GROWTH EFFECTS OF TAXATION UNDER ALTRUISM AND LOW ELASTICITY OF INTERTEMPORAL SUBSTITUTION*

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An increase in the tax rate on capital income may raise the rate of economic growth when the elasticity of intertemporal substitution is low and intergenerational transfers are absent. Since the strength of the bequest motive depends on tax rates, this paper provides conditions under which taxing capital income, and then reducing the labour income tax, is more growth enhancing than the classical policy of zero taxes on capital income, and vice versa.

This paper analyses the effects of different taxes on the rate of economic growth. We consider an economy in which public spending is a fixed fraction of the GNP and the government can obtain revenues from proportional taxes on both labour and capital income. Obviously, the growth effects of these two instruments will depend on the assumptions made about the economic environment.

In the standard overlapping generations (OLG) model with production (Diamond, 1965), an increase in capital income taxes allows a reduction in labour income taxes, and thus agents will enjoy more income when they are young. Since in the OLG model young agents must purchase the total stock of capital installed in the next period, and saving is increasing in young income, higher taxes on capital income may lead to faster capital accumulation. In order to complete the argument, we must ensure that the associated decrease in the after-tax interest rate does not lead to a reduction in saving which would outweigh the previous income effect. In other words, we need a sufficiently low elasticity of intertemporal substitution. The argument is thus similar to the one of Jones and Manuelli (1992), who have already pointed out that, if the technological environment makes sustained growth feasible, then taxing the old agents and subsidising the young ones may increase the rate of economic growth.

However, the situation is completely different if we consider instead an economic environment in which the life-cycle considerations are absent. For instance, in endogenous growth models with a representative agent (or dynasty) and infinite life-span, an increase in the capital income tax rate typically translates into lower growth (see, for instance, Sato, 1967; Feldstein, 1974; Stiglitz, 1978; and the more recent contributions of Lucas, 1990; and Rebelo, 1991).

A way to give a unified treatment to these two alternative models is by means

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of assuming altruistic preferences as in Barro (1974). So, the economy becomes dynastic if the bequest motive is strong enough, whereas the economy behaves like in the OLG model when the bequest motive is not too intense. In this respect, Weil (1987) considers a model of one-sided altruism (from parents to children) and provides a precise formula for the threshold level of the altruism factor below which intergenerational transfers are absent and thus the economy fails to be dynastic.\(^1\)

In this paper we suppose that intergenerational altruism is an important factor affecting capital accumulation. In fact, the importance of intergenerational transfers has been documented by several studies. For instance, Kotlikoff and Summers (1981; 1986) estimate that between 45% and 80% of the capital stock held by households in the United States arises from intergenerational transfers. Note that the pure life-cycle model without bequest motives would imply that the demand for annuities should be very strong. Only if we assumed away annuity markets, would intergenerational transfers appear as a consequence of the precautionary savings associated with uncertain life-spans. However, as Bernheim et al. (1985) convincingly argue, the empirical evidence suggests that the demand for annuities seems very weak even when such annuities are available on a fair basis. Thus, intergenerational altruism appears one of the most likely candidates for explaining such a substantial amount of intergenerational transfers. Of course, other explanations of the process leading to these transfers have been proposed.\(^2\) However, such alternative explanations are not mutually exclusive and the available empirical evidence is not conclusive either. In particular, bequest-motivated transfers seem to play an important role for individuals enjoying high levels of income and wealth (see Hurd, 1987).

In this context, the main point of the present paper is that the aforementioned threshold level of altruism depends on the tax structure. In particular, if taxes on labour income are high, then the economy tends to become dynastic since agents will foresee that their heirs’ disposable income will be low during youth. Conversely, the intergenerational links are broken when taxes on capital income are high, while those on labour are low, since agents are not altruistic enough to leave positive bequests in such a circumstance. Therefore, tax rates determine the regime in which the economy is operating and, hence, the desirability of taxing either capital or labour income depends on the previous selection of the regime. This means that an extensive tax reform could lead to an elimination of bequest-motivated transfers and, thus, to a reversal of the comparative statics effects of taxation on growth.

