Do aspirations reduce differences in wealth accumulation?*

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Abstract

We propose aspirations as a mechanism that allows reconciling the standard theoretical result stating that altruistic individuals accumulate more wealth than non-altruistic individuals with the empirical evidence showing that bequest motivated and non-bequest motivated individuals behave similarly concerning wealth accumulation. In particular, we show that although introducing aspirations at different ages displays different effects on wealth accumulation, both adult and old aspirations reduce the positive effect on wealth accumulation brought about by the bequest motive. Thus, bequest and non-bequest motivated individuals exhibit similar patterns of wealth accumulation, which also results in a decrease in the inequality caused by bequests. As a by-product of our analysis, we show that the introduction of aspirations raises the speed of convergence to the dynastic steady state.

JEL classification codes: D31, D9, E21
Keywords: Aspirations, Saving, Bequests, Wealth accumulation.

*Financial support from the Government of Spain and FEDER (grants PGC2018-094364-B-I00 and PID2019-111765GB-I00), the Generalitat of Catalonia (grant 2017-SGR-1765) and the Andalucía Government (grant PY18-4115) is gratefully acknowledged. The paper has benefited from comments by participants in the SAEe meeting (Palma de Mallorca), the Seventh Meeting of the Society for the Study of Economic Inequality (ECINEQ, New York), the XI Workshop on Public Policy Design (Girona) and two anonymous referees.
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1 Introduction

This paper analyzes the effects of aspirations on the difference in the pattern of wealth accumulation along the life cycle between individuals displaying a "joy-of-giving" motive for bequests and non-bequest motivated individuals. Specifically, we attempt to reconcile the theoretical models with empirical evidence regarding differences on wealth accumulation of altruistic and non-altruistic individuals. Following Hurd's (1987) work, several models indicate that the amount of wealth accumulation in all life stages should be higher for altruistic individuals than for non-altruistic ones as altruistic individuals wish to leave a bequest to their heirs, and this desire provides additional motivation for saving. This theoretical implication contradicts the empirical evidence suggesting that the patterns of saving and consumption and therefore of wealth accumulation of altruistic individuals do not differ substantially from non-altruistic individuals. Our analysis also contributes to the literature on wealth inequality. As there is non-conclusive evidence on the effect of bequests on wealth inequality, aspirations could reduce wealth inequality between altruistic and non-altruistic individuals as they behave more similar in terms of wealth accumulation.

A new feature introduced through our analysis is intergenerational transmission of preferences jointly with intergenerational transmission of wealth through be¬quests. The simplest way of generating intergenerational transmission of tastes is by introducing aspirations, that is, by assuming that an individual's utility depends on the comparison between his current amount of consumption and that of his parents. Effectively, the amount of consumption by the parents determines the standard of living concerning the consumption of their heirs (Becker, 1992; de la Croix, 1996; de la Croix and Michel, 1999, 2001; Alonso-Carrera et al., 2007; Caballe and Moro-Egido, 2014). We are not the first to introduce non-separable preferences to explain mismatches between the theoretical results concerning wealth accumulation and the corresponding empirical findings. However, to our knowledge, we are first in jointly considering aspirations and bequest motives to reconcile differences in the pattern of wealth accumulation along the life cycle between bequest and non-bequest motivated individuals.

The relevance of our analysis relies on the fact that bequests are viewed as one of the main factors of wealth accumulation (from Kotlikoff, 1988, to Palomino et al., 2020) and the persistence of high levels of wealth concentration (de Nardi, 2004;de Nardi and Fella, 2017).[1] Further, the characteristics of bequests within families are relevant for public policy as its effectiveness depends on the precise motives for intra-family transfers of income. Moreover, accounting for the links between the past and present has implications for the analysis of many microeconomic and macroeconomic problems. On the one hand, including the different ways the past influences current preferences may explain why parents attempt to shape their children's preferences (Becker, 1992). On the other hand, incorporating past experiences explains the existence of fluctuations in both output and employment (de la Croix, 2001). Finally, if the introduction of aspirations lowers the impact of bequests on the distribution of wealth between altruistic and non-altruistic individuals, then the relevance of inheritance taxes as an instrument for wealth

[1]While bequests may arise accidentally due to uncertain life spans, economists have mainly focused on models with voluntary bequests.

redistribution diminishes considerably.

We analyze theoretically whether including inherited tastes or aspirations could minimize the differences in the pattern of wealth accumulation along the life cycle between bequest and non-bequest motivated individuals. Some clarifications about our approach are required. First, our analysis will be conducted using the framework of an overlapping generations (OLG) economy where individuals display preferences of joy-of-giving altruism. This means that the utility of individuals will be an increasing function of the amount of bequest they leave to their children, like in Yaari (1965), Abel (1986), and Andreoni (1989).[2] Following the related literature, starting with Hurd (1987), we will identify those exhibiting altruism as those having offspring, whereas individuals with no children will be viewed as non-altruistic. As our main focus is on wealth variations along the life cycle rather than on wealth levels, we disregard the costs of child-rearing. Thus, we will assume that the amount of bequest collects all the intergenerational transfers taking place within a family and that the intensity of aspirations is given when individuals take decisions, such that aspirations are not endogenously determined.

We carry out our analysis for both the individual wealth accumulation choice and the steady-state allocation of wealth. In both cases, we show that introducing aspirations dampens the positive effect on wealth accumulation brought about by the bequest motive. Therefore, if aspirational concerns are present, bequest and non-bequest motivated individuals will exhibit more similar patterns of wealth accumulation at all life stages. This theoretical result explains the empirical evidence regarding the lack of substantial dif¬ference between the patterns of saving and consumption of bequest and non-bequest motivated individuals. Surprisingly, this dampening effect occurs even if the introduction of aspirations at different life stages does not have the same effect on wealth accumulation. Particularly, the presence of aspirations at an adult age results in a smaller amount of saving as adults want to mimic the consumption of their parents when they were also working. However, the introduction of aspirations at an old age raises the amount of saving of workers as they want to shift consumption to the age where they will be retired so as to replicate the standard of living of their retired parents.

Another interesting collateral result is that aspirations raise the speed of convergence to the dynastic steady state when individuals entertain a bequest motive. This is because aspirations introduce a sluggish response in bequests, which are the main factor giving rise to history dependence within a dynasty. As individuals want to mimic the consumption level of their parents, bequests cannot adjust with the same freedom as when aspirations were absent. Moreover, as in de la Croix (1996, 2001) and Fanti (2018, 2019), we can generate endogenous fluctuations with a sufficiently high intensity of aspirations.

Our paper is organized as follows: Section 2 presents relevant literature regarding bequests and aspirations, Section 3 introduces the general model with a bequest motive and aspirations, Section 4 analyzes the effect of both aspirations and the bequest motive on saving and inheritance, Section 5 analyzes the effect of aspirations on the transitional dynamics to the

[2]Several alternative motives leading to intergenerational transfers have been proposed. Among them and alongside joy-of-giving, we could mention strategic behavior (Bernheim et al., 1985), existence of incomplete annuity markets (Abel, 1985), and pure intergenerational altruism (Barro, 1974). However, the empirical evidence is not conclusive about why individuals make intergenerational transfers and the mechanism of intergenerational transmission of wealth is probably driven by a combination of motives.

steady-state equilibrium, Section 6 and 7 conduct comparative statics analysis to characterize the effects of changes in the intensities of aspirations and the bequest motive for an individual with exogenously given initial values of both aspirations and inheritance, Section 8 makes the comparison in terms of stationary equilibria, and Section 9 provides the conclusion. Finally, proofs, derivations of several mathematical expressions, and other additional material appear in the Appendix.

2 Related literature

In our model, the two features that drive intergenerational links are the transfers of wealth and tastes, modeled as bequests and aspirations, respectively.

2.1 Bequests

Studies on inheritance and wealth transmission generally fall into three broad categories tackling the importance of inheritance for wealth accumulation, wealth inequality, and intergenerational mobility in wealth. The literature, summarized in Davies and Shorrocks (2000), originated in the 1980s (Kotlikof and Summers, 1981; Modigliani, 1988; Laitner and Juster, 1996) and focuses on the contribution of bequests to aggregate wealth (or capital stock). In general, the findings are that inheritances account for 25%–50% of personal wealth. However, the results are sensitive to the way in which personal wealth is measured. Davies and Shorrocks (2000) concluded that the most reasonable estimate for the impact of inheritances ranges around 35%–45%. Similarly, Boserup et al. (2016) and Karagiannaki (2017) found that 27% of households older than 25 years in the UK received an inheritance.[3]

The evidence on the importance of inheritance for wealth inequality is inconclusive. Karagiannaki (2017) argues that bequests are a small fraction of overall wealth inequality. Boserup et al. (2016) and Elinder et al. (2018) found that inheritance reduce relative inequality, even if it increases the absolute dispersion of wealth. Fessler and Schiirz (2018) have shown that receiving an inheritance at any point lifts a household an average of 14 percentiles in wealth distribution. More recently, Palomino et al. (2020) have shown that intergenerational transfers contribute up to 30%, 26%, 24%, and 23% for France, the United States (US), Spain, and Britain, respectively. Considering the net effect of inheritance and social background and their joint interaction, the contribution to wealth inequality amounts to 48%, 46%, 44%, and 36% for the US, Spain, France, and Britain, respectively. As concluded by the authors, removing the differences in wealth associated with transfer receipts and parental background would account for nearly half of wealth inequality in some countries and more than one-third in all of them. De Nardi (2004) and de Nardi and Fella (2017) found that voluntary bequests can explain the concentration of wealth, while accidental bequests cannot do so and that adding the transmission of earnings ability from parents to children generates an even more concentrated wealth distribution. They also found that saving for precautionary purposes and retirement is

[3]Piketty and Zucman (2015) argued that the importance of inter-vivos transfers or gifts, as a fraction of total inheritances, has increased dramatically during the last 40 years, from about 20%–30% during the 1970s, to 40% in the 1980s, to 60% in the 1990s, and finally amounting to over 80% in the first decade of the 2000s.

the primary cause of wealth accumulation at the lower tail of the distribution, while saving for bequests significantly affects the upper tail.

Intergenerational wealth transfers are thus likely to be integral in determining the wealth accumulation process. However, whether such transfers enhance or reduce wealth inequality remains unresolved as different studies have different conclusions. However, this is an important issue for policy makers when tackling inequality. In this respect, this paper studies the effect of aspirations not only on wealth accumulation but also on wealth inequality between altruistic and non-altruistic individuals.

There is limited evidence available on intergenerational mobility of wealth, and the importance of inheritances in this process is largely unresolved. At the theoretical level, Alonso-Carrera et al. (2020) explored the role of borrowing constraints on intergenerational mobility of wealth. They found that not only the initial distribution of wealth, but also the distribution of the composition of wealth between bequests (i.e., initial physical wealth) and human capital, are important to characterize the degree of intergenerational mobility in socioeconomic status. Pfeffer and Killewald (2015) show that the main transmission of wealth occurs early in life through education rather than by bequests and inter-vivos transfers. However, Adermon et al. (2018) found that bequests and gifts are central in explaining intergenerational wealth correlations as they account for roughly half of the parent–child wealth correlation, whereas earnings and education may account for only a quarter.

A different strand of the literature has analyzed the mechanism behind the bequests motive. Altruism is the classical motive, as defined in Barro (1974) or Becker (1981): parents leave bequests because they earn utility from the economic resources of their children. Accidental bequests happen when old agents do not manage their wealth adequately and leave bequests unintentionally (see Hurd, 1987; Kopczuk and Lupton, 2007). Among others, Yaari (1965), Abel (1986), and Andreoni (1989) consider a joy-of-giving motive: parents get utility from the quantity bequeathed to their children, not from the amount the children actually receive or consume. Bernheim et al. (1985) found that children pay more attention to parents with bequeathable wealth. Evidence is not conclusive on the specific mechanism to explain intergenerational transfers. This paper selects the joy-of-giving motivation.

Finally, regarding how to model bequest motive, the standard procedure relies on the inclusion of a parameter that measures the intensity of altruism. However, other authors, such as Hurd (1987), classify those without living children as non-altruistic. Posterior empirical research has followed Hurd's suggested classification criterion. For instance, Haider et al. (2000) computed wealth differences associated with the variation in household characteristics such as marital status, level of retirement income, age, education level, and number of children. They found that households with children hardly behaved differently regarding savings than households without children. Similarly, Jiirges (2001) used data from West Germany to find a heterogeneity in the estimated wealth trajectories along the life cycle, especially when households are classified according to their declared bequest intentions. This result suggests that the bequest motive is relevant for Germany. However, when households are classified over whether they have children or not, differences in the age-wealth profiles diminish. Villanueva et al. (2005) aimed at detecting an operative bequest motive by comparing wealth accumulation across households with and without children. They found that in the three countries under consideration (Germany, the UK, and the US), the relationship between lifetime income of an individual and expected bequests does not depend on having children. Therefore, the marginal propensity to bequeath of individuals without children is very similar to that of individuals with at least one adult child. Moreover, Kopczuk and Lupton (2007) stated that although having children is not a definitive predictor of having a bequest motive, it is still a useful indicator, which provides support to Hurd's approach. These authors also showed that a significant portion of elderly households, either with or without children, exhibit a similar saving behavior. Related evidence regarding bequest intentions can be found in Blinder et al. (1990), who documented a positive but weak effect of the number of children on an estimate of planned bequests. Finally, Kazarosian (1997) reported a non-significant influence of a bequest intention dummy on the wealth to permanent income ratio. However, Scholz and Seshadri (2007) detected a negative relationship between the wealth level and the number of children in households. This is mainly because of the child-rearing costs. Thus, to properly adjust our theoretical model with the empirical evidence, we classify altruistic individuals as those having children.

