PS 3 - MICRO

3) 2F16

(a) $x(\alpha p, \alpha w) = \left(\frac{\cancel{\alpha} p_2}{\cancel{\alpha} p_3}, -\frac{\cancel{\alpha} p_1}{\cancel{\alpha} p_3}, \frac{\cancel{\alpha} w}{\cancel{\alpha} p_3} \right) = x(p, w)$

$p \cdot x(p, w) = p_1 \frac{p_2}{p_3} - p_2 \cdot \frac{p_1}{p_3} + p_3 \frac{w}{p_3} = w$

(b) Consider the following example

$(p, w) = ((1, 0, 1), 1) \Rightarrow x(p, w) = (0, -1, 1)$

$(p', w') = ((0, 1, 1), 0) \Rightarrow x(p', w') = (1, 0, 0)$

$x(p, w) \neq x(p', w')$ and $p \cdot x(p', w') = 1 = w$, namely $x(p', w')$ was affordable but not chosen.

However $p' \cdot x(p, w) = -1 + 1 = 0 = w'$, namely $x(p, w)$ was affordable as well when prices and wealth are (p', w')

WARP is violated.

1) Given that the demand satisfies Walras' law:

$p_1 x_1(p, w) + p_2 x_2(p, w) = w \iff$

$\cancel{p_1} \frac{\alpha w}{\cancel{p_1}} + p_2 x_2(p, w) = w \iff$

$x_2(p, w) = \frac{(1-\alpha) w}{p_2}$

We can now show that $x(p, w) = \left(\frac{\alpha w}{p_1}, \frac{(1-\alpha) w}{p_2} \right)$ is homogeneous of degree 0:

$\forall t > 0, \; x(tp, tw) = \left(\frac{\alpha \cancel{t} w}{\cancel{t} p_1}, \frac{(1-\alpha) \cancel{t} w}{\cancel{t} p_2} \right) = \left(\frac{\alpha w}{p_1}, \frac{(1-\alpha) w}{p_2} \right) = x(p, w)$

) $u(x_1, x_2) = -(x_1 - x_2)^q$ for $x_1, x_2 \geq 0$ and $q > 0$ even number

(q even number ensures that the domain is well defined and that $(x_1 - x_2)^q \geq 0 \quad \forall\, x_1, x_2$)

) We can derive equations for the indifference curves.

Note first that $u(x_1, x_2) \leq 0$, because $(x_1 - x_2)^q \geq 0$

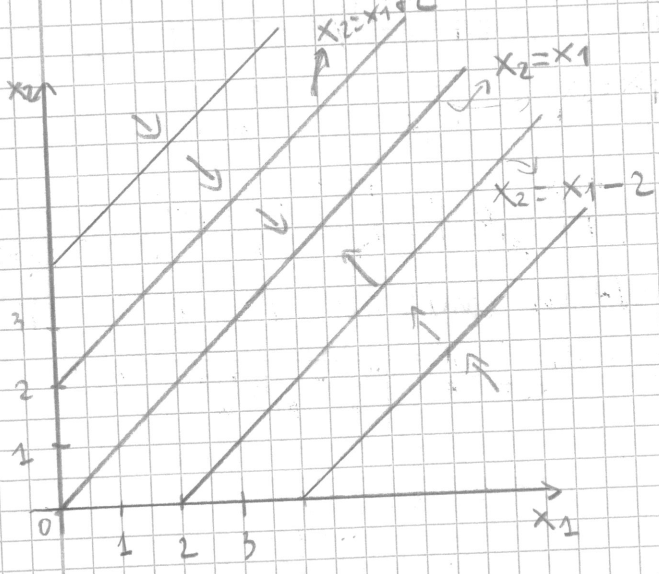
Let $u(x_1, x_2) = -k, \quad k \geq 0$

$$-(x_1 - x_2)^q = -k \iff (x_1 - x_2)^q = k \iff$$

$$\iff \quad x_1 - x_2 = \pm\left(k^{\frac{1}{q}}\right)$$

Summarizing $u(x_1, x_2) = -k \iff x_2 = x_1 - k^{\frac{1}{q}}$ or $x_2 = x_1 + k^{\frac{1}{q}}$

Thus for $k > 0$ the indifference curves are the union of two straight lines; for $k = 0$, it is the line $x_2 = x_1$.



Note that $\max\limits_{x_1, x_2} u(x_1, x_2) = 0$ and all the points on the line $x_2 = x_1$ are satiation points, therefore preferences are <u>not</u> locally non satiated

Moreover preferences are <u>not</u> weakly monotone. For instance $(3, 2) \geq (1, 1)$ but $u((3, 2)) = -(3-2)^q = -1^q = -1$ and $u((1,1)) = -(1-1)^q = 0$, namely $u((3,2)) < u((1,1))$.

b) No, the two underlying utilities are completely different,
Leontief preferences are both locally non satiated and weakly monotone.
The only similarity is that in both case for $p \gg 0$ the
consumer maximizes her utility choosing a bundle on
the 45° line, but for u any affordable bundle on the
45° line can be chosen, while the consumer with leontief
preference would choose the one with largest x.

Let $x(p,w)$ be the demand for u and $x'(p,w)$ the demand if preferences are U

$x(p,w) = \{(x,x) \mid x \geq 0,\ (p_1+p_2)x \leq w\}$ → It's a correspondence

$x'(p,w) = \{(x,x) \mid x \geq 0,\ (p_1+p_2)x = w\} = \left\{\left(\frac{w}{p_1+p_2}, \frac{w}{p_1+p_2}\right)\right\}$ → a function

For instance $(0,0) \in x((1,1),1)$ but $(0,0) \notin x'((1,1),1) = \left(\frac{1}{2},\frac{1}{2}\right)$

bis) Another counter example.

Recall: WARP $\Longleftrightarrow$ for any compensated price change and $\Delta x \neq 0$, $\Delta p \cdot \Delta x < 0$

$p = (1,1,1)$, $w = 3$ $\Rightarrow$ $x(p) = (1,-1,3)$

[Notice that basically the consumer is "selling" good 2 to buy good 1, and then he uses all his wealth to buy good 3.]

$p' = (1,2,1)$, i.e. good 2 is now more expensive.

w' that compensates for the change is $w' = p'x(p,w) = 1-2+3 = 2$

(i.e. decrease in wealth, because selling good 2 is now more profitable).

The new demand is $x(p',w') = (2,-1,2) \neq x(p,w)$

$(p'-p)\cdot(x(p')-x(p)) = (0,1,0)\cdot(1,0,-1) = 0$

Thus WARP is violated.