My results include an explicit formula for the altruism factor which divides the set of economies into two groups: the ones for which the growth maximis-

\(^1\) Caballé (1995a) characterises the critical level of altruism when bequest may also take the form of investment in child education.

\(^2\) Alternative explanations include the following: strategic behaviour, according to which intergenerational transfers arise as payments for child-provided services (Bernheim et al., 1985); ‘joy of giving’, which means that parents care about the size of the bequests they leave to their heirs (Yaari, 1966); or the existence of an incomplete annuity market at the family level (Kotlikoff and Spivak, 1981).

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ing policy consists of selecting a zero tax on capital income, and the ones for which the maximum growth is achieved by setting such a tax rate at the highest feasible level. The analysis is conducted for an economy displaying endogenous growth through an $A_k$ technology at the aggregate level, and with (possibly) spillover effects accruing from the average capital installed in the productive sector (see Romer, 1986).

The paper is organised as follows. Section 1 presents the model. Section 2 shows under which circumstances the economy is either dynastic or bequest constrained. Section 3 analyses the growth effects of taxing different sources of income. Section 4 concludes briefly the paper.

1. The Model
Consider an OLG economy in which individuals live for two periods and a new generation is born in each period. Each generation is composed of a continuum of identical agents and the gross rate of population growth is $n > 0$, that is, the number of children per parent is $n$. The utility derived from consumption of an agent of generation $t$ is represented by the function

$$U(c_t^1, c_{t+1}^2) = u(c_t^1) + \rho u(c_{t+1}^2), \quad \rho > 0,$$

with $u(c) = (c^{1-\gamma} - 1)/(1 - \gamma)$ and $\gamma > 0$, where $c_t^1$ and $c_{t+1}^2$ denote young and old consumption, respectively. The parameter $\gamma$ is the inverse of the (constant) elasticity of intertemporal substitution.

Agents are altruistic and derive also utility from the utility of their descendants as in Barro (1974). Therefore, the total utility of an individual of generation $t$ is

$$V_t = U(c_t^1, c_{t+1}^2) + \rho V_{t+1},$$

where the parameter $\beta > 0$ is the altruism factor and $V_{t+1}$ is the total utility of each of his heirs. We assume that parents can leave a non-negative bequest to their heirs. Such a non-negativity constraint is a consequence of both the one-sided altruism assumption and the lack of institutions or contracts to enforce liabilities on future generations. Let $\tilde{b}_t$ and $s_t$ denote the bequest received by each heir and the saving of a young agent in period $t$, respectively.

Agents work in the first period of their lives (youth) and are retired when they are old. A young agent supplies inelastically a unit of labour. The wage that a worker receives at time $t$ is $\hat{w}_t$, and the interest rate faced by an individual born at $t$ is $\hat{r}_{t+1}$. The government sets proportional taxes on both capital and labour income whose rates are $\tau_k \in [0, 1]$ and $\tau_l \in [0, 1]$, respectively. Hence, young consumption will be

$$c_t^1 = (1 - \tau_l) \hat{w}_t + \tilde{b}_t - s_t,$$

3 The utility function $U(c_t^1, c_{t+1}^2)$ has the above functional form if and only if it is twice continuously differentiable, strictly increasing, strictly concave, additively separable and homothetic (see theorem 2.4-4 in Katzner, 1970). The homotheticity of $U$ turns to be a necessary condition for the existence of a balanced growth path. Observe also that $u(c) = \ln(c)$ when $\gamma = 1$. © Royal Economic Society 1998
whereas old consumption will be

\[ c_{t+1}^2 = [1 + (1 - \tau_k) \hat{r}_{t+1}] s_t - n \hat{b}_{t+1}. \]

Therefore, each individual solves the following dynamic programming problem:

\[ V_{t}(\hat{b}_t) = \max_{s_t, \hat{b}_{t+1}} U\{ (1 - \tau_t) \hat{w}_t + \hat{b}_t - s_t, [1 + (1 - \tau_k) \hat{r}_{t+1}] s_t - n \hat{b}_{t+1} \} + \beta V_{t+1}(\hat{b}_{t+1}), \]

subject to \( \hat{b}_{t+1} \geq 0 \).

