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Market versus limit orders

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This paper compares two mechanisms of price formation for risky assets from the viewpoint of insiders' welfare. In one mechanism, prices are selected by competitive market makers who receive market orders from agents possessing inside information and from liquidity-constrained traders. In the second mechanism, prices are formed according to automatic market clearing and insiders submit limit orders. Since private information exhibits decreasing returns in both cases, the comparison of expected profits will depend on the ratio between precisions of private and public information.

1. Introduction

This paper compares two mechanisms of price formation for risky assets from the viewpoint of insiders' welfare. In one mechanism, prices are selected by competitive market makers who receive market orders from agents possessing inside information and from liquidity-constrained traders. In the second mechanism, prices are formed according to market clearing and insiders submit limit orders.

It will be shown that private information exhibits decreasing returns in both models of price formation. Too much private diverse information may be harmful to informed agents since their strategies become correlated, and this acts against the initial positive effect of having more private information. If private information is very precise compared to the public information, then expected profits of insiders are higher when the price is determined by competitive risk neutral market makers than when it is formed by automatic market clearing. The converse is true for liquidity-constrained traders. This result is a consequence of the decreasing returns associated with private information and the information sharing involved in the limit orders model.

The paper is organized as follows. Section 2 presents the model with market orders and competitive market makers and derives some of its properties. Section 3 conducts the same analysis for the mechanism with limit orders and automatic market clearing. I make a welfare comparison of those two mechanisms of price formation in section 4.

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2. The model with market orders and competitive market makers

The model of this section follows closely the one in Admati and Pfleiderer (1988). Let us consider a market with a single financial asset whose random payoff $\tilde{v}$ is normally distributed with mean $\bar{v}$ and precision (the inverse of the variance) equal to $\tau_v \in (0, \infty)$.

There are three kinds of traders in the market: informed traders, noise traders and market makers. All agents are assumed to be risk neutral. The noise traders provide a random demand $\tilde{z}$ of risky asset. These traders either buy or sell quantities of risky asset motivated by liquidity constraints which are not related to the payoff of the financial asset. This random demand is normally distributed with variance $\sigma_z^2 > 0$ and, without loss of generality, with zero mean.

There are N informed traders, indexed by n, in the market who trade on the basis of their private information about the future payoff of the risky asset. They know the parameters of the distribution of the total net demand $\tilde{z}$ for shares by the noise traders but they ignore the exact realization of that demand. Each informed trader receives a piece of private information which takes the form of a signal $\tilde{s}_n$ where $\tilde{s}_n = \tilde{v} + \tilde{\epsilon}_n$. The noise $\tilde{\epsilon}_n$ of the signal is also normally distributed with zero mean and precision $\tau_\epsilon > 0$ for all n. The random variables $\tilde{v}$, $\tilde{z}$, $\tilde{\epsilon}_1, \ldots,$ and $\tilde{\epsilon}_N$, are assumed to be mutually independent. The optimal demand of the informed trader n is denoted as $\tilde{x}_n = x_n(\tilde{s}_n)$. Note that demands are not allowed to be contingent on prices. This possibility is considered in the next section.

The model has two periods. In period 1 each trader submits a market order to a risk neutral market making sector. Market makers establish a price p for the risky asset after observing the total net quantity demanded by the investors. It is important to note that market makers observe only the aggregate demand. Thus, they cannot know if an order comes from an informed trader or from a noise trader. The parameters of the distributions of the random variables $\tilde{v}$, $\tilde{z}$ and $\tilde{\epsilon}_n$ are common knowledge for all agents.

Competition among market makers is assumed. This competition among price setters forces them to select a price such that they earn zero expected profits as in the Bertrand model of oligopolistic competition. Neither the market makers nor the informed agents have short-selling constraints. Thus, the market making sector must sell $\tilde{w}$ shares, where $\tilde{w}$ is the net total order flow

$$\tilde{w} = \sum_{n=1}^{N} x_n(\tilde{s}_n) + \tilde{z}. \tag{1}$$

The zero profit (or market efficiency) condition implies that the selected price equals the expected payoff conditional on all information available to market makers. Thus, the price $\tilde{p} = p(\tilde{w})$ is a random variable which is measurable with respect to order flows, and satisfies

$$p(\tilde{w}) = \mathrm{E}(\tilde{v} \mid \tilde{w}). \tag{2}$$

In period 2, the realization of $\tilde{v}$ is observed and each agent receives his payoff.