2.2 Aspirations

In most studies about intergenerational linkages, parents' roles in the formation of their children's future resource capacity has mainly focused on human capital transmission and wealth transfers. However, recent developments on formation and evolution of preferences suggest that the parental influence on the status of children goes beyond educational investment and inheritance. According to Becker (1998), preferences should broadly account for the formation of personal and/or social capital. Therefore, preferences should become non-separable across time and households. Concerning personal capital, each agent's own history influences their tastes and decisions. This effect is often referred to as "(intrinsic) habit formation." Among others, Carroll and Weil (1994), de la Croix and Michel (2001), Diaz et al. (2003), Alonso-Carrera et al. (2007), Caballe and Moro-Egido (2014), and Dalton et al. (2016) have analyzed the macroeconomic implications of habits.

Concerning social capital, the history of the society or social group that agents belong to influences their future tastes. In this case, individual preferences typically depend on the average consumption of the community or the overall economy. This is often referred to as "envy" (Varian, 1974), "catching up with the Joneses" (Abel, 1990), "keeping up with the Joneses" (Gali, 1994), "status" (Corneo and Jeanne, 2001), or "consumption externalities" (Liu and Turnovsky, 2005). At the theoretical level, these consumption externalities rationalize several departures from the predictions of the standard paradigm that assumes preferences are separable across households. Abel (1990) and Gali (1994) rely on interpersonal comparisons to account for the excess return on equity. Carroll et al. (2000) explores the implications of relative consumption for the process of capital accumulation. More¬over, a strand in the literature has studied the implications of aspirations on inequality, with special attention to poverty traps and aspiration failure (Ray, 2006; Dalton et al., 2016; Allen and Chakraborty, 2018). Genicot and Ray (2020) provide a review of the formation of social aspirations, the aspirations failure, and the implications of aspirations for individual decision-making. Maurer and Meier (2008) and Alvarez-

Cuadrado et al. (2016) estimated the relative importance of interdependence across both time and households. Their results provided strong support for preference specifications allowing for both types of non-separability. They reported two main findings. First, much of the co-movement of individual consumption within groups arises from correlated effects. Second, once they control for these effects, they still find substantial evidence of consumption externalities.

As de la Croix and Michel (1999) pointed out, a third type of habit naturally emerges; a habit within the family. According to this, children get accustomed to a certain standard of living while with their parents. This experience serves as a benchmark to evaluate the level of their own current consumption, once they become adults and work. Becker (1992) noticed that "the habits acquired as a child or young adult generally continue to influence behavior even when the environment changes radically." Similarly, Jellal and Wolf (2002) pointed out that besides human capital transmission and wealth transfers, another channel of parental transmission refers to the connection between childhood experiences and future behavior. Therefore, this literature relies on the fact that preferences, norms, and cultural attitudes are partly formed as the result of heritable genetic traits (see Heckman, 2006) and partly transmitted by a learning and socialization process or the imitation of role models (see Bisin and Verdier, 2011, for a survey of the literature).

Several papers have analyzed the economic implications of aspirations. For instance, de la Croix (1996) and de la Croix and Michel (1999) found that habit formation in consumption, which can (involuntarily) be passed across generations, may generate endogenous oscillations in a simple dynamic general equilibrium model. Alonso-Carrera et al. (2007) showed that an increase in the intensity of taste inheritance results in an increase in both savings and bequests in the steady state. These authors restrict the analysis to the case where the steady-state equilibrium is unique and saddle-path stable. Bethencourt and Kunze (2017) showed not only similar results but also that the ratio of inheritance to GDP may follow either an increasing or a U-shaped pattern throughout the transition toward the steady state and that the relationship between the size of an unfunded social security system and the long-run stock of per capita capital is non-linear (U-shaped). These results are consistent with those of Piketty (2011) and Piketty and Zucman (2015). Caballe and Moro-Egido (2014), among others, have studied the macroeconomic implications of aspirations and proved that the marginal introduction of aspirations reduces the value of intergenerational correlation of wealth such that the degree of mobility in wealth increases. Gori and Michetti (2016) and Kaneko et al. (2016) showed that aspirations may explain the declining fertility rate in developed economies. Galor and Oezak (2016) provided a well-documented example that inherited tastes do have persistent effects, affecting health, education, and savings of future generations. Lastly, Fanti et al. (2018, 2019) explained the existence and persistence of oscillations for the general class of utility with aspirations used by de la Croix and Michel (1999).

The empirical evidence on the existence of aspirations, which are associated with the involuntary transmission of consumption tastes across generations, is scarce. For instance, Mulligan (1998) showed that the intergenerational elasticity of consumption was roughly 7%–8%, while that of income was 6%–7% and that of earnings was 5%. Charles et al. (2014) found that intergenerational correlation in consumption ranges around 7%–9%. Chen and Cheung (2014) stated that a 1% increase in parental food consumption leads to an increase in offspring food consumption of about 0.58%–0.73%. Similarly, Bruze (2018) showed that the persistence of consumption across generations is higher than that of income and earnings. These findings imply the possibility that intergenerational linkages might take place through channels different from income. Moreover, Cavalli-Sforza and Feldman (1981) and Boyd and Richerson (1985) provided surveys regarding the evidence on intergenerational transmission of tastes. Waldkirch et al. (2004) estimated that parental preferences explain 5%–10% of the children's preferences after controlling for their respective incomes. Alvarez-Cuadrado et al. (2016) considered external and internal reference points, that is, the level of satisfaction derived from a given bundle of consumption depends not only on the consumption bundle itself but also on how it compares to the bundle of consumption of some reference group or the agent's own past consumption bundle. Their estimates suggest that households derive one-third of satisfaction from comparing their current and past consumptions and another third from comparing their consumption to their neighbors', with the final third being determined by current consumption choices.

Further, we mention two additional forms of aspirations. First, aspirations that parents have on their children's outcomes influence how they make decisions to fulfill them (Besley, 2017). This could work either through parents shaping their children's preferences or through parents' strategic investments. Secondly, Camacho et al. (2020) model positional bequest concerns such that individuals form aspirations on the number of bequests left, not on consumption.

3 The Model

3.1 The households

Let us consider a small, open OLG economy populated by a continuum of individuals, who live for three periods and where a new generation is born in each period. In our model, fertility is modeled as an exogenous shock: individuals of each generation are altruistic with probability π and they have an exogenous number $n \geq 1$ of children. With probability $1 - \pi$ individuals are non-altruistic with no children. We assume that these altruism/fertility shocks are identically and independently distributed across periods and dynasties. Therefore, in each period of this large economy, an exogenous fraction $\pi \in (0, 1)$ of individuals have offsprings in the second period of their life and a fraction $1 - \pi$ of individuals do not have children. Therefore, the rate of population growth is $n\pi > 0$. We follow the approach of Hurd (1987) and other authors by assuming that individuals with children are altruistic toward their children, while agents who have no offsprings are non-altruistic in our model[4] We use the superindex A to denote an altruistic agent and N for a non-altruistic individual. We assume that agents make economic decisions only in the last two periods of their lives. During the first period, individuals neither work nor consume but only observe the consumption of their parents. Each agent works and inelastically supplies one unit of labor in the second period of life (adult age) and retires in the

[4]There are some other models in which bequests are modeled as luxury goods (de Nardi, 2004) so the bequest motive is only active for rich individuals. Here, consumption and savings are not affected by the bequest motive for most parents. However, we choose Hurd's classification criterion to replicate the corresponding empirical evidence.

third period (old age). We index each generation by the period in which its members work, i.e., when they are adult.

There is a single commodity, which can be devoted to either consumption or saving. An adult of generation t and type i divides net labor income and inheritance between consumption and saving. The budget constraint faced in period t is

$$c_t^i + s_t^i = w_t + b_t, \qquad i = A, N, \tag{3.1}$$

where c_t^i is the amount of consumption (hereinafter, adult consumption), s_t^i is the amount of saving, w_t is the wage received and b_t is the amount of inheritance received from altruistic parents.

When individuals become old, they receive a return on their savings, which is divided between consumption and bequests for their children. Therefore, the budget constraint of an old altruistic individual belonging to generation t will be

$$R_{t+1} s_t^A = x_{t+1}^A + n b_{t+1}, \tag{3.2}$$

where R_{t+1} is the gross rate of return on saving, x_{t+1}^i is the amount of consumption of an old individual in period $t+1$ (hereinafter, old consumption) and b_{t+1} is the amount of bequest an individual leaves to each descendant (born in period t). Thus, we are implicitly making an equal-treatment assumption as all the direct descendants of the same altruistic individual receive the same amount of inheritance. The budget constraint of an old non-altruistic individual is simply

$$R_{t+1} s_t^N = x_{t+1}^N. \tag{3.3}$$

We will assume that in each period, individuals derive utility from comparing their own consumption with a consumption reference. As in de la Croix (1996), a generic member of the generation born in period $t-1$ inherits a certain level of aspirations a_t in period t. These aspirations are based on the standard of living achieved by the parents. We assume that the inherited aspiration a_t^c of an adult of generation t is

$$a_t^c = c_{t-1}^A, \tag{3.4}$$

where c_{t-1}^A is the parents' amount of consumption when they were in their second period of life. By definition, these parents are altruistic. We posit the following additive specification for the aspiration adjusted consumption $\hat{c}_t$ of an adult belonging to generation t:

$$\hat{c}_t^i = c_t^i - \delta_c a_t^c, \qquad \text{with } \delta_c \in [0,1), \qquad i = A, N, \tag{3.5}$$

where δ_c is a parameter that represents the intensity of aspirations when the individual is adult. Thus, adults who have acquired higher aspirations, due to their parents' experience of consumption, will require a larger amount of consumption to achieve the same level of utility. These aspirations arising when an individual is an adult/worker is dubbed adult aspirations.[5]

[5] An alternative functional form used to introduce past consumption references is the multiplicative (Abel, 1990; Diaz et al., 2003). The qualitative results of the model remain unchanged under this alternative formulation as both the additive and multiplicative forms exhibit the feature that a larger intensity of aspirations results in a larger marginal utility of own consumption.

Similarly, old individuals derive utility from comparing their consumption when old to the consumption of their parents' old consumption. Therefore, the aspiration adjusted consumption $\hat{x}_{t+1}$ of an old individual in period $t+1$ is given by

$$\hat{x}^i_{t+1} = x^i_{t+1} - \delta_x a^x_{t+1}, \qquad \text{with } \delta_x \in [0,1)\,, \qquad i = A, N, \tag{3.6}$$

where a^x_{t+1} is the inherited aspiration of an old individual of generation t. This aspiration satisfies

$$a^x_{t+1} = x^A_t, \tag{3.7}$$

where x^A_t is the parents' amount of consumption when the (altruistic) parents were old, and the value of the parameter δ_x measures the intensity of aspirations when an individual is old. The aspirations occurring when individuals are old/retired is dubbed old aspirations.

A generic altruistic individual belonging to generation t derives utility from aspiration adjusted adult consumption, aspiration adjusted old consumption, and the amount left as bequest. We posit the following separable utility function representing the preferences of altruistic individuals belonging to generation t:

$$U^A(\hat{c}^A_t, \hat{x}^A_{t+1}, b_{t+1}) = u_c(\hat{c}^A_t) + \beta u_x(\hat{x}^A_{t+1}) + \rho v(b_{t+1}), \tag{3.8}$$

where both β and ρ are strictly positive and the functions u_c, u_x, and v are twice-differentiable, strictly increasing, strictly concave, and satisfy the typical Inada conditions at zero and infinity. Note that the positive bequests are through a joy-of-giving motivation (as in Yaari, 1965; Abel, 1986; or Andreoni, 1989) such that the amount of bequests enters directly as an argument in the utility function. There are other motives for intergenerational transfers, such as altruistic preferences à la Barro (1974) and Becker (1981), where individuals derive utility from their children's indirect utility function or through paternalistic preferences where individuals care about their offspring's level of consumption (Pollak, 1988). Under altruistic preferences, the last term in the utility Equation (3.8) would be replaced by the indirect utility function of direct descendants, which is an increasing function of the amount of inheritance received by the descendants. If preferences were paternalistic, the last term in the utility Equation (3.8) would be replaced by the offspring's adult consumption, which would be an increasing function of the amount b_{t+1} of inheritance. . In both cases, the results would be qualitatively similar to those under our joy-of-giving specification. However, a problem posed by these two alternative preferences is the potential existence of corner solutions when the bequest motive is not operative, i.e., when the amount of bequest in equilibrium is equal to zero. We will avoid this problem by assuming joy-of-giving preferences displaying an Inada condition when the amount b_{t+1} of bequest tends to zero.

