likely explanations of these transfers (Cox, 1987; Hurd, 1987; Gale and Scholz, 1994).¹

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¹ Dynan *et al.* (2002) even assert that the life-cycle and bequest motives for savings overlap and cannot be distinguished under uncertainty.

At the same time, several theoretical studies have shown that bequests are operative only if the intensity of the altruism is strong enough. In particular, Weil (1987) showed that altruism is not strong enough to generate positive bequests in a dynamically inefficient economy, as defined by Cass (1972).² Some other studies have shown that Weil's conclusion may be affected by such other economic phenomena as the existence of externalities in the process of human capital accumulation (Caballé, 1995) and the incidence of fiscal policy (Caballé, 1998). Taking this further, we study the implications of the interaction of altruism with habit and aspiration formation. To this end, we should point out that habits and aspirations are intertemporal phenomena that display different features. Habit formation is intragenerational in nature, while aspiration formation is intergenerational. The introduction of altruism, which is another phenomenon featuring intergenerational links, affects the dynamic behaviour of the economy. In particular, while aspirations display intergenerational effects regardless of whether individuals are altruistic or not, habit formation only exhibits intergenerational effects through the potential altruistic linkages between individuals belonging to different generations. Therefore, the main target of our work is to analyse whether habit and aspiration formation affect the altruistic bequest motive from parents to children.

There is a strand of the literature that provides evidence in favour of time non-separable preferences and, in particular, of habit and aspiration formation. On the one hand, a large number of empirical studies show that individuals' past decisions affect the satisfaction derived from their current decisions (de la Croix and Urbain, 1998; Carrasco *et al.*, 2005). According to this evidence, the introduction of habit formation has recently been used to improve the predictions made under time separable preferences in different fields.³ For instance, Winder and Palm (1996) argue that 'ignoring habits or other forms of non-separability may explain the frequent rejection of the life cycle hypothesis'. On the other hand, apart from the intragenerational formation of habits, there is also empirical evidence of the existence of involuntary transmission of tastes from one generation to the next. As Becker (1992) notes, individual behaviour is influenced by habits acquired as a child, such that parental influence on children is not limited to resource transfers.⁴ This idea has been elaborated by several authors in studies of its implications on economic growth. In particular, de la Croix (1996, 2001), de la Croix and Michel (1999, 2001) and Artige *et al.* (2004) analyse several stability issues in an overlapping generations (OG) economy with aspirations, and they show that these intergenerational spillovers may be responsible for long cycles in economic variables. Moreover, de la Croix and Michel (2001) also find that altruism increases savings even when the bequest motive is inoperative if preferences display aspirations. Therefore, empirical studies that merely focus on intergenerational transfers to estimate the importance of bequest motivated savings may not be very useful for gauging the economic implications of altruism.

² See, among many others, the related papers by Abel (1987) and Aiyagari (1987).

³ See, among others, Abel (1990, 1999); Ljungqvist and Uhlig (2000); Carroll *et al.* (1997, 2000); Alonso-Carrera *et al.* (2004, 2005); Lahiri and Puhakka (1998); and Wendner (2002).

⁴ The evidence of intergenerational spillover is surveyed in papers by Cavalli-Sforza and Feldman (1981) and Boyd and Richerson (1985). Moreover, Cox *et al.* (2004) estimate that parental preferences explain between 5 to 10% of the preferences of their children after controlling for their respective incomes.

In this article, we aim to contribute to the literature on the economic implications of intergenerational altruism by analysing whether Weil's results (1987) regarding the Ricardian equivalence proposition (Barro, 1974) holds for time non-separable preferences. We show that time non-separable preferences affect the potential of altruism as a source of intentional bequests. In particular, the existence of habits raises the threshold value of the intergenerational discount factor above which altruistic bequests are positive, while aspiration formation pushes this value down. Furthermore, and in contrast with Weil's analysis, the dynamic inefficiency of an economy without altruism is not sufficient for preventing the bequest motive from being operative when individuals want to reach the standard of living of their parents. Finally, as a by-product of our analysis, we will characterise the long-run effects of both habits and aspirations on capital stock and on the amount of bequests when the bequest motive is operative. On the one hand, we show that an increase in habit intensity increases the amount of old-age consumption, since stronger habits reduce the overall utility accruing from early consumption. This can be achieved by either increasing savings or reducing the amount of bequests left to descendants. In particular, we show that if the bequest motive is operative, the amount of savings remains constant and a reduction in the amount of bequests occurs, whereas if the bequest motive is inoperative the amount of saving increases. On the other hand, an increase in aspiration intensity makes individuals raise the amounts of both savings and bequests in order to outweigh the negative effect of stronger aspirations for their descendants' utility.

Other papers that have conducted analyses related to ours are those by de la Croix and Michel (2001) and Jellal and Wolff (2002). The first paper analyses the differences in dynamic behaviour between an economy with aspiration formation and an operative bequest motive and an economy without bequests. Moreover, the authors present a numerical example that illustrates how aspirations affect the operativeness of the bequest motive. Our aim is to generalise their result and extend it to the case where habit formation is also present. In fact, we show that this generalisation is far from trivial and that the case analysed by de la Croix and Michel is highly particular. Furthermore, the introduction of habits seems natural since it would otherwise be difficult to argue that individuals take into account the past consumption level achieved by their parents but not their own past consumption experience. However, aspirations and habits have opposite effects on the operativeness of the bequest motive. Therefore, the overall effect of the presence of time non-separable preferences on the potential existence of positive bequests is ambiguous. Jellal and Wolff (2002) analyse how aspiration formation affects the amount of bequests left by parents to their children. In contrast with our work, these authors consider a finite horizon economy in which there are only two generations.

The article is organised as follows. Section 1 presents the general model with both habits and aspirations. Section 2 analyses the potential inoperativeness of the bequest motive in our general setup. In Sections 3 and 4 we conduct the comparative statics on the threshold level of altruism below (above) which bequests are zero (positive) of changes in the intensity of habits and aspirations, respectively. Section 5 contains some final comments and remarks. All the proofs appear in the Appendix.

1. The Model

We consider an OG model where a continuum of identical individuals live for three periods and a new generation is born in each period. Each individual has offspring in the second period of his life and the number of children per parent is $n \geq 1$. Agents make economic decisions during the last two periods of their lives only. In every period, the youngest individuals neither work nor consume. However, as in de la Croix (1996), the members of the youngest generation in period t inherit a certain level of aspirations a_{t+1} in period $t + 1$. These aspirations are based on the standard of living achieved by their parents. More precisely, we assume that

$$a_{t+1} = c_t^1, \quad (1)$$

where c_t^1 is their parents' amount of consumption when they (the parents) were adults (second period of life). Individuals are assumed to be altruistic towards their children and can leave bequests. Let b_t be the amount of bequests that an old individual (born in period $t - 2$) leaves to each of his direct descendants (who were born in period $t - 1$) in period t .

There is a single commodity, which can be devoted to either consumption or investment. Each agent inelastically supplies one unit of labour in the second period of life and is retired in the third period. Let us index each generation by the period in which its members work. An adult individual distributes his labour income and his inheritance between consumption and saving. The budget constraint faced by a worker (adult) in period t is

$$w_t + b_t = c_t^1 + s_t, \quad (2)$$

where w_t is the labour income and s_t is the amount saved. When individuals are old, they receive a return from their savings, which is distributed between consumption and bequests for their children. Therefore, the budget constraint of an old individual belonging to generation t will be

$$R_{t+1}s_t = c_{t+1}^2 + nb_{t+1}, \quad (3)$$

where R_{t+1} is the gross rate of return on saving and c_{t+1}^2 is the amount of consumption of an old individual in period $t + 1$. Note thus that the superscript 1 on consumption refers to an adult individual (worker), while the superscript 2 refers to an old individual.

