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Social rents, interest rates, and growth

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Abstract

We present a simple OLG model with a convex technology in which physical capital exhibits a productive externality, and workers receive part of the social rents generated by that externality. This setup allows for sustained growth without incurring poverty traps. Moreover, every equilibrium path displays constant interest rates as suggested by some empirical evidence.

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1. Introduction

Since the pathbreaking work of Romer (1986) and Lucas (1988), aiming at a theoretical explanation of sustained growth, many modern one-sector models of endogenous growth rely on two fundamental hypotheses on the economic environment: (1) the presence of externalities in the production process, and (2) the existence of increasing returns at the aggregate level.

Nevertheless, since the traditional business cycle theory has been mostly built upon classical economies with convex technologies and free of externalities, a natural question that arises is whether it is possible to obtain sustained growth in this classical environment. Within the infinite horizon growth model, Jones and Manuelli (1990) have answered positively this question, provided that real interest rates are uniformly bounded below by some appropriate magnitude. However, Boldrin (1992) and Jones and Manuelli (1992) have proved that sustained growth is impossible to achieve in the life-cycle, one-sector model [Diamond (1965)]. To get perpetual growth in the later model, Jones and Manuelli advocate income redistribution from old to young agents, whereas Boldrin appeals to the introduction of the aforementioned hypothesis on externalities and increasing aggregate returns. The latter author also assumes constant returns from the private inputs so that firms do not obtain profits.

In this paper we take Boldrin's model to analyze to what extent the assumption on increasing aggregate returns is important to obtain persistent growth. One of the reasons for dispensing with this assumption is that there is no clear empirical evidence on economies displaying increasing returns to scale at the aggregate level [see Backus et al. (1992)]. Moreover, an economy with both constant aggregate returns and positive external effects is a theoretical situation not explicitly

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explored in the current literature. In this situation there is an indeterminacy concerning who gets the surpluses generated by the external effect. Unlike Jones and Manuelli's redistribution policy, only those surpluses are distributed in our model and, therefore, old agents enjoy a disposable income greater or equal than the competitive return on saving.

Our analysis shows that sustained growth is impossible when the rents from the social factor are completely appropriated by the old consumers. However, if part of this surplus is given by young consumers, then sustained growth is possible under some additional conditions on the propensity to save and the magnitude of the external effects. Furthermore, if young consumers get a sufficiently high share of social rents, then it is also possible to rule out poverty traps. Finally, throughout the analysis we use a class of production functions for which all equilibrium paths exhibit constant real interest rates as some empirical evidence suggests.

2. The model

Let us consider an overlapping generations (OLG) economy with production in which individuals live two periods, and a new generation is born in each period. There is a single commodity which can be either consumed or invested.

We assume that all consumers have the same preferences which are represented by the utility function $U(c_t^{\mathrm{y}}, c_{t+1}^{\mathrm{o}})$, where c_t^{y} and c_{t+1}^{o} denote the young and old consumption, respectively, of an individual born at t. This utility function is twice continuously differentiable, strictly increasing, strictly quasi-concave, homothetic, and its indifference curves lie on the interior of the positive orthant.

Young consumers own a unit of labor which is inelastically offered to the firms, and there are N workers per firm in each period. Firms generate positive profits under the technological assumptions we will make below. We assume that workers might get part of those profits as a result, for instance, of a standard bargaining process between firms and unions which we do not model here. In this case, the equilibrium distribution of profits depends on the relative bargaining power of workers and capital owners, and it might be explicitly computed by applying the Nash bargaining solution, say, to this division problem. [1] Thus, the representative consumer solves the following maximization problem:

$$\max_{\{c_t^{\mathrm{y}}, c_{t+1}^{\mathrm{o}}, s_t\}} U(c_t^{\mathrm{y}}, c_{t+1}^{\mathrm{o}}),$$

$$\text{subject to } c_t^{\mathrm{y}} + s_t = w_t + \frac{\theta \pi_t}{N},$$

$$c_{t+1}^{\mathrm{o}} = R_{t+1} s_t + \frac{(1-\theta)\pi_{t+1}}{N},$$

$$c_t^{\mathrm{y}} \geq 0,\ c_{t+1}^{\mathrm{o}} \geq 0,\ s_t \geq 0,$$

where w_t is the real wage, $R_{t+1} \equiv 1 + r_{t+1}$ is the gross rate of return on saving at $t+1$, π_t denotes the profits of each firm at t, $\theta \in [0, 1]$ is the constant share of profits received by young consumers, and s_t is the saving of an agent born at t.