The utility function for non-altruistic individuals belonging to generation t is the equivalent to Equation (3.8) when $\rho = 0$,

$$U^N(\hat{c}^N_t, \hat{x}^N_{t+1}) = u_c(\hat{c}^N_t) + \beta u_x(\hat{x}^N_{t+1}). \tag{3.9}$$

3.2 Production

We assume that the good of this economy is produced by means of a production function displaying constant returns to scale in capital and labor. In our small open economy, capital is fully mobile, while labor is not. Under competitive input markets, this implies that the rental price of a unit of capital is constant and equal to its international level r. Therefore, the gross rate of return on savings satisfies $R_{t+1} = 1 + r \equiv R$ for all t. We will assume throughout that the interest rate r is strictly positive, i.e., $R > 1$. Moreover, the equilibrium capital to labor ratio becomes constant and, thus, the marginal productivity of a unit of labor (which is equal to the competitive real wage per unit of labor) is also constant, $w_t = w$ for all t.

4 Aspirations, saving, and bequests

Each period has two different types of individuals: altruistic and non-altruistic. Altruistic individuals (those with children) maximize Equation (3.8) for $\{c_t^A, x_{t+1}^A, b_{t+1}, s_t^A\}$ subject to Equations (3.1), (3.2), (3.5) and (3.6), taking as given a_t^c, a_{t+1}^x, b_t, w_t and R_{t+1}. If we plug the competitive rental prices of inputs $w_t = w$ and $R_{t+1} = R$ into the solution of this individual problem, we obtain the following first order conditions for the individual belonging to generation t:

$$u_c'(\hat{c}_t^A) = \beta R u_x'(\hat{x}_{t+1}^A) \tag{4.1}$$

and

$$n\beta u_x'(\hat{x}_{t+1}^A) = \rho v'(b_{t+1}), \tag{4.2}$$

where Equation (4.1) gives us the optimal allocation of consumption along the life cycle and Equation (4.2) gives us the optimal allocation of resources of an old individual's own old consumption and the amount of bequest left for each direct descendant. Using the aspiration formation Equations (3.4) and (3.7), we obtain the adjusted consumptions,

$$\hat{c}_t^A = w + b_t - s_t^A - \delta_c c_{t-1}^A \tag{4.3}$$

and

$$\hat{x}_{t+1}^A = R s_t^A - n b_{t+1} - \delta_x x_t^A. \tag{4.4}$$

To analyze the effect on the values s_t^A of saving and b_{t+1} of bequest of changes in the bequest intensity ρ and aspirations intensities δ_c and δ_x when adult and old, respectively, we make use of Equations (4.3) and (4.4) and implicitly differentiate the system of Equations (4.1) and (4.2) for the parameters ρ, δ_c and δ_x. The corresponding derivatives summarizing these comparative statics results for altruistic individuals are in Section A.1(Appendix). As expected, an increase in the intensity of the bequest motive, parameterized by the value of ρ, raises the amounts of both saving and bequest because of the shift of resources from adult consumption to the next generation. This effect of altruism is similar to Hurd (1987) and Bossmann et al. (2007) under pure altruism and joy-of-giving, respectively. Moreover, when the intensity δ_c of adult aspirations increases, the utility associated with adult consumption diminishes, while its marginal utility rises. Therefore, the optimal reaction of individuals is to increase their adult consumption c_t^A and reduce the values of the other arguments of their utility function, namely, old consumption x_{t+1}^A and bequests b_{t+1}. This shift from old to adult consumption results in a lower amount of saving. However, when the intensity δ_x of old aspirations rises, the utility associated with old consumption diminishes, while its marginal utility rises such that individuals optimally react by augmenting their old consumption, which implies an increase in saving and a decrease in the amount of bequest. While a stronger intensity of either adult or old aspirations result in lower bequests, the effect on saving depends on the type of aspirations that becomes stronger.

Concerning non-altruistic individuals, they can be viewed a special case of altruistic ones with $\rho = 0$ and $n = 0$However, non-altruistic individuals may receive inheritance, and the consumption of their parents affects their current decisions. We next describe their behavior.

Non-altruistic individuals (those without children) maximize Equation (3.9) for $\{c_t^N, x_{t+1}^N, s_t^A\}$ subject to Equations (3.1), (3.3), (3.5) and (3.6) taking as given $a_t^c, a_{t+1}^x, b_t, w_t$ and R_{t+1}. Substituting the competitive rental prices of inputs $w_t = w$ and $R_{t+1} = R$ in the solution of this individual problem, we obtain the first order condition for the individual belonging to generation t,

$$u_c'(\hat{c}_t^N) = \beta R u_x'(hatx_{t+1}^N) \tag{4.5}$$

where Equation (4.5) provides the optimal allocation of consumption along the life cycle of the individual. Using the aspiration formation Equations 3.4) and (3.7) to obtain the adjusted consumptions,

$$\hat{c}_t^N = w + b_t - s_t^N - \delta_c c_{t-1}^A \tag{4.6}$$

and

$$\hat{x}_{t+1}^N = R s_t^N - \delta_x x_t^A, \tag{4.7}$$

we can study the effect on the value of saving s_t^N of changes in the aspirations intensities δ_c and δ_x when adult and old, respectively. Making use of Equations (4.6) and (4.7) and implicitly differentiating Equation (4.5) for the parameters δ_c and δ_x, we get that an increase in the intensity δ_c of adult aspirations results in a lower amount of saving. However, saving increases when the intensity δ_x of old aspirations rises. Note again that the effect on savings varies on the type of aspirations being considered. As before, the corresponding derivatives are shown in Section A.1 (Appendix).

For simplicity, we will assume that the function $U^i(\hat{c}_t^i, \hat{x}_{t+1}^i, b_{t+1})$, for $i = A, N$, is additive separable and homothetic as in Abel (1986). Then, according to Katzner (1970, Theorem 2.4-4), the utility functions u_c, u_x and v must be isoelastic, i.e.,

$$u_c(z) = u_x(z) = v(z) = \begin{cases} \dfrac{z^{1-\sigma}}{1-\sigma} & \text{if } \sigma \neq 1 \\ \\ \ln z & \text{if } \sigma = 1, \end{cases} \tag{4.8}$$

with $\sigma > 0$. Under this parametric assumption, we can obtain the explicit equilibrium values of adult and old consumption, saving, and bequest for individuals with children:

$$c_t^A = \frac{1}{H^A} \left\{ R\,(w + b_t) + \left[(\beta R)^{\frac{1}{\sigma}} + n \left(\frac{\rho R}{n} \right)^{\frac{1}{\sigma}} \right] \delta_c c_{t-1}^A - \delta_x x_t^A \right\}, \tag{4.9}$$

$$x_{t+1}^A = \frac{1}{H^A} \left\{ R\,(\beta R)^{\frac{1}{\sigma}} \left(w + b_t - \delta_c c_{t-1}^A \right) + \left[R + n \left(\frac{\rho R}{n} \right)^{\frac{1}{\sigma}} \right] \delta_x x_t^A \right\}, \tag{4.10}$$

$$b_{t+1} = \frac{1}{H^A} R \left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} \left(w + b_t - \delta_c c_{t-1}^A - \frac{\delta_x x_t^A}{R}\right), \tag{4.11}$$

and

$$s_t^A = \frac{1}{H^A} \left\{ \left[(\beta R)^{\frac{1}{\sigma}} + n \left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} \right] \left(w + b_t - \delta_c c_{t-1}^A\right) + \delta_x x_t^A \right\}, \tag{4.12}$$

where

$$H^A = R + (\beta R)^{\frac{1}{\sigma}} + n \left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} > 0. \tag{4.13}$$

Note that the linearity of the previous functions concerning the state variables b_t, c_{t-1}, and x_t faced by an altruistic individual belonging to generation t is a direct consequence of the homotheticity of the utility function U^A and the assumed linearity of aspiration formation given in Equations (3.5) and (3.6). Moreover, we see that the optimal amount of bequest for an altruistic individual is strictly positive only if

$$w + b_t - \delta_c c_{t-1}^A - \frac{\delta_x x_t^A}{R} > 0. \tag{4.14}$$

As will be shown in Section 5, the previous inequality imposes a natural restriction on the initial values of bequest and consumption, b_0, c_{-1} and x_0.

Using the utility function Equation (4.8), we obtain the following explicit equilibrium values of adult and old consumption and saving for non-altruistic individuals:

$$c_t^N = \frac{1}{H^N} \left\{ R\left(w + b_t\right) + (\beta R)^{\frac{1}{\sigma}} \delta_c c_{t-1}^A - \delta_x x_t^A \right\}, \tag{4.15}$$

$$x_{t+1}^N = \frac{1}{H^N} \left\{ R\,(\beta R)^{\frac{1}{\sigma}} \left(w + b_t\right) - R\,(\beta R)^{\frac{1}{\sigma}} \delta_c c_{t-1}^A + R \delta_x x_t^A \right\}, \tag{4.16}$$

and

$$s_t^N = \frac{1}{H^N} \left\{ (\beta R)^{\frac{1}{\sigma}} \left(w + b_t\right) - (\beta R)^{\frac{1}{\sigma}} \delta_c c_{t-1}^A + \delta_x x_t^A \right\}, \tag{4.17}$$

where

$$H^N = R + (\beta R)^{\frac{1}{\sigma}}. \tag{4.18}$$

The homotheticity of the utility function U^N and the linearity of aspiration formation given in Equations (3.5) and (3.6) implies the linearity of the previous consumptions and saving concerning the variables b_t, c_{t-1}^A, and x_t^A that the non-altruistic individual belonging to the generation t takes as given.

The optimal values of adjusted adult and old consumption for altruistic and non-altruistic agents are easily calculated by using Equations (4.9), (4.10), (4.15), and (4.16) together with Equations (3.4)-(3.7). Those values are

$$\hat{c}_t^A = \frac{R}{H^A} \left(w + b_t - \delta_c c_{t-1}^A - \frac{\delta_x x_t^A}{R}\right), \tag{4.19}$$

$$\hat{x}_{t+1}^A = \frac{R\,(\beta R)^{\frac{1}{\sigma}}}{H^A} \left(w + b_t - \delta_c c_{t-1}^A - \frac{\delta_x x_t^A}{R}\right), \tag{4.20}$$

$$\hat{c}_t^N = \frac{R}{H^N}\left(w + b_t - \delta_c c_{t-1}^A - \frac{\delta_x x_t^A}{R}\right), \tag{4.21}$$

and

$$\hat{x}_{t+1}^N = \frac{R\left(\beta R\right)^{\frac{1}{\sigma}}}{H^N}\left(w + b_t - \delta_c c_{t-1}^A - \frac{\delta_x x_t^A}{R}\right). \tag{4.22}$$

These adjusted consumptions are strictly positive for both altruistic and non-altruistic individuals under condition (4.14).

5 Transitional dynamics

5.1 Convergence to the steady state

The evolution of consumption, saving, and intergenerational transfers of the dynasty under consideration is entirely governed by the sub-system of difference equations composed of Equations (4.9), (4.10), and (4.11), which refer to the dynamic evolution of consumptions and bequests for altruistic individuals. The dynamics of consumptions for non-altruistic individuals is a by product coming from Equations (4.15) and (4.16). Similarly, the dynamics of the amount of saving for both individuals is given by Equations (4.12) and (4.17).