We also impose the constraint that parents cannot force their children to give them gifts when they (the parents) are old,

$$b_t \geq 0. \quad (4)$$

The preferences of an individual belonging to generation t is given by the following utility function,

$$V_t = U(\hat{c}_t^1, \hat{c}_{t+1}^2) + \beta V_{t+1}, \quad (5)$$

where V_{t+1} represents the indirect utility of each of his descendants and the parameter $\beta \in [0,1)$ is the altruism factor.⁵ For tractability, let us assume that the function $U(\cdot, \cdot)$ is

⁵ We are implicitly assuming that each parent cares equally about the happiness of their n children. Thus, the intercohort utility discount β could be rewritten as $\beta = n\rho\beta'$, where ρ would be the temporal discount factor and β' is the pure interpersonal (from parents to children) discount factor.

twice continuously differentiable and additive in its two arguments as in Abel (1986) or Laitner (1988). Therefore, we use the following specification:

$$U(\hat{c}_t^1, \hat{c}_{t+1}^2) = u(\hat{c}_t^1) + \rho u(\hat{c}_{t+1}^2), \quad (6)$$

where $\rho > 0$ is the temporal discount factor. We assume that $u' > 0$, $u'' < 0$, $\lim_{\hat{c} \rightarrow 0} u'(\hat{c}) = \infty$ and $\lim_{\hat{c} \rightarrow \infty} u'(\hat{c}) = 0$.

The variables \hat{c}_t^1 and \hat{c}_{t+1}^2 represent the *effective consumption* in adult and old ages, respectively, of a representative individual belonging to generation t . We will assume that in each period individuals derive utility from the comparison of their consumption with a consumption reference. This consumption reference during adulthood will be given by the aspirations a_t inherited from one's parents. Taking into account (1), we posit the following additive specification for the effective consumption of an adult individual in period t :

$$\hat{c}_t^1 = c_t^1 - \gamma a_t = c_t^1 - \gamma c_{t-1}^1. \quad (7)$$

This additive formulation for effective consumption makes it possible to preserve the concavity of the objective function with respect to the consumption vector. Moreover, preferences exhibit habit formation and, hence, the consumption reference of an old individual is determined by the consumption level he has reached in the previous period. We thus assume that the effective consumption of an old individual in period $t + 1$ is given by the additive function

$$\hat{c}_{t+1}^2 = c_{t+1}^2 - \delta c_t^1. \quad (8)$$

In the previous formulae for effective consumption levels we make $\gamma \geq 0$ and $\delta \geq 0$ in order to be consistent with the notion of aspirations and habits, respectively. Moreover, to ensure that the utility function V_t is well defined we must impose that $\gamma \leq 1$ and

$$u'(\hat{c}_t^1) - \delta \rho u'(\hat{c}_{t+1}^2) - \gamma \beta u'(\hat{c}_{t+1}^1) \geq 0. \quad (9)$$

On the one hand, the former condition ensures that effective consumption when adult would take non-negative values in a steady state equilibrium. On the other hand, the latter condition ensures that the indifference curves will be downward sloping⁶ and that the effective consumption of old individuals will be positive in a steady state equilibrium.⁷ This condition imposes a restriction on the path of consumption in adult and old ages. That is, given the values of the parameters δ and γ , this condition restricts the initial values of capital stock, bequests and aspirations for which the dynamic equilibrium exists.

We assume that the good of this economy is produced by means of a neoclassical production function $\hat{F}(K_t, L_t)$, where K_t is the capital stock and L_t is the amount of labour used in period t . Capital depreciates every period at the rate $v \in [0, 1]$. The production function per capita is $\hat{f}(k_t)$, where k_t is the capital stock per capita. In order to simplify the exposition, we introduce the function $f(k_t) = \hat{f}(k_t) + (1 - v)k_t$. As

⁶ See Lahiri and Puhakka (1998) and Wendner (2002) for a detailed explanation of this fact.

⁷ Denote the left-hand side of (9) as $\Psi(\delta, \gamma)$ and note that the partial derivative of the function Ψ with respect to δ is negative. Moreover, observe that $\Psi(c_{t+1}^2/c_t^1, \gamma) = -\infty$ at a steady state for all $\gamma \leq 1$. Therefore, those values of δ ensuring $\Psi(\delta, \gamma) \geq 0$ imply that $c_{t+1}^2 \geq 0$ in a steady state equilibrium.

firms behave competitively, the rental prices of the two inputs equal their marginal productivities,

$$R_t = f'(k_t) \equiv R(k_t) \quad (10)$$

and

$$w_t = f(k_t) - f'(k_t)k_t \equiv w(k_t). \quad (11)$$

In equilibrium the capital stock installed in period $t + 1$ is equal to the aggregate saving in period t and, thus, we have

$$nk_{t+1} = s_t. \quad (12)$$

The maximisation of (5) with respect to $\{c_t^1, c_{t+1}^2, b_{t+1}\}$ subject to (2), (3), (4), (7) and (8), is equivalent to solving the following dynamic programming problem:

$$V_t(b_t, s_{t-1}) = \max_{\{s_t, b_{t+1}\}} \left\{ u \left[\underbrace{w_t + b_t - s_t - \gamma(w_{t-1} + b_{t-1} - s_{t-1})}_{\hat{c}_t^1} \right] + \rho u \left[\underbrace{R_{t+1}s_t - nb_{t+1} - \delta(w_t + b_t - s_t)}_{\hat{c}_{t+1}^2} \right] + \beta V_{t+1}(b_{t+1}, s_t) \right\}, \quad (13)$$

with $b_{t+1} \geq 0$, for w_t and R_{t+1} given for all t . The original state variables for each individual are the inheritance received and the aspirations acquired. Since aspirations coincide with parental consumption during adulthood and this consumption can be written as a function of parental saving (see (2)), the state variables for the individuals of generation t turn out to be the amount of bequests b_t and saving s_{t-1} of their parents.

Using the envelope theorem we obtain,

$$\frac{\partial V_{t+1}}{\partial b_{t+1}} = u'(\hat{c}_{t+1}^1) - \rho \delta u'(\hat{c}_{t+2}^2) \quad (14)$$

and

$$\frac{\partial V_{t+1}}{\partial s_t} = \gamma u'(\hat{c}_{t+1}^1). \quad (15)$$

Using (14) and (15), the first-order conditions of problem (13) corresponding to the derivatives with respect to s_t and b_{t+1} are

$$u'(c_t^1 - \gamma c_{t-1}^1) = \rho(R_{t+1} + \delta)u'(c_{t+1}^2 - \delta c_t^1) + \beta \gamma u'(c_{t+1}^1 - \gamma c_t^1), \quad (16)$$

and

$$n\rho u'(c_{t+1}^2 - \delta c_t^1) \geq \beta[u'(c_{t+1}^1 - \gamma c_t^1) - \delta \rho u'(c_{t+2}^2 - \delta c_{t+1}^1)], \quad (17)$$

where the last condition holds with equality if $b_{t+1} > 0$. Equation (16) yields the optimal allocation of consumption along the lifetime of an individual. Note that this

first-order condition reflects the introduction of habits and aspirations since individuals are aware of the effect of their adult consumption on both their children's utility and their own utility when old. Thus, habits and aspirations affect the marginal rate of substitution between adult and old consumption. Habits raise the consumption reference of old individuals and, thus, their willingness to save when adult. However, since aspirations raise both the consumption reference of adult individuals and the reference of their descendants, the effect of this phenomenon on saving is potentially ambiguous.⁸

Equation (17) characterises the optimal level of bequests. This equation tells that, when the bequest motive is operative ($b_{t+1} > 0$), the utility loss of parents arising from a larger amount of bequests must be equal to the discounted utility gain of their direct descendants. On the one hand, the left-hand side of this equation is the utility loss experienced by an individual who decreases his old consumption in order to marginally increase the amount of bequests left to their children. On the other hand, the right-hand side of (17) is the discounted utility gain obtained by the descendants due to a marginal increase in the amount of inheritances. Note that the utility gain is determined by two effects. First, bequests allow their recipients to increase their adult consumption. Second, bequests reduce the utility of their recipients when old because of habit formation.

