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Chaotic dynamics in credit constrained emerging economies

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Abstract

This paper analyzes the role of financial development as a source of endogenous instability in small open economies. By assuming that firms face credit constraints, our model displays a complex dynamic behavior (with high-period cycles or even chaotic dynamic patterns) for intermediate values of the parameter representing the level of financial development of the economy. We derive sufficient conditions for global stability and we prove that chaos appears via a border collision bifurcation. The basic implication of our model is that economies experiencing a process of financial development are more unstable than both very underdeveloped and very developed economies.

JEL classification: C62; O16

Keywords: Chaotic dynamics; Credit constraints; Financial development

1. Introduction

This paper considers a model where the process of financial development could be a source of endogenous instability in small open economies. Our basic macroeconomic

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model describes a dynamic open economy where firms face credit constraints. This means that the maximum amount entrepreneurs can borrow is proportional to the amount of their current level of wealth. Several authors have discussed the implications of borrowing constraints on the persistence of business cycles (Bernanke and Gertler, 1989) and on the emergence of cycles in closed economies (Azariadis and Smith, 1998; Kyotaki and Moore, 1997). Other authors have developed models where instability occurs at intermediate levels of financial development and, thus, these models provide support to the evidence that emerging markets are quite vulnerable (Gaytan and Ranciere, 2004; Daniel and Jones, 2001). Moreover, Aghion et al. (2000) showed that, if firms are credit constrained and there is debt issued both in domestic and in foreign currency, then currency crises could easily arise. Later on, Aghion et al. (2001) constructed a simple monetary model where currency crises are driven by the interplay between the credit constraints of domestic firms and the existence of nominal price rigidities, which in turn lead to multiple equilibria. We will use a similar formulation, although we will focus on the real side of the economy like in Aghion et al. (1999) and Aghion et al. (2004), where the authors provide numerical simulations showing the existence of a stable 2-period cycle in a model exhibiting a Leontieff technology. However, it does not follow from their work the existence of either cycles of higher period or chaotic dynamics. Although they show the existence of a first period doubling bifurcation in their model, they do not even show the existence of a canonical bifurcation cascade leading to chaos. In this paper we show instead the existence of a non-canonical route to chaos, which is associated to a border collision bifurcation.

By considering a more realistic Cobb–Douglas production function and using bifurcation analysis, we will prove that economies with either very developed or very undeveloped financial markets are globally structurally stable, while emerging markets (with intermediate levels of financial development) are unstable in the sense that they could exhibit high period cycles (with period larger than 2) or even chaotic dynamics and, thus, the evolution of the endogenous variables of the model turns out to be unpredictable. Therefore, when the economy is going through a phase of financial development, the dynamics of the economic system could change dramatically and evolve from a stable fixed point to a stable cycle and, finally, to an attractor displaying aperiodic dynamic behavior.

In our model complex dynamics arises because an increase in wealth has a positive effect on investment (via the credit constraint) but also a negative price effect due to the increase in the demand for the country specific input. The conclusion is that financial development could destabilize economies exhibiting an intermediate level of financial development, which agrees with the experience documented for several countries.[1]

The paper is organized as follows. In Section 2 we present the model. In Section 3 we perform the dynamic analysis in order to assess the plausibility of chaotic dynamics when entrepreneurs do not receive any exogenous income. In this case we

[1]See, among many others, De Melo et al. (1985), Galvez and Tybout (1985), and Petrei and Tybout (1985).

present some general properties of the attractors of the dynamic system depending on the parameter values of the model. In Section 4 we extend the analysis to the general case with positive exogenous income and we show the global stability of either very developed or very underdeveloped financial markets. Section 5 concludes our paper. The Appendix contains all the proofs.

2. The model

Let us consider a small open economy in discrete time. There are two types of individuals in this economy: the borrowers (or entrepreneurs), who own a production technology and may invest either in the production activity or in the international capital market, and the lenders (or families), who cannot directly invest in the production activity but they can either lend funds to the entrepreneurs or invest in the international capital market. The international gross rate of interest is constant and equal to $r>0$.

This economy produces a unique tradeable good. The production function of this good uses capital and a country specific input (like land, real estate, or a non-tradeable natural resource), which has a constant supply equal to Z. Moreover, the tradeable good can be consumed or accumulated as productive capital for the production in the next period. The output y_t of the tradeable good in each period is obtained through the following Cobb–Douglas gross production function[2]:

$$y_t = AK_t^{\rho} z_t^{1-\rho} \quad \text{with } \rho \in (0,1), \tag{1}$$

where z_t is the amount of the country specific input used in period t, K_t is the amount of capital, and A is the total factor productivity. We assume that $A>r$ since the entrepreneurs would do not find profitable to invest in the production activity otherwise. We assume that capital fully depreciates after one period.