The steady-state (or stationary) equilibrium values of adult consumption, old consumption, and bequest for the altruistic individuals can be found by making $c_t^A = c_{t-1}^A = c^A$, $x_{t+1}^A = x_t^A = x^A$, and $b_{t+1} = b_t = b$ in the dynamic sub-system of Equations (4.9)-(4.11) and then solving for the steady-state value of adult consumption c^A, old consumption x^A, and bequest b. Those steady-state values are:

$$c^A = \frac{\left(1-\delta_x\right)Rw}{J}, \tag{5.1}$$

$$x^A = \frac{\left(1-\delta_c\right)\left(\beta R\right)^{\frac{1}{\sigma}}Rw}{J}, \tag{5.2}$$

$$b = \frac{\left(1-\delta_c\right)\left(1-\delta_x\right)\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}Rw}{J}, \tag{5.3}$$

where

$$J = \left(1-\delta_x\right)R + \left(1-\delta_c\right)\left(\beta R\right)^{\frac{1}{\sigma}} + \left(1-\delta_c\right)\left(1-\delta_x\right)\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left(n-R\right). \tag{5.4}$$

Following the same procedure, using Equations (4.15) and (4.16) together with Equations (5.1)-(5.3), we provide the following steady-state amounts of consumption for non-altruistic individuals:

$$c^N = \frac{\left(1-\delta_x\right)\left[R + \left(\beta R\right)^{\frac{1}{\sigma}} + \left(1-\delta_c\right)n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]Rw}{\left(R + \left(\beta R\right)^{\frac{1}{\sigma}}\right)J} \tag{5.5}$$

and

$$x^N = \frac{(1-\delta_c)(\beta R)^{\frac{1}{\sigma}}\left[R + (\beta R)^{\frac{1}{\sigma}} + (1-\delta_x)\, n \left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right] Rw}{\left(R + (\beta R)^{\frac{1}{\sigma}}\right) J}. \tag{5.6}$$

Moreover, using Equations (4.12) and (4.17) together with Equations (5.1)-(5.3), we provide the stationary amounts of saving for altruistic and non-altruistic individuals, respectively,

$$s^A = \frac{(1-\delta_c)\left[(\beta R)^{\frac{1}{\sigma}} + (1-\delta_x)\, n \left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right] w}{J} \tag{5.7}$$

and

$$s^N = \frac{(1-\delta_c)(\beta R)^{\frac{1}{\sigma}}\left[R + (\beta R)^{\frac{1}{\sigma}} + (1-\delta_x)\, n \left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right] w}{\left(R + (\beta R)^{\frac{1}{\sigma}}\right) J}. \tag{5.8}$$

To have positive steady-state values c^A, x^A, b, c^N and x^N for consumption and bequest requires assuming that $J > 0$. Moreover, if we evaluate Equation (4.14) at the steady state by using Equations (5.1)-(5.3), we obtain

$$w + b - \delta_c c^A - \frac{\delta_x x^A}{R} = w (1-\delta_c)(1-\delta_x) \frac{H^A}{J}.$$

Therefore, condition (4.14) holds at the steady state only if $J > 0$. Note that, when the steady-state amounts of consumption are strictly positive, then all the steady-state levels of aspiration adjusted consumption for both types of individuals, that is, $\hat{c}^A = (1-\delta_c)\, c^A$, $\hat{x}^A = (1-\delta_x)\, x^A$, $\hat{c}^N = (1-\delta_c)\, c^N$, and $\hat{x}^N = (1-\delta_x)\, x^N$ are also strictly positive. Finally, the amount of saving for both individuals at the steady state is strictly positive when $J > 0$ (see (5.7) and (5.8)). The following Lemma, whose proof is immediate, provides the conditions that the parameters of the model must satisfy to have $J > 0$:

Lemma 1 *Assume that either one of these two conditions hold:*

(i) $n \geq R$,

(ii) $n < R$ *and* $\rho < g(\delta_c, \delta_x)$, *where*

$$g(\delta_c, \delta_x) \equiv \frac{n}{R}\left(\frac{1}{R-n}\left[\frac{R}{1-\delta_c} + \frac{(\beta R)^{\frac{1}{\sigma}}}{1-\delta_x}\right]\right)^{\sigma}. \tag{5.9}$$

Then, $J > 0$.

Since the function g defined in Equation (5.9) is increasing in its two arguments, a sufficient condition for having $J > 0$ when $n < R$ is that

$$\rho < \rho^* \equiv g(0,0) = \frac{n}{R}\left(\frac{1}{R-n}\left[R + (\beta R)^{\frac{1}{\sigma}}\right]\right)^{\sigma}. \tag{5.10}$$

The following proposition provides sufficient conditions for the monotonic stability of the steady-state equilibrium:

Proposition 1 *Assume that the initial values c_{-1}, x_0 and b_0 of consumptions and bequest are strictly positive and satisfy condition (4.14), i.e.,*

$$w + b_0 - \delta_c c_{-1}^A - \frac{\delta_x x_0^A}{R} > 0,$$

and either one of these conditions hold:

(i) $n \geq R$,

(ii) $n < R$ and $\rho < \rho^*$.

Then, for every ρ, there exists a strictly positive pair $\left(\bar{\delta}_c, \bar{\delta}_x\right)$ of aspiration intensities such that the dynamic system formed by Equations (4.9), (4.10) and (4.11) converges monotonically to the steady-state values of adult consumption, old consumption, and bequest for altruistic individuals given in Equations (5.1), (5.2) and (5.3), for $\delta_c \in \left(0, \bar{\delta}_c\right)$ and $\delta_x \in \left(0, \bar{\delta}_x\right)$.

Proof. Section A.5 (Appendix).

Condition (i) implies that if the gross return R of capital is not higher than the number n of children altruistic individuals have, then the path of wealth accumulation per capita is non-explosive. Under that condition, the gross return of wealth is divided among a larger population, which prevents the economy from displaying an unbounded growth of wealth per capita. Condition (ii) tells that even if the gross return of wealth is larger than the growth rate of population, unbounded wealth accumulation per capita is prevented if individuals exhibit a sufficiently weak bequest motive ($\rho < \rho^*$), as this implies a small amount of intergenerational transfer of wealth.

If the utility functions u_c, u_x and v are logarithmic ($\sigma = 1$), then the stability conditions of Proposition 1 collapse into:

$$\frac{\rho R}{n\left(1 + \beta + \rho\right)} < 1. \tag{5.11}$$

If $n \geq R$, then condition (5.11) holds automatically. If $n < R$, then the inequality $\rho < \rho^* \equiv n(1+\beta)/(R-n)$ becomes Equation (5.11). Again, the stability condition (5.11) suggests that the return to capital R and the intensity of the bequest motive ρ should not be very high to prevent the dynasty from accumulating wealth per capita unboundedly. A high rate of population growth, i.e., a large number of children per altruistic individual, will also eliminate the possibility of excessive accumulation of wealth per capita as the initial wealth of individuals will be small if family states are to be divided among many children.

5.2 Speed of convergence

Another related natural issue is the speed at which the steady-state values are approached by the endogenous variables of the model. As this is a by-product of our analysis and is not directly related to our main goal, we relegate all the equations, proofs, and further discussions to Section A.6 (Appendix). Here we only present the main proposition.

Proposition 2 *The speed of convergence around the steady state increases when aspirations at any age are marginally introduced.*

Proof. Section A.6 (Appendix).

To gain some intuition, note that if both bequests and aspirations are removed from our economy ($\rho = 0$, $\delta_c = 0$, and $\delta_x = 0$), convergence to the steady state is achieved instantaneously, given our assumption of fixed wages and interest rates. Introducing either a bequest motive for some individuals or an aspirational concern brings some inertia in the dynamic system such that the convergence is not instantaneous any longer. Bequests and aspirations both make individual decisions dependent on their parents' decisions. Even if these two preferences result in a lower speed of convergence when introduced separately, their interaction ends up displaying an offsetting effect. If aspirations are marginally introduced when individuals exhibit a bequest motive ($\rho > 0$), bequests remain the main driving force for local convergence. However, the strength of this bequest motive in governing the intergenerational linkage becomes weaker as now individuals condition the amount of bequest left to the achievement of a level of consumption adjusted to their aspirations. In fact, Equations (A.5) and (A.6) in the Appendix indicate that introducing either adult or old aspirations decreases the amount of bequests such that the intergenerational linkage through wealth transfer becomes weaker. This implies that the decisions from individuals of a given generation will be more independent of the previous generation's decisions, resulting in a faster convergence to the steady state.

Two additional features might also appear in our model as the intensity of aspirations becomes large. First, a high intensity of aspirations might be a source of endogenous fluctuations around the steady state, as discussed in de la Croix (1996), de la Croix and Michel (1999), Caballé and Moro-Egido (2014), and Fanti et al. (2018, 2019). Therefore, oscillations of wealth and consumption within a family could arise under strong aspirations. Note that the models of de la Croix (1996) and de la Croix and Michel (1999) do not display transmission of wealth through bequests but endogenous rental prices of labor and capital, while only considering adult aspirations. Here, adults consuming a lot to mimic their parents' consumption will save very little. Then, the next generation will receive a small labor income because of the small stock of capital installed in the economy, which will result in a low level of consumption relative to saving, causing consumption oscillations to appear. In our setup with bequests and exogenous rental prices, the mechanism for those oscillations under strong aspirations is even more straightforward. Consider a generation that consumes a lot for an aspirational motive (i.e., to achieve the same standard of living as their parents either when adult or old). This will cause a reduction in the amount left as bequests, which will reduce the lifetime income and thus the consumption of the next generation. Therefore, intergenerational oscillations of consumption will arise.

Second, for even larger values of the aspiration intensities, the speed of convergence decreases with aspirations. This is because when aspirations are very large, they become the dominating force driving the speed of convergence relative to the bequest motive. Here, a larger intensity of aspirations causes a larger inertia from the past and, hence, a lower speed of convergence.

Figure 1 depicts the values of the eigenvalues of the linear dynamic system under the conditions of Proposition 1. The figure shows that when there is a bequest motive ($\rho > 0$), the introduction of aspirations raises the speed of convergence and the transition remains monotonic. However, when the intensity of aspirations becomes larger, oscillations appear. For even larger values of the aspiration intensities, oscillations disappear and the convergence be¬comes monotonic at a local speed that is decreasing in the aspiration intensities.

[Insert Figure 1]

Also note that if aspirations are introduced in an economy where the bequest motive is absent, which corresponds to the vertical axis ($\rho = 0$) of Figure 1, then the economy does not exhibit any longer instantaneous convergence toward its steady state as the endogenous variables become history dependent. In this case, the speed of convergence decreases when aspirations are introduced, which contrasts with the situation where the bequest motive was initially present for some individuals.

In Figure 2, a numerical analysis is conducted to assess how the speed of convergence depends on aspiration intensities (either δ_c or δ_x) under the conditions of Proposition 2, that is, for low values of the aspiration intensities. We use $1 - \lambda_{\max}$, where $\lambda_{\max}$ is the largest eigenvalue of the characteristic polynomial of the dynamic linear system, as a standard measure of speed of convergence. We assume $\beta = 0.6$, $\rho = 0.3$, constant population ($n = 1$)), and we choose an interest rate of 4% per year for individual periods lasting for 30 years such that $R = (1.04)^{30} = 3.2434$. We have considered values for σ lower than, equal to, and larger than one. Our numerical exercises show that the increase in $1 - \lambda_{\max}$ brought about by either adult or old aspirations is sizable.

[Insert Figure 2]

6 Aspirations and wealth accumulation

In this section, we are going to analyze how the presence of aspiration and bequest motives modifies the pattern of wealth accumulation along the life cycle of an individual belonging to generation t. An individual belonging to generation t receives a given amount of inheritance b_t and is exposed to the given levels of aspirations c_{t-1}^A and x_t^A. We will consider two stages in the accumulation process. First, the individual's accumulation of wealth until the end of the adult period, i.e., until retirement (adult accumulation), which is summarized by the difference $s_t^A - b_t$ for altruistic and $s_t^N - b_t$ for non-altruistic individuals. Second, the wealth accumulation that occurs during retirement (old accumulation), which is collected by the difference $nb_{t+1} - s_t^A$ for altruistic and the amount of dissaving $-s_t^N$ for non-altruistic individuals. Lastly, considering the total accumulation of wealth along the life cycle, which is given by the difference between asset holdings at the end and beginning of an individual's life, $nb_{t+1} - b_t$, for altruistic individuals and just $-b_t$ for non-altruistic individuals as they consume all the received inheritance.

The effects of the bequest motive and aspirations on adult and total wealth accumulation are straightforward. For altruistic individuals, as the initial amount b_t of inheritance is received, adult accumulation $\left(s_t^A - b_t\right)$ of inheritance is received, adult accumulation $(nb_{t+1} - b_t)$ behave as s_t^A and b_{t+1}, respectively. Particularly, they increase with the intensity p of the bequest motive as saving s_t^A and bequest b_{t+1} do (see the derivatives (A.1) and (A.4) computed under general utility functions in the Appendix). Following the same argument, adult and total accumulation decrease with the intensity δ_c of adult aspirations, whereas the intensity δ_x of old aspirations raises adult accumulation and lowers total accumulation (see the derivatives (A.2), (A.3), (A.5) and (A.6)).

The analysis of the effect of the intensity of the bequest motive and aspirations on wealth accumulation by an old altruistic individual is more complex as the two terms of the difference $nb_{t+1} - s_t^A$ are endogenous. Concerning the effect of intensity ρ of the bequest motive, we find that old accumulation increases if $R > 1$ and condition (4.14) holds (see Equation (A.9) in the Appendix). Therefore, even if the intensity of the bequest motive raises the total amount of bequests nb_{t+1} and the amount s_t saved, the effect on nb_{t+1} is a direct effect that dominates the second-order effect on saving. Note, however, that the sign of the previous derivative crucially relies on there being a positive net return on saving, $R - 1 > 0$, as we have assumed.