The competitive equilibrium of this economy is thus given by the system of difference equations composed of (16) and (17), together with (2), (3), (4), (9), (10), (11), (12), and the transversality condition

$$\lim_{t \rightarrow +\infty} \beta^t u'(\hat{c}_t^1) b_t = 0.$$

The previous transversality condition states that the present value of bequests tends to zero. Note that, in contrast with the standard model with time separable preferences, the competitive equilibrium depends on the altruism factor β even if the bequest motive is not operative ($b_t = 0$). More precisely, observe that when $\gamma > 0$, the altruism factor β explicitly appears in the first-order condition (16). Therefore, the capital stock of an economy with aspiration formation and zero bequests, will depend on the altruism factor.

We are going to restrict our attention to stationary equilibria where all the endogenous variables remain constant. We will suppress the time subscripts when we refer to the steady state value of a variable. Therefore, the first-order conditions (16) and (17) evaluated at a steady state become

$$(1 - \beta\gamma)u'(c^1 - \gamma c^1) = \rho(R + \delta)u'(c^2 - \delta c^1), \quad (18)$$

and

$$\rho(n + \beta\delta)u'(c^2 - \delta c^1) \geq \beta u'(c^1 - \gamma c^1), \quad (19)$$

where the last condition holds with equality when $b > 0$.

From the budget constraints (2) and (3), the competitive rental prices (10) and (11), and the equilibrium condition (12), we obtain the following equations along the steady state:

⁸ Our article is therefore related to some degree with those studies that assume endogenous patience (Doepke and Zilibotti, 2005).

$$c^1 \equiv c^1(k, b) = w(k) + b - nk$$

and

$$c^2 \equiv c^2(k, b) = nkR(k) - nb.$$

Using these steady state consumption levels and noting that the aspiration level is equal to the adult consumption of the parent of the individual under consideration, (18) can be rewritten as

$$\begin{aligned} h(k, b) \equiv & (1 - \beta\gamma)u'\{(1 - \gamma)[w(k) + b - nk]\} \\ & - \rho[R(k) + \delta]u'\{n[kR(k) - b] - \delta[w(k) + b - nk]\} = 0. \end{aligned} \quad (20)$$

The previous equation implicitly defines a relationship between the steady state value of the capital stock and the amount of bequests, $k = K(b)$. In the following Sections, we will characterise this relationship as a first step geared towards determining when the bequest motive is operative.

2. Operative and Inoperative Bequest Motives

In this Section we extend Weil's (1987) analysis to the case of preferences exhibiting habit and aspiration formation. For the analysis of the operativeness of the bequest motive, Weil uses the economy with no bequests as a benchmark. That is, he considers the case with $\beta = 0$ and, hence, with $b_t = 0$ for all t . However, we should remember that, in contrast with the situation involving time separable preferences, the capital stock of an economy with an inoperative bequest motive is also affected by the discount factor when aspirations are present. This forces us to modify the procedure for analysing the operativeness of the bequest motive in our economy. We will thus look at the steady state properties of the economy with altruism ($\beta > 0$) and no bequests ($b_t = 0$) rather than the economy with no altruism ($\beta = 0$). Like Weil (1987), we will restrict our analysis to the case where the steady state equilibrium is unique and saddle-path stable. To this end we introduce the following assumption:

ASSUMPTION A. *The following conditions hold:*

$$\rho f''(k)u'(\hat{c}^2) + \rho n[f'(k) + \delta][f''(k)k + f'(k)]u''(\hat{c}^2) < 0, \quad (21)$$

and

$$f''(k)k + n \geq 0. \quad (22)$$

This assumption ensures the existence of a steady state equilibrium of the economy with zero bequests and, moreover, that this equilibrium is unique and saddle-path stable (see Appendix A).⁹ Condition (21) is equivalent to the condition appearing in Assumption 2 in de la Croix and Michel (2001) after allowing for habit formation.

⁹ When aspiration formation is absent, the existence and stability conditions of the model with habit formation coincide with those of Galor and Ryder (1989), which guarantee the uniqueness and stability of the steady state of the OG model with production (Diamond, 1965).

As pointed out by those authors, condition (21) is always satisfied when the production function is Cobb-Douglas. In particular, given the properties of the production and utility functions, the condition

$$f''(k)k + f'(k) \geq 0 \quad (23)$$

is sufficient for (21). It is plain to see that the sufficient condition (23) is always satisfied by a Cobb-Douglas production function. Moreover, condition (22) coincides with the sufficient condition provided by de la Croix and Michel (2001) for saddle-path stability of the steady state equilibrium after allowing for population growth. We will maintain Assumption A throughout the rest of the article.

The next Lemma establishes that the stationary value of the capital stock is an increasing function of the stationary amount of bequests:

LEMMA 1. $K'(b) > 0$.

Let \bar{k} be the steady state value of the capital stock per capita when there are no bequests. As we have already said, this stock depends on the altruism factor β so that we can write $\bar{k} = \bar{k}(\beta)$. The following Lemma characterises this relationship:

LEMMA 2. $\bar{k}'(\beta) > 0$ if $\gamma > 0$, whereas $\bar{k}'(\beta) = 0$ if $\gamma = 0$.

The intuition behind this Lemma is simple. Under aspiration formation, the consumption level of an adult individual negatively affects the utility of their descendants and, hence, thanks to the existence of altruism, this consumption generates disutility to this adult individual. Therefore, a rise in the intensity of altruistic sentiments results in a reduction in the marginal utility of adult consumption relative to that of old consumption. In fact, we see that, for a given value of the individual's consumption vector (c_t^1, c_{t+1}^2) , the marginal utility of adult consumption c_t^1 is equal to $u'(\hat{c}_t^1) - \beta\gamma u'(\hat{c}_{t+1}^1) - \rho\delta u'(\hat{c}_{t+1}^2)$, whereas the marginal utility of old consumption c_{t+1}^2 is $u'(\hat{c}_{t+1}^2)$ when there are no bequests. Therefore, an increase in the value of the altruism factor β makes adult individuals shift their current consumption to the future by means of a larger amount of saving. Finally, observe that in absence of aspiration formation the altruism factor β does not affect the stationary value of the capital stock \bar{k} with zero bequests even if habit formation is present.

We next provide necessary and sufficient conditions under which parents do not leave bequests to their children. It is simple to derive from (10), (18), and (19) the following weak inequality:

$$(1 - \beta\gamma)(n + \beta\delta) - \beta[f'(k) + \delta] \geq 0. \quad (24)$$

Clearly, if the left-hand side of (24) is strictly positive, individuals do not leave bequests. Note that, if the bequest motive is not operative, condition (24) implies that

$$f'(k) < \frac{n}{\beta} - \gamma(n + \beta\delta) < \frac{n}{\beta}.$$

Therefore, capital stock in the steady state equilibrium with an inoperative bequest motive is larger than the one associated with the modified golden rule.

From (24), we can implicitly define a threshold value of the altruism factor β , which will be the one solving the following equation:

$$G(\beta) \equiv (1 - \beta\gamma)(n + \beta\delta) - \beta[f'(\bar{k}(\beta)) + \delta] = 0. \quad (25)$$

This threshold value should determine in principle the value of the altruism factor β above which the bequest motive is operative, that is, for which altruism is strong enough to enable strictly positive bequests. However, a potential problem with (25) is that it could have multiple solutions or no solution for β .¹⁰ However, after characterising the previous equation, we can obtain a precise result for the operativeness of the bequest motive. Let $\bar{\beta}_i$, $i = 1, 2, \dots, I$, be the roots of (25) with $\bar{\beta}_i < \bar{\beta}_j$ if $j > i$.¹¹ The following Proposition characterises the operativeness of the bequest motive in terms of the threshold values $\bar{\beta}_i$ of the altruism factor. Its proof is omitted since it immediately follows on from the continuity of G , the fact that $G(0) > 0$, and condition (24).

PROPOSITION 1.