There is a productive sector with a continuum of competitive firms uniformly distributed on the interval \([0, 1]\) in which each firm has access to the same technology. We normalise the mass of individuals so that there is one firm for each \( N_t \) workers (young agents) in period \( t \). Since population grows at the gross rate \( n \), it follows that \( N_{t+1} = n N_t \) for all \( t \). Production takes place according to the gross production function \( F(K_{jt}, L_{jt}, \tilde{k}_t) \), where \( K_{jt} \) and \( L_{jt} \) denote the capital and the number of workers hired by firm \( j \in [0, 1] \) at time \( t \), respectively, and \( \tilde{k}_t \equiv \int_{[0,1]} K_{di} di/\int_{[0,1]} L_{di} di \) is the average capital-labour ratio of the economy. Therefore, the capital per worker displays a productive (or ‘learning-by-doing’) externality as in Romer (1986): each firm takes as given the value of \( \tilde{k}_t \) when deciding its demands for workers and capital. The function \( F \) is twice continuously differentiable and nondecreasing in all its arguments. The function \( F(\cdot, \cdot, \tilde{k}_t) \) defined on \( R_+^2 \) is concave and linearly homogeneous. Finally, in order to allow for balanced growth paths we assume that the single-variable function \( F(k, 1, k) \) is linear, i.e., \( F(k, 1, k) = Ak \) with \( A > 0 \).\(^4\) Let \( k_{jt} = K_{jt}/L_{jt} \) be the capital-labour ratio of firm \( j \). From the previous assumptions, it follows that the gross marginal productivity of private capital evaluated at \( \tilde{k}_t = k_{jt} \), \( \partial F(k_{jt}, 1, k_{jt})/\partial K_{jt} \), is a constant. We assume that capital depreciates at the constant rate \( \delta \in [0, \partial F(k_{jt}, 1, k_{jt})/\partial K_{jt}] \). Therefore, the net marginal productivity of capital, evaluated at \( \tilde{k}_t = k_{jt} \), is the constant \( r = [\partial F(k_{jt}, 1, k_{jt})/\partial K_{jt}] - \delta \geq 0 \). Finally, the marginal productivity of labour evaluated at \( \tilde{k}_t = k_{jt} \) is \( \partial F(k_{jt}, 1, k_{jt})/\partial L_{jt} = wk_{jt} \), where \( w = A - r - \delta \geq 0 \).

**Example 1.** If the gross production function is Cobb-Douglas,\n
\[ F(K_{jt}, L_{jt}, \tilde{k}_t) = A (K_{jt})^\alpha (L_{jt})^{1-\alpha} (\tilde{k}_t)^{1-\alpha}, \]

with \( \alpha \in [0, 1] \) and \( A > 0 \), then \( F(k, 1, k) = Ak \). Therefore, \( \partial F(k_{jt}, 1, k_{jt})/\partial K_{jt} = A \alpha \) and \( r = A \alpha - \delta \). Hence, \( w = A(1 - \alpha) \) and \( \partial F(k_{jt}, 1, k_{jt})/\partial L_{jt} = A(1 - \alpha) k_{jt} \).

The government finances the flow \( P_t \) of public spending per firm by getting revenues from proportional taxes on both capital income and wages. Thus, the government budget constraint is

\(^4\) The later assumption can be justified by assuming that the gross production function takes the form \( F(K_{jt}, L_{jt}, \tilde{k}_t) = \hat{F}(K_{jt}, L_{jt}) \), where \( L_{jt} \) are the efficiency units of labour, and \( \hat{F} \) is linearly homogeneous. The efficiency units of labour are defined as \( L_{jt} = \hat{B}_t L_{jt} \), where \( \hat{B}_t \) is a labour productivity parameter satisfying \( \hat{B}_t = \hat{b} k_t \). In this case, one sees immediately that the function \( F(k, 1, k) \equiv F(k, \hat{b} k) \) is linear.