Note that the intensity δ_c of aspirations on adult consumption pushes down both the amount of saving and bequests. The net effect on old accumulation is generally ambiguous. As the natural condition $R > 1$ holds, the sign of the effect would depend on the ratio β/ρ (see Equation (A.10) in the Appendix).). In particular, if we consider the ratio β/ρ as a measure of old consumption's importance relative to bequests in individual preferences, we see that the higher/(lower) the ratio's value, the higher/(lower) the impact on old accumulation. If old consumption is very important relative to bequests, then the amount of saving will be much larger than the bequest. Therefore, the negative impact of δ_c on saving will be larger than the negative impact on the already small amount of bequests. This results in a positive net effect on the accumulation of wealth $nb_{t+1} - s_t^A$ by an old individual. The converse result will hold when the ratio β/ρ is small, i.e., when bequest motives are more important than the appetite for consumption when old. In particular, old accumulation increases with the intensity of old aspirations when the intensity of the bequest motive is small (see Equation (A.11)). Here, the value of b_{t+1} converges to zero when ρ vanishes, while saving decreases with the intensity δ_c of adult aspirations.

Finally, a higher intensity δ_x of old aspirations lowers the level of wealth accumulation of old altruistic individuals as they prefer to consume more when old, which is achieved through more saving and less bequests, resulting in a lower value of $nb_{t+1} - s_t^A$.

Concerning the non-altruistic individuals, as the initial inheritance b_t of each individual is given, adult wealth accumulation $(s_t^N - b_t)$ behaves like savings (see Equations (A.7) and (A.8) in the Appendix), old wealth accumulation for non-altruistic individuals, $-s_t^N$, changes in the opposite direction of saving, and the pattern of lifetime accumulation for non-altruistic individuals, $-b_t$n is unaffected by changes in the intensities of aspirations.

Table 1 summarizes all the effects of the bequest motive and aspirations on the accumulation of capital along the life cycle for altruistic and non-altruistic individuals.

[Insert Table 1]

7 Comparing patterns of wealth accumulation

In this section, we will compare the patterns of wealth accumulation of altruistic and non-altruistic individuals and analyze how the difference between these two patterns are affected by the intensity of aspirations. The previous section showed that the intensity of the bequest motive tends to raise (decrease) the accumulation (disaccumulation) of wealth for adult and old altruistic agents. Next, we show that the existence of aspirations makes the pattern of accumulation of bequest and non-bequest motivated individuals more similar although the two aspirations display different effects on individual saving. This result may partially explain the empirical findings in Hurd (1987) and other papers concerning the insensitivity of wealth accumulation regarding altruism.

To perform our analysis, we need to compute the differences in wealth accumulation between altruistic and non-altruistic individuals within the same generation and family such that they have received the same amount of inheritance and experience the same level of aspirations. Thus, in our analysis, we take as given the same values of the initial wealth b_t and of aspirations, c_{t-1}^A and x_t^A, of a generic member of generation t as we compare family members. For the specific formulas used, see Section A.3 (Appendix). As expected, altruistic individuals will typically accumulate more wealth and those differences become larger as the intensity of the bequest motive ρ increases (see also Section A.3).

While an increase in the intensity of the bequest motive exacerbates the differences in the patterns of wealth accumulation at all lifetime stages between the two types of individuals, the opposite occurs when aspirations are introduced in an economy displaying a bequest motive. The following Proposition states how these differences are affected by the intensities of adult and old aspirations, parameterized by the values δ_c and δ_x, respectively:

Proposition 3 *The introduction of either adult or old aspirations reduces the differences in wealth accumulation patterns in all lifetime stages between altruistic and non-altruistic individuals.*

Proof. Section A.5 (Appendix).

If the intensity of aspirations associated with adult consumption increases then the value $c_t - \delta_c c_{t-1}$ of adjusted adult consumption appearing in the utility function becomes smaller. As the isoelastic utility function displays decreasing absolute risk aversion, the degree of concavity of utility u becomes higher *ceteris paribus*. In our non-stochastic environment, this translates into a lower willingness to change the level of adult consumption, resulting in a lower impact of the intensity of the bequest motive on saving and bequest. Moreover, adult aspirations dampen the positive effect of the bequest motive on the amount of wealth accumulation by old individuals even if this accumulation might increase the intensity of adult aspirations as we have seen in Section 6.

A higher strength of old aspirations lowers the positive impact of the bequest motive on saving, bequests, and wealth accumulation of adult and old individuals. As it happens under adult aspirations, parental consumption induces some inertia in the behavior of individuals, which results in a weaker response to the introduction of a bequest motive. Note that old aspirations dampen the positive effect of the bequest motive on the amount of wealth accumulation by adult individuals even if old aspirations raise the wealth accumulation of both types of old individuals (see Equations (A.3) and (A.8)).

Table 2 summarizes the signs of the derivatives obtained in the proof of the Proposition and the ones referred to the derivatives with respect to ρ. Note that both types of aspirations (adult and old) exhibit different effects on individual saving but both result in similar patterns of wealth accumulation for the two types of individuals.

[Insert Table 2]

8 Steady-state effects

This section compares the allocation of saving and bequest of a generation in the steady state, where $c_t^i = c^i$, $x_{t+1}^i = x$, $s_t^i = s^i$, and $b_{t+1} = b$ for all t, $i = A, N$, with the allocation under different values of the parameters characterizing the intensities of the bequest motive and aspirations once the dynasty has reached the new steady state.

We start with the effects on the stationary value of saving, bequests, and wealth accumulation at all life stages of the intensity p of the bequest motive and aspirations δ_c and δ_x for altruistic and non-altruistic individuals. The corresponding derivatives summarizing the comparative statics are in Section A.4 (Appendix). In general, the sign of the partial derivatives is identical to Sections 4 and 6 for the non-stationary values of bequest, savings, and wealth accumulation. Table 3 summarizes all the signs of the comparative statics exercises on wealth and wealth accumulation at the stationary equilibrium. However, some differences appear when performing the comparative statics exercise on the steady-state equilibrium values. The most relevant ones concern non-altruistic individuals. The intensity ρ of the bequest motive affects the amount of saving of non-altruistic agents and, thus, their wealth accumulation, as the intensity of altruism determines the amount of inheritance that both altruistic and non-altruistic individuals receive at the steady state. However, this effect is ambiguous unless we restrict our analysis to low values of the intensities of adult and old aspirations (see Equations (A.25), (A.28) and (A.29) in the Appendix). Another difference with the results from Section 6 is that the intensities of adult and old aspirations exert a positive effect on total wealth accumulation of non-altruistic individuals as the steady-state value of inheritances decreases with both types of aspirations.

[Insert Table 3]

Moreover, total accumulation along the life cycle of altruistic individuals does not vary with the intensity of the bequest motive and aspirations if $n = 1$. However, if $n > 1$, total accumulation behaves as in Section 6, where the initial value of the amount of bequest is not endogenously adjusted to be equal to its new steady-state value (see Equations (A.20), (A.23) and (A.24)). The higher the rate of population growth, the larger the amount of wealth individuals must accumulate to endow their children with the stationary amount of inheritance. We also find that adult accumulation $s^A - b$ for altruistic individuals increases (decreases) with the intensity of the bequest motive in a steady state if $n > (<) R$ (see Equation (A.19)). If the gross return R from saving is small, savings must increase relative to bequest per children to generate a transfer increase to the next generation. Similarly, if the number n of children is high, savings should also increase significantly so as to raise the amount of bequest per child.

Finally, for some effects to have the same sign as their non-stationary counterparts requires a low level of the bequest motive ρ. TThis happens when analyzing the effect of old aspirations on saving and adult wealth accumulation for both types of individuals (see Equations (A.18), (A.21), (A.27) and (A.31)). Furthermore, the analysis of the effect on old wealth accumulation of adult aspirations for altruistic individuals and of old aspirations for non-altruistic individuals also requires a small value of the bequest motive (see Equations (A.22) and (A.32)). Like the non-stationary case, we also find that wealth accumulation of altruistic individuals is larger at all life stages (see Equations (A.33)-(A.35)) and the differences increase with the intensity of the bequest motive ρ (see Equations (A.36)-(A.38)).

The following proposition shows that the dampening effect also holds in the steady state when aspirations are marginally introduced in the individuals' preferences:

Proposition 4 *The marginal introduction of either adult or old aspirations reduces the differences in wealth accumulation patterns at all ages between altruistic and non-altruistic individuals in the steady state.*

Proof. Section A.5 (Appendix).

Thus, the presence of either adult or old aspirations dampens the effect of the bequest motive on the stationary pattern of wealth accumulation. Therefore, we have extended the conclusion from Section 7 regarding the individual decision with exogenous initial values of bequest and aspirations to the stationary patterns of capital accumulation. The previous comparative statics exercise is summarized in Table 4.

[Insert Table 4]

9 Conclusion

We developed a simple OLG model that enabled us to study the discrepancies in the individual pattern of wealth accumulation between bequest and non-bequest motivated individuals. Our results show that aspirations at different life stages display different effects on the amount of asset holdings for given initial values of bequest and aspirations. However, adult and old aspirations make the pattern of accumulation between the two types of individuals more similar. Therefore, under aspirations, the two types of individuals will behave more similarly than when they do not exhibit aspirational concerns. Moreover, the dampening effect of aspirations prevails when we make the comparison in terms of stationary allocations. This result provides a theoretical explanation to the empirical findings about the lack of significant difference between the pattern of wealth accumulation between individuals with children and those without children.

As a by-product of our analysis, we have also shown that marginally introducing aspirations at any age makes a dynasty converge faster toward its steady state as intergenerational transfers play a less relevant role and this makes individual decisions less dependent of ancestral decisions.

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A Appendix

A.1 Effects on saving and bequests.

Altruistic individuals. Making use of Equations (4.3) and (4.4) and implicitly differentiating the system of Equations (4.1) and (4.2) regarding the parameters ρ, δ_c and δ_x, we get the following partial derivatives, where we have suppressed the arguments of the functions to ease the notation:

$$\frac{ds_t^A}{d\rho} = -\frac{n\beta R u_x'' v'}{M} > 0, \tag{A.1}$$

$$\frac{ds_t^A}{d\delta_c} = -\frac{u_c''\left(n^2\beta u_x'' + \rho v''\right)}{M} c_{t-1}^A < 0, \tag{A.2}$$

$$\frac{ds_t^A}{d\delta_x} = \frac{\beta\rho R u_x'' v''}{M} > 0, \tag{A.3}$$

$$\frac{db_{t+1}}{d\rho} = -\frac{\left(u_c'' + \beta R^2 u_x''\right) v'}{M} > 0, \tag{A.4}$$

$$\frac{db_{t+1}}{d\delta_c} = -\frac{n\beta R u_c'' u_x''}{M} c_{t-1}^A < 0, \tag{A.5}$$

$$\frac{db_{t+1}}{d\delta_x} = -\frac{n\beta u_c'' u_x''}{M} x_t^A < 0, \tag{A.6}$$

where $M = n^2\beta u_c'' u_x'' + \rho u_c'' v'' + \beta\rho R^2 u_x'' v'' > 0$.

Non-altruistic individuals. Using Equations (4.6) and (4.7) and implicitly differentiating Equation (4.5) for the parameters δ_c and δ_x, we get the following partial derivatives, where we have again suppressed the arguments of the functions:

$$\frac{ds_t^N}{d\delta_c} = \frac{-u_c''}{u_c'' + \beta R^2 u_x''} c_{t-1}^A < 0 \tag{A.7}$$

and

$$\frac{ds_t^N}{d\delta_x} = \frac{\beta R u_x''}{u_c'' + \beta R^2 u_x''} x_t^A > 0. \tag{A.8}$$

A.2 Effects on wealth accumulation

Altruistic individuals. The analysis of the effect of the intensity of the bequest motive and aspirations on wealth accumulation by an old altruistic individual has to consider that the two terms of the difference $nb_{t+1} - s_t^A$ are endogenous. The amounts of saving s_t and bequest b_{t+1} are given in Equations (4.12) and (4.11), respectively. Concerning the effect of the intensity ρ of the bequest motive, we can compute the following partial derivative:

$$\frac{\partial\left(nb_{t+1} - s_t^A\right)}{\partial\rho} = \frac{nR\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left[R - 1 + (\beta R)^{\frac{1}{\sigma}}\right]}{\rho\sigma\left(H^A\right)^2}\left(w + b_t - \delta_c c_{t-1}^A - \frac{\delta_x x_t^A}{R}\right) > 0, \tag{A.9}$$

where the positive sign arises because $R > 1$ and Equation (4.14).