The total investment I_t in period t is devoted to purchase both capital and country specific input. For a given level of investment, the optimal demands for the country specific input z_t and for capital K_t in each period t arise from the maximization of the profit function subject to the budget constraint

$$I_t = K_t + p_t z_t,$$

where p_t is the price of the country specific input measured in units of the tradeable good. The first-order condition for the profit maximization problem immediately yields

$$z_t = \left(\frac{1-\rho}{p_t}\right) I_t \tag{2}$$

and

$$K_t = \rho I_t. \tag{3}$$

[2]Aghion et al. (2004) consider instead a Leontieff production function, which prevents any kind of substitution among the different inputs.

Therefore, the country specific input equilibrium price is obtained by equating the country specific input demand (2) with its constant supply Z,

$$p_t = \left(\frac{1-\rho}{Z}\right) I_t. \tag{4}$$

Finally, substituting (2) and (3) into (1), we may write the total equilibrium output y_t in terms of the level of investment I_t and the price p_t,

$$y_t = G(p_t) I_t, \tag{5}$$

with

$$G(p_t) = \frac{A\rho^{\rho}(1-\rho)^{1-\rho}}{p_t^{1-\rho}}. \tag{6}$$

Note that $G(p_t)$ can be viewed as the gross return of a unit of investment.

We assume that the credit market operates imperfectly due to, say, either adverse selection or moral hazard problems. In particular, we assume that the entrepreneur's wealth serves as a collateral for the loan and, hence, an entrepreneur with wealth W_t may borrow the amount L_t with $L_t \leqslant \mu W_t$.[3] Therefore, the investment in each period is bounded above by $(1+\mu)W_t$. The proportional coefficient $\mu \geqslant 0$ can be viewed as a credit multiplier reflecting the level of financial development of the domestic economy.

The dynamics of the model is described as follows. In period t entrepreneurs decide the amount of borrowing (and, thus, of investment) and pay the cost of the country specific input. Hence, at period $t+1$ the entrepreneurs receive the corresponding profits and pay the cost of debt rL_t. We assume here that entrepreneurs save a constant fraction $(1-\alpha)$ of their total wealth at the end of each period, where α is the constant propensity to consume.[4] Therefore, the dynamics of the entrepreneurs' wealth is given by

$$W_{t+1} = (1-\alpha)(e + y_t - rL_t), \tag{7}$$

with $L_t \leqslant \mu W_t$, and where $e \geqslant 0$ is an exogenous income in terms of tradeable good.

Let us consider first the case where $G(p_t) \geqslant r$, which means that the productive investment return exceeds the international capital market return and, hence, the entrepreneurs will invest in the productive project the largest amount they can borrow,

$$I_t = (1+\mu) W_t. \tag{8}$$

Thus, from (5), total output will be given by

$$y_t = G(p_t)(1+\mu) W_t. \tag{9}$$

[3] This is the type of constraint found in Bernanke and Gertler (1989).

[4] This simple saving rule could be derived under the assumption that entrepreneurs maximize the discounted sum of instantaneous utilities when these utilities are logarithmic and $e = 0$ (see Woodford, 1989).

Substituting (8) in (4) and then inserting the resulting equation in (6), we can rewrite (9) as

$$y_t = \xi W_t^{\rho} \quad \text{with } \xi = A\rho^{\rho}(1+\mu)^{\rho} Z^{1-\rho}, \tag{10}$$

which gives us the total output in period t as a function of wealth. As follows from (7), the dynamics of entrepreneurs' wealth is thus

$$W_{t+1} = (1-\alpha)(e + y_t - r\mu W_t), \tag{11}$$

which, after using (10), becomes

$$W_{t+1} = (1-\alpha)[e + W_t^{\rho}(\xi - r\mu W_t^{1-\rho})]. \tag{12}$$

The previous equation describes the dynamics of the entrepreneurs' wealth as long as $G(p_t) \geqslant r$. Combining (4), (6), and (8), the previous inequality holds whenever $W \leqslant W^m$, where the critical value of wealth W^m is given by

$$W^m = \left(\frac{A}{r}\right)^{1/(1-\rho)} \left(\frac{Z}{1+\mu}\right) \rho^{\rho/(1-\rho)}. \tag{13}$$

As it can be immediately seen from (12), the impact of a change in the volume of current wealth on the next period wealth could be ambiguous. This is so because, even if an increase in wealth raises the investment, the amount of invested wealth depends negatively on the price p_t of the country specific input, and this price depends positively on current wealth.