- (i) If $\beta \in (0, \bar{\beta}_1)$, then $b = 0$.
- (ii) If $\beta \in (\bar{\beta}_i, \bar{\beta}_{i+1})$ where i is an odd integer, then $b > 0$.
- (iii) If $\beta \in (\bar{\beta}_i, \bar{\beta}_{i+1})$ where i is an even integer, then $b = 0$.

It should be noticed that the dynamic inefficiency of the economy without altruism is not sufficient for preventing the bequest motive from being operative when individuals are altruistic towards their descendants. Analogously, the dynamic efficiency of the economy with $\beta = 0$ is no longer a necessary condition for positive bequests when parents are altruistic towards their children. Since \bar{k} increases with β , we have $f'[\bar{k}(0)] > f'[\bar{k}(\beta)]$ for all $\beta > 0$. Therefore, if the economy without an altruistic motive ($\beta = 0$) is dynamically inefficient ($f'[\bar{k}(0)] < n$), then it holds that $f'(\bar{k}) < n$. However, as can be seen from (25), this is not sufficient for having $\bar{\beta} > 1$ in an economy with aspiration formation, which means that positive bequests could appear even if the economy with $\beta = 0$ is dynamically inefficient. Note that this result is in stark contrast with the one obtained by Weil (1987) for time separable preferences.

In the next two Sections, we analyse how the strength of habit and aspiration formation affect the operativeness of the bequest motive, that is, for which values of the altruism factor β bequests are strictly positive. For simplicity, we will focus on two extreme cases. First, we will assume that only habit formation is present ($\delta > 0$ and $\gamma = 0$). Second, we will look at the case where individuals form aspirations but no habits ($\delta = 0$ and $\gamma > 0$). We also analyse these two cases separately because the two phenomena leading to time non-separable preferences are of quite a different nature. Habit formation is not an intergenerational but an intragenerational phenomenon, while aspiration formation is of a clearly intergenerational nature. Therefore, the

¹⁰ In Section 4 we will show the existence of two solutions for numerical examples where the utility and the production function are logarithmic and Cobb-Douglas, respectively. The largest of these solutions lies above unity, while the smallest can lie either below or slightly above unity.

¹¹ We disregard the nongeneric roots where $G(\beta) = 0$ and $G'(\beta) = 0$ as any marginal perturbation in the parameter values makes them disappear or become a pair of generic solutions.

interaction of these two phenomena with altruism (which is also of an intergenerational nature) could give rise to a different dynamic behaviour of the economy. In particular, while aspirations always have direct intergenerational effects, regardless of the degree of altruism of individuals, habit formation only has indirect effects through the altruistic links from parents to children.

We will also study how the intensities of habits and aspirations affect the level of capital stock and the amount of bequests (provided they are positive). This is a qualitative rather than a quantitative question since we would like to know if, when the intensity of habits and aspirations changes, individuals react by adjusting only the amount of bequests they leave or by adjusting the amount of savings (i.e., the relative distribution between adult and old consumption).

3. Habit Formation and the Operativeness of the Bequest Motive

We assume in this Section that individuals form habits but not aspirations ($\delta > 0$ and $\gamma = 0$). Therefore, the capital stock \bar{k} of the economy with zero bequests is independent of the altruism factor and, hence, (25) has a unique solution $\bar{\beta}$ when $\gamma = 0$. This solution is given by

$$\bar{\beta} = \frac{n}{f'(\bar{k})}. \quad (26)$$

Obviously, bequests are positive if and only if $\beta > \bar{\beta}$.

To see how the threshold value $\bar{\beta}$ of the altruism factor depends on the habit intensity δ we must first establish the effect of δ on the capital stock \bar{k} corresponding to the economy with zero bequests. The next result, already proved by Wendner (2002), shows that, if the steady state equilibrium is stable, then \bar{k} depends positively on δ .

LEMMA 3. $\frac{d\bar{k}}{d\delta} > 0$.

The intuition lying behind the previous result is the standard in models of habit formation (Alonso-Carrera *et al.*, 2004). When habits are present, individuals do not obtain utility only from the total consumption in each period but also from the comparison of the amount of old consumption with a reference based on their own adult consumption. An increase in habit intensity reduces the marginal rate of substitution between adult and old consumption, that is, the marginal valuation of adult consumption decreases relative to that of old consumption. Therefore, when δ increases, individuals raise their amount of saving and this results in a larger capital stock in equilibrium.

We can now perform the comparative statics exercise relating δ with the threshold value $\bar{\beta}$ of the altruism factor.

PROPOSITION 2. Assume that $\gamma = 0$. Then $\frac{d\bar{\beta}}{d\delta} > 0$.

This Proposition shows that habits increase the critical value of the altruism factor above which the bequest motive is operative. Therefore, the introduction of habits makes the existence of positive voluntary bequests more difficult. On the one hand, when habits are present, old individuals use their asset holdings to reach the standard

of living achieved when they were adults so that their willingness to leave bequests will decrease. That is, since marginal utility is decreasing in consumption, (17) tells us that, as habit intensity increases, the utility loss associated with leaving bequests becomes larger. On the other hand, a larger value for δ means that the consumption reference increases for the old individuals who have received an inheritance when adult and, therefore, this reduces the utility gain accruing from a larger amount of inheritances. As the two effects we have just discussed point in the same direction, the final outcome is that an increase in habit intensity results in a lower willingness to leave bequests.

In order to illustrate the previous result, we compute the critical value $\bar{\beta}$ of the altruism factor for different values of the parameter δ measuring habit intensity. To this end, we parametrise the economy as in Weil (1987) in order to make the comparison easier. In particular, let us consider a logarithmic utility, $u(c_t) = \ln c_t$, and a Cobb-Douglas production function, $\hat{f}(k_t) = Ak_t^\alpha$. Under logarithmic preferences the propensity to save out of labour income is $h = \rho/(1 + \rho)$. In our simulations we consider three alternative values for h : 0.25, 0.5 and 0.75. We also assume that the share of capital income out of total income is 25% (i.e., $\alpha = 0.25$) and that capital does not depreciate (i.e., $\nu = 0$). Finally, let us assume that the period of adulthood lasts for 35 years so that, if m is the annual gross rate of population growth, then $n = m^{35}$. We will use four alternative values for m : 1.01, 1.03, 1.05 and 1.07.

Table 1 shows the simulated threshold values $\bar{\beta}$ of the altruism factor for alternative values of the parameter δ . The results suggest that the bequest motive is operative only if the altruism factor takes a very high value, which amounts to a fairly low intergenerational discount rate. Moreover, habit formation substantially raises the value of the

Table 1
The Critical Value $\bar{\beta}$ as a Function of δ , h and m

δ	m			
	1.01	1.03	1.05	1.07
$h = 0.25$				
0	0.49	0.59	0.66	0.70
0.05	0.51	0.61	0.67	0.71
0.15	0.54	0.64	0.69	0.72
0.25	0.57	0.67	0.71	0.73
0.5	0.64	0.74	0.77	0.77
0.75	0.69	0.82	0.83	0.80
$h = 0.5$				
0	0.73	0.98	1.18	1.32
0.05	0.74	0.99	1.19	1.32
0.15	0.75	1.01	1.21	1.34
0.25	0.77	1.03	1.23	1.35
0.5	0.80	1.07	1.27	1.38
0.75	0.82	1.11	1.32	1.42
$h = 0.75$				
0	0.87	1.25	1.60	1.86
0.05	0.87	1.26	1.60	1.86
0.15	0.88	1.26	1.61	1.87
0.25	0.88	1.27	1.62	1.88
0.5	0.89	1.29	1.65	1.90
0.75	0.90	1.31	1.67	1.92

critical altruism factor. For instance, when $h = 0.25$ and $m = 1.01$, the threshold value $\bar{\beta}$ of the altruism factor under habit formation when $\delta = 0.15$ is 10.20% larger than when there is no habit formation ($\delta = 0$). Moreover, the impact of habit formation on the threshold value $\bar{\beta}$ is smaller for larger values of the annual gross rate m of population growth and of the temporal discount factor ρ . Note that the introduction of a sufficiently strong level of habit formation allows $\bar{\beta}$ to become larger than unity, which means that the stationary bequests cannot be positive in this case. Moreover, for the values of $\bar{\beta}$ larger than unity appearing in Table 1 it follows that the corresponding OG economy without altruism is dynamically inefficient, since in this case it holds that $f'(\bar{k}) < n$ as dictated by (26).