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Since all firms are *ex-ante* equal, they will choose the same capital-labour ratio, that is, $k_j = k_t$ for all $j$ and $t$. In equilibrium the aggregate saving of young individuals must be equal to the capital installed in the productive sector, i.e.,

$$L_{t+1}k_{t+1} = N_t s_t, \quad \text{for all } t,$$

(4)

Furthermore, equilibrium requires that

$$L_t = N_t \quad \text{and} \quad k_t = \bar{k}_t, \quad \text{for all } t.$$  

(5)

Finally, perfect competition in the input markets implies that each input is paid according to its private marginal productivity, that is,

$$\hat{r}_t = r \quad \text{and} \quad \hat{w}_t = wk_t, \quad \text{for all } t.$$  

(6)

Dividing the government budget constraint (3) by $N_1k_1$, and after using (4), (5) and (6), such a constraint becomes

$$p_t = \tau_k r + \tau_l w,$$

(7)

where $p_t = P_t/N_tk_t$ is the public spending-capital ratio, which is in turn proportional to the public spending-output ratio in equilibrium. We assume that government purchases are fixed as an exogenously given proportion of the national income, i.e., $p_t = p$ for all $t$.

If an individual born at $t$ receives a bequest $\hat{b}_t$ from her parent and leaves to each of her $n$ heirs a bequest $b_{t+1}$, then her endowment in her first period of life will be $(1 - \tau_l)\hat{w}_t + \hat{b}_t$, whereas her endowment when she is old will be $-nb_{t+1}$. From the assumptions on both the utility function and the government fiscal policy, the optimal saving of such an individual must satisfy

$$s_t = \phi[(1 - \tau_k)\hat{r}_{t+1}][(1 - \tau_l)\hat{w}_t + \hat{b}_t] + \left\{\frac{1 - \phi[(1 - \tau_k)\hat{r}_{t+1}]}{1 + (1 - \tau_k)\hat{r}_{t+1}}\right\}nb_{t+1},$$

(8)

where $\phi(x) = 1 - \left[1 + \rho^{1/\gamma}(1 + x)^{(1-\gamma)/\gamma}\right]^{-1}$. It can be readily seen that $\text{sign}\left[\phi'(x)\right] = \text{sign}(1 - \gamma)$. We can combine the market equilibrium conditions (4) and (5) with the competitive payments to inputs (6) and the optimal saving (8) to obtain for all $t$,

$$N_{t+1}k_{t+1} = N_t\left(\phi[(1 - \tau_k)r][(1 - \tau_l)wk_t + \hat{b}_t] + \left\{\frac{1 - \phi[(1 - \tau_k)r]}{1 + (1 - \tau_k)r}\right\}nb_{t+1}\right).$$

(9)

After dividing both sides of (9) by $N_tk_t$, we get

$$n(1 + g_{t+1}) = \phi[(1 - \tau_k)r][(1 - \tau_l)w + b_t] + \left\{\frac{1 - \phi[(1 - \tau_k)r]}{1 + (1 - \tau_k)r}\right\}n(1 + g_{t+1})b_{t+1},$$

(10)

where $g_{t+1} = (k_{t+1}/k_t) - 1$ is the rate of growth of capital per worker, which

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obviously coincides with the rate of growth of output per capita in equilibrium; and $b_t = \hat{b}_t/k_t$ is the bequest-capital ratio, which is in turn proportional to the bequest-output ratio. Note that $\hat{b}_t = 0$ if and only if $b_t = 0$.

2. The Strength of the Bequest Motive

Applying the envelope theorem, we obtain the following first order conditions for a solution to problem (2) evaluated at equilibrium:

$$\beta(c_{t+1})^{-\gamma} = \rho[1 + (1 - \tau_k)r](c_{t+1})^{-\gamma}, \quad (11)$$

$$\hat{b}_{t+1}[\beta(c_{t+1})^{-\gamma} - \rho n(c_{t+1})^{-\gamma}] = 0, \quad \text{with } \hat{b}_{t+1} \geq 0 \text{ and } \beta(c_{t+1})^{-\gamma} \leq \rho n(c_{t+1})^{-\gamma}. \quad (12)$$