The effects of adult aspirations δ_c on the accumulation of wealth by an altruistic old individual is summarized by the following derivative

$$\begin{aligned}\frac{\partial\left(nb_{t+1} - s_t^A\right)}{\partial\delta_c} &= \frac{(\beta R)^{\frac{1}{\sigma}}\left[1 + (1 - R)\,n\left(\frac{\rho}{n\beta}\right)^{\frac{1}{\sigma}}\right]}{H^A}c_{t-1}^A \gtrless 0 \\ &\Longleftrightarrow \frac{\beta}{\rho} \gtrless \frac{(R-1)^\sigma}{n^{1-\sigma}}.\end{aligned} \tag{A.10}$$

In particular, for a sufficiently small bequest motive, we have

$$\lim_{\rho\to 0}\frac{\partial\left(nb_{t+1} - s_t^A\right)}{\partial\delta_c} = \frac{(\beta R)^{\frac{1}{\sigma}}}{H^A}c_{t-1}^A > 0. \tag{A.11}$$

A.3 Differences in accumulation patterns

Under condition (4.14) and the preferences given in Equation (4.8), we know that the difference in adult accumulation between altruistic and non-altruistic individuals satisfies

$$\begin{aligned}\left(s_t^A - b_t\right) - \left(s_t^N - b_t\right) &= s_t^A - s_t^N \\ &= \frac{Rn}{H^A H^N}\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left(w + b_t - \delta_c c_{t-1}^A - \frac{\delta_x x_t^A}{R}\right) > 0,\end{aligned} \tag{A.12}$$

where the second equality follows from Equations (4.12), (4.17) and the inequality follows from condition (4.14). The derivative of the difference in adult accumulation regarding bequest motive is

$$\frac{\partial\left(s_t^A - s_t^N\right)}{\partial\rho} = \frac{\partial s_t^A}{\partial\rho} > 0, \tag{A.13}$$

where the equality is because the amount s_t^N of saving for the non-altruistic individuals is unaffected by changes in the parameter ρ and the inequality comes from Equation (A.1).

The difference in old wealth accumulation between altruistic and non-altruistic individuals is

$$\begin{aligned}\left(nb_{t+1} - s_t^A\right) - \left(-s_t^N\right) &= nb_{t+1} - s_t^A + s_t^N \\ &= \frac{Rn}{H^A H^N}\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left(R - 1 + (\beta R)^{\frac{1}{\sigma}}\right)\left(w + b_t - \delta_c c_{t-1}^A - \frac{\delta_x x_t^A}{R}\right) > 0,\end{aligned} \tag{A.14}$$

where the second equality follows from Equations (4.11), (4.12) and (4.17) and the inequality follows from condition (4.14) and the fact that $R > 1$. Again, as non-altruistic individuals are not affected by the bequest motive, we know from Equation (A.9) that

$$\frac{\partial\left(nb_{t+1} - s_t^A + s_t^N\right)}{\partial\rho} = \frac{\partial\left(nb_{t+1} - s_t^A\right)}{\partial\rho} > 0, \tag{A.15}$$

Finally, the difference in lifetime wealth accumulation between altruistic and non-altruistic individuals is simply

$$\begin{aligned} &(nb_{t+1} - b_t) - (-b_t) = nb_{t+1} \\ &= \frac{Rn}{H^A}\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left(w + b_t - \delta_c c_{t-1}^A - \frac{\delta_x x_t^A}{R}\right) > 0, \end{aligned} \tag{A.16}$$

where the second equality follows from Equation (4.11)) and the inequality from condition Equation (4.14). We know from Equation (A.4) that

$$\frac{\partial\left(nb_{t+1}\right)}{\partial\rho} = n\frac{\partial b_{t+1}}{\partial\rho} > 0. \tag{A.17}$$

A.4 Steady-state effects

Altruistic individuals. We start with the effects on the stationary value of saving and bequests of the intensity of the bequest motive ρ and aspirations δ_c and δ_x. Using Equations (5.3) and (5.7), we can compute the following derivatives, which summarize the comparative statics effects on bequest and saving at the steady state:

$$\frac{\partial s^A}{\partial\rho} = \frac{\left(1-\delta_c\right)\left(1-\delta_x\right)\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left[\left(1-\delta_x\right)n + \left(1-\delta_c\right)\left(\beta R\right)^{\frac{1}{\sigma}}\right]Rw}{\sigma\rho J^2} > 0,$$

$$\frac{\partial b}{\partial\rho} = \frac{\left(1-\delta_c\right)\left(1-\delta_x\right)\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left[\left(1-\delta_x\right)R + \left(1-\delta_c\right)\left(\beta R\right)^{\frac{1}{\sigma}}\right]Rw}{\sigma\rho J^2} > 0,$$

$$\frac{\partial s^A}{\partial\delta_c} = -\frac{\left(1-\delta_x\right)\left[\left(\beta R\right)^{\frac{1}{\sigma}} + \left(1-\delta_x\right)n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]Rw}{J^2} < 0,$$

$$\frac{\partial b}{\partial\delta_c} = -\frac{\left(1-\delta_x\right)^2\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}R^2 w}{J^2} < 0,$$

$$\frac{\partial s^A}{\partial\delta_x} = \frac{\left(1-\delta_c\right)\left(\beta R\right)^{\frac{1}{\sigma}}\left[1 - \left(1-\delta_c\right)\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]Rw}{J^2} \gtreqless 0,$$

$$\lim_{\rho\to 0}\frac{\partial s^A}{\partial\delta_x} = \frac{\left(1-\delta_c\right)\left(\beta R\right)^{\frac{1}{\sigma}}Rw}{\left[\left(1-\delta_x\right)R + \left(1-\delta_c\right)\left(\beta R\right)^{\frac{1}{\sigma}}\right]^2} > 0, \tag{A.18}$$

$$\frac{\partial b}{\partial \delta_x} = -\frac{(1-\delta_c)^2 (\beta R)^{\frac{1}{\sigma}} \left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} Rw}{J^2} < 0.$$

where J is defined in Equation (5.4).

Next, we analyze how wealth accumulation at each life stage varies with the intensities of the bequest motive and both adult and old aspirations. Using Equations (5.3) and (5.7), we obtain the following effects of the bequest motive:

$$\frac{\partial \left(s^A - b\right)}{\partial \rho} = \frac{(1-\delta_c)(1-\delta_x)^2 \left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} Rw}{\sigma \rho J^2} (n - R) \gtrless 0, \tag{A.19}$$

$$\frac{\partial \left(nb - s^A\right)}{\partial \rho} = \frac{(1-\delta_c)(1-\delta_x)\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} \left[n(1-\delta_x)(R-1)+(\beta R)^{\frac{1}{\sigma}}(1-\delta_c)(n-1)\right] Rw}{\sigma \rho J^2} > 0,$$

$$\frac{\partial \left[(n-1)\, b\right]}{\partial \rho} = (n-1)\frac{\partial b}{\partial \rho} \geq 0. \tag{A.20}$$

Concerning the effects of both types of aspirations on wealth accumulation at each life stage, we find the following signs of the corresponding derivatives:

$$\frac{\partial \left(s^A - b\right)}{\partial \delta_c} = \frac{(1-\delta_x)\left[(1-\delta_x)(R-n)\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} - (\beta R)^{\frac{1}{\sigma}}\right] Rw}{J^2} \gtrless 0,$$

$$\lim_{\rho \to 0} \frac{\partial \left(s^A - b\right)}{\partial \delta_c} = -\frac{(1-\delta_x)\, Rw}{\left[(1-\delta_x)\, R + (1-\delta_c)(\beta R)^{\frac{1}{\sigma}}\right]^2} < 0, \tag{A.21}$$

$$\frac{\partial \left(nb - s^A\right)}{\partial \delta_c} = \frac{(1-\delta_x)\left[(\beta R)^{\frac{1}{\sigma}} - (1-\delta_x)\, n \left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} (R-1)\right] Rw}{J^2} \gtrless 0,$$

$$\lim_{\rho \to 0} \frac{\partial \left(nb - s^A\right)}{\partial \delta_c} = \frac{(1-\delta_x)(\beta R)^{\frac{1}{\sigma}} Rw}{\left[(1-\delta_x)\, R + (1-\delta_c)(\beta R)^{\frac{1}{\sigma}}\right]^2} > 0, \tag{A.22}$$

$$\frac{\partial \left[(n-1)\, b\right]}{\partial \delta_c} = (n-1)\frac{\partial b}{\partial \delta_c} \geq 0. \tag{A.23}$$

$$\frac{\partial \left(s^A - b\right)}{\partial \delta_x} = \frac{(1-\delta_c)(\beta R)^{\frac{1}{\sigma}} Rw}{J^2} > 0,$$

$$\frac{\partial \left(nb - s^A\right)}{\partial \delta_x} = -\frac{(1-\delta_c)(\beta R)^{\frac{1}{\sigma}}\left[1 + \left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} (1-\delta_c)(n-1)\right] Rw}{J^2} < 0,$$

$$\frac{\partial\left[(n-1)\,b\right]}{\partial\delta_x} = (n-1)\,\frac{\partial b}{\partial\delta_x} \geq 0. \tag{A.24}$$

where J is defined in Equation (5.4).

Non-altruistic individuals. Using the steady-state values of the amounts of inheritance received and saving for non-altruistic individuals, Equations (5.3) and (5.8), respectively, we obtain the effects on the stationary values of saving and bequests of the intensity ρ of the bequest motive and of aspirations δ_c and δ_x. The following derivatives summarize the results:

$$\frac{\partial s^N}{\partial\rho} = \frac{(1-\delta_c)(1-\delta_x)\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}(\beta R)^{\frac{1}{\sigma}}\left[\left(R+(\beta R)^{\frac{1}{\sigma}}\right)(1-\delta_c)+n(\delta_c-\delta_x)\right]Rw}{\sigma\rho\left(R+(\beta R)^{\frac{1}{\sigma}}\right)J^2} \gtrless 0$$

$$\lim_{\delta_c\to 0,\delta_x\to 0}\frac{\partial s^N}{\partial\rho} = \frac{\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}(\beta R)^{\frac{1}{\sigma}}Rw}{\sigma\rho\left(R+(\beta R)^{\frac{1}{\sigma}}+\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}(n-R)\right)^2} > 0 \tag{A.25}$$

$$\frac{\partial s^N}{\partial\delta_c} = -\frac{(1-\delta_x)(\beta R)^{\frac{1}{\sigma}}\left[R+(\beta R)^{\frac{1}{\sigma}}+(1-\delta_x)\,n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]Rw}{\left(R+(\beta R)^{\frac{1}{\sigma}}\right)J^2} < 0, \tag{A.26}$$

$$\frac{\partial s^N}{\partial\delta_x} = \frac{(1-\delta_c)(\beta R)^{\frac{1}{\sigma}}\left[R+(\beta R)^{\frac{1}{\sigma}}-\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}(1-\delta_c)\left(R-n+(\beta R)^{\frac{1}{\sigma}}\right)\right]Rw}{\left(R+(\beta R)^{\frac{1}{\sigma}}\right)J^2} \gtrless 0,$$

$$\lim_{\rho\to 0}\frac{\partial s^N}{\partial\delta_x} = \frac{(1-\delta_c)(\beta R)^{\frac{1}{\sigma}}Rw}{\left[(1-\delta_x)R+(1-\delta_c)(\beta R)^{\frac{1}{\sigma}}\right]^2} > 0. \tag{A.27}$$

The effects on wealth accumulation at each life stage of changes in the intensity of the bequest motive and both adult and old aspirations are given as

$$\frac{\partial\left(s^N-b\right)}{\partial\rho} = -\frac{(1-\delta_c)(1-\delta_x)\left[R\left(R+(\beta R)^{\frac{1}{\sigma}}\right)(1-\delta_x)+n(\beta R)^{\frac{1}{\sigma}}(\delta_x-\delta_c)\right]\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}Rw}{\sigma\rho\left(R+(\beta R)^{\frac{1}{\sigma}}\right)J^2} \gtrless 0,$$

$$\lim_{\delta_c\to 0,\delta_x\to 0}\frac{\partial\left(s^N-b\right)}{\partial\rho} = -\frac{\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}R^2w}{\sigma\rho\left(R+(\beta R)^{\frac{1}{\sigma}}+\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}(n-R)\right)^2} < 0, \tag{A.28}$$

$$\frac{\partial\left(-s^N\right)}{\partial\rho} = -\frac{\partial\left(-s^N\right)}{\partial\rho} \gtrless 0,$$

$$\lim_{\delta_c\to 0,\delta_x\to 0}\frac{\partial\left(-s^N\right)}{\partial\rho} = -\lim_{\delta_c\to 0,\delta_x\to 0}\frac{\partial s^N}{\partial\rho} < 0, \tag{A.29}$$

$$\frac{\partial\left[-b\right]}{\partial\rho} = -\frac{\partial b}{\partial\rho} < 0. \tag{A.30}$$

$$\frac{\partial\left(s^N - b\right)}{\partial\delta_c} = -\frac{(1-\delta_x)\left[(\beta R)^{\frac{1}{\sigma}}\left(R+(\beta R)^{\frac{1}{\sigma}}\right)-\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}(1-\delta_x)\left((R-n)(\beta R)^{\frac{1}{\sigma}}+R^2\right)\right]Rw}{\left(R+(\beta R)^{\frac{1}{\sigma}}\right)J^2} \gtrless 0,$$