At this point, it is obvious that an increase in the intensity δ of habit formation results in a reduction in the amount of bequests. However, we would like to know if the adjustment brought about by the variation in δ affects the amount of saving and not just the amount of bequests. We will now provide an answer to this question.

As the first approach to the question we have just posed, note that individuals would like to reduce their adult consumption when the habit intensity δ increases. This is so for two reasons. First, by reducing the amount of consumption, habits become less important for the following period. Second, by reducing adult consumption there are more resources available for old consumption, which allows individuals to overcome the negative effect of habits due to the increase in the marginal utility of old consumption triggered by habits. When the bequest motive is inoperative ($\beta < \bar{\beta}$), then the previous effect on saving will occur as shown in Lemma 3. However, when the bequest motive is operative ($\beta > \bar{\beta}$), individuals have another strategy at their disposal for accommodating the increase in habit intensity. This strategy consists of decreasing the amount of bequests left to their descendants in order to increase old consumption without modifying the amount of saving and, thus, leaving the stock of capital unchanged. We are thus left asking which of these two possible strategies are followed by individuals when the bequest motive is operative. Note that the cost of reducing the amount of savings and keeping bequests at the same level is the reduction in the utility accruing from personal consumption. The cost of reducing the amount of bequests and keeping the amount of saving invariant arises because altruistic individuals internalise the decrease in the initial endowment of their descendants. Depending on the relative magnitudes of these two costs we can provide an answer to our previous question.

PROPOSITION 3. *Assume that $\gamma = 0$ and $\beta > \bar{\beta}$. Then, $\frac{dk}{d\delta} = 0$ and $\frac{db}{d\delta} < 0$.*

This Proposition tells us that an increase in habit intensity results in less bequests but the amount of saving remains unchanged. This means that the induced increase in old consumption is reached thanks to the reduction in the amount of bequests left to the descendants. Obviously, if habit intensity increases sharply so that $\beta < \bar{\beta}$, then the bequest motive will not be operative any longer. In this case, the decrease in the amount of bequests will be insufficient for absorbing the impact of stronger habits and, therefore, some increase in the amount of saving will also be necessary.

4. Aspirations and the Operativeness of the Bequest Motive

In this Section we will assume that individuals only form aspirations ($\gamma > 0$ and $\delta = 0$). To determine how the likelihood of positive bequests varies with aspiration intensity, we must first establish the effect of γ on the capital stock \bar{k} of the economy with no bequests. De la Croix (1996) finds that $d\bar{k}/d\gamma < 0$ in an economy without altruism ($\beta = 0$) when the utility and the production functions are logarithmic and Cobb-Douglas, respectively. We will next generalise this result for an economy with altruism ($\beta > 0$) but with zero bequests. Implicitly differentiating condition (20) with $\delta = 0$ and $b = 0$ we get

$$\frac{d\bar{k}}{d\gamma} = \frac{-\beta[u'(\hat{c}^1) - \gamma c^1 u''(\hat{c}^1)] - c^1 u''(\hat{c}^1)}{(1-\gamma)(1-\beta\gamma)[\bar{k}f''(\bar{k}) + n]u''(\hat{c}^1) + \rho f''(\bar{k})u'(\hat{c}^2) + \rho n f'(\bar{k})[\bar{k}f''(\bar{k}) + f'(\bar{k})]u''(\hat{c}^2)}. \quad (27)$$

The denominator of the derivative is negative by Assumption A. However, the sign of the numerator could be ambiguous since aspiration formation affects the individuals' decisions in two ways. On the one hand, when aspirations are present, an individual does not obtain utility from his adult consumption but from its comparison with his parents' adult consumption. Thus, an increase in aspiration intensity forces adult individuals to increase their current consumption to outweigh the effect of greater aspirations. In other words, an increase in the value of γ raises the marginal value of adult consumption relative to that of old consumption. This first effect is collected by the second term of the numerator of (27) and corresponds to the one obtained by de la Croix (1996).¹² On the other hand, aspiration formation decreases the utility associated with adult consumption of an altruistic individual since the reference of his children is raised accordingly. Therefore, aspirations reduce the relative marginal value of adult consumption. Note that this effect, which is only present when individuals are altruistic, is collected by the first term of the numerator of (27). The previous two effects act in opposite directions, but the next result solves the potential ambiguity in the sign of $d\bar{k}/d\gamma$ by means of an appropriate restriction. Let us thus define the index of relative risk aversion $\sigma(c) = -cu''(c)/u'(c)$. The index $\sigma(c)$ is a measure of the curvature of the utility function and is equal to the inverse of the elasticity of intertemporal substitution.

The following Lemma follows directly from (27):

LEMMA 4. $\frac{d\bar{k}}{d\gamma} < 0$ if and only if $\sigma(\hat{c}^1) > \frac{\beta}{1-\beta\gamma}$.

The previous result establishes that an increase in aspiration intensity results in a smaller capital stock provided the elasticity of intertemporal substitution is sufficiently low. A small value for this elasticity means a low willingness to accept an increase in the relative marginal value of adult consumption induced by stronger inherited aspirations. In this case, individuals will raise their adult consumption in order to keep that relative marginal value. Moreover, note that the ratio $\beta/(1-\beta\gamma)$ is inversely related to the individuals' willingness to accept a reduction in the relative marginal value of adult

¹² Note that $d\bar{k}/d\gamma < 0$ whenever $\beta = 0$.

consumption as this willingness is affected by the interaction between altruism and aspiration formation.

In order to assess the empirical content of Lemma 4, consider the case of isoelastic preferences with $\sigma(c) = \sigma$ for all c . Observe that $\sigma > \beta/(1 - \beta\gamma)$ if the value of the aspiration parameter γ is not very large and the value σ of the inverse of the elasticity of intertemporal substitution is relatively large. Note in this respect that the vast majority of estimated values of the inverse of the elasticity of intertemporal substitution found in the literature are significantly larger than one (Mehra and Prescott, 1985).¹³

We can now conduct a comparative statics exercise to find the effect of changes in the aspiration intensity γ on the threshold value $\bar{\beta}$ of the altruism factor. The following Proposition provides sufficient conditions for an explicit characterisation:

PROPOSITION 4. *Assume that $\delta = 0$ and $\sigma(\hat{c}^1) > \beta/(1 - \beta\gamma)$. Then, an increase in the aspiration intensity γ enlarges the range of values of the altruism factor β for which the bequest motive is operative.*

From Proposition 1 it is easy to see that the previous results mean that, when the elasticity of intertemporal substitution is sufficiently low, aspirations make positive bequests more likely in equilibrium. The intuition behind this result is clear: under altruism, individuals take into account the fact that their adult consumption determines the standard of living of their descendant. Therefore, the increase in aspiration intensity makes altruistic individuals raise the fraction of their saving devoted to bequests for a given level of altruism. Note that, as established in Lemma 4, since adult consumption increases with aspiration intensity when $\sigma(\hat{c}^1) > \beta/(1 - \beta\gamma)$, it is obvious that individuals must leave a larger amount of bequests to outweigh the negative effect of aspirations on their descendants' utility. Therefore, as aspiration intensity rises, the minimal degree of altruism needed for positive bequests goes down.