[1] Salinger (1984) reports evidence on unionized workers capturing most of the profits accruing from the monopoly power enjoyed by firms in the goods market. In our model, positive profits appear instead from the assumption on decreasing returns associated with private inputs.

From the consumer's problem, and under the previous assumptions on the utility function U, we obtain the following saving function:

$$s_t = \hat{s}(R_{t+1})\left(w_t + \frac{\theta\pi_t}{N}\right) - (1 - \hat{s}(R_{t+1}))\left(\frac{(1-\theta)\pi_{t+1}}{NR_{t+1}}\right), \tag{1}$$

where $\hat{s}(R_{t+1})$ is the propensity to save out of first-period income. Note that, from the homotheticity of preferences, $\hat{s}(R_{t+1})$ is a function of the rate of return only. From normality and the boundary condition, we have that $0 < \hat{s}(R_{t+1}) < 1$.

We assume that there are competitive firms in each period. The net production function of each firm is

$$F(K_t, L_t, \bar{K}_t) = \mu(K_t, \bar{K}_t) + \nu(L_t, \bar{K}_t), \tag{2}$$

where K_t and L_t denote respectively the capital and the labor used by the firm, and $\bar{K}_t$ is the average capital installed in the economy in period t. We have thus the typical externality from capital as in Romer (1986). Obviously, since all firms have access to the same technology, we will have $K_t = \bar{K}_t$ in equilibrium, for all t. The functions μ and ν are twice continuously differentiable, linearly homogeneous, and concave. The function μ is strictly increasing in the average capital $\bar{K}_t$, and ν is strictly increasing in its two arguments. The single-variable function $\psi(K) \equiv \mu(K, K)$ is also strictly increasing. We also assume that $\mu(0,0) = \nu(L, 0) = 0$, i.e. capital is essential for positive production. Finally, we assume that

$$\lim_{\bar{K}\to 0} (\nu_{\bar{K}}(L, \bar{K})) = \infty, \tag{3}$$

$$\lim_{\bar{K}\to 0} (\nu_{L\bar{K}}(L, \bar{K})) = \infty, \tag{4}$$

$$\epsilon(L, \bar{K}) \equiv \frac{\nu_{L\bar{K}}(L, \bar{K})L}{\nu_{\bar{K}}(L, \bar{K})} \le 1, \tag{5}$$

where the subindexes in capital letters denote the variable with respect to which the partial derivative is taken. Assumptions (3) and (4) are somewhat standard Inada conditions, whereas condition (5) on the elasticity $\epsilon(L, \bar{K})$ will guarantee the existence of equilibrium paths. Obviously, the Cobb–Douglas functions satisfy (3)–(5).[2]

The choice of the functional form (2) not only simplifies our computations but also has some features which are worth emphasizing. Since we are interested in obtaining competitive paths displaying sustained growth without imposing increasing aggregate returns, it is necessary to assume that the non-reproducible input (in this case, labor) is not essential for production.[3] Hence, we need

$$F(K, 0, K) > 0, \quad \text{for all } K > 0. \tag{6}$$

Moreover, there is strong empirical evidence in favor of constant interest rates abstracting from business cycle fluctuations [see King and Rebelo (1990) and the abundant references therein]. This reported lack of trend in the interest rates series applies not only to economies in a steady state,

[2] More generally, condition (5) holds when, besides satisfying its aforementioned properties, the function ν can be written as $\nu(L, \bar{K}) = \kappa(L) \cdot \lambda(\bar{K})$, with κ and λ being concave functions.