The objective of an insider trader is to maximize expected profits conditional on his information. The optimal demand for risky asset of an informed agent n is

$$x_n(s_n) = \underset{x \in \mathbb{R}}{\operatorname{argmax}}\ \mathrm{E}\big[(\tilde{v} - p(\tilde{w}))x \mid s_n\big].$$

Definition. The equilibrium of the model with market orders and competitive market makers is a set of N strategies $x_n(\tilde{s}_n)$, $n = 1, \ldots, N$, which maximize the expected profits for each informed trader given the observed signal, and a price function $p(\tilde{w})$ which makes the expected profits of market makers equal to zero for each realization of the order flow.

For reasons of tractability, I restrict attention to symmetric and linear equilibria, i.e., equilibria in which $x(\tilde{s}_n) = x_n(\tilde{s}_n)$ for all n, and both $x(\tilde{s}_n)$ and $p(\tilde{w})$ are linear functions. The equilibrium of the proposed game is given in the next proposition.

Proposition 2.1. There exists a unique symmetric, linear equilibrium with diverse private information which is given by

$$\tilde{x}_n = x(\tilde{s}_n) = \beta(\tilde{s}_n - \bar{v}), \quad n = 1, \ldots, N,$$

$$\tilde{p} = p(\tilde{w}) = \bar{v} + \lambda \tilde{w},$$

where

$$\beta = \left(\frac{\sigma_z^2}{N(1/\tau_v + 1/\tau_\epsilon)} \right)^{1/2} \quad \textit{and} \quad \lambda = \frac{1}{2(\tau_v/\tau_\epsilon) + N + 1} \left(\frac{N(1/\tau_v + 1/\tau_\epsilon)}{\sigma_z^2} \right)^{1/2}.$$

Proof. The computation of the equilibrium follows the same steps as the proofs of Lemmas 1 and 3 in Admati and Pfleiderer (1988). However, we must replace $\sigma_v^2 = 1$ by $1/\tau_v$ in their proof.

The coefficient λ is the inverse of the depth of the market which is defined as the order flow required to change the price of the risky asset by one dollar.

The following Corollaries will be useful in order to conduct the welfare analysis of section 4.

Corollary 2.2. Expected profits for informed traders are

$$\mathrm{E}(\pi^n) = \frac{1}{2(\tau_v/\tau_\epsilon) + N + 1} \left(\frac{\sigma_z^2(1/\tau_v + 1/\tau_\epsilon)}{N} \right)^{1/2}. \tag{3}$$

Proof. Compute the total (negative) profits $\sum_{t=1}^{T} \mathrm{E}(\pi^t) = -\lambda \sigma_z^2$ of noise traders. This, together with the zero profit condition for the market maker, implies that total expected profits for insiders are equal to the negative of total expected profits of liquidity traders. Therefore, the expected profits of each insider are equal to $\lambda \sigma_z^2 / N$, which is equivalent to (3).

Evaluating the derivative of $\mathrm{E}(\pi^n)$ with respect to τ_ϵ, and defining the private–public information ratio $a \equiv \tau_\epsilon / \tau_v$, we obtain

Corollary 2.3. Expected profits of informed agents are strictly decreasing (increasing) in τ_ϵ iff $((N-3)/2)a > (<)\ 1$. The opposite is true for liquidity-constrained traders.

Corollary 2.3 tells us that informed traders might be worse off when more precise private information becomes available. There is an initial positive effect for the insiders of having more precise private information: their informational position with respect to the market maker improves. However, when private information becomes more precise, private signals become similar since they cannot differ too much from the true payoff $\tilde{v}$. This increases the correlation among the demands submitted by insiders and, therefore, there will be less noise in the order flow due to trade crossed among insiders with different signals. This in turn implies that the market maker will be able to predict more accurately $(\sum_{n=1}^{N} s_n)/N$, which is the sufficient statistic for all the private information in the market. This negative effect on the price rule acts against the initial positive effect when the private–public information ratio a is high enough. Note that if $N < 3$, expected profits are always increasing in τ_ϵ: when insiders do not face too much competition, they are always better off by being better informed.