Let us now consider the case where $G(p_t) < r$. In this case the entrepreneurs have no incentives to borrow up to the credit limit because the return from the productive investment is lower than the return from the international capital market. Hence, they borrow until the level where the productive investment return is equal to the international capital market return, that is, until $y_t - rL_t = rW_t$. Therefore, substituting the previous equation into (7), we find the dynamic equation for wealth when $W > W^m$,

$$W_{t+1} = (1-\alpha)(e + rW_t). \tag{14}$$

The asymptotic behavior of wealth is thus determined by the iterates of the following function (see (12) and (14)):

$$f(W_t) = \begin{cases} (1-\alpha)[e + W_t^{\rho}(\xi - r\mu W_t^{1-\rho})] \equiv f^l(W_t) & \text{if } 0 \leqslant W_t \leqslant W^m, \\ (1-\alpha)(e + rW_t) \equiv f^r(W_t) & \text{if } W_t > W^m. \end{cases} \tag{15}$$

The iterates of f describe a one-dimensional discrete dynamic system. In the next sections we will analyze the general dynamic properties of the system depending on the parameters of the model and we will determine the cases where complex (or even chaotic) dynamics could appear. Note that the dynamic behavior of the other endogenous variables of the model, output y_t, investment I_t, and price of the country specific input p_t, is entirely determined by the behavior of the entrepreneurs' wealth W_t, as follows immediately from the previous analysis.

It is straightforward to see that, for any value of the parameters, $f^l(W_t)$ is a single peaked function with a maximum at

$$W^M = \left(\frac{r\mu}{\xi\rho}\right)^{1/(\rho-1)}, \tag{16}$$

while $f^r(W_t)$ is linear. We notice, however, that $f(W_t)$ has a local maximum at W^M only if $W^M < W^m$.

3. Dynamic analysis without exogenous income

In this section we describe the long run dynamics of the model when the exogenous income e is equal to zero. This analysis will be helpful in the next section, where we study the case with positive exogenous income, $e > 0$. For the rest of the paper we will assume the empirically plausible inequality $(1-\alpha)r < 1$, which ensures the existence of a positive fixed point for the function f.

In the next lemma we summarized the dynamic properties of the map f given by (15) when $e = 0$ and the parameter value μ is relatively small. Recall that a high value of μ corresponds to a financially developed economy, while low values are associated with financial underdevelopment.

Lemma 1. *Let* $e = 0$.

(a) *If* $\mu \in [0, \rho/(1-\rho))$, *then* $W^M > W^m$ *and* f *is a strictly increasing function. Moreover* f *has a unique positive, globally stable fixed point* $W^\star \in (0, W^m)$.
(b) *If* $\mu \in [\rho/(1-\rho), \rho/r(1-\alpha)(1-\rho))$, *then* $W^M \leqslant W^m$ *and* f *has a unique positive, globally stable fixed point* $W^\star \in (0, W^M)$.
(c) *If* $\mu \in [\rho/r(1-\alpha)(1-\rho), (1+\rho)/r(1-\alpha)(1-\rho))$, *then* $W^M \leqslant W^m$ *and* f *has a unique positive, globally stable fixed point* $W^\star \in [W^M, W^m)$.
(d) *A* 2*-period bifurcation occurs at* $\mu_2 = (1+\rho)/r(1-\alpha)(1-\rho)$ *and the derivative of* f *at the corresponding parabolic fixed point* $W^\star$ *is* -1. *Hence, a* 2*-period globally stable cycle exists if* $\mu \in (\mu_2, \mu_2 + \delta)$ *for* δ *sufficiently small.*
(e) *For any value of* μ, $W = 0$ *is a repelling fixed point.*

This lemma essentially reproduces the dynamic conclusions of Aghion et al. (2004) and it points out that, in order to obtain rich or chaotic dynamics for the entrepreneurs' wealth, the value of the parameter μ must be sufficiently high. In other words, the previous lemma states the global stability for economies with low values of μ, that is, for economies exhibiting a low level of financial development. As we will see in Section 4, this result also holds for the case with positive exogenous income. Moreover, part (d) of the previous lemma gives explicit conditions for the first period doubling bifurcation, which seems to announce a route to chaos through a period doubling bifurcation cascade.[5] However, in the next discussion it will

[5] See Devaney (1992).

become clear that such a bifurcation cascade does not exist although we will be able to obtain chaos through a non-canonical bifurcation route.

