MICRO - PS 2

Ex 2E5 . Recall:

Homogeneity of degree 0: $x(\alpha p, \alpha w) = x(p,w)$

Walras' law: $\forall p \gg 0,\ w > 0 \quad p \cdot x = w \quad \forall x \in x(p,w)$.

We assume: ① $x(p, \alpha w) = \alpha x(p,w)$

② $\dfrac{\partial x_\ell(p,w)}{\partial p_k} = 0 \quad \forall k, \ell \quad k \neq \ell$

We want to show that this implies $\forall \ell,\ x_\ell(p,w) = \alpha_\ell \dfrac{w}{p_\ell}$, $\alpha_\ell \geq 0$ constant.

Lemma: If $x(p,w)$ is homogeneous of degree 0, and $x(p,w)$ is homogeneous of degree 1 in w, then $x(p,w)$ is homogeneous of degree -1 in p.

proof: $x(\alpha p, w) = x\left(\alpha p, \dfrac{\alpha w}{\alpha}\right) = x\left(p, \dfrac{w}{\alpha}\right) = \dfrac{1}{\alpha} x(p,w)$.

$x(p,w) = (x_1(p,w), .., x_n(p,w)) = (w\, x_1(p,1), .., w\, x_n(p,1))$,

i.e. $x_\ell(p,w) = w\, x_\ell(p,1)$.

Moreover $\dfrac{\partial x_\ell}{\partial p_k} = 0 \quad \forall k \neq \ell$, thus $x_\ell(p,w) = x_\ell(p_\ell, w) = w\, x_\ell(p_\ell, 1)$

From the lemma, $x(\alpha p, w) = \dfrac{1}{\alpha} x(p,w)$, but $x(\alpha p, w) =$

$= (x_1(\alpha p_1, w), .., x_n(\alpha p_n, w)) = \left(\dfrac{1}{\alpha} x_1(p_1, w), .., \dfrac{1}{\alpha} x_n(p_n, w)\right)$

i.e. $x_\ell(\alpha p_\ell, w) = \dfrac{1}{\alpha} x_\ell(p_\ell, w)$, x_ℓ is homogeneous of degree -1 in p_ℓ for each ℓ.

$x_\ell(p_\ell, w) = x_\ell(1,1) \cdot \dfrac{w}{p_\ell} = \alpha_\ell \dfrac{w}{p_\ell}$

$\alpha_\ell \geq 0$, moreover for some ℓ it should be $\alpha_\ell > 0$, because, by walras law, for $p \gg 0$ it holds the following:

$p \cdot x = w \iff \sum p_\ell \alpha_\ell \dfrac{w}{p_\ell} = w \iff$

$\iff \sum \alpha_\ell = 1 \implies \alpha_\ell > 0$ for some ℓ.

F3 Recall weak axiom of revealed preferences:
Walrasian demand function $x(p,w)$ satisfies weak axiom if
$\forall (p,w), (p',w')$
$p \cdot x(p',w') \leq w$ and $x(p',w') \neq x(p,w) \Rightarrow p' \cdot x(p,w) > w'$

Thus it is violated if there exists $(p,w), (p',w')$ s.t.
$p \cdot x(p',w') \leq w$, $x(p',w') \neq x(p,w)$ and $p' \cdot x(p,w) \leq w'$

In the exercise $(p,w) = ((100,100), 100\times100 + 100\times100) = ((100,100), 20000)$
$(p',w') = ((100,80), 100\cdot120 + 80y)$
$x(p,w) = (100,100) \neq (120,y) = x(p',w')$
Thus the axiom is violated if:

$$\begin{cases} 100\cdot120 + 100\cdot y \leq 20000 \\ 100\cdot100 + 80\cdot100 \leq 120\cdot100 + y\cdot80 \end{cases} \Rightarrow \begin{cases} y \leq 80 \\ y \geq 75 \end{cases}$$

$$\boxed{y \in [75,80]}$$

b) It is revealed preferred if $(120,y)$ was affordable but not chosen.
Moreover the weak axiom should hold:

$$\begin{cases} y \notin [75,80] \\ 100\cdot120 + 100 y \leq 20000 \end{cases} \Rightarrow y < 75$$

c) If $(100,100)$ is affordable but not chosen, and weak axiom satisfied:

$$\begin{cases} y \notin [75,80] \\ 100\cdot100 + 80\cdot100 \leq 120\cdot100 + y\cdot80 \end{cases} \Rightarrow y > 80$$

d) a), b), c) together cover all $\mathbb{R}^+$, we always have enough information.

Recall Inferior good: demand rises when income decreases.

e) We assume $y \notin [75,80]$
Intuition: for $y < 75$, the consumer has less total wealth, but demand for good 1 increases.

$y < 75 \Rightarrow 120 \cdot 100 + 80 \cdot y < 18000 < 20000$

The consumer is poorer in year 2, but he buys more of good 1, ~~absolute~~ price is the same and relative price increase; then good 2 is inferior

More formally:

$W = 120 \cdot 100 + y \cdot 80 < 120 \cdot 100 + y \cdot 75 = 18000 < 20000$

$x_1((100,80), w) = 120$

Suppose good 1 is normal: then $x_1((100,80), 18000) \geq 120$

Notice that $((100,80), 18000)$ is a compensated price change.