3. The model with limit orders and market clearing

I now consider a different mechanism of price formation in this section. This mechanism resembles the one used in the traditional models of noisy rational expectations [Hellwig (1980)], and its structure is based on Kyle (1989). The equilibrium prices will be formed by automatic market clearing and the quantities demanded by informed agents will be conditional on prices (limit orders). When agents are allowed to submit limit orders, the information sharing among informed agents increases greatly. Since insiders are now able to condition their demands on prices, they are able to infer part of others' information from the prices at which the transactions are carried out.

After observing their signals, informed agents will select a continuous demand function, i.e., a function which specifies for each price the number of shares that they are willing to buy. Let us denote X_n as the strategy of agent n. The strategy X_n is a map from $\mathbb{R}$ (the domain of private signals) into the set of functions from $\mathbb{R}$ (prices) to $\mathbb{R}$ (quantities). Therefore, $X_n(\cdot;\tilde{s}_n)$ is a random variable that takes values in the set of continuous functions. We can also define the function $x_n(\cdot,\cdot)$ from $\mathbb{R}^2$ to $\mathbb{R}$ as $x_n(p, s_n) \equiv X_n(p; s_n)$, which gives us the quantity of asset demanded by an agent, given a particular realization of $\tilde{s}_n$ and the price p.

Prices are formed according to an automatic market clearing rule. Thus, we assume that there is a computerized system that receives all the limit orders plus noise. The aggregate net demand function is defined by

$$D(\cdot) = \sum_{n=1}^{N} x_n(\cdot, \tilde{s}_n) + \tilde{z}.$$

The equilibrium price $\tilde{p}$ satisfies $D(\tilde{p}) = 0$.

It is also important to note that $\tilde{p}$ is a function of all the strategies used by informed agents. Therefore, I can write

$$\tilde{p} = p\big(X_1,\dots,X_N;\ \tilde{z},\tilde{s}_1,\dots,\tilde{s}_N\big).$$

The profits for the informed agent n are

$$\hat{\pi}^n(X_1,\dots,X_N) = (\tilde{v} - \tilde{p})X_n(\tilde{p};\tilde{s}_n).$$

Definition. The equilibrium of the model with limit orders and market clearing is the set of strategies $X_1^*,\dots,X_N^*$ such that

$$\mathrm{E}(\hat{\pi}^n(X_1^*,\dots,X_n^*,\dots,X_N^*)) \geqslant \mathrm{E}(\hat{\pi}^n(X_1^*,\dots,X_n,\dots,X_N^*)), \quad n = 1,\dots,N,$$

$$\text{for all } X_n.$$

For reasons of tractability, I am going to restrict the space of demand schedules to the space of linear demand functions, $X_n(p;\, s_n) = A_n(s_n) + C_n(s_n)p$. Furthermore, I assume that $A_n(s_n) = \hat{\alpha}_n + \hat{\beta}_n s_n$ and $C_n(s_n) = \hat{\mu}_n$, for all $s_n \in \mathbb{R}$. I will also restrict attention to symmetric equilibria, i.e., $\hat{\alpha}_n = \hat{\alpha}$, $\hat{\beta}_n = \hat{\beta}$ and $\hat{\mu}_n = \hat{\mu}$ for all n.

Proposition 3.1. When $N > 2$, there exists a unique symmetric and linear equilibrium with limit orders and market clearing. This equilibrium is given by

$$x_n = \hat{\alpha} + \hat{\beta} s_n - \hat{\mu} p, \quad n = 1, \ldots, N,$$

where

$$\hat{\alpha} = \frac{2\tau_v \bar{v}}{N^2}\left(\frac{N(N-2)\sigma_z^2}{(N-1)\tau_\epsilon}\right)^{1/2}, \quad \hat{\beta} = \left(\frac{(N-2)\sigma_z^2\tau_\epsilon}{N(N-1)}\right)^{1/2}$$

and

$$\hat{\mu} = \frac{2\tau_v + N\tau_\epsilon}{N^2}\left(\frac{N(N-2)\sigma_z^2}{(N-1)\tau_\epsilon}\right)^{1/2}.$$

Proof. The computation of the equilibrium follows the same steps as the proof of Theorem 5.1 in Kyle (1989). When informed traders are risk neutral, it is possible to obtain the explicit solution given in the statement of the proposition. The complete derivation can be found in Caballé (1991).