We now provide an example illustrating the asymptotic dynamics of wealth with special attention to relatively high values of the degree of financial development parametrized by μ. Although our approach is mainly numerical, it allows us to provide a complete description of the dynamics of the model when $e = 0$. Let us thus consider the empirically plausible parameter values $\rho = 1/3,\ \alpha = 0.8,\ r = 1.02$. We have checked that the qualitative properties of the dynamics of the model are preserved for a grid of values lying on a neighborhood of the proposed parameter value configuration. Moreover, we also consider the arbitrary parameter values $A = 3/2$ and $Z = 100$. It can be seen from the equations of the previous section that, when $e = 0$, the parameters A and Z are just scaling parameters that do not affect the qualitative properties of the model. Under these parameter values, the function f given by (15) becomes

$$f = \begin{cases} (W_t)^{1/3}\left[\sqrt[3]{900}(1+\mu) - \dfrac{51}{250}(W_t)^{2/3}\mu\right] \equiv f^l(W_t) & \text{for } 0 \leqslant W \leqslant W^m, \\ \dfrac{51}{250} W \equiv f^r(W_t) & \text{for } W > W^m, \end{cases}$$

where $W^m = 12\,500/17\sqrt{51}(1+\mu)$. Notice that f^l is a single peaked concave function with the local maximum at the point

$$W^M = \frac{12\,500(1+\mu)^{1/2}}{153\sqrt{17}\mu^{3/2}}.$$

When the value of the financial development parameter μ belongs to the interval $[0, 25.239)$ we obtain a simple dynamics for which the entrepreneurs' wealth is asymptotically given by a stable fixed point or a 2-period stable cycle. The dynamics of wealth for those values of the parameter μ obviously agrees with Lemma 1 and, hence, when $\mu = \mu_2 \equiv (1+\rho)/r(1-\alpha)(1-\rho) = 9.804$, we obtain the first period doubling bifurcation.

Now let us move the value of μ beyond 25.239. Once we cross that value of μ the asymptotic dynamics (that is, the window where the dynamics occurs in the long run) belongs to the interval $[f(W^m), f^2(W^m)]$. Thus, we first compose the function

$$f : [f(W^m), f^2(W^m)] \to [W^m, f^2(W^m)]$$

with an homeomorphism h so that $g = h(f(h^{-1}))$ is a function from $[0,1]$ to $[0,1]$.[6] We define the new variable $X_t = h(W_t)$ and we can see the function $g(X)$ displayed in Panel (a) of Fig. 1. Since g is a continuous map of $[0,1]$ into itself and the steady state is unstable, the wealth never grows without bound nor converges to a steady state (for almost all initial conditions). Hence, it must fluctuate forever, either converging to a cycle or following an aperiodic path.

[6]The function h is simply a linear transformation depending on μ that assigns the value 0 to $f(W^m)$ and the value 1 to $f^2(W^m)$ for each μ. The advantage of working with the function g instead of f lies on the fact that the window where the asymptotic dynamics occurs is $[0, 1]$, which is independent of μ. Obviously, the dynamics associated with f and g display the same qualitative properties.

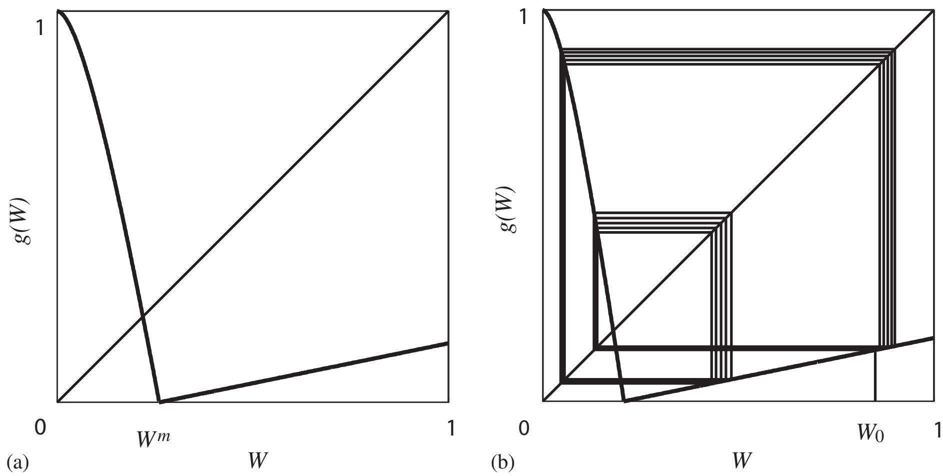

Fig. 1. (a) Function g for an arbitrary value of $\mu > 25.239$. (b) 4-period attracting cycle ($\mu = 50$).