Equation (11) is the typical first order condition which gives the optimal allocation of consumption along the life cycle, whereas condition (12) refers to the optimal intergenerational transfers. The latter condition becomes

$$\beta(c_{t+1})^{-\gamma} = \rho n(c_{t+1})^{-\gamma}, \quad (13)$$

when $\hat{b}_{t+1} > 0$, which means that an optimal amount of positive bequest must equalise the marginal utilities of parents and sons. If $\beta(c_{t+1})^{-\gamma} < \rho n(c_{t+1})^{-\gamma}$, then $\hat{b}_{t+1} = 0$, and the economy is bequest constrained since the optimal bequest is given by a corner solution.

Using (7), the capital market equilibrium condition (10) becomes

$$n(1 + g_{t+1})$$

$$= \varphi[(1 - \tau_k)r](w - p + \tau_k r + b_t) + \left\{ \frac{1 - \varphi[(1 - \tau_k)r]}{1 + (1 - \tau_k)r} \right\} n(1 + g_{t+1})b_t. \quad (14)$$

On a balanced growth path (BGP), both young and old consumption, the bequest $\hat{b}_t$, and the capital-labour ratio $k_t$ grow at the same constant rate $g$ for all $t$. This means that the ratio $b_t = \hat{b}_t/k_t$ is constant at a BGP, i.e., $b_t = b_t$. Moreover, when the economy is bequest constrained ($b_t = 0$ for all $t$), the rate of growth $g_{t+1}$ displays no transition and is always equal to

$$g_c(\tau_k) = \frac{\varphi[(1 - \tau_k)r](w - p + \tau_k r)}{n} - 1, \quad (15)$$

as follows from solving $g_{t+1}$ in (14). Similarly, when the bequest motive is operative ($b_t > 0$ for all $t$), we can divide (11) by (13) to find that the rate of growth of $c_t$ is

$$g_a(\tau_k) = \left\{ \frac{\beta[1 + (1 - \tau_k)r]}{n} \right\}^{1/\gamma} - 1, \quad (16)$$

for all $t$. As follows from (11), $g_a(\tau_k)$ is also the rate of growth of $c_t$, which in turn implies that the saving, the capital-labour ratio $k_t$, and the output per capita grow without transition at the rate $g_a(\tau_k)$. Moreover, the bequest-capital
ratio \( b_t \) is then also constant. Summing up, regardless of the regime in which the economy is operating (either bequest constrained or unconstrained) the economy is always at a BGP. This implies that, if a change in fiscal policy affects the stationary values of both the growth rate \( g \) and the bequest-capital ratio \( b \), the adjustment to the new BGP takes place in just one period without transition.

When the economy is bequest unconstrained, we must also impose the transversality condition \( \beta [1 + g_u (\tau_k)]^{1-\gamma} < 1 \) which makes the utility of the dynasties bounded above. Using (16), such a condition becomes

\[
\beta < \left[ \frac{n}{1 + (1 - \tau_k) r} \right]^{1-\gamma}. \tag{17}
\]

For a given tax rate \( \tau_k \) on capital income, define \( \bar{\beta}(\tau_k) \) as the level of altruism which makes the non-negativity constraint on bequests just binding in equilibrium. In other words, if \( \beta = \bar{\beta}(\tau_k) \), then \( b_t = 0 \) for all \( t \), and the equilibrium consumption satisfies (13),

\[
\bar{\beta}(\tau_k)(a_{t+1}^1)^{-\gamma} = \rho n(c_{t+1}^2)^{-\gamma}. \tag{18}
\]

Dividing (18) by (11), we obtain

\[
\frac{\bar{\beta}(\tau_k)(a_{t+1}^1)^{-\gamma}}{(c_t^1)^{-\gamma}} = \frac{n}{1 + (1 - \tau_k) r}. \tag{19}
\]