$$\lim_{\rho\to 0}\frac{\partial\left(s^N - b\right)}{\partial\delta_c} = \lim_{\rho\to 0}\frac{\partial\left(s^A - b\right)}{\partial\delta_c} = -\frac{\left(1-\delta_x\right)\left(\beta R\right)^{\frac{1}{\sigma}} Rw}{\left[\left(1-\delta_x\right)R + \left(1-\delta_c\right)\left(\beta R\right)^{\frac{1}{\sigma}}\right]^2} < 0, \tag{A.31}$$

$$\frac{\partial\left(-s^N\right)}{\partial\delta_c} = -\frac{\partial\left(s^N\right)}{\partial\delta_c} > 0,$$

$$\frac{\partial\left[-b\right]}{\partial\delta_c} = -\frac{\partial b}{\partial\delta_c} > 0,$$

$$\frac{\partial\left(s^N - b\right)}{\partial\delta_x} = \frac{\left(1-\delta_c\right)\left(\beta R\right)^{\frac{1}{\sigma}}\left[R + \left(\beta R\right)^{\frac{1}{\sigma}} + \left(1-\delta_c\right)n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]Rw}{\left(R + \left(\beta R\right)^{\frac{1}{\sigma}}\right)J^2} > 0,$$

$$\frac{\partial\left(-s^N\right)}{\partial\delta_x} = -\frac{\left(1-\delta_c\right)\left(\beta R\right)^{\frac{1}{\sigma}}\left[R + \left(\beta R\right)^{\frac{1}{\sigma}} - \left(1-\delta_c\right)\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left(R - n + \left(\beta R\right)^{\frac{1}{\sigma}}\right)\right]Rw}{\left(R + \left(\beta R\right)^{\frac{1}{\sigma}}\right)J^2} \gtrless 0,$$

$$\lim_{\rho\to 0}\frac{\partial\left(-s^N\right)}{\partial\delta_x} = \lim_{\rho\to 0}\frac{\partial\left(-s^A\right)}{\partial\delta_x} = -\frac{\left(1-\delta_c\right)\left(\beta R\right)^{\frac{1}{\sigma}} Rw}{\left[\left(1-\delta_x\right)R + \left(1-\delta_c\right)\left(\beta R\right)^{\frac{1}{\sigma}}\right]^2} < 0, \tag{A.32}$$

$$\frac{\partial\left[-b\right]}{\partial\delta_x} = -\frac{\partial b}{\partial\delta_x} > 0.$$

Differences in patterns of wealth accumulation

The expressions for the difference between the stationary patterns of wealth accumulation at different life stages between altruistic and non-altruistic individuals are:

$$s^A - s^N = \frac{\left(1-\delta_c\right)\left(1-\delta_x\right)n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} Rw}{\left(R + \left(\beta R\right)^{\frac{1}{\sigma}}\right)J} > 0, \tag{A.33}$$

$$nb - s^A + s^N = \frac{\left(1-\delta_c\right)\left(1-\delta_x\right)n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left(R - 1 + \left(\beta R\right)^{\frac{1}{\sigma}}\right)Rw}{\left(R + \left(\beta R\right)^{\frac{1}{\sigma}}\right)J} > 0, \tag{A.34}$$

and

$$nb = \frac{n\left(1-\delta_c\right)\left(1-\delta_x\right)\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} Rw}{J} > 0. \tag{A.35}$$

The expression defining J in Equation (5.4) depends also on the intensities of the bequest motive and aspirations, ρ and δ_c, δ_x. Thus, we conclude that altruistic individuals accumulate more wealth at all life stages.

From expressions (A.33)-(A.35), we can compute the following partial derivatives for the intensity of the bequest motive ρ:

$$\frac{\partial\left(s^A-s^N\right)}{\partial\rho}=\frac{\left(1-\delta_c\right)\left(1-\delta_x\right)n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left[\left(1-\delta_x\right)R+\left(1-\delta_c\right)\left(\beta R\right)^{\frac{1}{\sigma}}\right]Rw}{\left[R+\left(\beta R\right)^{\frac{1}{\sigma}}\right]\sigma\rho J^2}>0, \tag{A.36}$$

$$\frac{\partial\left(nb-s^A+s^N\right)}{\partial\rho}=\frac{(1-\delta_c)(1-\delta_x)\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left(R-1+(\beta R)^{\frac{1}{\sigma}}\right)\left[(1-\delta_x)R+(1-\delta_c)(\beta R)^{\frac{1}{\sigma}}\right]Rw}{\left[R+(\beta R)^{\frac{1}{\sigma}}\right]\sigma\rho J^2}>0, \tag{A.37}$$

$$\frac{\partial\left(nb\right)}{\partial\rho}=n\frac{\partial b}{\partial\rho}>0. \tag{A.38}$$

An increase in altruism for individuals with children results in a higher discrepancy in the patterns of accumulation at all lifetime stages between altruistic and non-altruistic individuals in the stationary equilibrium.

A.5 Proofs

Proof of Proposition 1. We can rewrite the system composed of the difference Equations (4.9), (4.10), and (4.11) in matrix form as

$$\begin{bmatrix} c_t^A \\ x_{t+1}^A \\ b_{t+1} \end{bmatrix}=\mathbb{P}\times\begin{bmatrix} c_{t-1}^A \\ x_t^A \\ b_t \end{bmatrix}+\frac{1}{H^A}\begin{bmatrix} R \\ R\left(\beta R\right)^{\frac{1}{\sigma}} \\ R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} \end{bmatrix}w,$$

where H^A is given in Equation (4.13) and the coefficient matrix $\mathbb{P}$ is

$$\mathbb{P}=\frac{1}{H^A}\begin{bmatrix} \left[(\beta R)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_c & -\delta_x & R \\ -R\left(\beta R\right)^{\frac{1}{\sigma}}\delta_c & \left[R+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_x & \left(\beta R\right)^{\frac{1}{\sigma}}R \\ -R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\delta_c & -\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\delta_x & R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} \end{bmatrix}. \tag{A.39}$$

The characteristic polynomial of the matrix $\mathbb{P}$ is

$$P\left(\lambda\right)=\lambda^3-\mu_1\lambda^2+\mu_2\lambda-\mu_3, \tag{A.40}$$

where

$$\mu_1=\frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}+\left[(\beta R)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_c+\left[R+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_x}{H^A},$$

$$\mu_2=\frac{\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left(R\delta_c+R\delta_x+n\delta_c\delta_x\right)}{H^A},$$

$$\mu_3 = \frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\delta_c\delta_x}{H^A}.$$

Moreover, if both δ_c and δ_x converge to zero, the characteristic polynomial converges to the following:

$$P(\lambda) = \lambda^3 - \left[\frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}}{H^A}\right]\lambda^2,$$

whose eigenvalues are

$$\lambda_1 = \frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}}{R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}} > 0, \quad \lambda_2 = 0 \text{ and } \lambda_3 = 0.$$

As the eigenvalues are continuous functions of the parameters δ_c and δ_x, the three eigenvalues will lie in the interior of the unit circle for sufficiently small values of the parameters measuring the intensity of aspirations if $\lambda_1 < 1$. If $n \geq R$, the latter condition is automatically satisfied, whereas, if $n < R$ and $\rho < \rho^*$, where ρ^* is defined in Equation (5.10), then $\lambda_1 < 1$ also holds.

Under either condition (i) or (ii) in the proposition, it holds that $J > 0$ such that consumptions, bequest, savings, and adjusted consumptions are all strictly positive at a stationary equilibrium. Moreover, if the initial values of consumptions and bequest c_{-1}^A, x_0^A and b, satisfy condition (4.14) and are all strictly positive, then the equilibrium values of c_{t-1}^A, x_t^A and b_t, for $t = 1, 2, 3, ...$, are also strictly positive and satisfy condition (4.14) along the dynamic equilibrium. This is so because, first, the set Φ of strictly positive vectors $(c_{t-1}^A, x_t^A, b_t$ satisfying (4.14) is convex. Second, the vector of steady-state values (c^A, x^A, b) belongs to Φ. Third, the convergence from $\left(c_{-1}^A, x_0^A, b\right)$ to $\left(c^A, x^A, b\right)$ takes place along a one-dimension linear manifold as the system of difference Equations ((4.9)-(4.11)) is linear. Therefore, this manifold must belong to Φ. *Q.E.D.*

Proof of Proposition 3. The effect of the aspiration intensity δ_c associated with adult consumption on the differences of wealth accumulation in all life stages is given by the following derivatives, which immediately follow from Equations (A.12), (A.14), and (A.16):

$$\frac{\partial\left(s_t^A - s_t^N\right)}{\partial\delta_c} < 0, \quad \frac{\partial\left(nb_{t+1} - s_t^A + s_t^N\right)}{\partial\delta_c} < 0, \text{ and } \frac{\partial\left(nb_{t+1}\right)}{\partial\delta_c} < 0.$$

Similarly, the impact of the intensity δ_x of aspirations associated with old consumption is given by

$$\frac{\partial\left(s_t^A - s_t^N\right)}{\partial\delta_x} < 0, \quad \frac{\partial\left(nb_{t+1} - s_t^A + s_t^N\right)}{\partial\delta_x} < 0, \text{ and } \frac{\partial\left(nb_{t+1}\right)}{\partial\delta_x} < 0.$$

To compute the derivatives, we use the fact that both H^A and H^N are independent of the aspiration intensities δ_c and δ_x (see Equations (4.13) and (4.18)). Moreover, we have taken as given the same values for the initial wealth b_t and aspirations, c_{t-1}^A and x_t^A, of a generic member of the generation t as given because we compare family members.

The result in the proposition follows from the fact that altruistic individuals accumulate more wealth in all stages of their life. *Q.E.D.*

Proof of Proposition 4. To find the effect of higher aspiration intensities δ_c and δ_x, we only need to compute the derivatives of the expressions (A.33)-(A.35):

$$\frac{\partial\left(s^A - s^N\right)}{\partial \delta_c} = -\frac{n\left(1-\delta_x\right)^2\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} R^2 w}{\left(R + (R\beta)^{\frac{1}{\sigma}}\right) J^2} < 0$$

$$\frac{\partial\left(nb - s^A + s^N\right)}{\partial \delta_c} = -\frac{n\left(1-\delta_x\right)^2\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} \left[R + (R\beta)^{\frac{1}{\sigma}} - 1\right] R^2 w}{\left(R + (R\beta)^{\frac{1}{\sigma}}\right) J^2} < 0$$

$$\frac{\partial\left(nb\right)}{\partial \delta_c} = -\frac{n\left(1-\delta_x\right)^2\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} R^2 w}{J^2} < 0$$

$$\frac{\partial\left(s^A - s^N\right)}{\partial \delta_x} = -\frac{n\left(1-\delta_c\right)^2 (R\beta)^{\frac{1}{\sigma}}\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} R w}{\left(R + (R\beta)^{\frac{1}{\sigma}}\right) J^2} < 0$$

$$\frac{\partial\left(nb - s^A + s^N\right)}{\partial \delta_x} = -\frac{n (R\beta)^{\frac{1}{\sigma}}\left(1-\delta_c\right)^2\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} \left[R + (R\beta)^{\frac{1}{\sigma}} - 1\right] R w}{\left(R + (R\beta)^{\frac{1}{\sigma}}\right) J^2} < 0$$

and

$$\frac{\partial\left(nb\right)}{\partial \delta_x} = -\frac{n\left(1-\delta_c\right)^2 (R\beta)^{\frac{1}{\sigma}}\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} R w}{J^2} < 0$$

As all the signs of the previous derivatives are negative, we conclude that the differences in the patterns of wealth accumulation at all ages between altruistic and non-altruistic individuals become smaller. *Q.E.D.*

A.6 Speed of convergence

If we write the system formed by the linear difference Equations (4.9), (4.10), and (4.11) in vector form (see the proof of Proposition 1), we see that the coefficient matrix $\mathbb{P}$ defined in Equation (A.39) has three-eigenvalues, λ_1, λ_2 and λ_3. Therefore, the solution of the linear dynamic system will be

$$z_t = A_{z,1}\lambda_1^t + A_{z,2}\lambda_2^t + A_{z,3}\lambda_3^t + z \qquad t = 0, 1, ... \tag{A.41}$$

for $z_t = c_t^A, x_{t+1}^A, b_{t+1}$, and where z is the stationary value of z_t and the transpose vector $(A_{c,j}, A_{x,j}, A_{b,j})'$, for $j = 1, 2, 3$, is equal to $\kappa_j \cdot (m_{c,j}, m_{x,j}, m_{b,j})'$, where $(m_{c,j}, m_{x,j}, m_{b,j})'$ is an eigenvector associated with the eigenvalue λ_j of the matrix $\mathbb{P}$. The constants κ_j, $j = 1, 2, 3$, are pinned down by the initial values of c_{-1}, x_0 and b_0. Then, the speed of convergence of the economy could be measured by the fraction of the distance between the value of the generic variable z_t in period t and the stationary value z that the system travels in one period,

$$\frac{z_{t+1} - z_t}{z - z_t}.$$

If $\lambda_{\max}$ is the largest eigenvalue, then

$$\lim_{t\to\infty} \frac{z_{t+1} - z_t}{z - z_t} = 1 - \lambda_{\max},$$

such that the value of the largest eigenvalue of the matrix $\mathbb{P}$ is inversely related to the speed of convergence around the steady state. The main result presented as Proposition 2 characterizes the effect of introducing aspiration on the speed of convergence. Particularly, it shows that the speed of convergence around the steady state increases when aspirations at any age are marginally introduced. We next provide its proof.