[3] See Romer (1989) and Jones and Manuelli (1990) for the proof.

but also to the ones that are in the transition towards a steady state, or that are perpetually growing. In this respect, our technological assumption resembles that of the 'Ak' models [Rebelo (1991)]. It is easy to show that the functions belonging to the class described in (2) are the unique ones that simultaneously satisfy (6), display externalities from capital, are linearly homogeneous, and exhibit constant interest rates for $K_t = \bar{K}_t$.

The profit maximization objective for each firm, taking the externality $\bar{K}_t$ as given, yields the following first-order conditions for all t:

$$r_t = F_K(K_t, L_t, \bar{K}_t) = \mu_K(K_t, \bar{K}_t) , \tag{7}$$

$$w_t = F_L(K_t, L_t, \bar{K}_t) = \nu_L(L_t, \bar{K}_t) . \tag{8}$$

Thus, the positive profits of a firm at time t are

$$\pi_t = \mu(K_t, \bar{K}_t) + \nu(L_t, \bar{K}_t) - \mu_K(K_t, \bar{K}_t)K_t - \nu_L(L_t, \bar{K}_t)L_t . \tag{9}$$

These profits can be dubbed social rents since they are in fact equal to the marginal productivity of the average capital installed in the economy.

The equilibrium conditions for the labor and capital markets in each period are, for all t,

$$L_t = N , \tag{10}$$

$$K_{t+1} = Ns_t . \tag{11}$$

Notice also that, in equilibrium,

$$\bar{K}_t = K_t , \tag{12}$$

and therefore $r_t = \mu_K(K_t, \bar{K}_t)$ is constant regardless of the value of K_t. Let us denote by $\bar{r}$ this constant interest rate, and let $\bar{R} \equiv 1 + \bar{r}$ be the constant gross rate of return on capital. The partial derivative $\mu_{\bar{K}}(K_t, \bar{K}_t)$ is also constant along an equilibrium path, and $\mu_{\bar{K}}$ denotes such a derivative.

Combining (1) and (7)–(12) and using the homotheticity of the utility function, which makes the saving function linearly homogeneous in the income profile, we get the following difference equation:

$$\begin{aligned} K_{t+1} &+ (1 - \hat{s}(\bar{R}))\Big(\frac{(1-\theta)}{\bar{R}}(\mu(K_{t+1}, K_{t+1}) + \nu(N, K_{t+1}) - \bar{r}K_{t+1} - \nu_L(N, K_{t+1})N)\Big) \\ &= \hat{s}(\bar{R})(\theta(\mu(K_t, K_t) + \nu(N, K_t) - \bar{r}K_t) + (1-\theta)\nu_L(N, K_t)N) . \end{aligned} \tag{13}$$

A non-negative sequence $\{K_t\}_{t=0}^{\infty}$ solving (13) defines a competitive equilibrium path of capital accumulation. It is obvious that if the LHS of (13) has a derivative with respect to K_{t+1} uniformly bounded below by a positive number, then the existence of a unique equilibrium path for every given initial capital $K_0 \geq 0$ is guaranteed. This is indeed the case since that derivative is equal to

$$1 + (1 - \hat{s}(\bar{R}))\Big(\frac{(1-\theta)}{\bar{R}}(\mu_{\bar{K}} + \nu_{\bar{K}}(N, K_{t+1}) - \nu_{L\bar{K}}(N, K_{t+1})N)\Big) ,$$

which, by virtue of (5), turns out to be always greater than one.

3. Poverty traps and sustained growth

Equation (13) implicitly defines a function ϕ such that $K_{t+1} = \phi(K_t)$. This equation satisfies $\phi(0) = 0$. Implicitly differentiating (13) we can compute the derivative of such a function:

$$\phi'(K_t) = \frac{\hat{s}(\bar{R})(\theta(\mu_{\bar{K}} + \nu_{\bar{K}}(N, K_t)) + (1-\theta)\nu_{L\bar{K}}(N, K_t)N)}{1 + (1 - \hat{s}(\bar{R}))\left(\frac{(1-\theta)}{\bar{R}}(\mu_{\bar{K}} + \nu_{\bar{K}}(N, K_{t+1}) - \nu_{L\bar{K}}(N, K_{t+1})N)\right)}. \tag{14}$$

This derivative is always positive and, therefore, the equilibrium path is monotonic so that equilibrium cycles do not exist.