Using the numerical example of the previous Section, we compute the threshold values $\bar{\beta}_i$ of the altruism factor for different values γ of the aspiration intensity parameter. In this numerical example there are two threshold values $\bar{\beta}_1$ and $\bar{\beta}_2$, with $\bar{\beta}_1 < \bar{\beta}_2$. Moreover, in all the simulations we have conducted $\bar{\beta}_2$ is larger than unity, whereas $\bar{\beta}_1$ could be larger or smaller than 1. Table 2 shows the simulated values for the altruism factor $\bar{\beta}_1$. The results suggest that aspiration formation significantly reduces that threshold value. For instance, when $h = 0.25$ and $m = 1.01$, the value of $\bar{\beta}_1$ for an economy with aspiration formation with $\gamma = 0.2$ is 14.29% smaller than in an economy without aspiration formation. Moreover, the impact of aspiration intensity in the threshold value of the altruism factor $\bar{\beta}_1$ is larger for larger values of the annual gross rate m of population growth and of the temporal discount factor ρ . Note that in some cases aspiration formation makes the value of $\bar{\beta}_1$ smaller than 1 while that value was larger than 1 with no aspirations.

Recall that, in contrast to the economy without aspiration formation, the dynamic inefficiency of the economy with no altruism is no longer sufficient to prevent bequests from being positive in an economy with altruistic agents. Similarly, the dynamic

¹³ For instance, under the empirically plausible value $\sigma = 3$, we have $\sigma > \beta/(1 - \beta\gamma)$ for $\beta = 1$ whenever $\gamma < 2/3$. This upper bound for the value of γ rises as the value of β decreases.

Table 2
The Critical Value $\bar{\beta}$ as a Function of γ , h and m

		<i>m</i>			
	γ	1.01	1.03	1.05	1.07
<i>h</i> = 0.25	0	0.49	0.59	0.66	0.70
	0.05	0.47	0.58	0.63	0.67
	0.1	0.45	0.54	0.60	0.64
	0.15	0.44	0.52	0.57	0.61
	0.2	0.42	0.50	0.55	0.58
<i>h</i> = 0.5	0	0.73	0.98	1.18	1.32
	0.05	0.70	0.93	1.12	1.24
	0.1	0.67	0.89	1.06	1.17
	0.15	0.65	0.85	1.00	1.11
	0.2	0.62	0.81	0.95	1.05
<i>h</i> = 0.75	0	0.87	1.25	1.60	1.86
	0.05	0.83	1.19	1.49	1.71
	0.1	0.80	1.11	1.39	1.60
	0.15	0.77	1.05	1.30	1.47
	0.2	0.74	1.00	1.22	1.37

efficiency of the economy without altruism is no longer a necessary condition for the operativeness of the bequest motive in the economy with altruism when $\delta = 0$.

We have already pointed out that an increase in the aspiration intensity γ results in larger bequests for the empirically relevant case ($\sigma(\hat{c}^1) > \beta/(1 - \beta\gamma)$). However, we have to determine whether individuals adjust only the amount of bequests they leave or also adjust the amount of saving. We now turn to this question.

As we have already seen, an increase in aspiration intensity has two opposite effects on the individual's welfare. On the one hand, it raises the marginal utility of adult consumption. On the other hand, it raises the marginal utility associated with their descendants' welfare. Therefore, the final effect will depend on which of the previous two opposite effects dominates. When there are no bequests, we showed that the final effect depended on the index of relative aversion evaluated at the equilibrium level of adult consumption. However, the effects of a change in aspiration intensity could be quite different if the bequest motive is operative. In this case, individuals can not only modify the capital stock for accommodating the aspiration shock but can also modify the amount of bequests left to their descendants. The following result characterises the effects of aspiration intensity on capital stock and on bequests:

PROPOSITION 5. *Assume that $\delta = 0$ and $b > 0$. Then,*

- (i) $\frac{dk}{d\gamma} > 0$.
- (ii) $\frac{db}{d\gamma} > 0$ if $\sigma(\hat{c}^1) > \frac{\beta}{1 - \beta\gamma}$.

This result tells us that when bequests are positive individuals accommodate the increase in aspiration intensity by increasing their saving levels. This allows altruistic individuals to keep the aspirations of their descendants to a moderate level. Moreover,

under the condition $\sigma(\hat{c}^1) > \beta/(1 - \beta\gamma)$, individuals leave larger bequests to their descendants in order to outweigh the effect of stronger aspirations.

5. Concluding Remarks

We have analysed an OG model where the members of a family are connected by altruistic links and individuals form habits and aspirations. Therefore, the standard of living parameterised by past consumption experiences is relevant in evaluating the utility from current consumption. We have seen that aspirations (based on the standard of living of parents) make the existence of positive bequests easier, whereas habits (one's own past consumption) make it difficult for the bequest motive to be operative.

Since the joint introduction of aspirations and habits appears naturally in our model, we must conclude that the final effect of the presence of time non-separable preferences on the operativeness of the bequest motive is ambiguous. To the best of our knowledge, there is no evidence of the relative strength of these two phenomena. However, we can still obtain some insights into this ambiguity by means of certain numerical simulations. Note that the two phenomena share the same foundations, namely, that past consumption is used as a reference with respect to which current personal consumption is compared. The weight attached by old individuals to their past adult consumption was given by the value of the parameter δ . In fact, aspirations can also be viewed as a manifestation of habits. To this end, it should be remembered that individuals in their first period of life do not make any economic decision. Moreover, we can now assume that their parents choose the total amount of consumption of the household and that this consumption is distributed equally among adult parents and their children. In this case, under habit formation, when those children become adults, they could also attach a weight δ to their young consumption, which in fact coincides with their parents' consumption. Hence, it is not unrealistic to consider a benchmark economy where the strengths of habit and aspiration formation coincide, i.e., $\delta = \gamma$. Under this assumption, we can also use the numerical example of the previous two Sections to compute the threshold value $\bar{\beta}$ of the altruistic factor for alternative identical values of δ and γ . We have found that the effect of an increase in the intensity γ of aspiration formation dominates the effect of the increase in the value of the parameter δ measuring habit intensity in all the corresponding simulated steady state equilibria.

Note that in our model there is a consumption externality brought about by aspirations formation since adult consumption determines the consumption reference of the next generation. However, when altruism is present and the bequest motive is operative, adult individuals do internalise the effect on their descendants' welfare and, therefore, the decentralised solution is already efficient. In this case, taxation policy plays no role in the maximisation of social welfare.¹⁴

If the altruism factor lies on the interval where bequests are absent ($\beta < \bar{\beta}$), then a clear inefficiency appears, since individuals do not internalise the effect of aspirations.

¹⁴ In a model where consumption externalities are more pervasive, Abel (2005) analyses the optimal taxes in an OG without altruism when each individual's utility depends on the weighted average consumption by others.

In particular, there is an excess of adult consumption and, thus, a suboptimally low level of saving. An optimal tax policy should consist of a subsidy to physical capital investment or a tax on adult consumption. De la Croix and Michel (1999) have already analysed the optimal subsidy for investment geared towards the internalisation of aspirations. This fiscal policy induces an adjustment to adult consumption and the achievement of a saving rate that is consistent with the modified golden rule.

The inoperativeness of the bequest motive implies that the neutrality of public debt, resulting from the Ricardian equivalence proposition, no longer holds. An issue left for future research is an analysis of the size of the crowding-out effect generated by public deficits when the bequest motive is inoperative. This size will obviously depend on habit and aspiration intensity. However, this task turns out to be infeasible in our general model and, thus, the analysis should be conducted under specific functional forms and parametric restrictions.

Appendix

A. Properties of the Steady State Equilibrium of the Economy with Zero Bequests

Existence and Uniqueness

By imposing $b = 0$, the steady state equilibrium of the economy with an inoperative bequest motive is defined by the values of k solving (20). However, the solutions to this equation must satisfy two constraints to be a steady state equilibrium. On the one hand, the condition (9) on δ imposes a restriction on the attainable capital. On the other hand, from the non-negative constraint on c^1 it follows that there is an upper bound \tilde{k} to the attainable capital, which is given by $f(\tilde{k}) = \tilde{k}[f'(\tilde{k}) + n]$.