Solving for \( \bar{\beta}(\tau_k) \) in (19), we get

\[
\bar{\beta}(\tau_k) = \frac{n(1 + \bar{g})^\gamma}{1 + (1 - \tau_k) r}. \tag{20}
\]

where \( \bar{g} \) is the rate of growth associated with the altruism factor \( \bar{\beta}(\tau_k) \). Moreover, the growth rate \( \bar{g} \) also satisfies (15) since \( b_t = 0 \) for all \( t \). Thus, the expression (20) can be rewritten as

\[
\bar{\beta}(\tau_k) = \frac{n^{1-\gamma} \{ \phi[(1 - \tau_k) r] (w - \bar{b} + \tau_k r) \}^\gamma}{1 + (1 - \tau_k) r}. \tag{21}
\]

The next proposition shows that \( \bar{\beta}(\tau_k) \) is the threshold level of the altruism factor above (below) which the bequest motive is (is not) operative.

**Proposition 1**  
(a) If \( \beta \leq \bar{\beta}(\tau_k) \), then \( b = 0 \).  
(b) If \( \beta > \bar{\beta}(\tau_k) \), then \( b > 0 \).

**Proof** (a) We will proceed by contradiction. Assume that \( b > 0 \) when the altruism factor is \( \beta \), with \( \beta \leq \bar{\beta}(\tau_k) \). Then, the rate of growth \( g \) will be given by (16). On the other hand, the rate of growth associated with the altruism factor \( \bar{\beta}(\tau_k) \) is, by assumption,

\[
\bar{g} = \left\{ \frac{\bar{\beta}(\tau_k) [1 + (1 - \tau_k) r]}{n} \right\}^{1/\gamma} - 1, \tag{22}
\]

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as follows from solving for $\bar{g}$ in (20). Clearly $g_u(\tau_k) \leq \bar{g}$ since $\beta \leq \overline{\beta}(\tau_k)$. Therefore,

$$n[1 + g_u(\tau_k)] \leq n(1 + \bar{g}) = \phi[(1 - \tau_k)r](w - p + \tau_k r) <$$

\[
\phi[(1 - \tau_k)r](w - p + \tau_k r + b) + \left\{ \frac{1 - \phi[(1 - \tau_k)r]}{1 + (1 - \tau_k)r} \right\} n[1 + g_u(\tau_k)]b,
\]

where the equality is just the stationary capital market equilibrium equation (14) when the altruism level is $\overline{\beta}(\tau_k)$, and thus $b = 0$; and the last strict inequality is a consequence of the fact that saving is increasing in first period endowment and decreasing in second period endowment. Therefore, the equilibrium condition (14) is violated on a BGP when the altruism factor is $\beta$ and $b > 0$.

(b) (By contradiction). If $b = 0$, the market clearing condition (14) on a BGP becomes

$$n[1 + g_c(\tau_k)] = \phi[(1 - \tau_k)r](w - p + \tau_k r). \tag{23}$$

On the other hand, if the altruism factor is $\overline{\beta}(\tau_k)$, then $b = 0$ and the corresponding market clearing condition is

$$n(1 + \bar{g}) = \phi[(1 - \tau_k)r](w - p + \tau_k r). \tag{24}$$

Equations (23) and (24) imply that $g_c(\tau_k) = \bar{g}$. However, combining (11) with (12), we obtain

$$g_c(\tau_k) \geq \left\{ \frac{\beta[1 + (1 - \tau_k)r]}{n} \right\}^{1/\gamma} - 1. \tag{25}$$

Note then that (25) readily implies that the rate of growth $\bar{g}$ given in (22), which corresponds to the altruism factor $\overline{\beta}(\tau_k)$, is strictly lower than $g_c(\tau_k)$ since $\beta > \overline{\beta}(\tau_k)$. Thus, this contradicts our previous result which established that $g_c(\tau_k) = \bar{g}$.