We say that the economy displays a poverty trap if there exists a positive number K^* such that, for every initial capital level $K_0 \in (0, K^*)$, the equilibrium path of capital converges to zero. It is clear that we can rule out poverty traps when $\lim_{K_t \to 0}(\phi'(K_t)) > 1$. Let us define $\epsilon^* \equiv \lim_{\bar{K} \to 0}(\epsilon(L, \bar{K}))$, where the elasticity $\epsilon(L, \bar{K})$ is defined in (5). Using the properties of the production function, the limit of the derivative (14) turns out to be

$$\lim_{K_t \to 0}(\phi'(K_t)) = \frac{\bar{R}\hat{s}(\bar{R})[(1-\epsilon^*)\theta + \epsilon^*]}{(1 - \hat{s}(\bar{R}))(1-\theta)(1-\epsilon^*)}. \tag{15}$$

It is straightforward to see that (15) is strictly increasing in θ. Moreover, this limit becomes infinite for $\theta = 1$. Therefore, if workers (young consumers) receive a sufficiently high share of the social rents, then poverty traps fail to exist. However, it is possible to eliminate poverty traps even if $\theta < 1$: a sufficient condition that makes (15) greater than one for all $\theta \in [0, 1]$ is that

$$\frac{\bar{R}\hat{s}(\bar{R})}{1 - \hat{s}(\bar{R})} > \frac{1-\epsilon^*}{\epsilon^*}. \tag{16}$$

The LHS of (16) is strictly increasing in $\bar{R}$ if saving is increasing in the interest rate, that is, if young and old consumption are gross substitutes. In such a case, (16) holds if the gross rate of return $\bar{R}$ is high enough. We also arrive at the same conclusion if we assume instead that the utility function is additively separable:

$$U(c_t^{\mathrm{y}}, c_{t+1}^{\mathrm{o}}) = u(c_t^{\mathrm{y}}) + \delta u(c_{t+1}^{\mathrm{o}}), \quad \text{with } \delta > 0. \tag{17}$$

Then, from the other properties of $U(\cdot, \cdot)$ and Theorem 2.4-4 in Katzner (1970), $u(\cdot)$ is isoelastic, i.e. it takes the following functional form:

$$u(z) = \frac{z^{1-\gamma}}{1-\gamma}, \quad \text{with } \gamma > 0. \tag{18}$$

In this case, (16) becomes $(\delta\bar{R})^{1/\gamma} > (1-\epsilon^*)/\epsilon^*$ so that, as before, high interest rates are sufficient to rule out poverty traps even if the social rents are completely appropriated by the old agents.

We say that the economy displays sustained growth if there exists a non-negative number K^{**} such that, for every initial capital level $K_0 > K^{**}$, the equilibrium path of capital tends to infinity. It is also clear that sustained growth is achieved when $\lim_{K_t \to \infty}(K_{t+1}/K_t) > 1$.

If $\theta = 0$, we see from (13) that

$$\lim_{K_t\to\infty}\left(\frac{K_{t+1}}{K_t}\right)\le\lim_{K_t\to\infty}\left(\frac{\nu_L(N,K_t)N}{K_t}\right)=0\,,$$

where the last equality comes from the linear homogeneity and concavity of the function ν. Hence, sustained growth is impossible when workers do not capture any of the positive social rents generated by the externality, and happens in the convex versions of the models considered by Boldrin (1992) and Jones and Manuelli (1992). In this case wages do not grow fast enough to buy an unboundedly increasing amount of physical capital.