As mentioned before, the asymptotic dynamics of g in $[0, 1]$ when the parameter μ has just crossed the value 25.239 is governed by an attracting 2-period cycle with negative multiplier. As the value of μ increases the multiplier approaches -1 and, when it becomes equal to -1, we have a new period doubling bifurcation from a 2-period cycle to a 4-period cycle. After this bifurcation the asymptotic dynamics is given by an attracting 4-period cycle with positive multiplier (see Panel (b) of Fig. 1). When μ reaches the value 53.953 the critical point X^m of the function g belongs to the 4-period cycle and we get what is known as a border collision bifurcation.[7] From Nusse and Yorke (1995) and performing a careful numerical analysis, it is possible to see that the non-canonical border collision bifurcation cascade passes through cycles with period 8, 16, 32 and, finally, enters into a chaotic attractor of full Legesgue measure in $[0, 1]$.[8]

Another evidence of the existence of values of μ for which we obtain chaotic asymptotic dynamics comes from the fact that, if $\mu = 54.925$, then the critical point X^m of the function g is pre-periodic as $g(X^m) = 0$, $g(0) = 1$, and $g(1) = X^{\star}$, where $X^{\star}$ is the fixed point of g. Thus, for this value of μ, no attracting cycles exist since the basin of attraction of these cycles cannot contain the critical point X^m.[9] Note that

[7]This non-canonical bifurcation has been mainly studied in the context of piecewise linear maps. For instance, Hommes and Nusse (1991) showed that a 'period three to period two bifurcation' occurs for a class of piecewise linear maps. More recently, Nusse and Yorke (1995) have conducted a deeper analysis of these bifurcations and described the very rich dynamics arising from them.

[8]Notice that, even though we have a bifurcation cascade from 1 to 32-period attracting cycles only the bifurcations from 1 to 2, 2 to 4, and 8 to 16-period attracting cycles are period doubling bifurcations, while from the 4 to 8, and 16 to 32 are border collision bifurcations.

[9]Moreover, the topological entropy of the map g for $\mu = 54.925$ is positive, which confirms the existence of a chaotic attractor (see Alsedà et al., 2001, Corollary 4.4.9). Recall that the topological entropy is given by $\lim_{n\to\infty} \ln N_n/n$ where N_n is the number of distinct n-period cycles.

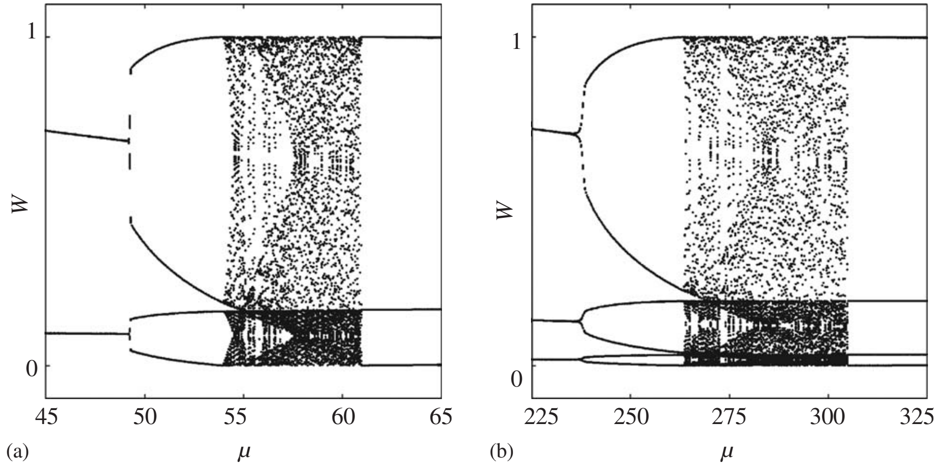

Fig. 2. Bifurcation diagrams for different intervals of the parameter μ.

the fact that the interval [0, 1] is a chaotic attractor under g implies immediately that the chaotic attractor under the original map f has positive Lebesgue measure when $\mu = 54.925$.

When $\mu = 60.936$, the image of $X = 1$ is equal to X^m and, hence, we obtain a 3-period cycle. This parameter value of μ is the initial point of an interval for which an attracting 3-period cycle exists (see the bifurcation diagram displayed in Panel (a) of Fig. 2). It can be shown that the existence of those two distinct values of μ, one implying chaotic dynamics and the other initiating simple stable dynamics, repeats over and over as μ increases (see the two panels of Fig. 2).[10] The first values of these two sequences $\{\mu_k\}$ and $\{\mu'_k\}$ are

$$\begin{aligned} &\mu_1 = 54.925,\ \mu_2 = 298.793,\ \mu_3 = 1493.594,\ \mu_4 = 7350.175, \ldots \\ &\mu'_1 = 60.936,\ \mu'_2 = 304.676,\ \mu'_3 = 1499.480,\ \mu'_4 = 7356.363, \ldots \end{aligned} \tag{17}$$

respectively.

Summing up, in this section we have reformulated the dynamic analysis of Aghion et al. (2004) with a Cobb–Douglas production function and we have seen how the dynamics of wealth evolves as μ increases. In particular, we have shown that there are intervals of this parameter value for which the long run dynamics is governed by periodic attracting patterns while there are other intervals where the long run dynamics is aperiodic (or chaotic). Moreover, this pattern repeats over and over as μ increases and, hence, there is not a value of μ above which stability holds in the long run. As we will prove in the following section, this last result does not longer hold for the case $e > 0$.