Then (assuming homogeneity of degree 0 and walras law) given that $x(p,w)$ satisfies the weak axiom:

$$((100,100) - (100,80)) \cdot ((100,100) - (x_1, x_2)) < 0$$

$$\Rightarrow \quad 20 \cdot (100 - x_2) < 0 \quad \Rightarrow \quad x_2 > 100$$

But then $100 \cdot x_1 + 80 \cdot x_2 > 100 \cdot 120 + 80 \cdot 100 = 20000 > 18000$, contraddiction. It can't be $x_1((100,80), 18000) \geq 120$

Thus good 1 is inferior.

8) Take $80 < y < 100$

$100 \cdot 120 + 100 \cdot y > 100 \cdot 100 + 80 \cdot 100 = 18000 \Rightarrow$ real wealth increases.

relative price of good 2 decreases, but the demand for good 2 decreases. A more formal argument can be derived as in point e)

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More formally:

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8) Take $80 < y < 100$

$100 \cdot 120 + 100 \cdot y > 100 \cdot 100 + 80 \cdot 100 = 18000 \Rightarrow$ real wealth increases. relative price of good 2 decreases, but the demand for good 2 decreases. A more formal argument can be derived as in point e)

11 Show that for $L=2$, $S(p,w)$ is always symmetric.

$$S(p,w) = \begin{bmatrix} s_{11}(p,w) & s_{12}(p,w) \\ s_{21}(p,w) & s_{22}(p,w) \end{bmatrix} = \begin{bmatrix} \frac{\partial x_1(p,w)}{\partial p_1} + \frac{\partial x_1(p,w)}{\partial w} x_1(p,w) & \frac{\partial x_1}{\partial p_2} + \frac{\partial x_1}{\partial w} x_2 \\ \frac{\partial x_2}{\partial p_1} + \frac{\partial x_2}{\partial w} x_1 & \frac{\partial x_2}{\partial p_2} + \frac{\partial x_2}{\partial w} x_2 \end{bmatrix}$$

We assume x differentiable, homogeneous of degree zero, satisfies Walras' law.

By prop 2F3 $\quad p S(p,w) = (0,0)$, $S(p,w) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ for any (p,w)

$$(p_1, p_2) \begin{pmatrix} s_{11} \\ s_{21} \end{pmatrix} = p_1 s_{11} + p_2 s_{21} = 0 \implies s_{21} = -\frac{p_1}{p_2} s_{11}$$

$$(s_{11} \;\; s_{12}) \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = p_1 s_{11} + p_2 s_{12} = 0 \implies s_{12} = -\frac{p_1}{p_2} s_{11}$$

From 1 and 2, it follows $s_{12} = s_{12}$. Thus the matrix is symmetric.

2F17 $\quad x_k(p,w) = \dfrac{w}{\left(\sum_{\ell=1}^{L} p_\ell\right)}$ for $k = 1, \dots, L$

a) $$x(\alpha p, \alpha w) = \left(\frac{\alpha w}{\left(\sum_{\ell=1}^{L} \alpha p_\ell\right)} \right)_{\ell = 1,\dots,L} = \left(\frac{\alpha}{\alpha} \cdot \frac{w}{(\sum p_\ell)} \right)_\ell = x(p,w)$$

Yes, it is

b) $$p\, x(p,w) = \sum_{\ell=1}^{L} \frac{p_\ell w}{\sum_{\ell=1}^{L} p_\ell} = \frac{w}{\sum_{\ell=1}^{L} p_\ell} \cdot \sum_{\ell=1}^{L} p_\ell = w$$

Yes, it does

c) Recall: $p\, x(p', w') \leq w$ and $x(p',w') \neq x(p,w) \implies p'\, x(p,w) > w'$

$$p\, x(p', w') = \sum_{\ell=1}^{L} p_\ell \frac{w'}{\sum p'_\ell} \leq w$$

$$\iff \quad w' \frac{\sum p_\ell}{\sum p'_\ell} \leq w \quad \iff \quad \frac{\sum p'_\ell}{\sum p_\ell} w \geq w'$$

Now, if equality holds, then $\frac{w}{\sum p_e} = \frac{w'}{\sum p'_e}$ $\Leftrightarrow$ $x_k(p,w) = x_k(p',w')$
$\forall k$, i.e. $x(p,w) = x'(p,w)$, weak axiom is satisfied.

If the inequality holds strictly, $x(p,w) \neq x'(p,w)$ and
$p' x(p,w) > w'$
We can conclude that the weak axiom is satisfied.

d) $$s_{ek} = \frac{\partial x_e}{\partial p_k} + \frac{\partial x_e}{\partial w} x_k =$$

$$= -\frac{w}{(\sum p_e)^2} + \frac{1}{(\sum p_e)} \cdot \frac{w}{(\sum p_e)} =$$

$$= -\frac{w}{(\sum p_e)^2} + \frac{w}{(\sum p_e)^2} = 0$$

Thus $S(p,w) = 0$, it is the matrix of 0, thus symmetric, negative semidefinite, but not negative definite.

Intuition: s_{ek} measures the differential change in consumption of good e due to a differential change in price of good k, if wealth is adjusted so that consumer can just afford the original bundle. But in this case, given the symmetry of demand, she would just choose the same bundle, i.e. <u>no change</u> in consumption of good e.