Proof of Proposition 2. As we are only interested in the effect of introducing aspirations, we will consider the marginal introduction of adult and old aspirations separately. First, let us consider the introduction of adult aspirations from a situation with no aspirations. Thus, we take the characteristic polynomial Equation (A.40) and make $\delta_x = 0$ to get

$$P(\lambda) = \lambda^3 - \left[\frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} + \left[(\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_c}{R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}}\right]\lambda^2 + \left[\frac{\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} R\delta_c}{R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}}\right]\lambda.$$

Here, one of the eigenvalues equals zero as the parental old consumption x_t is not a state variable for an individual of generation t and, hence, the value of the initial condition x_0 is irrelevant. The other two eigenvalues λ_1 and λ_2 are equal to the conjugate pair

$$\frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} + \left[(\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_c \pm \left[\left(R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} + \left[(\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_c\right)^2 - 4\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} R\delta_c\left[R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\right]^{\frac{1}{2}}}{2\left[R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]}.$$

If we take the largest of this two eigenvalues, λ_1 say, and perform the derivative with respect to the aspiration intensity δ_c and then we evaluate the derivative when $\delta_c = 0$, we obtain

$$\lim_{\delta_c\to 0, \delta_x\to 0} \frac{d\lambda_1}{d\delta_c} = -\frac{R}{R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}} < 0. \tag{A.42}$$

Similarly, we can replicate the argument for the introduction of aspirations on old consumption. The characteristic polynomial in Equation (A.40) with $\delta_c = 0$ becomes

$$P(\lambda) = \lambda^3 - \left[\frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} + \left[R + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_x}{R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}}\right]\lambda^2 + \left[\frac{\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}} R\delta_x}{R + (\beta R)^{\frac{1}{\sigma}} + n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}}\right]\lambda.$$

Here, one of the eigenvalues is again equal to zero as the parental adult consumption c_t is not a state variable for an individual of generation t and, hence, the value of the initial condition c_{-1}

does not affect decisions. The other two eigenvalues, λ_1 and λ_2 are equal to the conjugate pair

$$\frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}+\left[R+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_x\pm\left[\left(R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}+\left[R+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\delta_x\right)^2-4\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}R\delta_x\left[R+(\beta R)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]\right]^{\frac{1}{2}}}{2\left[R+(\beta R)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right]}.$$

If we take the largest of the two eigenvalues, say λ_1, and perform the derivative for the aspiration intensity δ_x and then evaluate the derivative when $\delta_x = 0$, we obtain

$$\lim_{\delta_c\to 0,\delta_x\to 0}\frac{d\lambda_1}{d\delta_x} = -\frac{(\beta R)^{\frac{1}{\sigma}}}{R+(\beta R)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}} < 0. \tag{A.43}$$

Therefore, the introduction of aspirations either on adult or old consumption increases the speed of convergence around the steady state. *Q.E.D.*

To better understand the proposition, let us consider an economy where the agents' preferences do not exhibit aspirations ($\rho > 0,\ \delta_c = 0,\ \delta_x = 0$). IHere, the three eigenvalues of the dynamic system formed by the difference Equations (4.9), (4.10), and (4.11) are

$$\lambda_1 = \frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}}{R+(\beta R)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}} \in (0,1),\ \ \lambda_2 = 0,\ \ \lambda_3 = 0.$$

Thus, the largest eigenvalue λ_1 determines the speed of convergence around the steady state and remains the largest eigenvalue when aspirations are marginally introduced. As indicated in the proof of Proposition 2, this eigenvalue decreases when introducing aspirations (see Equations (A.42) and (A.43)). Therefore, the introduction of either adult or old aspirations result in faster local convergence.

In our model, a high intensity of aspirations might be a source of endogenous fluctuations around the steady state. Particularly, if there are only adult aspirations (i.e., $\delta_x = 0$)), it can be proven that the eigenvalues of the matrix $\mathbb{P}$ given in Equation (A.39) are real and positive, and λ_1 is the dominating eigenvalue when

$$\delta_c \in \left[0, \frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left[2R+(\beta R)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}-2R^{\frac{1}{2}}\left(R+(\beta R)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right)^{\frac{1}{2}}\right]}{\left((\beta R)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right)^2}\right).$$

Similarly, when only old aspirations are present (i.e., $\delta_c = 0$)), the eigenvalues of the matrix $\mathbb{P}$ are real and positive, and λ_1 is the dominating eigenvalue when

$$\delta_x \in \left[0, \frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left[R+2(\beta R)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}-2(\beta R)^{\frac{1}{2\sigma}}\left(R+(\beta R)^{\frac{1}{\sigma}}n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right)^{\frac{1}{2}}\right]}{\left(R+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right)^2}\right).$$

Moreover, the upper values of the previous two intervals are bifurcations where the eigenvalues become complex. Therefore, oscillations arise in the economy under stronger aspirations.

Finally, for even larger values of the aspiration intensities, the eigenvalues become real again but the dominating eigenvalue is decreasing in the aspiration intensity such that the speed of convergence decreases with aspirations. In particular, these new bifurcations associated with large values of aspiration intensities occur when

$$\delta_c = \frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left[2R+(\beta R)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}+2\left(R+(\beta R)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right)^{\frac{1}{2}}R^{\frac{1}{2}}\right]}{\left((\beta R)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right)^2} \quad \text{if } \delta_x = 0$$

and

$$\delta_x = \frac{R\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\left[R+2(\beta R)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}+2(\beta R)^{\frac{1}{2\sigma}}\left(R+(\beta R)^{\frac{1}{\sigma}}+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right)^{\frac{1}{2}}\right]}{\left(R+n\left(\frac{\rho R}{n}\right)^{\frac{1}{\sigma}}\right)^2} \quad \text{if } \delta_c = 0.$$

Note that if the bequest motive were absent ($\rho = 0$), then the previous two bifurcations would not appear because all the bifurcation values for the aspiration intensities become equal to zero.

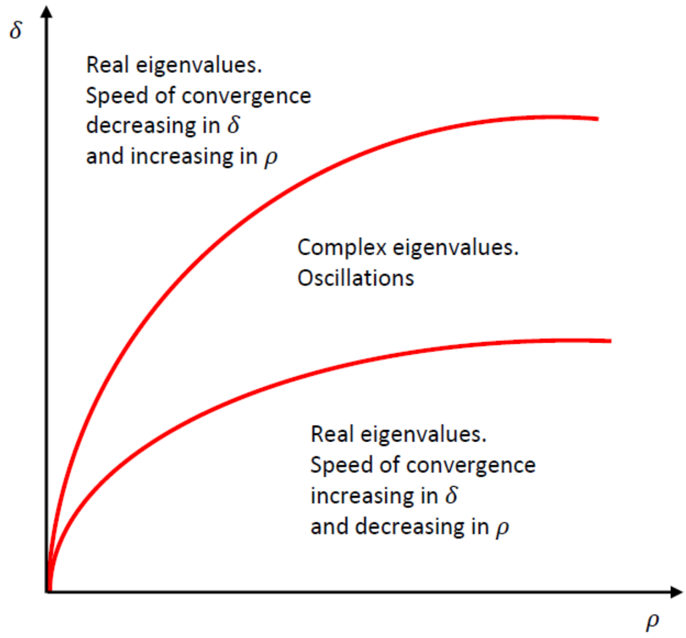

Figure 1. Eigenvalues for combinations of the bequest motive and aspiration intensities under the assumptions of Proposition 1.

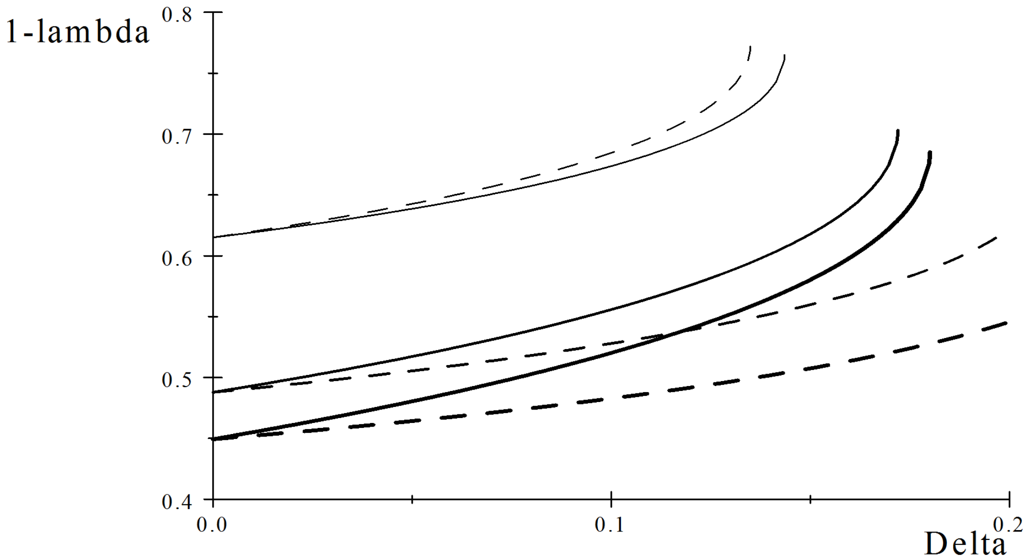

Figure 2. The effects of aspirations (δ) on the speed of convergence $1 - \lambda_{\max}$.
Solid line to δ_c. Thick line: $\sigma = 3/2$. Medium line: $\sigma = 1$. Thin line: $\sigma = 1/2$.
Dash line to δ_x. Thick line: $\sigma = 3/2$. Medium line: $\sigma = 1$. Thin line: $\sigma = 1/2$.

	Altruistic individuals					Non-altruistic individuals			
	s_t^A	b_{t+1}	$s_t^A - b_t$	$nb_{t+1} - s_t^A$	$nb_{t+1} - b_t$	s_t^N	$s_t^N - b_t$	$-s_t^N$	$-b_t$
$\frac{\partial}{\partial \rho}$	> 0	> 0	> 0	> 0	> 0				
$\frac{\partial}{\partial \delta_c}$	< 0	< 0	< 0	> 0 $\big(\text{if } \rho \to 0\big)$	< 0	< 0	< 0	> 0	0
$\frac{\partial}{\partial \delta_x}$	> 0	< 0	> 0	< 0	< 0	> 0	> 0	< 0	0

Table 1. Comparative statics of saving, bequest, and wealth accumulation.

	$s_t^A - s_t^N$	$nb_{t+1} - s_t^A + s_t^N$	nb_{t+1}
$\frac{\partial}{\partial \rho}$	> 0	> 0	> 0
$\frac{\partial}{\partial \delta_c}$	< 0	< 0	< 0
$\frac{\partial}{\partial \delta_x}$	< 0	< 0	< 0

Table 2. The exacerbating effect of the bequest motive and dampening effect of aspirations.

	Altruistic individuals					Non-altruistic Individuals			
	s^A	b	$s^A - b$	$nb - s^A$	$(n-1)\,b$	s^N	$s^N - b$	$-s^N$	$-b$
$\dfrac{\partial}{\partial \rho}$	> 0	> 0	$\gtreqless 0$ (if $n \lesseqgtr R$)	> 0	$\gtreqqless 0$	> 0 $\left(\text{if } {\delta_c \to 0 \atop \delta_x \to 0}\right)$	< 0 $\left(\text{if } {\delta_c \to 0 \atop \delta_x \to 0}\right)$	< 0 $\left(\text{if } {\delta_c \to 0 \atop \delta_x \to 0}\right)$	< 0
$\dfrac{\partial}{\partial \delta_c}$	< 0	< 0	< 0 (if $\rho \to 0$)	> 0 (if $\rho \to 0$)	$\lesseqqgtr 0$	< 0	< 0 (if $\rho \to 0$)	> 0	> 0
$\dfrac{\partial}{\partial \delta_x}$	> 0 (if $\rho \to 0$)	< 0	> 0	< 0	$\lesseqqgtr 0$	> 0 (if $\rho \to 0$)	> 0	< 0 (if $\rho \to 0$)	> 0

Table 3. Comparative statics of stationary saving, bequest, and wealth accumulation.

	$s^A - s^N$	$nb - s^A + s^N$	nb
$\dfrac{\partial}{\partial \rho}$	> 0	> 0	> 0
$\dfrac{\partial}{\partial \delta_c}$	< 0	< 0	< 0
$\dfrac{\partial}{\partial \delta_x}$	< 0	< 0	< 0

Table 4. The exacerbating effect of the bequest motive and the dampening effect of aspirations in the steady state