### 3. Growth and Taxes

In this section, we will assume that the saving function, $\phi[(1 - \tau_k)r] \times (w - p + \tau_k r)$, is increasing in $\tau_k$. Obviously, when the parameter $\gamma$ (the inverse of the elasticity of substitution) is greater or equal than 1, saving is increasing in the capital income tax. This is so because an increase in capital income taxes allows a relief on labour income taxation, which in turn allows workers to enjoy more disposable income, $w - p + \tau_k r$. On the other hand, the propensity to save out of young income, $\phi(\cdot)$, is a nonincreasing function if and only if $\gamma \geq 1$, and thus $\phi[(1 - \tau_k)r]$ is nondecreasing in $\tau_k$. Moreover, even when $\gamma < 1$, saving may be increasing in the capital income tax if the elasticity of intertemporal substitution is not too high: we just need an income effect that dominates the substitution effect. In this respect we should mention that most estimates of $\gamma$ found in the literature are either significantly above

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one or not significantly below one (see Hall, 1988; Blinder, 1981; Bosworth and Burtless, 1992; among many others).

Therefore, when saving is increasing in $\tau_k$, the threshold level of altruism $\beta(\tau_k)$ is also increasing in $\tau_k$, as can be readily seen from (21). The following proposition provides the fiscal policies that implement the highest feasible growth rate depending on the altruism factor $\beta$:

**PROPOSITION 2** Let $\beta \equiv n^{1-\gamma}[\phi(0)(A-\delta-p)]^{\gamma}/(1+r)$, where $\phi(0) = \rho^{1/\gamma}/(1+\rho^{1/\gamma})$, and assume that saving is increasing in $\tau_k$. Then, if $\beta > \beta(1)$, the fiscal policy which maximises economic growth consists of a zero tax rate on capital income. On the other hand, if $\beta < \beta$, the growth maximising tax system implies the selection of $\tau_k = 1$.

Proof. First, observe that $\beta$ is also equal to $n^{1-\gamma}[\phi(0)(w+r-p)]^{\gamma}/(1+r)$ since $w + r = A - \delta$, as follows from the assumptions on the production function made in Section 1. Note also that $\tilde{\beta}(1) \geq \beta \geq \tilde{\beta}(0)$, where

$$\tilde{\beta}(1) = n^{1-\gamma}[\phi(0)(w+r-p)]^{\gamma}$$

and

$$\tilde{\beta}(0) = \frac{n^{1-\gamma}[\phi(r)(w-p)]^{\gamma}}{1+r},$$

as can be seen from (21) and because saving is increasing in $\tau_k$ and $r \geq 0$. To prove the proposition we consider the following three possible cases:

(i) $\beta > \beta(1)$. In this case the economy is always bequest unconstrained $(b > 0)$ for all $\tau_k \in [0, 1]$ since $\tilde{\beta}(\tau_k)$ is an increasing function. Then, the rate of growth is given by (16), which is clearly decreasing in $\tau_k$. This proves the optimality of $\tau_k = 0$, from the growth viewpoint.

(ii) $\beta(0) > \beta$. Here the economy is always bequest constrained for all the feasible tax rate levels. As a consequence, the rate of growth is given by (15), which is increasing in $\tau_k$ when saving is increasing in $\tau_k$. Therefore, selecting $\tau_k = 1$ is the growth maximising policy.

(iii) $\beta(1) \geq \beta \geq \beta(0)$. In this case the economy is bequest constrained when $\tau_k = 1$, whereas it remains unconstrained when $\tau_k = 0$. Hence, by virtue of Proposition 1 and expressions (15) and (16), we just have to compare the growth rates associated with these two extreme policies. When $\tau_k = 1$, the rate of growth is, according to (15),

$$g_e(1) = \frac{\phi(0)(w+r-p)}{n} - 1. \quad (26)$$

On the other hand, if $\tau_k = 0$, the rate of growth is, according to (16),

$$g_u(0) = \left[\frac{\beta(1+r)}{n}\right]^{1/\gamma} - 1.$$
It is then straightforward to check that \( g_a(0) > (<) g_a(1) \) if and only if \( \beta > (<) \hat{\beta} \).

The previous proposition tells us that for high values of the altruism factor \( \beta \), the classical policy of zero taxes on capital income is the one that delivers faster economic growth. This policy implies that the economy is bequest unconstrained and, therefore, behaves like a dynastic economy with infinite horizon. In this setup, the policy of zero taxes on capital income is also the most desirable in terms of dynamic efficiency, as has been argued by Chamley (1986) and Lucas (1990).