As Kyle (1989) notes, the requirement of $N > 2$ is similar to the one in the theory of Cournot competition. There, the equilibrium fails to exist if the demand is infinitely inelastic. Here, liquidity traders have an infinitely inelastic demand, and having more than two informed agents suffices to have bounded demands for the insiders.

From the equilibrium given in Proposition 3.1, the equilibrium price satisfies

$$\tilde{p} = \frac{N\hat{\alpha} + \hat{\beta}\sum_{n=1}^{N}\tilde{s}_n + \tilde{z}}{N\hat{\mu}}. \tag{4}$$

The following corollary gives the expected profits of insiders submitting limit orders:

Corollary 3.2. Expected profits of informed traders are

$$\mathrm{E}(\hat{\pi}^n) = \frac{1}{2(\tau_v/\tau_\epsilon) + N}\left(\sigma_z^2(N-1)/N(N-2)\tau_\epsilon\right)^{1/2}. \tag{5}$$

Proof. Compute

$$\mathrm{E}(\hat{\pi}^n) = \mathrm{E}((\tilde{v} - \tilde{p}) \cdot x_n(\tilde{p}, \tilde{s}_n)),$$

and use (4) and the equilibrium values of $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\mu}$ in Proposition 3.1 to obtain (5).

Corollary 3.3. Expected profits of informed agents are strictly decreasing (increasing) in τ_ϵ iff $(N/2)a > (<)1$.

Proof. As in Corollary 2.4.

Corollary 3.3 tells us that private information exhibits the same decreasing returns as we have seen in section 2.

4. A comparison result

I have introduced two mechanisms of price formation. These two mechanisms are the most popular in the finance literature and it seems natural to ask which mechanism delivers higher expected profits for the insiders. In our zero-sum context this is equivalent to asking which mechanism delivers the lowest cost of trading for the liquidity-constrained traders. The result of the comparison follows from the discussion about the value of private information. Too much private information is harmful for informed agents in both mechanisms of price formation. As we have said, the limit orders model involves more information sharing than the market orders model because the equilibrium price reveals others' private information. Therefore, when private information is very precise, the informed traders in a regime with limit orders and automatic market clearing would prefer to switch to a regime with market orders in which there is no information sharing through prices, and in which all agents become less informed. The converse argument applies when private information is very imprecise.

The following Proposition confirms the previous intuitive argument.

Proposition 4.1. For $N > 2$, there exists a $\tau_\epsilon^* \in (0, \infty)$ such that $\mathrm{E}(\pi^n) > (<)\, \mathrm{E}(\hat{\pi}^n)$ iff $\tau_\epsilon > (<)\, \tau_\epsilon^*$.

Proof. Using the expressions for the expected profits of insiders in both regimes given in (3) and (5), compute their ratio and simplify to obtain

$$Q(a) = \frac{\mathrm{E}(\hat{\pi}^n)}{\mathrm{E}(\pi^n)} = \left[\frac{(N-1)}{(N-2)(1+a)}\right]^{1/2}\left(1 + \frac{a}{2+Na}\right),$$

where $a \equiv \tau_\epsilon/\tau_v$. It can be proved that $\partial Q(a)/\partial a < 0$ when $N > 2$. Also

$$\lim_{a\to 0} Q(a) = \left(\frac{N-1}{N-2}\right)^{1/2} > 1 \quad \text{and} \quad \lim_{a\to\infty} Q(a) = 0.$$

Therefore, by continuity of $Q(\cdot)$, there exists an $a^* \in (0, \infty)$ such that $Q(a^*) = 1$. Then, $\tau_\epsilon^* = a^*\tau_v$ is the desired threshold that equates both expected profits.

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