From (20) we observe that $h(\tilde{k}, 0) = +\infty$ and $h(\hat{k}, 0) < 0$, where \hat{k} denotes a value of k for which condition (9) holds with equality.¹⁵ Moreover, we obtain that

$$\begin{aligned} \frac{\partial h(k, 0)}{\partial k} = & -(1 - \beta\gamma)(1 - \gamma)[kf''(k) + n]u''(\hat{c}^1) - \rho f''(k)u'(\hat{c}^2) \\ & - \rho[f'(k) + \delta]\{n[kf''(k) + f'(k)] + \delta[kf''(k) + n]\}u''(\hat{c}^2). \end{aligned}$$

From Assumption A and the properties of the utility and production functions, we find that the previous derivative is positive. We have thus proved that $\hat{k} < \tilde{k}$, and that there is a unique value k on the interval (\hat{k}, \tilde{k}) that solves the equation $h(k, 0) = 0$. Moreover, this solution satisfies condition (9) since this condition holds when $h(k, 0) = 0$. Hence, the steady state equilibrium both exists and is unique.

Stability

In order to obtain a saddle-path stable steady state, we need to have exactly two eigenvalues with modulus smaller than 1 since the model has two state variables. We thus extend the analysis made by de la Croix and Michel (2001) to the case where individuals form habits in addition to aspirations.

The equilibrium dynamics can be written as the following third-order difference equation, which is obtained immediately from (16), the individuals' budget constraint, and the competitive rental prices:

$$-u'(c_t^1 - \gamma c_{t-1}^1) + \rho[R(k_{t+1}) + \delta]u'[nR(k_{t+1})k_{t+1} - \delta c_t^1] + \beta\gamma u'(c_{t+1}^1 - \gamma c_t^1) = 0,$$

where

¹⁵ If preferences do not display habit formation ($\delta = 0$), then $\hat{k} = 0$.

$$c_{t+i}^1 = w(k_{t+i}) - nk_{t+i+1}, \text{ for } i = -1, 0, 1.$$

Consider a steady state $\{\bar{c}^1, \bar{k}\}$ satisfying $(1 - \beta\gamma)(n + \beta\delta) - \beta[f'(k) + \delta] > 0$, which means that in this steady state individuals do not leave bequests to their descendants (24). Linearising around this steady state we get

$$-[(1 + \beta\gamma^2)u''(\hat{c}^1) - g_{\bar{c}^1}(\bar{k}, \bar{c}^1)]dc_t^1 + \gamma u''(\hat{c}^1)dc_{t-1}^1 + \beta\gamma u''(\hat{c}^1)dc_{t+1}^1 + g_{\bar{k}}(\bar{k}, \bar{c}^1)dk_{t+1} = 0, \quad (\text{A.1})$$

where

$$\begin{aligned} dc_{t+i}^1 &= w'(k_{t+i})dk_{t+i} - ndk_{t+i+1}, \quad i = -1, 0, 1, \\ g(\bar{k}, \bar{c}^1) &= \rho[R(\bar{k}) + \delta]u'[nR(\bar{k})\bar{k} - \delta\bar{c}^1], \\ g_{\bar{c}^1}(\bar{k}, \bar{c}^1) &= -\rho\delta[R(\bar{k}) + \delta]u''(\hat{c}^2) > 0, \\ g_{\bar{k}}(\bar{k}, \bar{c}^1) &= \rho R'(\bar{k})u'(\hat{c}^2) + \rho n[R(\bar{k}) + \delta][R(\bar{k}) + R'(\bar{k})\bar{k}]u''(\hat{c}^2), \\ \hat{c}^1 &= \bar{c}^1 - \gamma\bar{c}^1, \end{aligned}$$

and

$$\hat{c}^2 = nR(\bar{k})\bar{k} - \delta\bar{c}^1.$$

Note that condition (21) in Assumption A is equivalent to assuming that

$$g_{\bar{k}}(\bar{k}, \bar{c}^1) < 0. \quad (\text{A.2})$$

Substituting dc_{t+i}^1 by $w'(k_{t+i})dk_{t+i} - ndk_{t+i+1}$ in (A.1), we obtain the characteristic polynomial

$$-[(1 + \beta\gamma^2)u''(\hat{c}^1) - g_{\bar{c}^1}(\bar{k}, \bar{c}^1)](w'\lambda - n\lambda^2) + \gamma u''(\hat{c}^1)(1 + \beta\lambda^2)(w' - n\lambda) + g_{\bar{k}}\lambda^2 = 0,$$

where we have substituted dk_{t+i} by λ^{i+1} , with $i = -1, 0, 1, 2$, and used the notation $w' = w'(\bar{k})$, $R = R(\bar{k})$ and $g_{\bar{k}} = g_{\bar{k}}(\bar{k}, \bar{c}^1)$. Dividing the previous polynomial by $-\gamma u''(\hat{c}^1)$ we get the following:

$$P(\lambda) = (n\lambda - w')Q(\lambda) - \psi\lambda^2, \quad (\text{A.3})$$

with

$$Q(\lambda) = \beta\lambda^2 - \phi\lambda + 1,$$

$$\psi = \frac{g_{\bar{k}}}{\gamma u''(\hat{c}^1)} > 0,$$

and

$$\phi = \frac{(1 + \beta\gamma^2)u''(\hat{c}^1) - g_{\bar{c}^1}(\bar{k}, \bar{c}^1)}{\gamma u''(\hat{c}^1)} > 0.$$

LEMMA A.1. *The polynomial $Q(\lambda)$ has two positive real roots, μ_1 and μ_2 , and they satisfy $0 < \mu_1 \leq 1 < 1/\beta \leq \mu_2$.*

Proof. For $\lambda = 1$ we have

$$Q(1) = 1 + \beta - \phi = \frac{-(1 - \gamma)(1 - \beta\gamma)u''(\hat{c}^1) - g_{\bar{c}^1}(\bar{k}, \bar{c}^1)}{\gamma u''(\hat{c}^1)} \leq 0.$$

Since $Q(0) = 1 > 0$, $Q(\lambda)$ is equal to zero at a point μ_1 of the interval $(0, 1]$. The characteristic polynomial $Q(\lambda)$ can be written as

$$Q(\lambda) = \beta(\lambda^2 - \frac{\phi}{\beta}\lambda + \frac{1}{\beta}),$$

and we know that

$$Q(\lambda) = \beta[\lambda^2 - (\mu_1 + \mu_2)\lambda + (\mu_1\mu_2)].$$

Therefore, the product of the two roots is equal to $1/\beta$ so that $\mu_2 = 1/(\beta\mu_1) \geq 1/\beta$.

LEMMA A.2. *The largest eigenvalue λ_1 satisfies $\lambda_1 > 1/\beta$ and $\lambda_1 > w'/n$.*

Proof. Note that $\lim_{\lambda \rightarrow \infty} P(\lambda) = \infty$. Moreover, $P(\mu_2) = -\psi(\mu_2)^2 < 0$ as $Q(\mu_2) = 0$ by definition. Therefore, $\lambda_1 > \mu_2 \geq 1/\beta$. Similarly, $P(w'/n) = -\psi(w'/n)^2 < 0$. We thus conclude that $\lambda_1 > w'/n$.

The previous result rules out the possibility of having three eigenvalues with modulus lying on the interval $(-1, 1)$, which means that the stationary equilibrium cannot be locally indeterminate.

The other two eigenvalues λ_2 and λ_3 are real or conjugate complex. The polynomial $P(\lambda)$ can be written as

$$P(\lambda) = -n\beta \left[-\lambda^3 + \left(\frac{\beta w' + n\phi + \psi}{n\beta} \right) \lambda^2 - \left(\frac{n + \phi w'}{n\beta} \right) \lambda + \frac{w'}{n\beta} \right].$$

Therefore,

$$\lambda_1 \lambda_2 \lambda_3 = \frac{w'}{n\beta},$$

and

$$\lambda_1 + \lambda_2 + \lambda_3 = \frac{\beta w' + n\phi + \psi}{n\beta},$$

so that λ_2 and λ_3 are the roots of the polynomial

$$\lambda^2 - \left(\frac{\beta w' + n\phi + \psi}{n\beta} - \lambda_1 \right) \lambda + \frac{w'}{n\beta \lambda_1} = 0.$$

LEMMA A.3. *The steady state equilibrium is saddle-path stable if and only if $P(1) < 0$ and $\lambda_1 > w'/n\beta$.*

Proof. (Necessity) On the one hand, since $P(\mu_2) < 0$ and $\lim_{\lambda \rightarrow \infty} P(\lambda) = \infty$, if $P(1) \geq 0$, then there exists a second real eigenvalue λ_2 satisfying $\lambda_2 \geq 1$ and the steady state is thus unstable. Therefore, stability imposes the condition $P(1) < 0$. On the other hand, stability implies that $1 > \lambda_2 \lambda_3 = w'/n\beta \lambda_1$ and, hence, the condition $\lambda_1 > w'/n\beta$ is also necessary.