However, for low levels of the altruism factor \( \beta \), the policy leading to faster growth consists of taxing the income accruing from the return on saving at the highest feasible rate. In this second setup, the economy behaves like the standard OLG model with production (Diamond, 1965). Note that such a policy means that the retired agents pay more taxes and workers benefit from a tax relief. As Jones and Manuelli (1992) argue, such a ‘reverse social security’ scheme is growth enhancing since it provides workers with additional disposable income, which enables them to buy capital at an increasing pace.

Since \( \hat{\beta} \) is clearly decreasing in the equilibrium interest rate \( r \), we can also conclude that an economy that displays large enough externalities (\( r \) is low for a given value of \( A - \delta \)) will exhibit a growth rate which is increasing in the capital income tax rate \( \tau_k \).

**Example 2** Assume that the economy has a Cobb-Douglas technology as in Example 1, and that preferences are logarithmic (\( \gamma = 1 \)). Then,

\[
\hat{\beta} = \frac{\rho(A - \delta - \rho)}{(1 + \rho)(1 + \Lambda \alpha - \delta)},
\]

and Fig. 1 displays the combinations of the technology parameter \( \alpha \) and the altruism factor \( \beta \) for which the two extreme policies \( \tau_k = 1 \) and \( \tau_k = 0 \) are growth maximising. Note that in this case the transversality condition (17) becomes simply \( \hat{\beta} < 1 \).

4. Conclusion

It should be remarked that this paper only contains a positive analysis of the growth effects of proportional taxation. Although a normative analysis, including the design of an optimal fiscal policy package, lies outside of the main focus of the paper, we can make some comments about efficiency issues.

The model of this paper has two potential sources of inefficiency: the externality from capital and the typical dynamic inefficiency of OLG models due to capital overaccumulation. The inefficiency caused by the spillover effects can be solved by means of subsidies on capital acquisition so as to internalise the external effects that are not taken into account by the firms when they determine their demands for capital. On the other hand, the problem of capital overaccumulation may appear in bequest constrained economies (see Weil, 1987), and its solution would involve intergenerational

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lump-sum transfers from young to old agents which will reduce the equilibrium saving. Of course, the exact amount of such transfers would depend on the weights assigned to each generation in the social planner objective function.

The model we have considered is extremely simple so that there is room for many extensions. For instance, we have disregarded the role of public spending as a productive input since we have concentrated the analysis on the effects of different financing policies. Introducing productive public spending will generate a new relation between growth rates and the public spending-output ratio. This new relation is typically inverted U-shaped (see Barro, 1990).

There are also alternative ways of allowing for sustained growth in a perfectly competitive economy, like, for instance, through the accumulation of human capital (as in Caballé and Santos, 1993). The main results of the present paper carry over to the BGP of such a two-sector model. However, the equilibrium path would exhibit a non-instantaneous transition towards its steady state. Therefore, an analysis of the effects of fiscal policy on transitional dynamics could be undertaken within that more complex framework.

A final extension would be to allow for fiscal deficits so that public spending could be financed by means of issuing public debt. Note that, when the policy leading to faster growth involves zero taxes on capital income, government deficits are neutral since the economy is bequest unconstrained and the Ricardian equivalence proposition will hold in such a case (see Barro, 1974). However, the introduction of public debt is not neutral when the economy is bequest constrained. The introduction of public debt allows in principle a relief on labour taxes, which in turn would allow an increase in the speed of capital accumulation. However, it can be easily shown (see Caballé, 1995b)
that, when the economy is bequest constrained and the elasticity of intertemporal substitution is smaller than 1, the package of fiscal policies that maximises the rate of economic growth, for a given feasible level of public spending, consists of running a balanced budget in all periods and taxing the capital income at the highest feasible rate. Public debt reduces the growth rate since it directly absorbs resources which could have been used to acquire productive capital. Therefore, the negative effect of government debt on the capital level found in Diamond (1965) carries over to the growth rate in the model considered in this paper.

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