[10]The proof is available from the authors upon request.

4. Dynamic analysis with positive exogenous income

Let us now study the dynamics of the general model defined by function (15) for the case with positive exogenous income, $e > 0$. Panel (a) of Fig. 3 illustrates the changes of the function f as the exogenous income e increases for a fixed value of μ. Obviously, if they exist, the values of the critical point W^m and of the maximum W^M are the same as when $e = 0$ (see (13) and (16)). However, both the existence and the qualitative properties of the attracting sets are affected by the fact that e is strictly positive.

The main result of this section is that, for any positive exogenous income e, we obtain stable dynamics for the variable W_t (i.e., a globally stable fixed point exists) for low as well as for high levels of financial development. Moreover, for any positive e, there are intermediate levels of financial development (i.e., a bounded interval of values of μ) for which the dynamics is complex or even chaotic and follows the same pattern as in the previous section. The following proposition summarizes the main result of this section:

Proposition 1. *Let* $e > 0$.

(a) $W = 0$ *is not a fixed point.*
(b) *The function* $f(W_t)$ *has a unique positive, globally stable fixed point* $W^\star$ *for each* $\mu \in [0, \rho/(1-\rho))$.
(c) *The function* $f(W_t)$ *has a globally stable fixed point* $W^\star$ *with* $W^\star \geqslant W^m$ *for each* $\mu \in [\mu^M(e), \infty)$, *where*

$$\mu^M(e) \equiv \left(\frac{A}{r}\right)^{1/(1-\rho)} Z\rho^{\rho/(1-\rho)} \left[\frac{1-(1-\alpha)r}{(1-\alpha)e}\right] - 1.$$

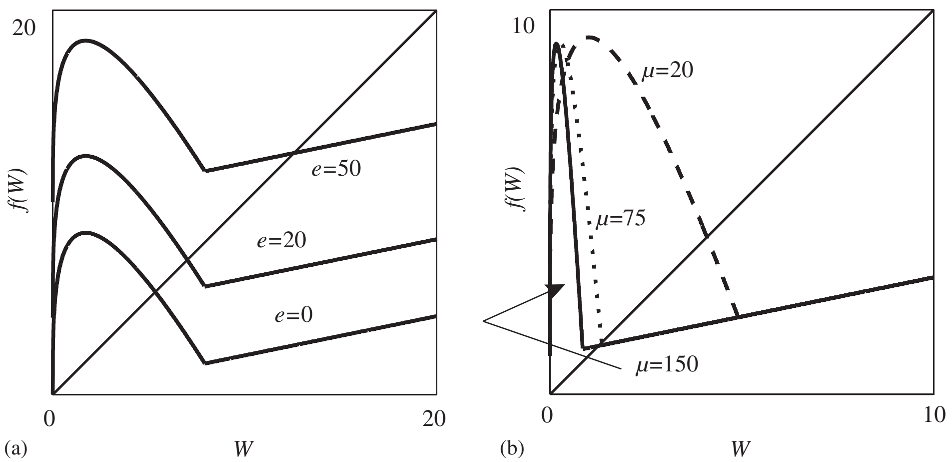

Fig. 3. (a) Vertical shift of f for different positive values of e ($\mu = 12$). (b) The minimum point passing from the right to the left of the diagonal when μ increases ($e = 5$).

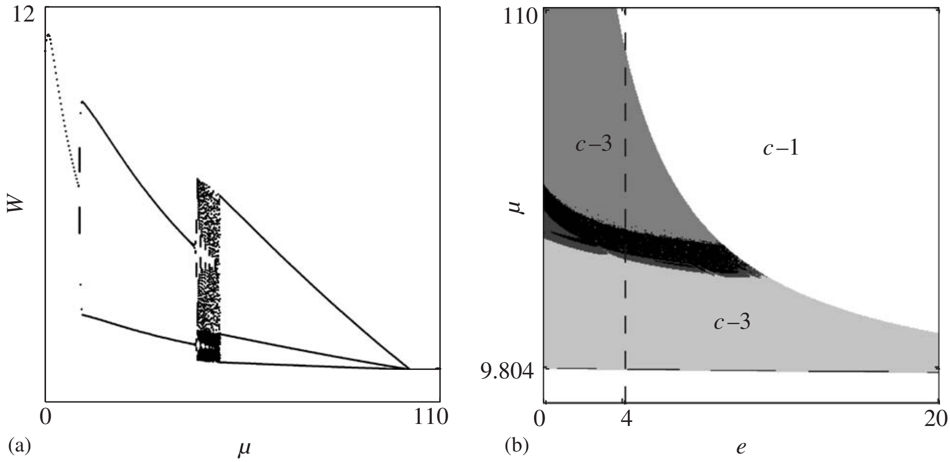

Fig. 4. (a) Bifurcation diagram with respect to μ when $e = 4$. (b) The parameter plane (e, μ).