The situation is quite different when $\theta = 1$. In this case,

$$\lim_{K_t\to\infty}\left(\frac{K_{t+1}}{K_t}\right)=\hat{s}(\bar{R})(\mu_{\bar{K}}+\lim_{K_t\to\infty}(\nu_K(N,K_t)))\ge\hat{s}(\bar{R})\mu_{\bar{K}}\,.$$

It follows that, when workers capture a sufficiently high share of social rents, sustained growth may be achieved if

$$\hat{s}(\bar{R})\mu_{\bar{K}}>1\,. \tag{19}$$

A continuity argument allows us to conclude that, if (19) holds and the share θ is close enough (but not necessarily equal) to one, then the economy also displays sustained growth. Thus, the share of social rents provides young agents with an additional income which might allow them to purchase capital at an increasing pace.

We can compute condition (19) explicitly by means of an example. Let the function μ be Cobb–Douglas with depreciation,

$$\mu(K_t,\bar{K}_t)=AK_t^{\alpha}\bar{K}_t^{1-\alpha}-\rho K_t\,,\quad\text{with } A>0,\ \alpha\in(0,1)\text{ and }\rho\in(0,A)\,,$$

and the utility function be isoelastic [see (17) and (18)]. For this example, (19) becomes

$$A(1-\alpha)[1-(1+\delta^{1/\gamma}[1+A\alpha-\rho]^{(1-\gamma)/\gamma})^{-1}]>1\,. \tag{20}$$

If $\gamma\le 1$ [i.e. if $\hat{s}(\bar{R})$ is non-decreasing in $\bar{R}$], then the LHS of (20) is strictly increasing in A. In this case, we may also conclude that sufficiently high values of both the scaling factor A and the share θ will make perpetual growth possible.

4. Conclusion

In this paper we have considered a very stylized convex model of growth with decreasing returns from private inputs, and constant returns at the aggregate level. The average capital of the economy plays the role of a social productive input. This technological assumption allows the model to exhibit two empirically relevant features: constant interest rates along every equilibrium path, and positive profits accruing from the externality (social rents).

In this framework our analysis highlights the importance of the distribution of those social rents for both ruling out poverty traps and achieving perpetual growth. These two properties of the accumulation path are thus compatible with input owners getting a disposable income greater than or equal to the corresponding marginal productivities.

To have workers (young agents) getting most of the social rents is sufficient to eliminate poverty traps. However, this is not a necessary condition to rule out these traps: high interest rates will typically suffice to avoid asymptotic starvation even if all social rents go to old agents.

On the other hand, a strictly positive share of social rents going to workers is necessary to obtain sustained growth. In this case, we also need to have both a high saving propensity and large external effects from capital.

A consequence of our OLG model is that the existence of social institutions (like unions, redistributing policies, etc.), which enable workers to capture some of the firms' profits, may give rise to accumulation paths with appealing properties.

References

Backus, D., P. Kehoe and T. Kehoe, 1992, In search of scale effects in trade and growth, Journal of Economic Theory 58, 377–409.

Boldrin, M., 1992, Dynamic externalities, multiple equilibria, and growth, Journal of Economic Theory 58, 198–218.

Diamond, P., 1965, National debt in a neoclassical growth model, American Economic Review 55, 1126–1150.

Jones, L. and R. Manuelli, 1990, A convex model of economic growth: Theory and policy implications, Journal of Political Economy 98, 1008–1038.

Jones, L. and R. Manuelli, 1992, Finite lifetimes and growth, Journal of Economic Theory 58, 171–197.

Lucas, R., 1988, On the mechanics of economic development, Journal of Monetary Economics 22, 3–42.

Katzner, D., 1970, Static demand theory (The Macmillan Company, New York).

King, R. and S. Rebelo, 1990, Public policy and economic growth: Developing neoclassical implications, Working Paper, University of Rochester.

Rebelo, S., 1991, Long run policy analysis and long run growth, Journal of Political Economy 99, 500–521.

Romer, P., 1986, Increasing returns and long-run growth, Journal of Political Economy 94, 1002–1035.

Romer, P., 1989, Increasing returns and new developments in the theory of growth, Working Paper, University of Chicago.

Salinger, M., 1984, Tobin's q, unionization and the concentration–profits relationship, Rand Journal of Economics 15, 159–170.