(Sufficiency) (a) If λ_2 and λ_3 are real, then they are positive since $P(\lambda) < 0$ when $\lambda \leq 0$. Moreover, in this case, $P(1) < 0$ implies that both λ_2 and λ_3 are either larger or smaller than 1. Note that, if only one of the eigenvalues is smaller than 1, then $P(1) > 0$ as $\lim_{\lambda \rightarrow \infty} P(\lambda) = \infty$. Since $\lambda_2 \lambda_3 = w'/n\beta \lambda_1 < 1$, both eigenvalues are smaller than 1.

(b) When λ_2 and λ_3 are complex we have $|\lambda_2|^2 = |\lambda_3|^2 = \lambda_2 \lambda_3 = w'/n\beta \lambda_1 < 1$.

PROPOSITION A.4. *If Assumption A holds, then the steady state equilibrium is saddle-path stable.*

Proof. Consider the conditions stated in Lemma A.3. Condition $P(1) < 0$ can be written as

$$(n - w')(\beta - \phi + 1) - \psi < 0. \quad (\text{A.4})$$

Note that $Q(1) = 1 + \beta - \phi$ is non-positive and ψ is positive by condition (21) and the properties of the utility function. Therefore, thanks to condition (22) in Assumption A, which is equivalent to $w' \leq n$, the inequality (A.4) holds. Moreover, (22) and $n \geq 1$ imply that

$$\frac{w'}{n\beta\lambda_1} \leq \frac{1}{\beta\lambda_1} < 1,$$

where the second inequality holds because of Lemma A.2.

B. Proofs

Proof of Lemma 1. Applying the implicit function theorem to (20) we get

$$K'(b) = -\frac{h_b}{h_k},$$

where

$$h_b = (1 - \gamma)(1 - \gamma\beta)u''(\hat{c}^1) + \rho(n + \delta)[f'(k) + \delta]u''(\hat{c}^2)$$

and

$$\begin{aligned} h_k = & -(1 - \gamma)(1 - \gamma\beta)[kf''(k) + n]u''(\hat{c}^1) \\ & - \rho f''(k)u'(\hat{c}^2) - \rho[f'(k) + \delta]\{n[kf''(k) + f'(k)] + \delta[kf''(k) + n]\}u''(\hat{c}^2) \end{aligned}$$

are the partial derivatives of $h(k, b)$ with respect to b and k , respectively. From Assumption A and the properties of the utility and production functions, we know that $h_b < 0$ and $h_k > 0$, which proves the result.

Proof of Lemma 2. The proof follows directly from the definition of the steady state. The stationary value \bar{k} of the capital stock is implicitly given by (20) when we impose $b = 0$. Implicitly differentiating the previous equation, we get

$$\frac{d\bar{k}}{d\beta} = \frac{\gamma u'(\hat{c}^1)}{h_k}, \quad (\text{B.1})$$

where h_k is the partial derivative of $h(k, b)$ with respect to k . The numerator of (23) is clearly positive while the denominator is also positive as dictated by Assumption A.

Proof of Lemma 3. This result follows directly from the definition of the steady state capital of the economy with zero bequests. This capital is implicitly given by (20) after imposing $b = 0$ and $\gamma = 0$. Implicitly differentiating that equation we obtain

$$\frac{d\bar{k}}{d\delta} = \frac{-\rho\{u'(\hat{c}^2) - [f'(\bar{k}) + \delta]c^1 u''(\hat{c}^2)\}}{[\bar{k}f''(\bar{k}) + n]u''(\hat{c}^1) + \rho f''(\bar{k})u'(\hat{c}^2) + \rho[f'(\bar{k}) + \delta]\{n[\bar{k}f''(\bar{k}) + f'(\bar{k})] + \delta[\bar{k}f''(\bar{k}) + n]\}u''(\hat{c}^2)}.$$

The numerator of the previous derivative is negative by the properties of u , while it is obvious that the denominator is also negative as a consequence of Assumption A when $\gamma = 0$.

Proof of Proposition 2. From (26), we obtain

$$\frac{d\bar{\beta}}{d\delta} = -\frac{\bar{\beta}f''(\bar{k})(d\bar{k}/d\delta)}{f'(\bar{k})} > 0,$$

where the inequality comes from the properties of the function $f(k)$ and the fact that $d\bar{k}/d\delta > 0$ (see Lemma 3).

Proof of Proposition 3. The steady state value of capital k and bequests b for the economy with habits, positive bequests and no aspirations is given by the following two equations (see (20) and (24)):

$$\begin{aligned} u'[f(k) - kf'(k) + b - nk] \\ - \rho[f'(k) + \delta]u'\{n[kf'(k) - b] - \delta[f(k) + b - kf'(k) - nk]\} = 0 \end{aligned}$$

and

$$n - \beta f'(k) = 0.$$

The second equation tells us that the capital stock k does not depend on the habit intensity δ . Using this fact, we can implicitly differentiate the first equation above to get

$$\frac{db}{d\delta} = \frac{\rho\{u'(\hat{c}^2) - [f'(k) + \delta]c^1u''(\hat{c}^2)\}}{u''(\hat{c}^1) + \rho(n + \delta)[f'(k) + \delta]u''(\hat{c}^2)}.$$

From the properties of the production and utility functions it is obvious that the previous derivative is negative.

Proof of Proposition 4. Note that

$$\frac{dG(\beta)}{d\gamma} = -\beta \left[n + f''(\bar{k}) \left(\frac{d\bar{k}}{d\gamma} \right) \right],$$

which is negative when $\sigma(\hat{c}^1) > \beta/(1 - \beta\gamma)$ by Lemma 4. Hence, the desired result follows directly from Proposition 1.

Proof of Proposition 5. The steady state value of capital k and bequests b for the economy with aspirations, positive bequests and no habits is given by the following two equations (see (20) and (24)):

$$(1 - \beta\gamma)u'\{(1 - \gamma)[f(k) - kf'(k) + b - nk]\} - \rho f'(k)u'\{n[kf'(k) - b]\} = 0,$$

and

$$n(1 - \beta\gamma) - \beta f'(k) = 0.$$

Implicitly differentiating the second equation we get the following derivative:

$$\frac{dk}{d\gamma} = -\frac{n}{f''(k)},$$

which is clearly positive. Using the previous derivative, we can implicitly differentiate the first equation to obtain

$$\frac{db}{d\gamma} = -\frac{\Theta}{(1 - \gamma)(1 - \beta\gamma)u''(\hat{c}^1) + \rho f'(k)u''(\hat{c}^2)},$$

where the denominator is clearly negative and the numerator is

$$\Theta = -(1 - \beta\gamma) \left[\frac{\beta}{1 - \beta\gamma} - \sigma(\hat{c}^1) \right] u'(\hat{c}^1) \\ - \{ (1 - \gamma)(1 - \beta\gamma)[kf''(k) + n]u''(\hat{c}^1) + \rho f''(k)u'(\hat{c}_2) + \rho n f'(k)[kf''(k) + f'(k)]u''(\hat{c}_2) \} \frac{dk}{d\gamma}.$$

The second term in Θ is positive because of both the stability conditions imposed by Assumption A and the sign of $dk/d\gamma$. Moreover, the first term in Θ is positive if and only if $\sigma(\hat{c}^1) > \beta/(1 - \beta\gamma)$.

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