The previous proposition shows that, for each positive value of e, there exists a large enough value $\mu^M(e)$ of the parameter μ guaranteeing the stability of financially developed economies. Intuitively, this is due to the fact that as $\mu \to \infty$ the critical point W^m tends to 0 so that it has to pass from the right to the left of the 45°-degree line (see Panel (b) of Fig. 3). Therefore, according to this observation and the existence of the infinite sequences given in (17) the stability of financially developed economies is obtained if and only if e is positive.

To illustrate the above arguments, we draw in Panel (a) of Fig. 4, the bifurcation diagram with respect to μ when $e = 4$. Moreover, in Panel (b) of Fig. 4 we draw the different asymptotic dynamics on the plane (e, μ), where $c - n$ means that, for this combination of values of e and μ, the asymptotic dynamics is governed by a n-period cycle. To see the logic behind these figures, let us fix a strictly positive value of e (for instance, $e = 4$). Then for small values of μ we have a globally stable fixed point lying below W^m and, for high values of μ (i.e., when $\mu \geqslant \mu^M(4)$), we have also a globally stable fixed point lying above W^m. For intermediate values of financial development we may find complex or even chaotic dynamics. It should be noted finally that the potential complexity for those intermediate values depends on how many terms of the sequence given by (17) are smaller than $\mu^M(e)$. In particular, for high values of e the value of $\mu^M(e)$ is so small that the attracting set is given by either a fixed point or a low period cycle, while for small values of e the value of $\mu^M(e)$ is large enough so that emerging economies could display chaotic behavior.

In order to understand the resulting dynamics of our model note that, on the one hand, for low values of μ the credit constraint faced by entrepreneurs is so strong that investment is insensitive to the amount of current wealth. On the other hand, for very high values of μ the amount of current wealth is also irrelevant for investment since entrepreneurs are not credit constrained. Only for intermediate values of the parameter representing the degree of financial development, the current level of funds are relevant for the determination of investment and, thus, for future wealth.

Note that in the chaotic region the path of the entrepreneur's wealth is sensitive to the initial conditions, as is implied by the aperiodicity of the resulting dynamics. Therefore, small transitory shocks end up having permanent effects in the long run.

5. Conclusions

We have studied the asymptotic dynamics of entrepreneurs' wealth in a small open economy with credit constraints under a Cobb–Douglas technology. A similar model has been considered in Aghion et al. (2004) using a Leontieff technology. These authors only showed that financial underdeveloped, as well as very developed economies, present stable fixed points, whereas intermediate levels of financial development could be a source of instability. They obtain the instability result by showing that, for some range of parameter values, a 2-period cycle appears.

Our analysis differs from that of Aghion et al. (2004) in the following aspects:

- We show that financially developed economies present complex dynamics when there is no exogenous income of the tradeable good. However, very developed economies are stable when entrepreneurs enjoy positive exogenous income.
- We derive sufficient conditions on the parameter values of the model in order to obtain global stability.
- We show that economies with an intermediate level of financial development could present dynamics more complex than a 2-period cycle since they could display cycles with a higher period or even chaotic dynamics. Those cycles are robust as they remain under perturbations of the parameter μ representing the level of financial development. In this case, even though the dynamics becomes complex, it is still predictable. This predictability does not longer hold in the chaotic region. Obviously, when the economy displays chaotic dynamics, small temporary shocks turn out to have permanent effects.

Note that when there is no exogenous income, complex dynamics occurs for arbitrarily large values of the parameter μ. However, the range of values of μ for which complexity arises is bounded when the exogenous income is positive. Thus, in such a case, there is a sufficiently high level of financial development that guarantees stability in the long run.

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Appendix

Proof of Lemma 1. Let $e=0$. It is easy to see that

$$W^M - W^m = Z\left(\frac{r}{A}\right)^{1/(\rho-1)}(\rho(1+\mu))^{\rho/(1-\rho)}\left[\left(\frac{\mu}{\rho}\right)^{1/(\rho-1)} - (1+\mu)^{1/(\rho-1)}\right].$$

Therefore, if $\mu<\rho/(1-\rho)$, then $W^m - W^M>0$ and f is a strictly increasing function. Moreover, since $f^r(W^m)=f^l(W^m)<W^m$, we obtain the existence of a unique positive, globally stable fixed point given by

$$W^\star = \left(\frac{1+r\mu(1-\alpha)}{\xi(1-\alpha)}\right)^{1/(\rho-1)} \in (0, W^m).$$

This concludes the proof of part (a).

When $\mu=\rho/(1-\rho)$, then $W^M = W^m$ and, as $(1-\alpha)r<1$, we have $W^\star \in (0, W^M)$. If $\mu>\rho/(1-\rho)$, then $W^M - W^m<0$ and

$$f^l(W^M) - W^M = W^M\left[\frac{(1-\alpha)r(1-\rho)\mu}{\rho} - 1\right].$$

Thus, if $\mu<\rho/r(1-\alpha)(1-\rho)$, we conclude that $f^l(W^M) - W^M<0$ and so, the unique positive fixed point is globally attracting and satisfies $W^\star \in (0, W^M)$. This finishes the proof of part (b).

When $\mu=\rho/r(1-\alpha)(1-\rho)$ the positive stable fixed point corresponds precisely to the maximum of f^l, that is, when $W^\star = W^M$ it holds that $f'(W^\star)=0$. As the parameter μ increases, the derivative at the fixed point is given by $f'(W^\star)=(1-\alpha)(\rho/(1-\alpha)-\mu r(1-\rho))$. Therefore, this derivative goes from zero to negative and it takes the value -1 when

$$\mu = \frac{1+\rho}{r(1-\alpha)(1-\rho)} \equiv \mu_2.$$

Therefore, for the values of μ belonging to the interval $[\rho/r(1-\alpha)(1-\rho), (1+\rho)/r(1-\alpha)(1-\rho))$, there is a locally attracting fixed point belonging to the interval $[W^M, W^m)$. Obviously, we have a period doubling bifurcation at $\mu=\mu_2$ and the local stability of the fixed point applies now to the locally attracting 2-period cycle for $\mu \in (\mu_2, \mu_2+\delta)$, with δ sufficiently small.

To finish the proof of (c) and (d) we claim that the local stability is indeed global. To see this, it is enough to show that $f(W^M) - W^m<0$ for $\mu \in (\rho/r(1-\alpha)(1-\rho), \mu_2+\delta)$, or equivalently, that after a finite number of iterates all the dynamics is concentrated on the interval $(0, W^m)$. Clearly, the claim holds for $\mu=\rho/r(1-\alpha)(1-\rho)$ since $f(W^M)=W^M<W^m$. From the monotonicity of W^M and W^m with respect to the parameter μ, we only need to check whether the inequality still holds for $\mu=\mu_2$. From a straightforward computation we get $f(W^M) - W^m<0$

if and only if

$$\Psi(\mu) = r(1-\alpha)(1-\rho)\rho^{\rho/(1-\rho)}\left(\frac{1+\mu}{\mu^{\rho}}\right)^{1/(1-\rho)} < 1.$$

Hence, we need to check that $\Psi(\mu_2) < 1$. A direct substitution leads to

$$\begin{aligned}\Psi(\mu_2) &= r(1-\alpha)(1-\rho)\rho^{1/(1-\rho)}\left(\frac{1+\rho}{r(1-\alpha)(1-\rho)}\right)^{\rho/(\rho-1)} \\ &\quad\times\left(1+\frac{1+\rho}{r(1-\alpha)(1-\rho)}\right)^{1/(1-\rho)} \\ &= \rho^{1/(1-\rho)}(1+\rho)^{\rho/(1-\rho)}(r(1-\alpha)(1-\rho)+1+\rho)^{1/(1-\rho)} \\ &< \left[\frac{2\rho}{(1+\rho)^{\rho}}\right]^{1/(1-\rho)} < 1,\end{aligned}$$

for all $\rho \in [0,1)$, which is the desired result.

Part (e) is straightforward. □

Proof of Proposition 1.

(a) Obvious.
(b) We just have to notice that if $\mu \in [0, \rho/(1-\rho))$ the function f is strictly increasing. Moreover, $f(0) = e > 0$ and $f^{r}(W_t)$ is a linear function with slope smaller than 1. Hence, there exists a $W^{\star}$ such that $f(W^{\star}) = W^{\star}$, with $0 < f'(W^{\star}) < 1$.
(c) From the functional form of the function f we see that, for sufficiently large values of μ, the function f has local maximum at W^M and a local minimum at W^m with $W^M < W^m$. Moreover, it is easy to see from (13) that W^m tends to 0 as μ goes to infinity. Clearly, for a given $e > 0$, a sufficient condition for having a globally stable fixed point is $f(W^m) \geqslant W^m$, where the fixed point $W^{\star}$ satisfies $W^m \leqslant W^{\star}$. A direct computation shows that the last inequality is equivalent to

$$W^m \leqslant \frac{(1-\alpha)}{1-(1-\alpha)r},$$

which holds for large enough values of μ since W^m tends to zero as μ tends to infinity. The solution to the equation $W^m = (1-\alpha)/(1-(1-\alpha)r)$ gives us the value of $\mu^M(e)$ appearing in the statement of the proposition. □

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